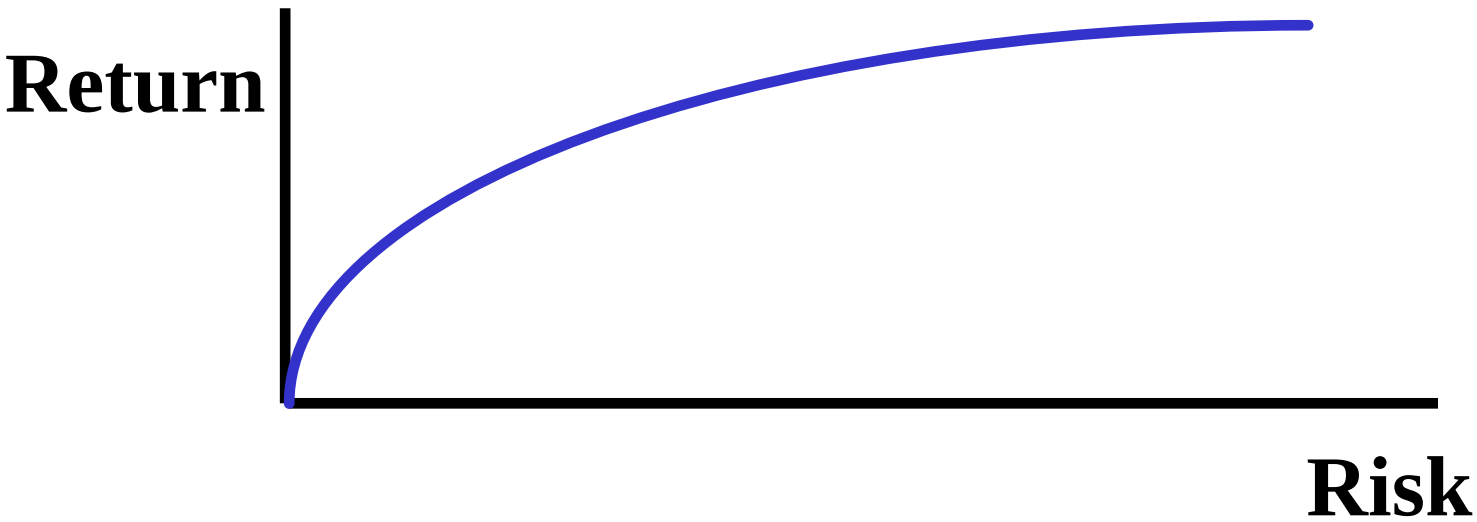
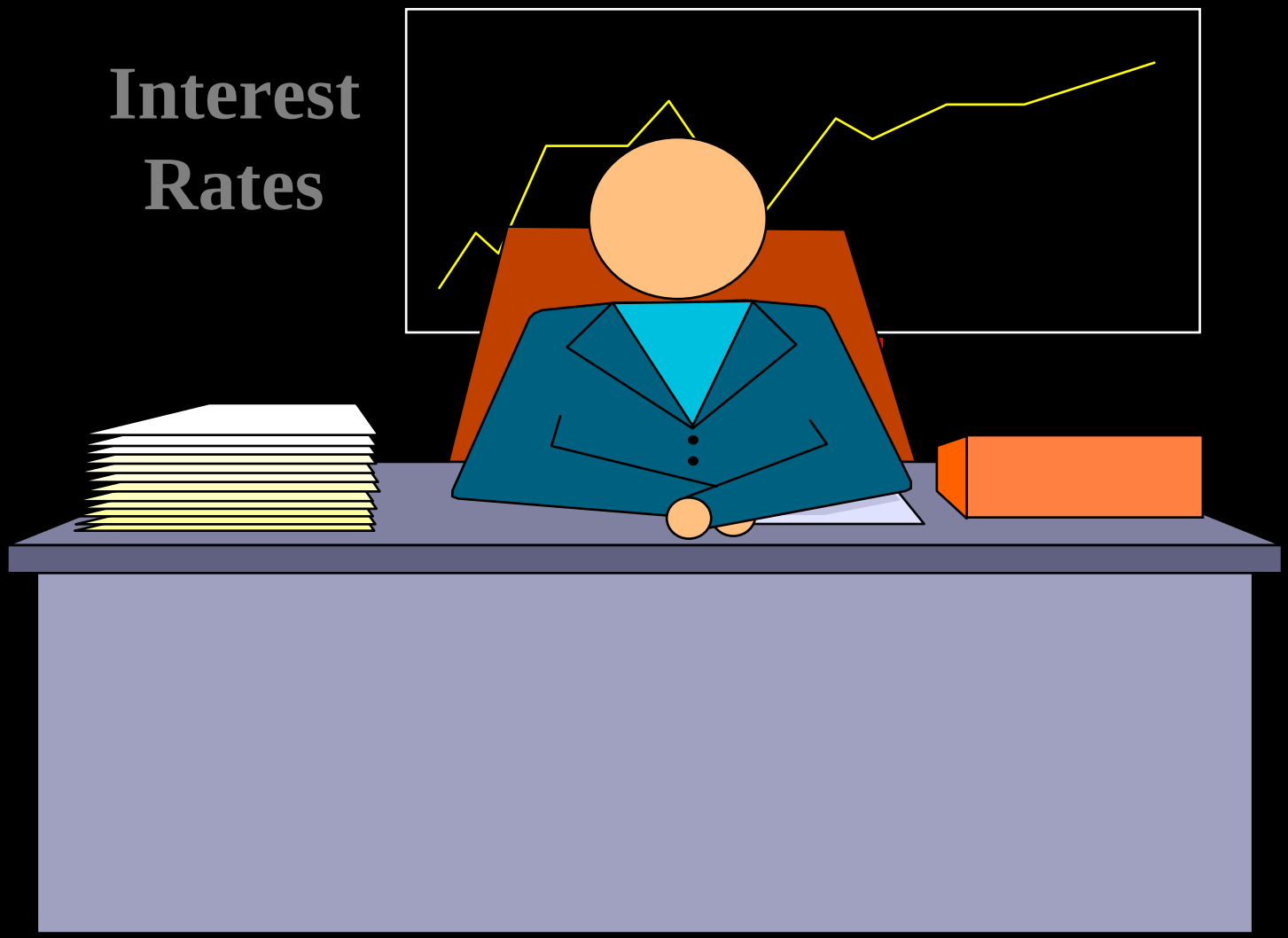
Risk and Return



M. En C. Eduardo Bustos Farías

Inflation, Rates of Return, and the Fisher Effect

Interest Rates



Interest Rates

Conceptually:

Nominal risk-free Interest Rate k_{rf}	=	Real risk-free Interest Rate k^*	+	Inflation-risk premium IRP

Mathematically:

$$(1 + k_{rf}) = (1 + k^*)(1 + IRP)$$

This is known as the "Fisher Effect"

Interest Rates

- **Suppose the real rate is 3%, and the nominal rate is 8%. What is the inflation rate premium?**

$$(1 + k_{rf}) = (1 + k^*)(1 + IRP)$$

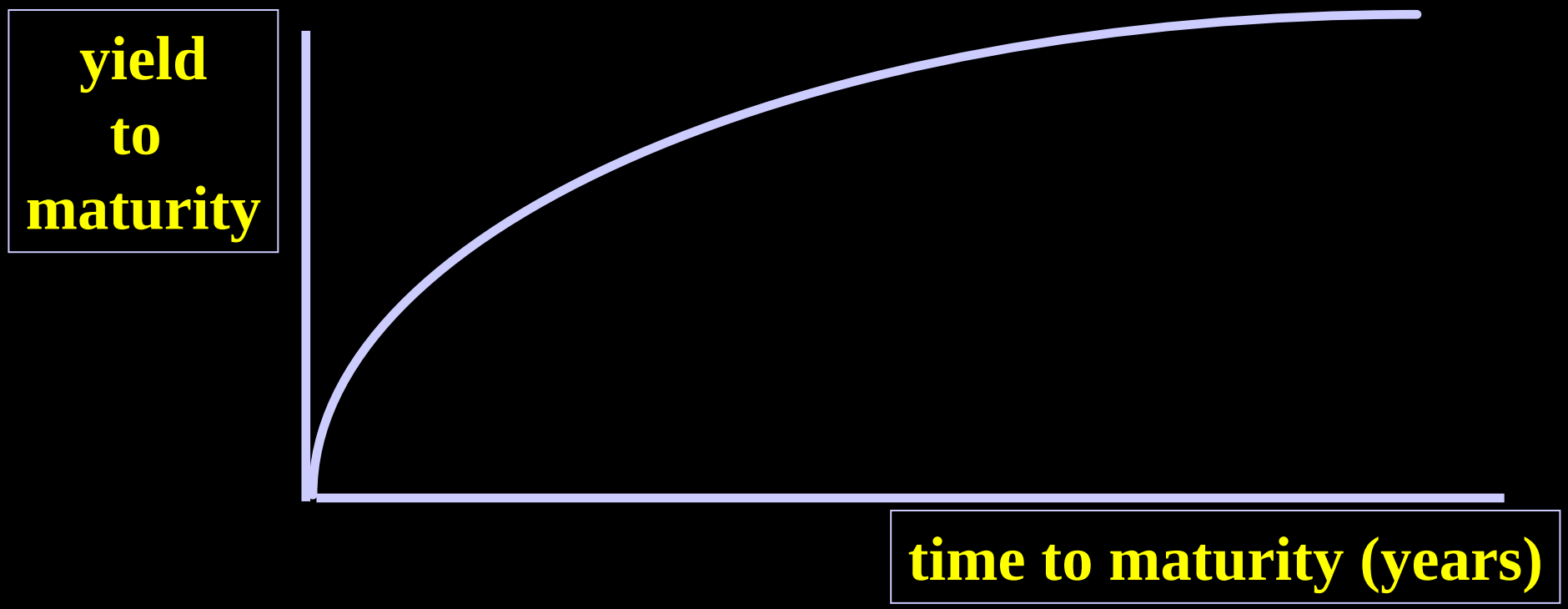
$$(1.08) = (1.03)(1 + IRP)$$

$$(1 + IRP) = (1.0485), \text{ so}$$

$$IRP = 4.85\%$$

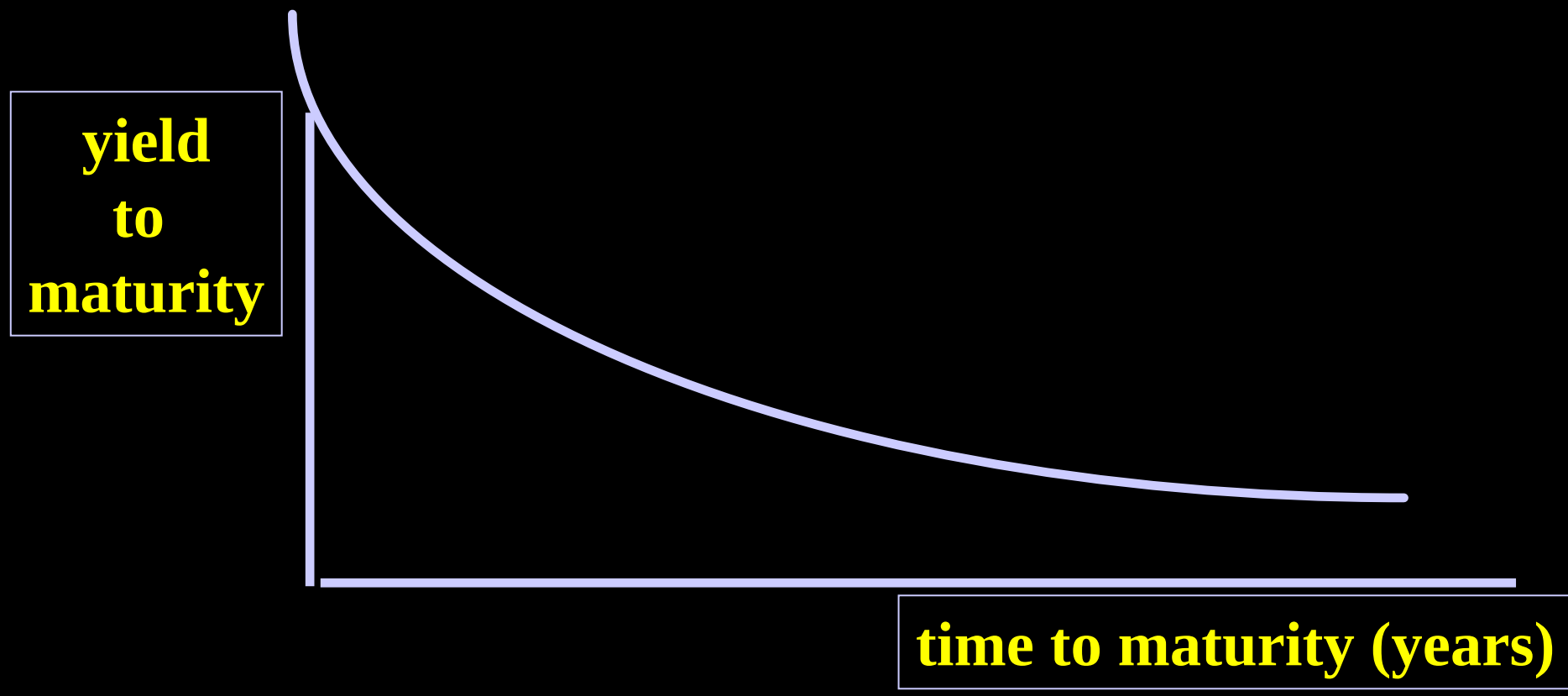
Term Structure of Interest Rates

- The pattern of rates of return for debt securities that differ only in the length of time to maturity.



Term Structure of Interest Rates

- The yield curve may be downward sloping or "inverted" if rates are expected to fall.



For a Treasury security, what is the required rate of return?

Required rate of return	=	Risk-free rate of return

Since Treasuries are essentially free of default risk, the rate of return on a Treasury security is considered the "risk-free" rate of return.

For a corporate stock or bond, what is the required rate of return?

Required rate of return	=	Risk-free rate of return	+	Risk premium

Returns

Total dollar return

The return on an investment measured in dollars that accounts for all cash flows and capital gains or losses.

Example

Total dollar return on stock	=	Dividend income	+	Capital gain (or loss)

Returns

Total percent return

The return on an investment measured as a % of the originally invested sum that accounts for all cash flows and capital gains or losses.

It is the return for *each dollar* invested.

Example

$$\text{Percent return on stock} = \text{Dividend yield} + \text{Capital gains yield}$$

$$\text{or } \frac{\text{Total dollar return}}{\text{Beginning stock price}}$$

Returns

Example: Calculating Returns

- Suppose you invested $1,000 in a stock at $25 per share. After one year, the price increases to $35. For each share, you also received $2 in dividends.

✧ Dividend yield = $2 / $25 = 8%

✧ Capital gains yield = ($35 − $25) / $25 = 40%

✧ Total percentage return = 8% + 40% = 48%

✧ Total dollar return = 48% of $1,000 = $480

✧ At the end of the year, the value of your $1,000 investment is $1,480.

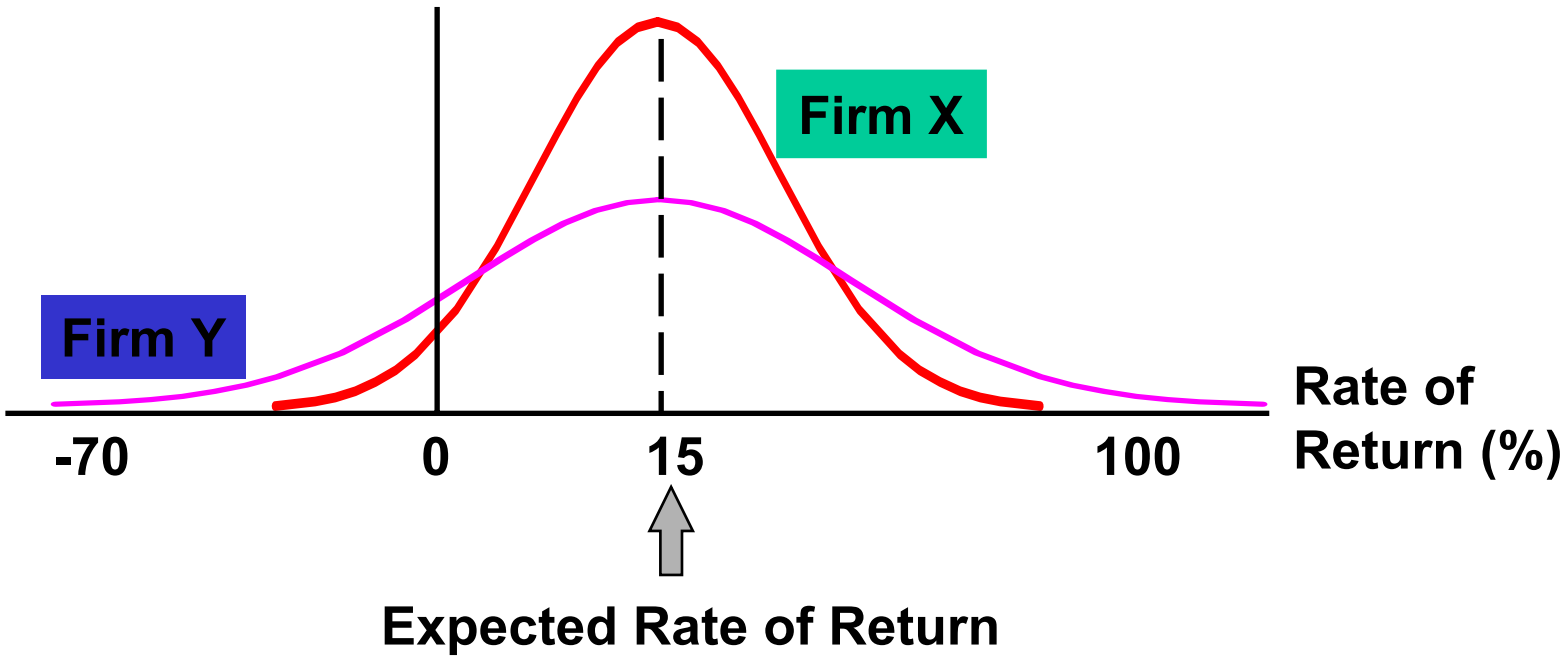
Probability distributions

- Probability is defined as the chance that an event will occur.
- Probability Distribution is a listing of all possible outcomes, or events, with a probability (chance of occurrence) assigned to each outcome.

Outcome	Probability
Rain	0.4 = 40%
No rain	0.6 = 60%

Probability distributions

- A listing of all possible outcomes, and the probability of each occurrence.
- Can be shown graphically.



Probability

Example

Let's think about a Game. If you draw a dice, the dealer will pay you some cash according to the following table.

ω		X
1	→	\$ 100
2	→	\$ 100
3	→	\$ 200
4	→	\$ 200
5	→	\$ 300
6	→	\$ 300

Then, the cash flow, X, after you draw the dice, can be \$100, 200, or \$300 depending on the outcomes.

$\Omega = \{1,2,3,4,5,6\}$ → **Sample Space**

$\omega = 1,2,3,4,5, or 6$ → **Sample Point**

$X : \Omega \rightarrow 100,200, or 300$ → **X: Random Variable**

$X(\omega) = 100,200, or 300$

$X(1) = 100$

$X(2) = 100$

$X(3) = 200$

$X(4) = 200$

$X(5) = 300$

$X(6) = 300$

$$\Pr(X = 100) \equiv \Pr(\{1,2\}) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

$$\Pr(X = 200) \equiv \Pr(\{3,4\}) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

$$\Pr(X = 300) \equiv \Pr(\{5,6\}) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

Pr(X=100) : Probability that your cash flow will be \$100.

Pr({1,2}) : Probability that the outcome will be one or two.

Probability – Expectation

- the *sample space*, a set, Ω , whose elements, ω , corresponds to the possible outcomes of an experiment;
- *Sample point* is an element in the sample space;
- a *random variable* (function) assigns a value to a sample point;
- or a *random variable* is one whose value is subject to uncertainty.

Expectation

$$E(X) = \sum_{i=1}^{n} X_i p_i = X_1 p_1 + X_2 p_2 + \bullet\bullet\bullet + X_n p_n$$

ω		Y
1	→	\$ 50
2	→	\$ 100
3	→	\$ 150
4	→	\$ 200
5	→	\$ 300
6	→	\$ 400

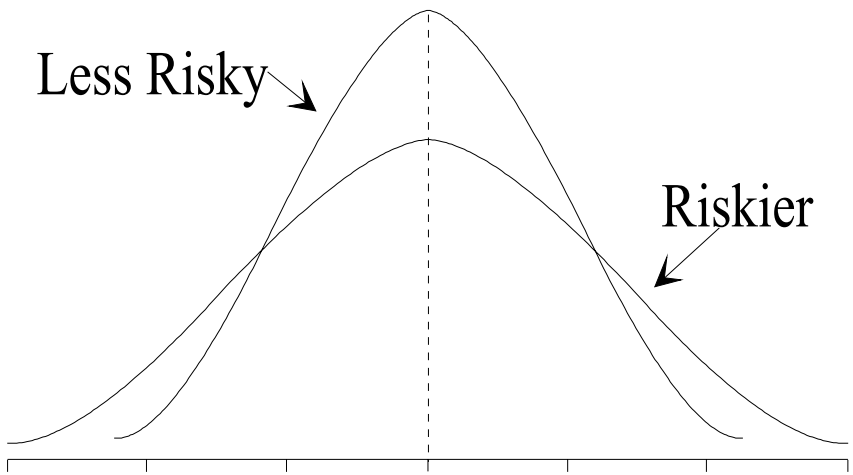
What is expected value of Y?

The Variance & Standard Deviation

- The variance and standard deviation describe the dispersion (spread) of the potential outcomes around the expected value

resultado

- Greater dispersion generally means greater uncertainty and therefore higher risk



$$\sigma_R^2 = \sum_{t=1}^{N} \rho_t \left(R_t - \overline{R} \right)^2$$

$$\sigma_R = \sqrt{\sigma_R^2}$$

Probability – Variance, Standard Deviation and CV

Variance, σ^2

$$\sigma^2 = \sum_{i=1}^{n} (X_i - E(X))^2 p_i = (X_1 - E(X))^2 p_1 + (X_2 - E(X))^2 p_2 + \bullet\bullet\bullet + (X_n - E(X))^2 p_n$$

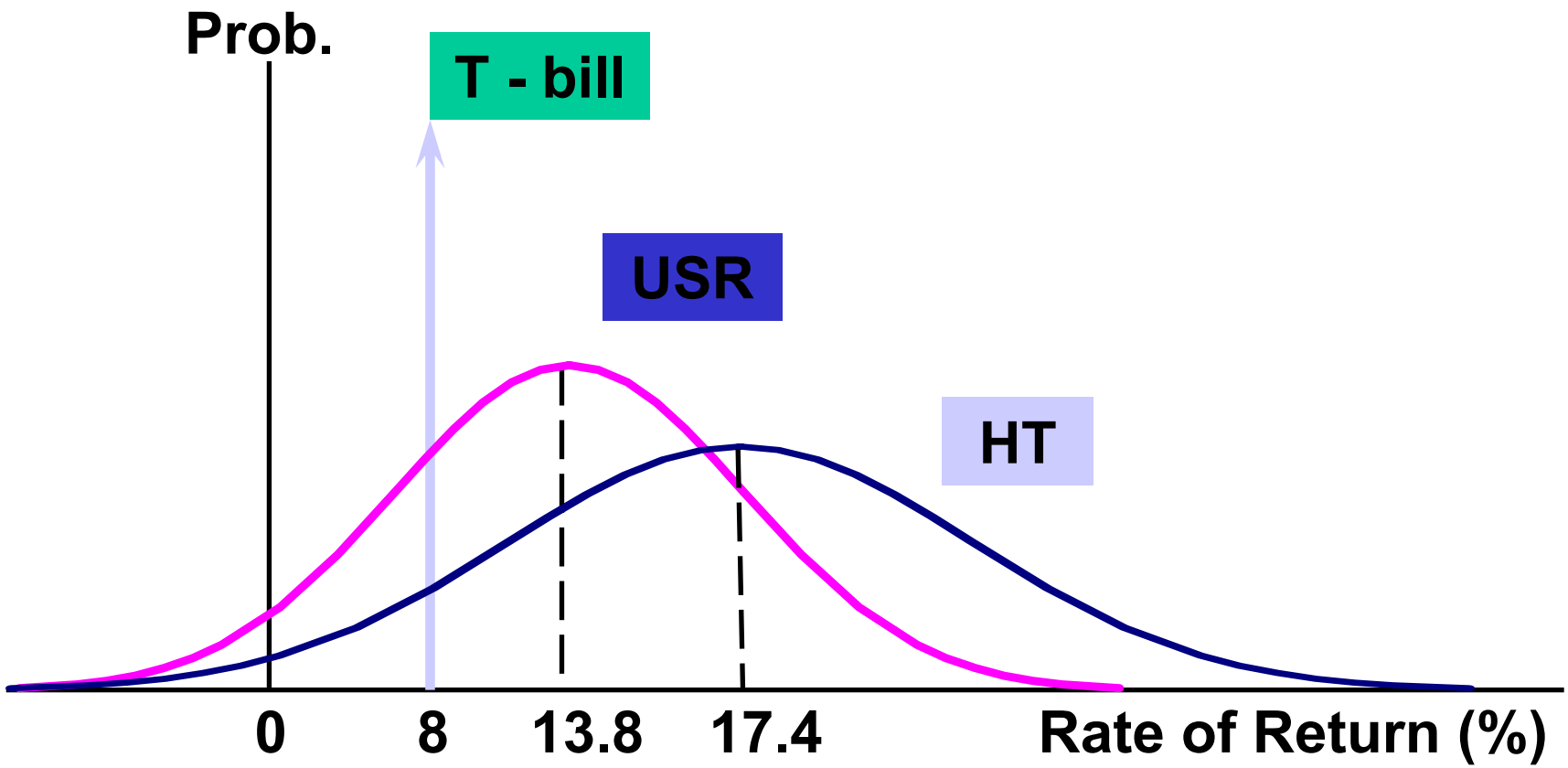
Standard Deviation, σ

$$\sigma = \sqrt{\sigma^2} = \sqrt{\sum_{i=1}^{n} (X_i - E(X))^2 p_i}$$

Coefficient of Variation, CV

$$CV = \frac{\sigma}{E(X)}$$

Comparing standard deviations



Probability –X and Y

Expected Value : The weighted average of possible values, with weights being the probabilities of occurrence.

Variance : The weighted average of square of possible deviations from its mean, with weights being the probabilities of occurrence.

Standard Deviation : A measure of the variability of a distribution around its mean. It is the square root of the variance.

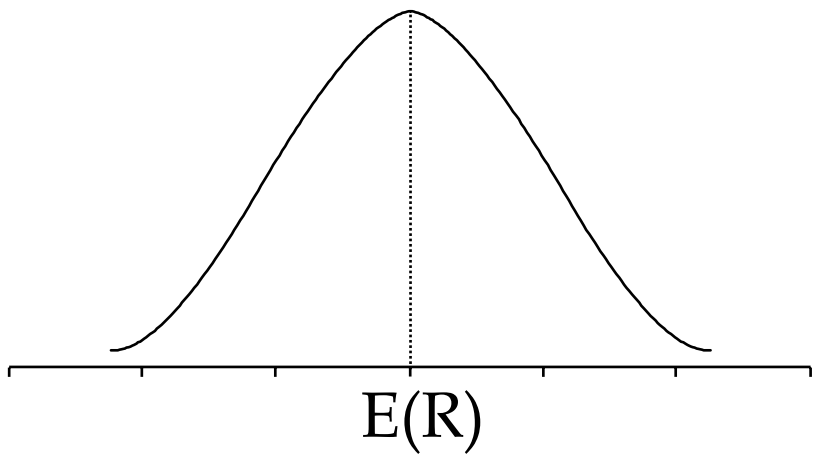
Coefficient of Variation : The ratio of the standard deviation of a distribution to the mean of that distribution. It is a measure of relative variability of a distribution.

	X	Y
Expectation	$200	$200
Variance	6,666.67	14,166.67
Standard Deviation	81.65	119.02
Coefficient of Variation	0.41	0.60

1. **We can expect the same amount of cash flow from the both games. ← same expected value**
2. **If we play the game X, the average deviation of the cash flow from the mean ($200) will be $81.65, and if we play the game Y, the average deviation of the cash flow from the mean ($200) will be $119.02.**
3. **So, we may say that the game X and Y, will provide the same expected cash flows, the game Y is riskier.**
4. **When you are pricing the games, you should think about the variability of the cash flows as well as the expected cash flows!!!**

The Expected Value

- The expected value of a distribution is the most likely outcome
- For the normal dist., the expected value is the same as the arithmetic mean
- All other things being equal, we assume that people prefer higher expected returns



$$E(R) = \sum_{t=1}^{N} \rho_t R_t$$

The Expected Return: An Example

- Suppose that a particular investment has the following probability distribution:
  - 25% chance of -5% return
  - 50% chance of 5% return
  - 25% chance of 15% return
- This investment has an expected return of 5%

$$E(R_i) = 0.25(-0.05) + 0.50(0.05) + 0.25(0.15) = 0.05$$

Calculating σ^2 and σ : An Example

- Using the same example as for the expected return, we can calculate the variance and standard deviation:

$$\sigma_i^2 = 0.25(-0.05-0.05)^2 + 0.50(0.05-0.05)^2 + 0.25(0.15-0.05) = .005$$

$$\sigma_i = \sqrt{0.25(-0.05-0.05)^2 + 0.50(0.05-0.05)^2 + 0.25(0.15-0.05)} = 0.071$$

- Note: In this example, we know the probabilities. However, often we have only historical data to work with and don't know the probabilities. In these cases, we assume that each outcome is equally likely so the probabilities for each possible outcome are 1/N or (more commonly) 1/(N-1).

Returns

- **Expected Return - the return that an investor expects to earn on an asset, given its price, growth potential, etc.**

- **Required Return - the return that an investor requires on an asset given its risk and market interest rates.**

Expected Return

State of Economy	Probability (P)	Return	
		Orl. Utility	Orl. Tech
Recession	.20	4%	-10%
Normal	.50	10%	14%
Boom	.30	14%	30%

For each firm, the expected return on the stock is just a *weighted average*:

$$\hat{k} = P(k_1)*k_1 + P(k_2)*k_2 + \ldots + P(k_n)*k_n$$

Expected Return

State of Economy	Probability (P)	Return	
		Orl. Utility	Orl. Tech
Recession	.20	4%	-10%
Normal	.50	10%	14%
Boom	.30	14%	30%

$$k = P(k_1)*k_1 + P(k_2)*k_2 + \ldots + P(k_n)*k_n$$

$$\hat{k}\ (OU) = .2\ (4\%) + .5\ (10\%) + .3\ (14\%) = 10\%$$

Expected Return

State of Economy	Probability (P)	Return	
		Orl. Utility	Orl. Tech
Recession	.20	4%	-10%
Normal	.50	10%	14%
Boom	.30	14%	30%

$$k = P(k_1)*k_1 + P(k_2)*k_2 + \ldots + P(k_n)*k_n$$

$$\hat{k}_{(OT)} = .2\,(-10\%) + .5\,(14\%) + .3\,(30\%) = 14\%$$

The Scale Problem

- The variance and standard deviation suffer from a couple of problems
- The most tractable of these is the scale problem:
  - Scale problem - The magnitude of the returns used to calculate the variance impacts the size of the variance possibly giving an incorrect impression of the riskiness of an investment

The Scale Problem: an Example

	Potential Returns	
Prob	**ABC**	**XYZ**
10%	-12%	-24%
15%	-5%	-10%
50%	2%	4%
15%	9%	18%
10%	16%	32%
E(R)	2.0%	4.0%
Variance	0.00539	0.02156
Std. Dev.	7.34%	14.68%
C.V.	3.6708	3.6708

Is XYZ really twice as risky as ABC?

No!

Probability –Another game Z

Example

Let's think about another game which pays half of the X's cash flows and half of Y's cash flows.

ω		X	Y	X/2	Y/2	Z
1	→	\$ 100	\$ 50	\$ 50	\$ 25	\$ 75
2	→	100	100	50	50	100
3	→	200	150	100	75	175
4	→	200	200	100	100	200
5	→	300	300	150	150	300
6	→	300	400	150	200	350

$$E(Z) = 75\times\frac{1}{6}+100\times\frac{1}{6}+175\times\frac{1}{6}+200\times\frac{1}{6}+300\times\frac{1}{6}+350\times\frac{1}{6}=200$$

or

$$E(Z) = W_X E(X) + W_Y E(Y)$$

$$\sigma_Z^2 = (75-200)^2\times\frac{1}{6}+(100-200)^2\times\frac{1}{6}+(175-200)^2\times\frac{1}{6}+(200-200)^2\times\frac{1}{6}+(300-200)^2\times\frac{1}{6}+(350-200)^2\times\frac{1}{6}=9791.67$$

$$\sigma_Z = \sqrt{\sigma_Z^2} = 98.95$$

$$\sigma_Z \le 0.5\sigma_X + 0.5\sigma_Y = 0.5\times 81.65 + 0.5\times 119.02 = 100.34$$

Probability –Covariance and Correlation Coefficient

Covariance : A statistical measure of the degree to which two random variables move together. A positive value means that, on average, they move in the same direction.

Correlation Coefficient : A standardized statistical measure of the linear relation between two variables. Its range is from –1.0 (perfect negative correlation), through 0 (no correlation), to +1.0 (perfect positive correlation)

ω		X	Y	X/2	Y/2	Z
1	→	$ 100	$ 50	$ 50	$ 25	$ 75
2	→	100	100	50	50	100
3	→	200	150	100	75	175
4	→	200	200	100	100	200
5	→	300	300	150	150	300
6	→	300	400	150	200	350

$$COV(X,Y) = \sigma_{XY} = \sum_{i=1}^{n} (X_i - E(X))(Y_i - E(Y))p_i$$

COV(X,Y) **: Covariance between X and Y**

r_{XY} **: Correlation Coefficient**

$$COV(X,Y) = \sigma_{XY} = \sigma_X \sigma_Y r_{XY}$$

$$r_{XY} = \frac{COV(X,Y)}{\sigma_X \sigma_Y}$$

$$\sigma_Z^2 = W_X^2 \sigma_X^2 + W_Y^2 \sigma_Y^2 + 2W_X W_Y \sigma_X \sigma_Y r_{XY}$$

$$\sigma_Z = \sqrt{W_X^2 \sigma_X^2 + W_Y^2 \sigma_Y^2 + 2W_X W_Y \sigma_X \sigma_Y r_{XY}}$$

Probability –Summary

$$E(X) = \sum_{i=1}^{n} X_i p_i = X_1 p_1 + X_2 p_2 + \bullet\bullet\bullet + X_n p_n$$

$$\sigma^2 = \sum_{i=1}^{n} (X_i - E(X))^2 p_i = (X_1 - E(X))^2 p_1 + (X_2 - E(X))^2 p_2 + \bullet\bullet\bullet + (X_n - E(X))^2 p_n$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{\sum_{i=1}^{n} (X_i - E(X))^2 p_i}$$

$$CV = \frac{\sigma}{E(X)}$$

$$E(Z) = W_X E(X) + W_Y E(Y)$$

$$COV(X,Y) = \sigma_{XY} = \sum_{i=1}^{n} (X_i - E(X))(Y_i - E(Y)) p_i$$

$$r_{XY} = \frac{COV(X,Y)}{\sigma_X \sigma_Y}$$

$$\sigma_Z^2 = W_X^2 \sigma_X^2 + W_Y^2 \sigma_Y^2 + 2 W_X W_Y \sigma_X \sigma_Y r_{XY}$$

$$\sigma_Z = \sqrt{W_X^2 \sigma_X^2 + W_Y^2 \sigma_Y^2 + 2 W_X W_Y \sigma_X \sigma_Y r_{XY}}$$

Have you considered RISK?

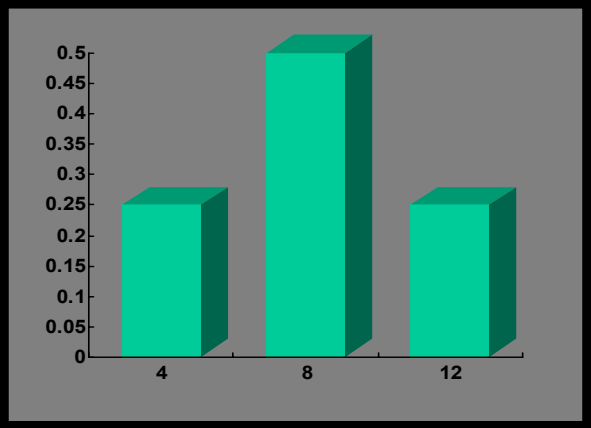
What is Risk?

- **The possibility that an *actual* return will differ from our *expected* return.**
- **Uncertainty in the distribution of possible outcomes.**

What is Risk?

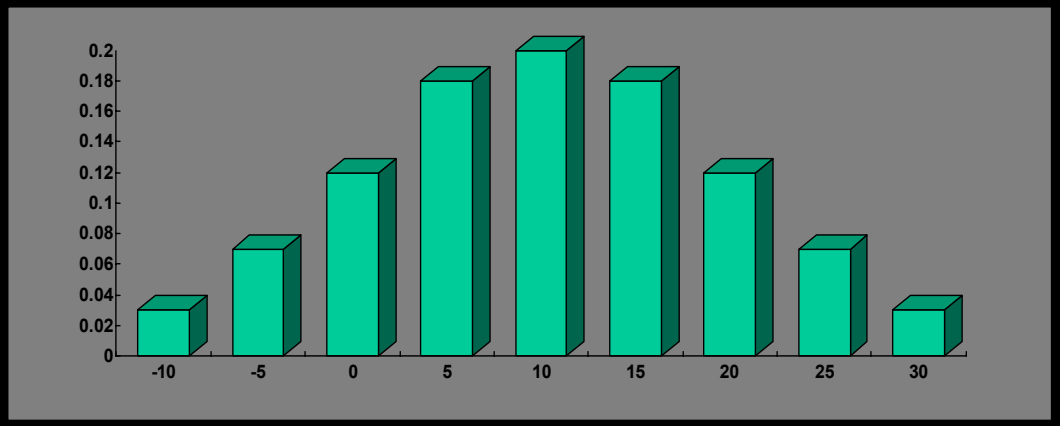
- **Uncertainty in the distribution of possible outcomes.**

Company A



return

Company B



return

What is Risk?

- A risky situation is one which has some probability of loss
- The higher the probability of loss, the greater the risk
- The riskiness of an investment can be judged by describing the probability distribution of its possible returns

Sources of Risk

- Changing Economic Conditions
- Changing Conditions of the Security Issuer

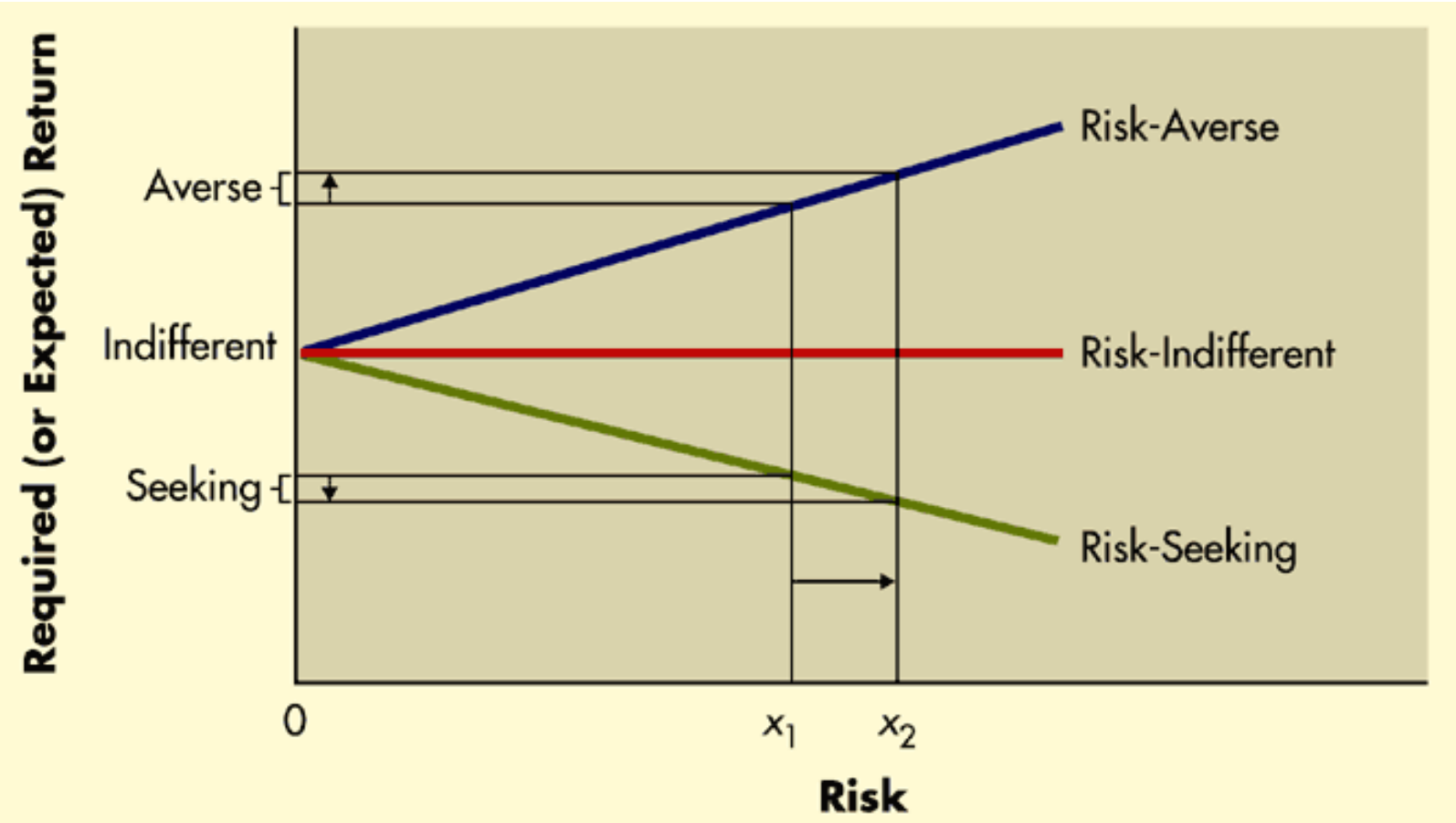
Risk and Changing Economic Conditions

- Inflation Risk--Inflation Increases and the Return on Your Investment Does Not Keep Pace
- Business Cycle Risk--Your Investment's Return Fluctuates in Tandem with the Overall Business Cycle
- Interest-Rate Risk--Newly-Issued Bonds Offer Higher Rates than Your Bonds

Risk and Changing Conditions of the Security Issuer

- Management Risk--The Company in Which You Invested Has Poor Managers
- Business Risk--Risks Associated with a Company's Product/Service Lines
- Financial Risk--The Risk of Insolvency Because the Company Has Borrowed Too Much

Risk Preferences



Risk and Return

Return : Income received on an investment plus any change in market price, usually expressed as a percent of the beginning market price of the investment.

For common stock, one-period return would be

$$R = \frac{D_t + (P_t - P_{t-1})}{P_{t-1}}$$

R : return for one period (from t-1 to t)
D_t : cash dividend at t
P_t : the stock's price at t
P_{t-1} : the stock's price at t-1
$P_t - P_{t-1}$: capital gain

Example

You have 100 shares of XYZ common stock. You bought the stock for $100 per share one year ago. The stock is currently trading at $106 per share, and you just received $7 cash dividends per share. What return was earned for the past one year?

Risk : The variability of returns from those that are expected.

It can be measured by standard deviation of the returns or coefficient variation of the returns.

Risk and Return (Probability Distribution)

The actual rate of return can be viewed as a random variable subject to a probability distribution. Then, as we did in previous section, we can analyze the future return in terms of expected return and standard deviation of the returns.

Example

ABC common stock has the following distribution of possible one-year returns;

Probability of occurrence	0.10	0.20	0.40	0.20	0.10
Possible Return	- 0.15	- 0.03	0.09	0.21	0.33

What is the expected return and standard deviation of the return on this stock?

P_i	R_i	$R_i \times P_i$	$(R_i - \overline{R})^2 \times P_i$
0.10	- 0.15	- 0.015	0.00576
0.20	- 0.03	- 0.006	0.00288
0.40	0.09	0.036	0.00000
0.20	0.21	0.042	0.00288
0.10	0.33	0.033	0.00576
Sum	**1.00**	$\overline{R}$ **0.090**	σ_R^2 **0.01728**
		σ_R	0.1315 or 13.15%

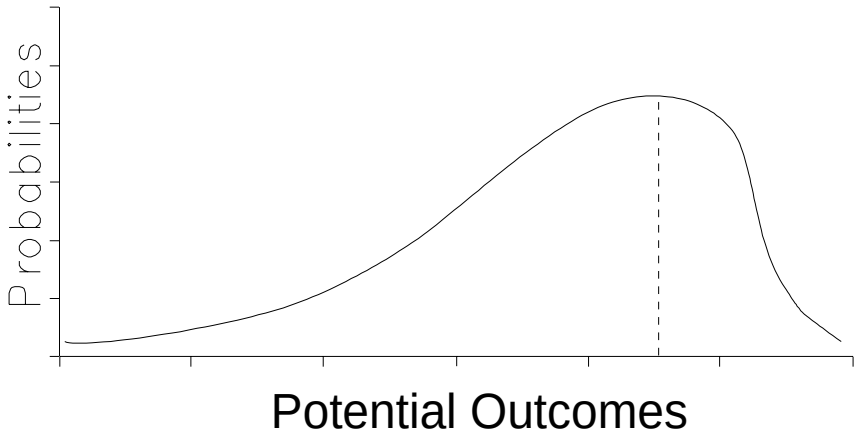
$$E(R) = \overline{R} = \sum_{i=1}^{n} R_i P_i = R_1 P_1 + R_2 P_2 + \bullet\bullet\bullet + R_n P_n$$

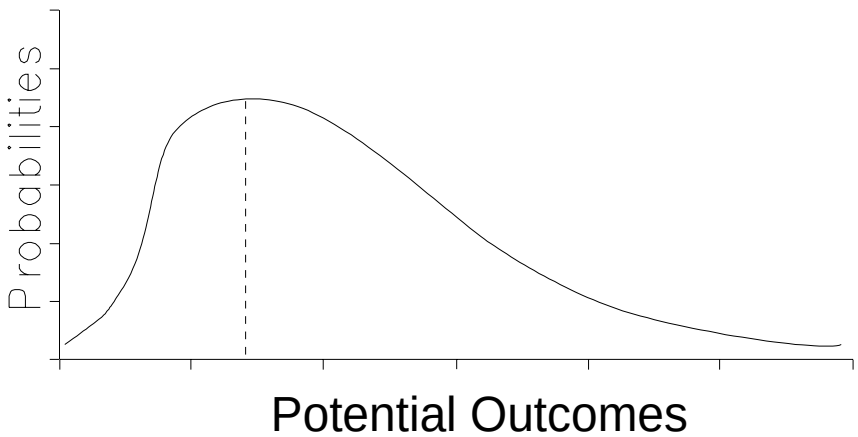
$$\sigma_R^2 = \sum_{i=1}^{n} (R_i - \overline{R})^2 P_i = (R_1 - \overline{R})^2 P_1 + (R_2 - \overline{R})^2 P_2 + \bullet\bullet\bullet + (R_n - \overline{R})^2 P_n$$

$$\sigma_R = \sqrt{\sigma_R^2}$$

Probability Distributions

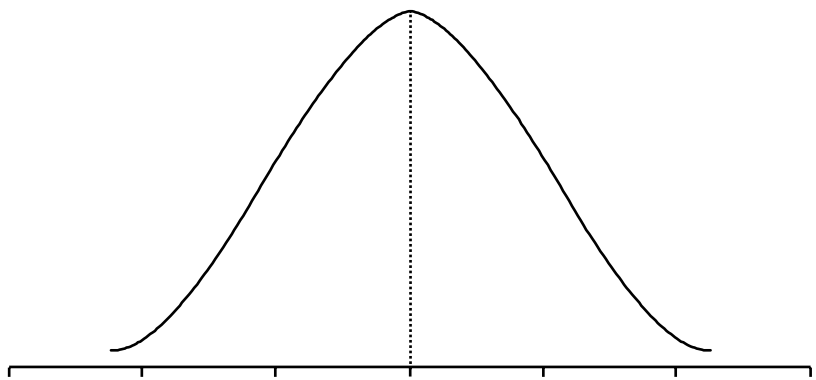
- A probability distribution is simply a listing of the probabilities and their associated outcomes
- Probability distributions are often presented graphically as in these examples





The Normal Distribution

- For many reasons, we usually assume that the underlying distribution of returns is normal
- The normal distribution is a bell-shaped curve with finite variance and mean



The Coefficient of Variation

- The coefficient of variation (CV)provides a scale-free measure of the riskiness of a security
- It removes the scaling by dividing the standard deviation my the expected return (risk per unit of return):

$$CV = \frac{\sigma_R}{E(R)}$$

  - In the previous example, the CV for XYZ and ABC are identical, indicating that they have exactly the same degree of riskiness

Risk and Return (Coefficient of Variation)

Coefficient of Variation : The ratio of the standard deviation of a distribution to the mean of that distribution. It is a measure of relative risk.

Example

Consider two investment opportunities, A and B, whose probability distributions of one-year returns have the following characteristics:

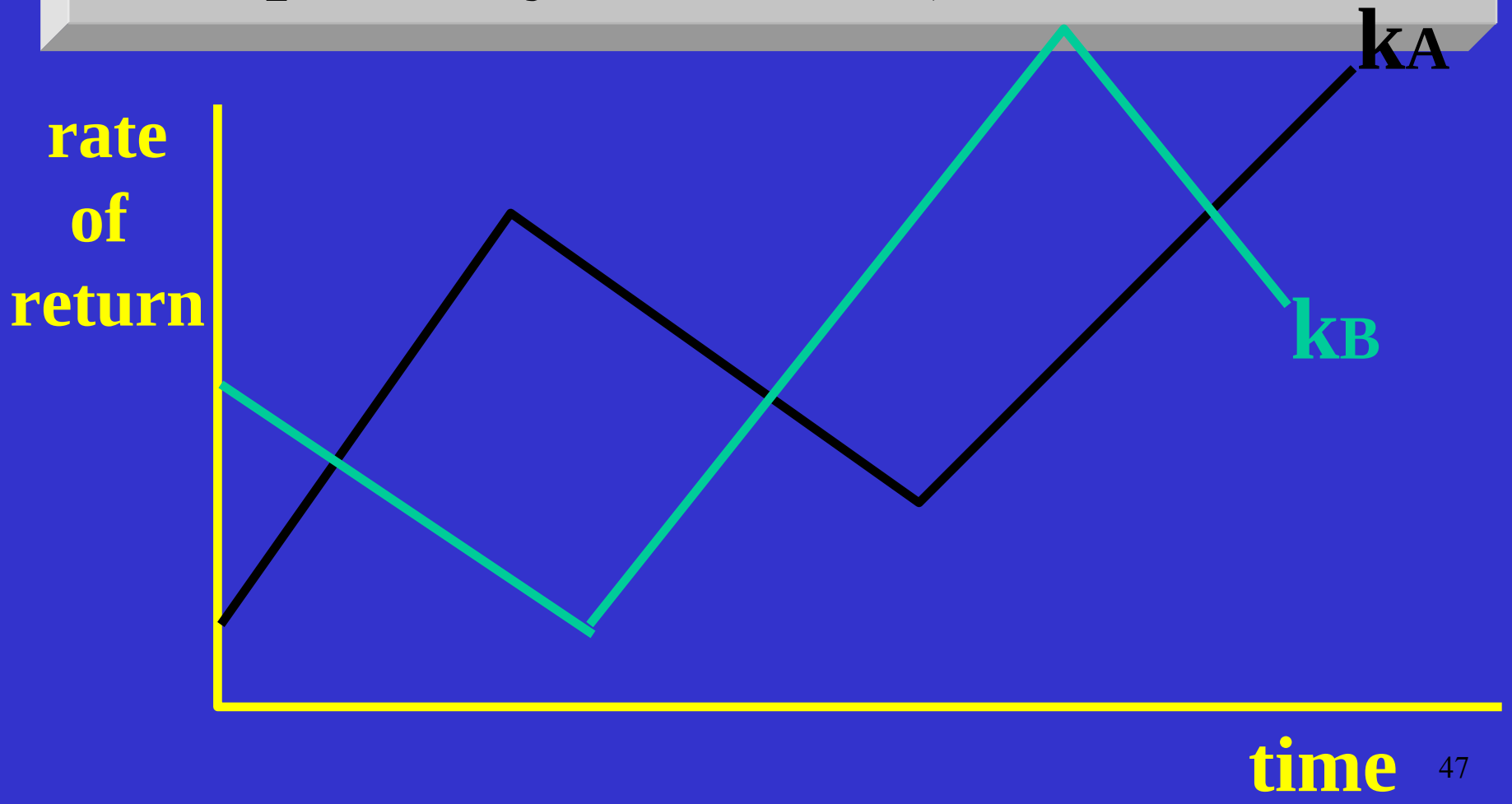
	Investment A	**Investment B**
Expected Return	0.08	0.24
Standard Deviation	0.06	0.08
Coefficient of Variation	0.75 (0.06/0.08)	0.33 (=0.08/0.24)

1. **If our measure of riskiness of the investment is only standard deviation, we should conclude that investment B is riskier than investment A because the standard deviation of B is larger than that of A.**
2. **However, relative to the size of expected return, investment A has greater variation.**
3. **The coefficient of variation is a measure of relative dispersion (risk) – a measure of risk per unit of expected return.**
4. **The larger CV, the larger the relative risk of the investment.**
5. **Using the CV as our risk measure, investment A is viewed riskier than investment B.**

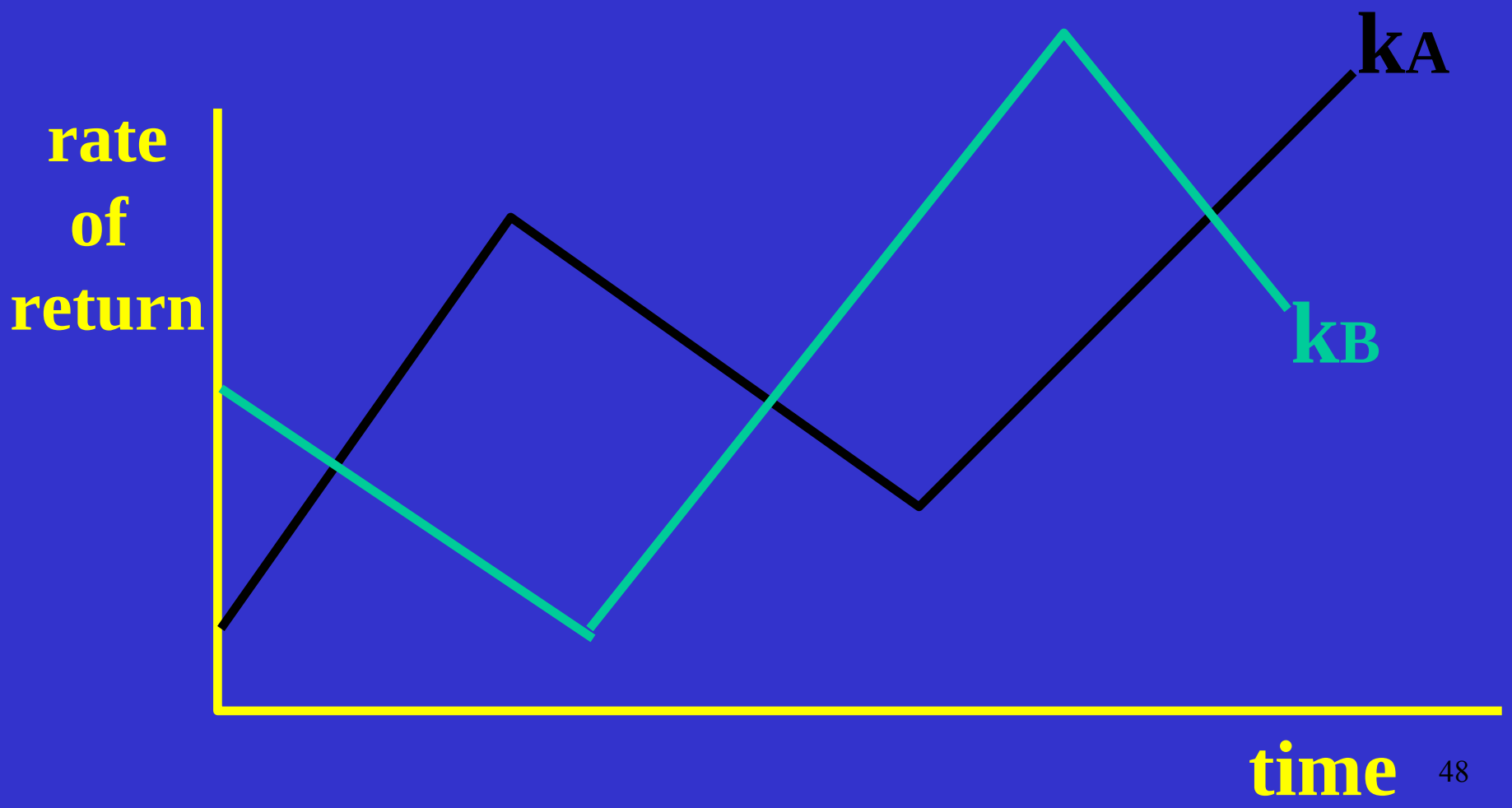
Portfolios

- Combining several securities in a portfolio can actually reduce overall risk.
- How does this work?

Suppose we have stock A and stock B. The returns on these stocks do not tend to move together over time (they are not perfectly correlated).



What has happened to the variability of returns for the portfolio?



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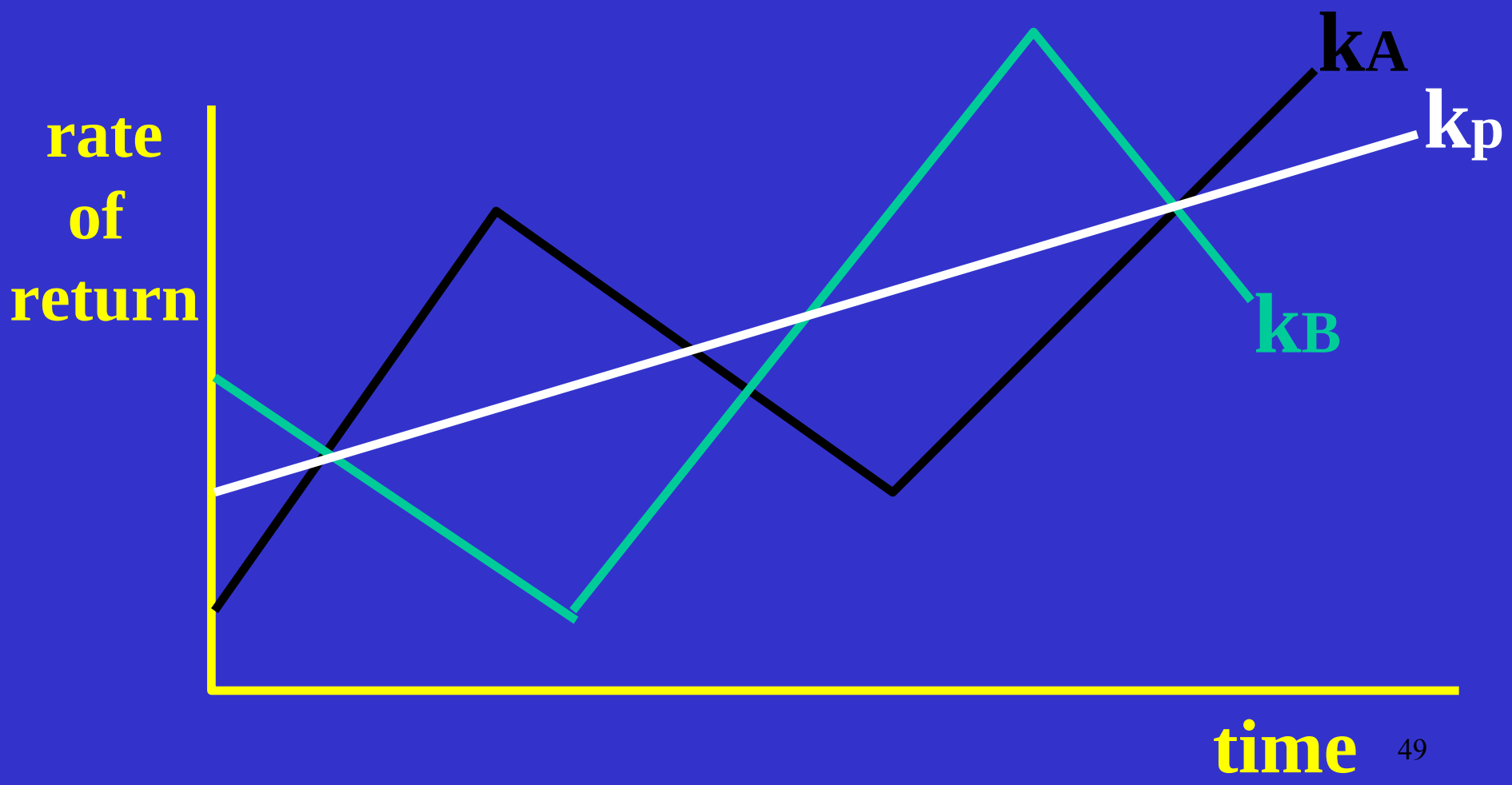
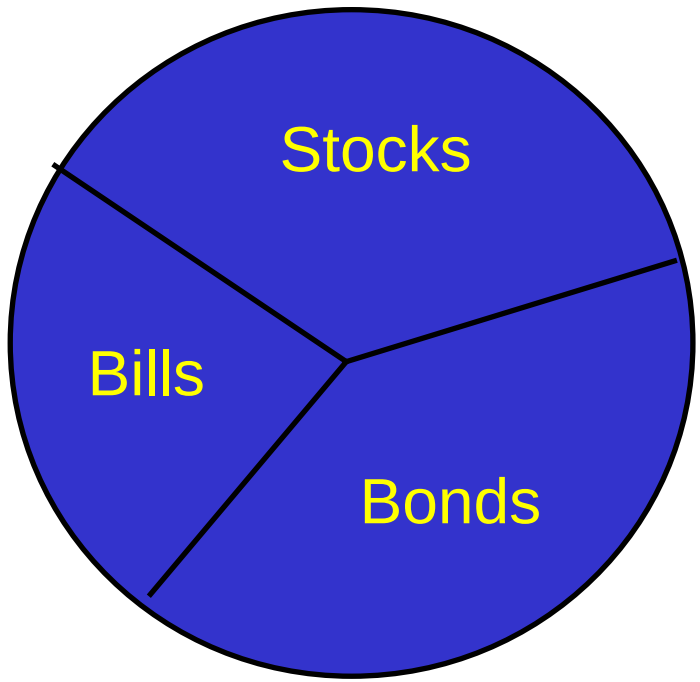


A Portfolio

A Portfolio is Simply a Group of Assets Held at the Same Time



Portfolio

Portfolio : A combination of two or more securities or assets.

Problem

You are creating a portfolio of Stock D and Stock ABC (from earlier). You are investing $2,000 in Stock ABC and $3,000 in Stock D. Remember that the expected return and standard deviation of Stock ABC is 9% and 13.15%, respectively. The expected return and standard deviation of Stock D is 8% and 10.65%, respectively. The correlation coefficient between returns of ABC and D is 0.75.

What is the expected return, standard deviation and coefficient of variation of the portfolio?

	ABC	**D**
Weights	0.4 (=2,000/5,000)	0.6 (=3,000/5,000)
Expected Return	9%	8%
Standard Deviation	13.15%	10.65%
Coefficient of Variation	1.46	1.33
Correlation coefficient	0.75	

Formula you need to know

$$\overline{R}_P = W_1\overline{R}_1 + W_2\overline{R}_2$$

$$\sigma_P^2 = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2\sigma_{12} = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2\sigma_1\sigma_2 r_{12}$$

$$\sigma_P = \sqrt{\sigma_P^2}$$

$\overline{R}_P$: Expected return on portfolio

W_i : Weight on asset *i*

$\overline{R}_i$: Expected return on asset *i*

r_{12} : Correlation coefficient between returns of asset *1* and *2*

σ_{12}: Covariance between returns of asset *1* and *2*

Portfolio

	ABC	D
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W_i : Weight on asset *i*

$\overline{R}_i$: Expected return on asset *i*

r_{12} : Correlation coefficient between returns of asset *1* and *2*

σ_{12}: Covariance between returns of asset *1* and *2*

$\overline{R}_P =$

$\sigma_P^2 =$

$\sigma_P =$

$CV_P =$

Diversification

- Investing in more than one security to reduce risk.
- If two stocks are perfectly positively correlated, diversification has no effect on risk.
- If two stocks are perfectly negatively correlated, the portfolio is perfectly diversified.

- **If you owned a share of every stock traded on the NYSE and NASDAQ, would you be diversified?**

  <u>YES!</u>

- **Would you have eliminated all of your risk?**

  <u>NO!</u> Common stock portfolios still have risk.

Some risk can be diversified away and some cannot.

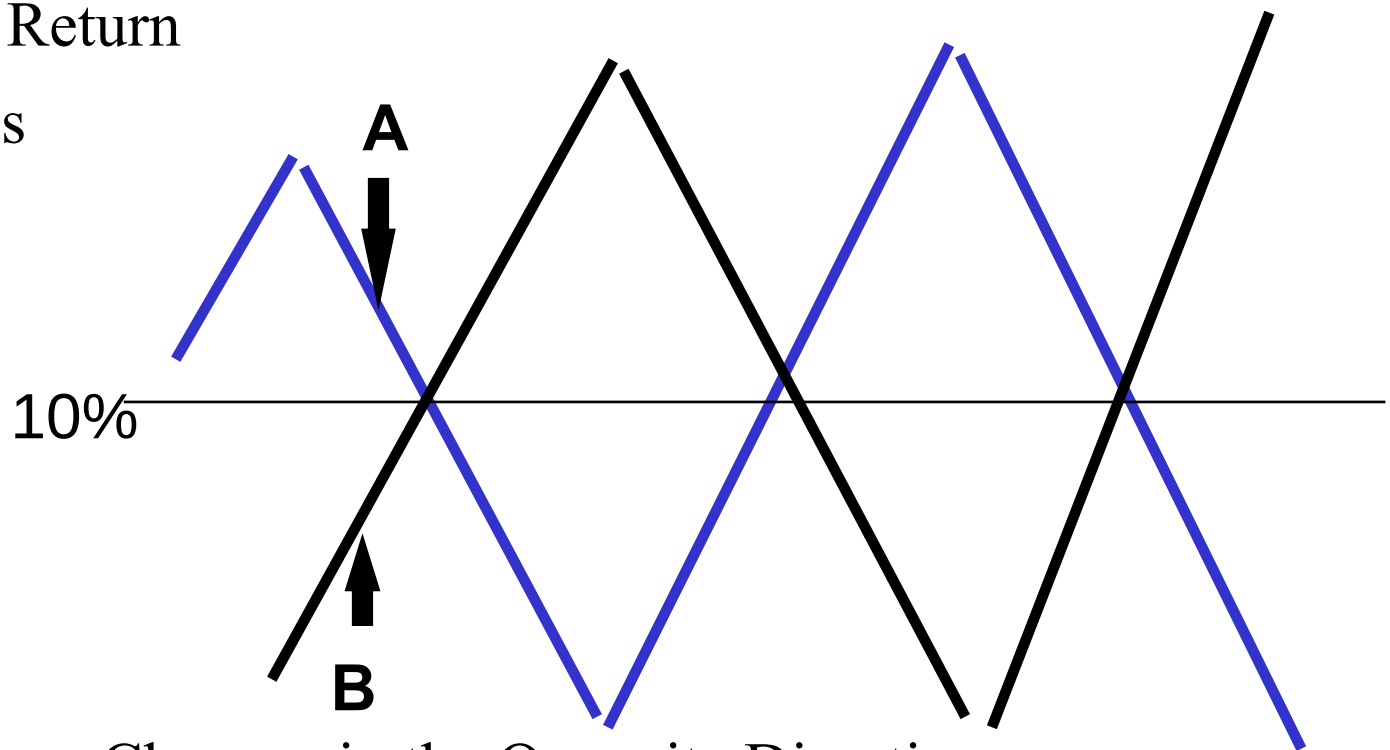
- **<u>Market risk</u> (*systematic risk)* is *nondiversifiable.* This type of risk cannot be diversified away.**
- **<u>Company-unique risk</u> *(unsystematic risk)* is *diversifiable*. This type of risk can be reduced through diversification.**

Diversification

- Diversification Lowers Investment Risk
- It Accomplishes this Goal Because Asset Returns Are Poorly Correlated
- Diversification is Not Effective if Asset Returns Are Strongly, Positively Correlated
- The Return Correlations Among Stocks, Bonds, and Bills Are Low; Holding These Investments in a Portfolio is Effective

An Example of Negative Return Correlation

As A's Return Changes



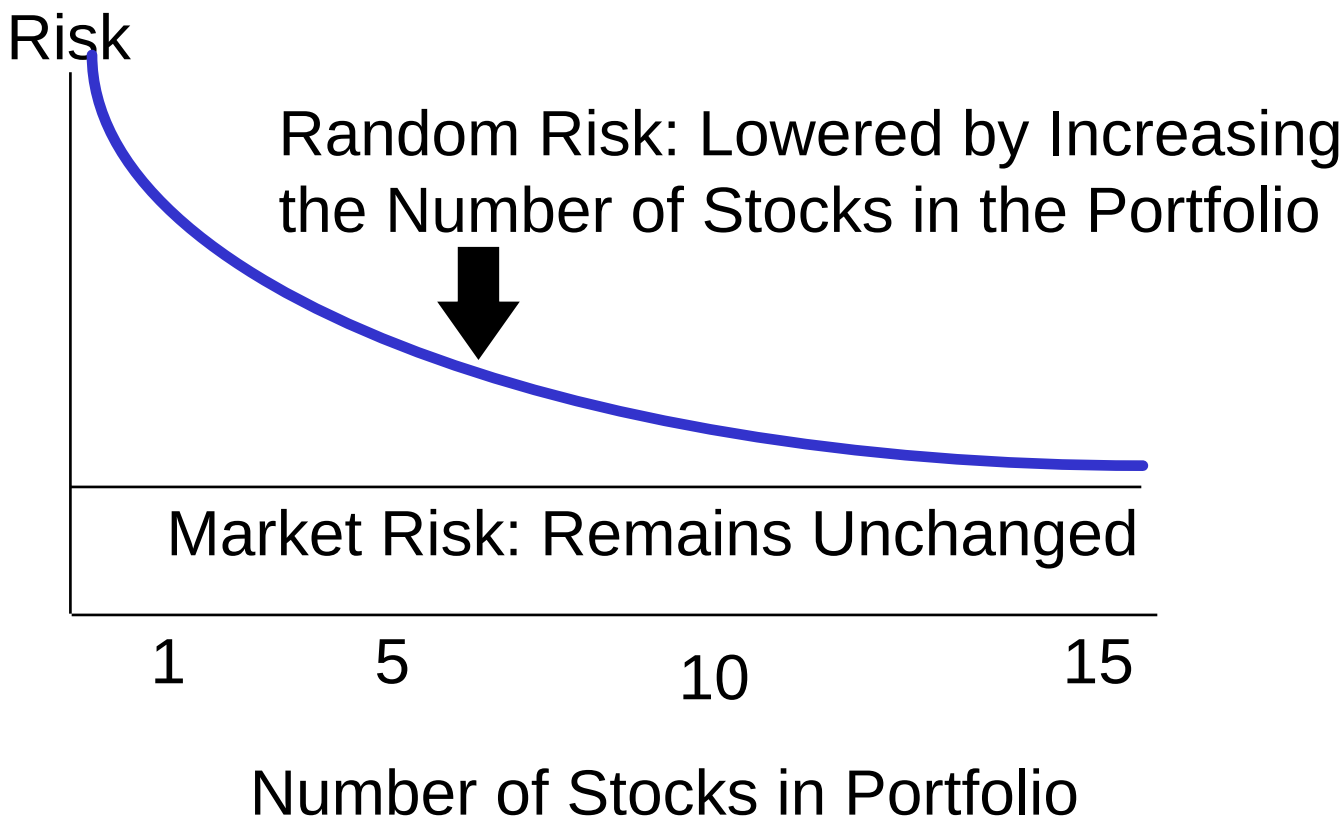
B's Return Changes in the Opposite Direction

Holding Each Gives a 10% Constant Return

Diversification Guidelines

- Diversify Among Intangibles and Tangibles
  - Remember: A House Is a Major Tangible
- Diversify Globally
  - Invest in Foreign Securities
- Diversify within Asset Groups
  - Own a Variety of Common Stocks

Portfolio Risk and the Number of Stocks Held



Portfolio and Diversification

	ABC	D	Portfolio
Expected Return	9%	8%	8.4%
Standard Deviation	13.15%	10.65%	10.91%
Coefficient of Variation	1.46	1.33	1.30

From stock ABC and D, we have made a portfolio whose expected return, standard deviation and coefficient of variation are 8.4%, 10.91% and 1.30 respectively.

Notice that the portfolio's relative risk measured by coefficient of variation is lowest. This is the reason that we need diversification. ("Don't put all your eggs in one basket.")

Through creating portfolios (diversification), we can make more favorable expected return and risk profile.

If we combine securities that are not perfectly, positively correlated, the risk of the portfolio decreases.

Mathematically,

$$\sigma_P = \sqrt{W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2\sigma_1\sigma_2 r_{12}}$$

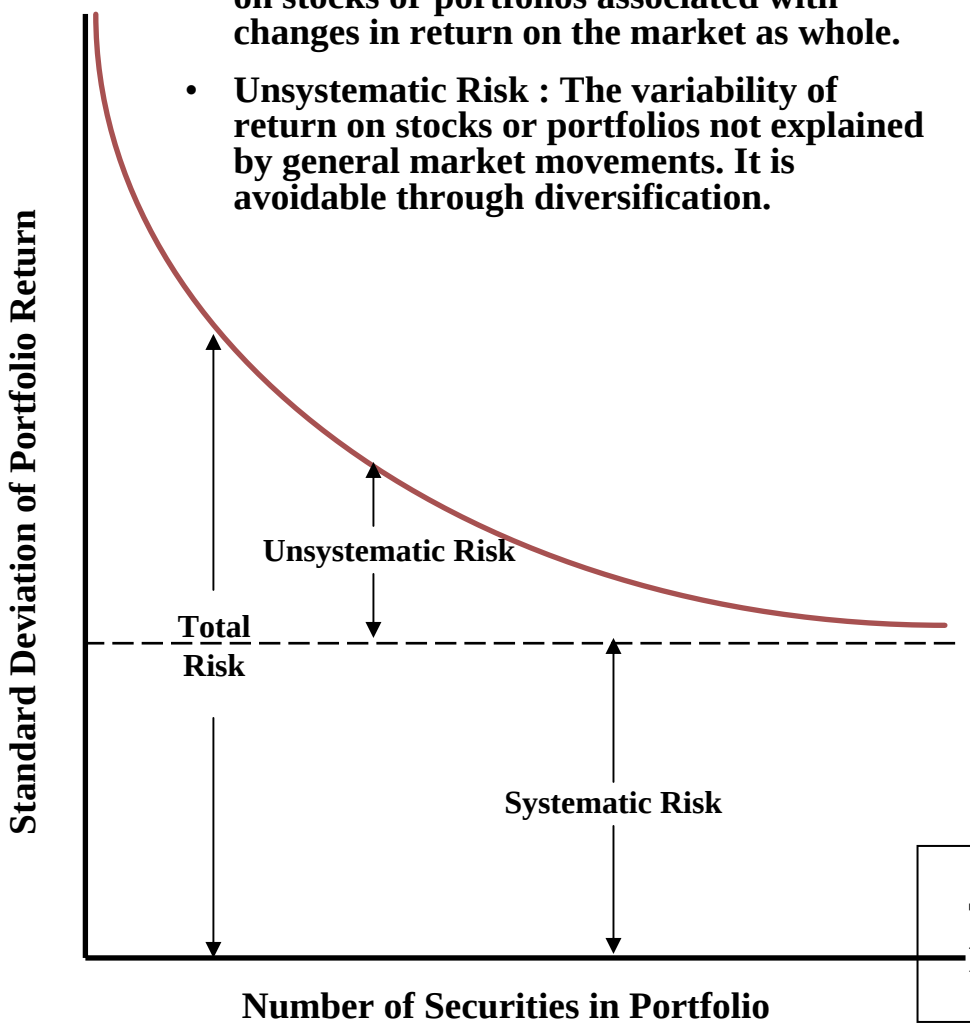
$$Since_r_{12} < 1$$

$$\sigma_P < \sqrt{W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2\sigma_1\sigma_2} = \sqrt{(W_1\sigma_1 + W_2\sigma_2)^2}$$

$$\sigma_P < W_1\sigma_1 + W_2\sigma_2$$

Systematic and Unsystematic Risk

- **Systematic Risk : The variability of return on stocks or portfolios associated with changes in return on the market as whole.**
- **Unsystematic Risk : The variability of return on stocks or portfolios not explained by general market movements. It is avoidable through diversification.**



1. **When we begin with a single stock, the risk of the portfolio is the standard deviation of that one stock.**
2. **As the number of randomly selected stocks held in portfolio is increase the total risk of the portfolio is reduced.**
3. **Such a reduction is at a decreasing rate.**
4. **Thus, a substantial proportion of the portfolio risk can be eliminated with a relatively moderate amount of diversification (15 to 20 randomly selected stocks).**
5. **Even if we hold all of the risky assets in the market, the portfolio still have some degree of riskiness due to risk factors that affect the overall market. → unavoidable through diversification → systematic risk**
6. **We can diversify away some risk factors, which is unique to a particular company or industry. → avoidable through diversification → unsystematic risk**

Total Risk	=	**Systematic Risk (nondiversifiable or unavoidable)**	+	**Unsystematic Risk (diversifiable or avoidable)**

Correlation and Portfolio

If you invest half of your money in stock *W* and the remainder in stock *M*, then your weight on *W* and *M* is 0.5 for each stock.

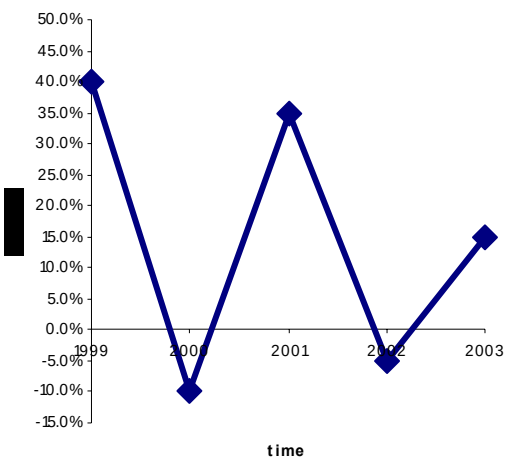
$$E(R_P) = W_W E(R_W) + W_M E(R_M) = 0.5 \times 15.0\% + 0.5 \times 15.0\% = 15.0\%$$

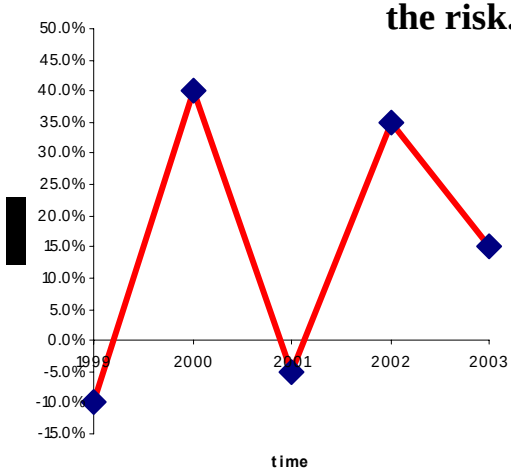
$$\sigma_P = \sqrt{W_W^2 \sigma_W^2 + W_M^2 \sigma_M^2 + 2 W_W W_M r_{WM} \sigma_W \sigma_M} = 0.0\%$$

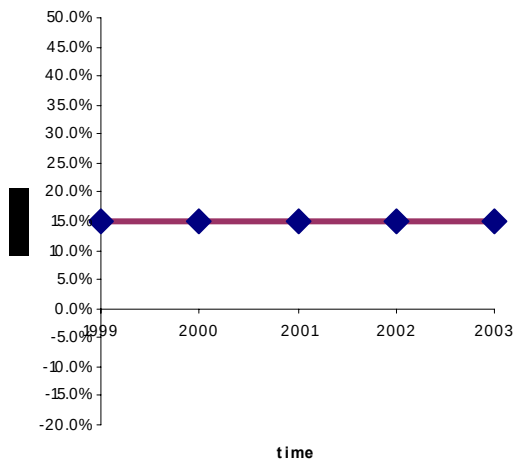
In case that r_{WM} = -1,

	Return		
Year	Stock W	Stock M	Portfolio WM
1999	40.0%	-10.0%	15.0%
2000	-10.0%	40.0%	15.0%
2001	35.0%	-5.0%	15.0%
2002	-5.0%	35.0%	15.0%
2003	15.0%	15.0%	15.0%
Average Return	**15.0%**	**15.0%**	**15.0%**
Standard Deviation	**22.6%**	**22.6%**	**0.0%**

- The two stocks would be quite risky if they were held in isolation.
- When they are combined to form Portfolio WM, they are not risk at all.
- The returns on stocks W and M are *perfectly negatively correlated*, with r_{WM} = -1.
- It is theoretically possible to combine stocks that are individually quite risky and to form a portfolio which is completely riskless, with σ_P = 0.
- **In this case, diversification completely eliminate the risk.**







Correlation and Portfolio

If you invest half of your money in stock *M* and the remainder in stock *M'*, then your weight on *M* and *M'* is 0.5 for each stock.

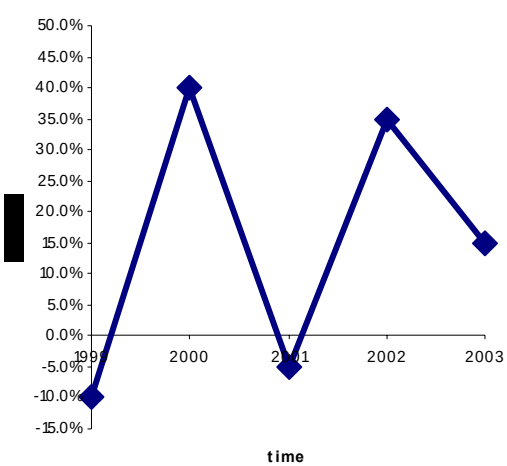
In case that $r_{MM'} = 1$,

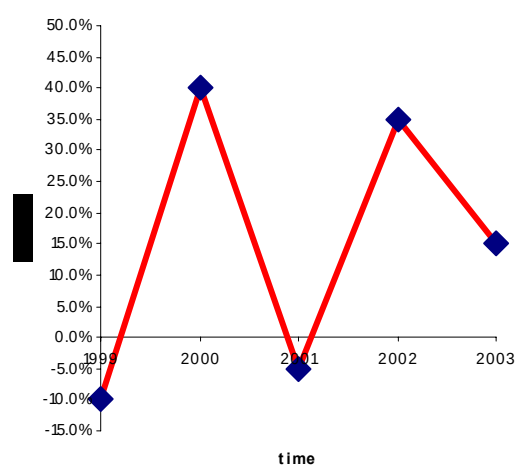
Year	Return		
	Stock M	Stock M'	Portfolio MM'
1999	-10.0%	-10.0%	-10.0%
2000	40.0%	40.0%	40.0%
2001	-5.0%	-5.0%	-5.0%
2002	35.0%	35.0%	35.0%
2003	15.0%	15.0%	15.0%
Average Return	**15.0%**	**15.0%**	**15.0%**
Standard Deviation	**22.6%**	**22.6%**	**22.6%**

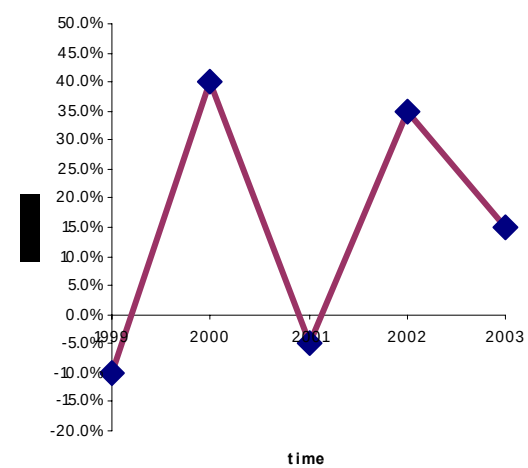
$$E(R_P) = W_M E(R_M) + W_{M'} E(R_{M'}) = 0.5 \times 15.0\% + 0.5 \times 15.0\% = 15.0\%$$

$$\sigma_P = \sqrt{W_M^2 \sigma_M^2 + W_{M'}^2 \sigma_{M'}^2 + 2 W_M W_{M'} r_{MM'} \sigma_M \sigma_{M'}} = 22.6\%$$

- The returns on stocks M and M' are perfectly (positively) correlated, with $r_{MM'} = 1$.
- In this case, the riskiness is just weighted average of the riskiness of the individual assets in the portfolio.
- Diversification does nothing to reduce risk *if the portfolio consists of perfectly positively correlated stocks.*







Correlation and Portfolio

If you invest half of your money in stock *W* and the remainder in stock *Y*, then your weight on *W* and *Y* is 0.5 for each stock.

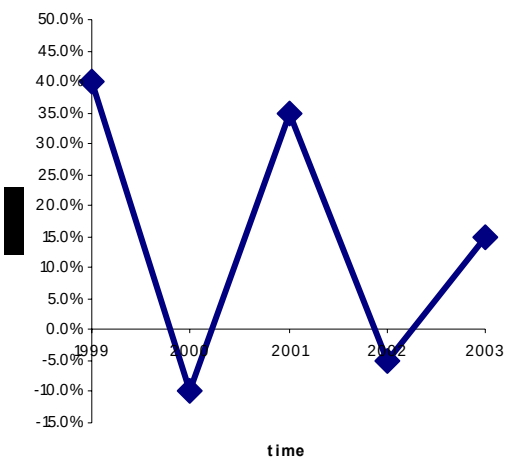
In case that r_{WY} = 0.67

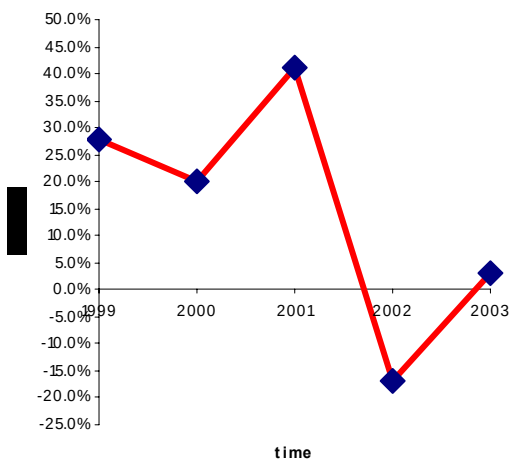
Year	Return		
	Stock W	Stock Y	Portfolio WY
1999	40.0%	28.0%	34.0%
2000	-10.0%	20.0%	5.0%
2001	35.0%	41.0%	38.0%
2002	-5.0%	-17.0%	-11.0%
2003	15.0%	3.0%	9.0%
Average Return	**15.0%**	**15.0%**	**15.0%**
Standard Deviation	**22.6%**	**22.6%**	**20.6%**

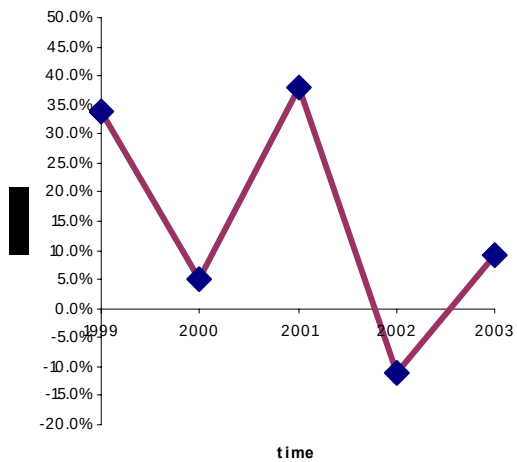
$$E(R_P) = W_W E(R_W) + W_Y E(R_Y) = 0.5 \times 15.0\% + 0.5 \times 15.0\% = 15.0\%$$

$$\sigma_P = \sqrt{W_W^2 \sigma_W^2 + W_Y^2 \sigma_Y^2 + 2 W_W W_Y r_{WY} \sigma_W \sigma_Y} = 20.6\%$$

- In reality, most stocks are positively correlated , but not perfectly so.
- Correlation coefficient is 0.67. So, the portfolio's standard deviation is 20.6%, which is less than the standard deviation of either stock.
- Diversification reduces the portfolio's risk, but not eliminate it completely.







Correlation and Portfolio

What would happen if we included more than two stocks in the portfolio?

→ *As a rule, the riskiness of a portfolio will decline as the number of stocks in the portfolio increases.*

If we added enough partially correlated stocks, could we completely eliminate risk?

→ *In general, the answer is no, but the extent to which adding stocks to a portfolio reduces its risk depends on the degree of correlation among the stocks.*

→ *The smaller the positive correlation coefficients, the lower the risk in a large portfolio.*

→ *In the real world, where the correlations among the individual stocks are generally positive but less than +1.0, some, but not all, risk can be eliminated.*

Would you expect to find higher correlations between the returns on two companies in the same or in different industries?

→ *In general, the correlations between the returns on companies in same industry are higher.*

→ *For example, Ford's and GM's returns have a correlation coefficient of about 0.9 with one another, but their correlation is only about 0.6 with that of AT&T.*

→ *A two-stock portfolio consisting of Ford and GM would be less well diversified than a two-stock portfolio of Ford and AT&T.*

Risk-free security : A security whose return over the holding period is known with certainty.

Frequently, the rate on short- to intermediate-term Treasury securities is used as a proxy for the risk-free rate.

Market Risk

- **Unexpected changes in interest rates.**
- **Unexpected changes in cash flows due to tax rate changes, foreign competition, and the overall business cycle.**

Company-unique Risk

- **A company's labor force goes on strike.**
- **A company's top management dies in a plane crash.**
- **A huge oil tank bursts and floods a company's production area.**

As you add stocks to your portfolio, company-unique risk is reduced.

Do some firms have more market risk than others?

Yes. For example:

Interest rate changes affect all firms, but which would be more affected:

a) Retail food chain

b) Commercial bank

Do some firms have more market risk than others?

Yes. For example:

Interest rate changes affect all firms, but which would be more affected:

a) Retail food chain

b) Commercial bank

- **Note**

As we know, the market compensates investors for accepting risk - but only for market risk. Company-unique risk can and should be diversified away.

So - we need to be able to measure market risk.

This is why we have Beta.

Beta: a measure of market risk.

- Specifically, beta is a measure of how an individual stock's returns vary with market returns.

- It's a measure of the "sensitivity" of an individual stock's returns to changes in the market.

Capital Asset Pricing Model (CAPM)

- If investors are mainly concerned with the risk of their portfolio rather than the risk of the individual securities in the portfolio, how should the risk of an individual stock be measured?
  - In important tool is the CAPM.
  - CAPM concludes that the relevant risk of an individual stock is its contribution to the risk of a well-diversified portfolio.
  - CAPM specifies a linear relationship between risk and required return.
- The equation used for CAPM is as follows:

$$K_i = K_{rf} + \beta_i(K_m - K_{rf})$$

- Where:
  - K_i = the required return for the individual security
  - K_{rf} = the risk-free rate of return
  - β_i = the beta of the individual security
  - K_m = the expected return on the market portfolio
  - $(K_m - K_{rf})$ is called the market risk premium
- This equation can be used to find any of the variables listed above, given the rest of the variables are known.

CAPM (Capital Asset Pricing Model)

Investors demand a higher expected return for bearing higher risk. (risk aversion)

If investors are primarily concerned with the riskiness of their portfolios rather than the risk of the individual securities in the portfolio, how should the riskiness of an individual stock be measured?

- *The relevant riskiness of an individual stock is its contribution to the riskiness of a well-diversified portfolio.*
- *In this context, the well-diversified portfolio means a portfolio that does not have unsystematic risk (diversifiable risk), in other words, the well-diversified portfolio has only systematic risk (nondiversifiable risk) –* ***Market portfolio***

Are all stocks equally risk in the sense that adding them to the market portfolio would have the same effect on the portfolio's riskiness?

- *No.*
- *Different stocks will affect the portfolio differently, so different securities have different degree of relevant risk.*

How can the relevant risk of an individual stock be measured?

- *All risk except that related to broad market movements can be diversified away.*
- *The risk that remains after diversifying is systematic risk, or the risk that is inherent in the market.*
- *It can be measured by the degree to which a given stock tends to move up or down with the market.*

Beta

- Measures a stock's market risk, and shows a stock's volatility relative to the market.
- Indicates how risky a stock is if the stock is held in a well-diversified portfolio.

Calculating betas

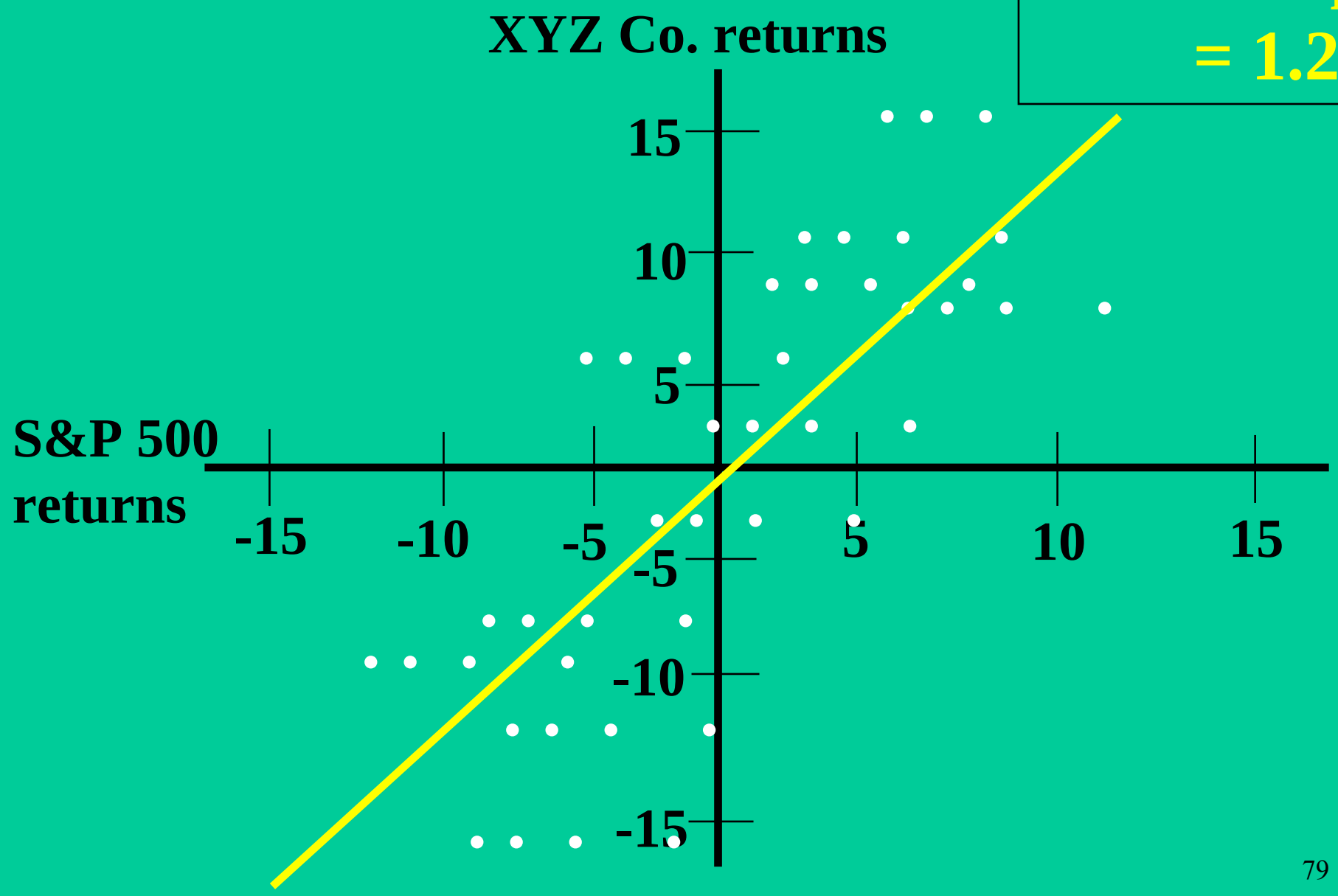
- Run a regression of past returns of a security against past returns on the market.
- The slope of the regression line (sometimes called the security's characteristic line) is defined as the beta coefficient for the security.

The market's beta is 1

- A firm that has a beta = 1 has average market risk. The stock is no more or less volatile than the market.
- A firm with a beta > 1 is more volatile than the market.
  - (ex: technology firms)
- A firm with a beta < 1 is less volatile than the market.
  - (ex: utilities)

Calculating Beta

Beta = slope
= 1.20



Sample Beta Values

- America Online 1.6
- AT&T 0.7
- Battle Mountain Gold 0.3
- Gillette 0.9
- Intel 1.3
- Southwest Airlines 1.5
- Texaco 0.6

Risk of a Portfolio

Capital Asset Pricing Model (CAPM)

TABLE 5.11 Beta Coefficients for Selected Stocks (March 8, 2002)

Stock	Beta	Stock	Beta
Amazon.com	1.95	Int'l Business Machines	1.05
Anheuser-Busch	.60	Merrill Lynch & Co.	1.85
Bank One Corp.	1.25	Microsoft	1.20
Daimler Chrysler AG	1.25	NIKE, Inc.	.90
Disney	1.05	PepsiCo, Inc.	.70
eBay	2.20	Qualcomm	1.30
Exxon Mobil Corp.	.80	Sempra Energy	.60
Gap (The), Inc.	1.60	Wal-Mart Stores	1.15
General Electric	1.30	Xerox	1.25
Intel	1.30	Yahoo! Inc.	2.00

Source: Value Line Investment Survey (New York: Value Line Publishing, March 8, 2002).

CAPM - Beta

Beta Coefficient, β

- A measure of the extent to which the returns on a given stock move with the stock market.

β = 1 means

- *If the market moves up by 10 percent, the stock will also move up by 10 percent, while if the market falls by 10 percent, the stock will fall by 10 percent.*
- *It will be just as risky as the average (market).*
- *Beta of the market portfolio is 1.*

β = 0.5 means

- *The stock is only half as risky as the the market.*

β = 2 means

- *The stock is twice as risky as the market.*

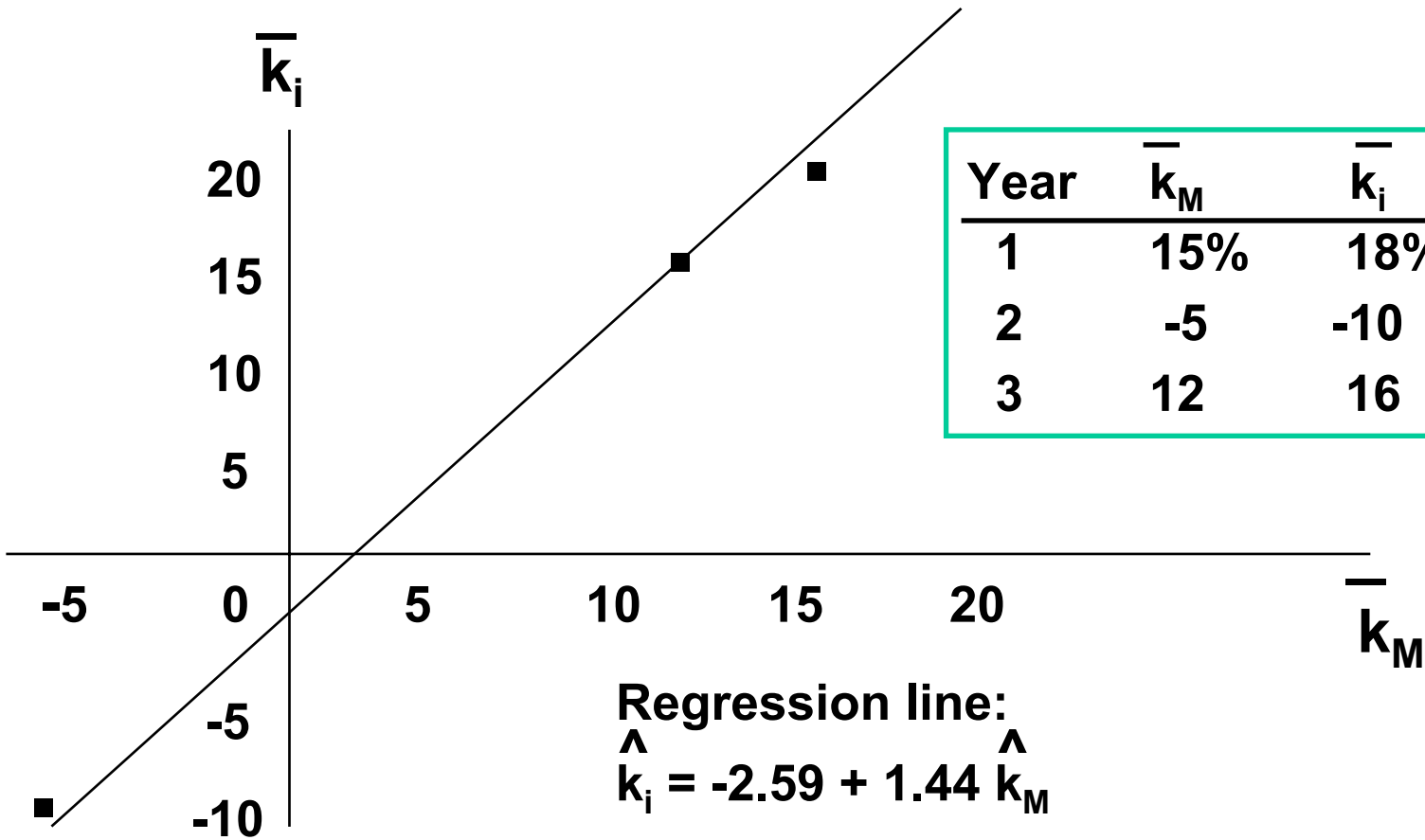
If a stock whose beta is greater than 1.0 is added to a β = 1 portfolio,

- *then the portfolio's beta, and consequently its riskiness, will increase.*

If a stock whose beta is less than 1.0 is added to a β = 1 portfolio,

- *then the portfolio's beta, and consequently its riskiness, will decrease.*
- ***Since a stock's beta measures its contribution to the riskiness of a portfolio, beta is the theoretically correct measure of the stock's riskiness.***

Illustrating the calculation of beta



Year	$\bar{k}_M$	$\bar{k}_i$
1	15%	18%
2	-5	-10
3	12	16

Regression line:

$$\hat{k}_i = -2.59 + 1.44\,\hat{k}_M$$

CAPM - Beta

Stock H, high risk: $\beta = 2$

Stock A, average risk: $\beta = 1$

Stock L, low risk: $\beta = 0.5$

	Return			
Year	Stock H	Stock A	Stock L	market portfolio
2001	10%	10%	10%	10%
2002	30%	20%	15%	20%
2003	-30%	-10%	0%	-10%

CAPM – Beta, Summary

1. A stock's risk consists of two components, systematic risk and unsystematic risk.
2. Unsystematic risk can be eliminated by diversification. We are left, then, with only systematic risk. Systematic risk is the only relevant risk to a investor.
3. Investors must be compensated for bearing risk. However, compensation is required only for risk which cannot be eliminated by diversification.
4. The systematic risk of a stock is measured by its beta coefficient, which is an index of the stock's relative volatility.
5. Since a stock's beta coefficient determines how the stock affects the riskiness of a diversified portfolio, beta is the most relevant measure of any stock's risk.

CAPM Example

- Find the required return on a stock given that the risk-free rate is 8%, the expected return on the market portfolio is 12%, and the beta of the stock is 2.

- $K_i = K_{rf} + \beta_i(K_m - K_{rf})$
- $K_i = 8\% + 2(12\% - 8\%)$
- $K_i = 16\%$

- Note that you can then compare the required rate of return to the expected rate of return. You would only invest in stocks where the expected rate of return exceeded the required rate of return.

Another CAPM Example

- Find the beta on a stock given that its expected return is 12%, the risk-free rate is 4%, and the expected return on the market portfolio is 10%.

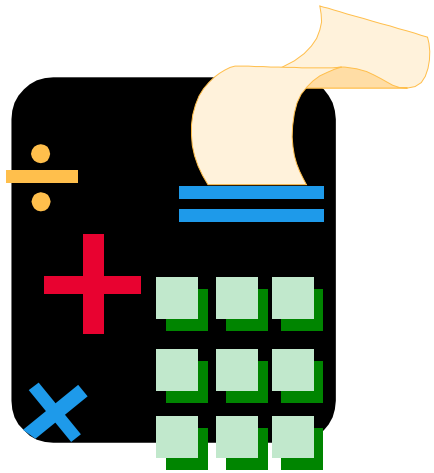
- $12\% = 4\% + \beta_i(10\% - 4\%)$

$\beta_i = \frac{12\% - 4\%}{10\% - 4\%}$

$\beta_i = 1.33$

- Note that beta measures the stock's volatility (or risk) relative to the market.

Average Returns: The First Lesson



- Average annual return $= \dfrac{\Sigma \text{ yearly returns}}{\text{number of years}}$

Average Returns: The First Lesson

Average Annual Returns: 1926–99

Investment	Average Return	Maximum	Minimum
Large stocks	13.3%	53.99%	−43.34%
Small stocks	17.6	142.87	−58.01
Long-term corporate bonds	5.9	43.79	−8.09
Long-term government bonds	5.5	40.35	−9.19
U.S. Treasury bills	3.8	14.71	−0.02
Inflation	3.2	13.31	−10.30

Source: *Stocks, Bonds, Bills, and Inflation Yearbook*™, Ibbotson Associates, Inc., Chicago (annually updates work by Roger G. Ibbotson and Rex A. Sinquefield). All rights reserved.

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Average Returns: The First Lesson

Risk-free rate

The rate of return on a riskless investment.

Risk premium

The extra return on a risky asset over the risk-free rate; the reward for bearing risk.

Average Returns: The First Lesson

Average Annual Returns and Risk Premiums: 1926–99		
Investment	**Average Return**	**Risk Premium**
Large stocks	13.3%	9.5%
Small stocks	17.6	13.8
Long-term corporate bonds	5.9	2.1
Long-term government bonds	5.5	1.7
U.S. Treasury bills	3.8	0.0

Source: *Stocks, Bonds, Bills, and Inflation Yearbook*™, Ibbotson Associates, Inc., Chicago (annually updates work by Roger G. Ibbotson and Rex A. Sinquefield). All rights reserved.

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Return Variability

Variance

A common measure of volatility.

Standard deviation

The square root of the variance.

Normal distribution

A symmetric, bell-shaped frequency distribution that is completely defined by its average and standard deviation.

Return Variability

Variance of return

$$Var(R) = \sigma^2 = \frac{\sum_{i=1}^{N}\left(R_i - \overline{R}\right)^2}{N-1}$$

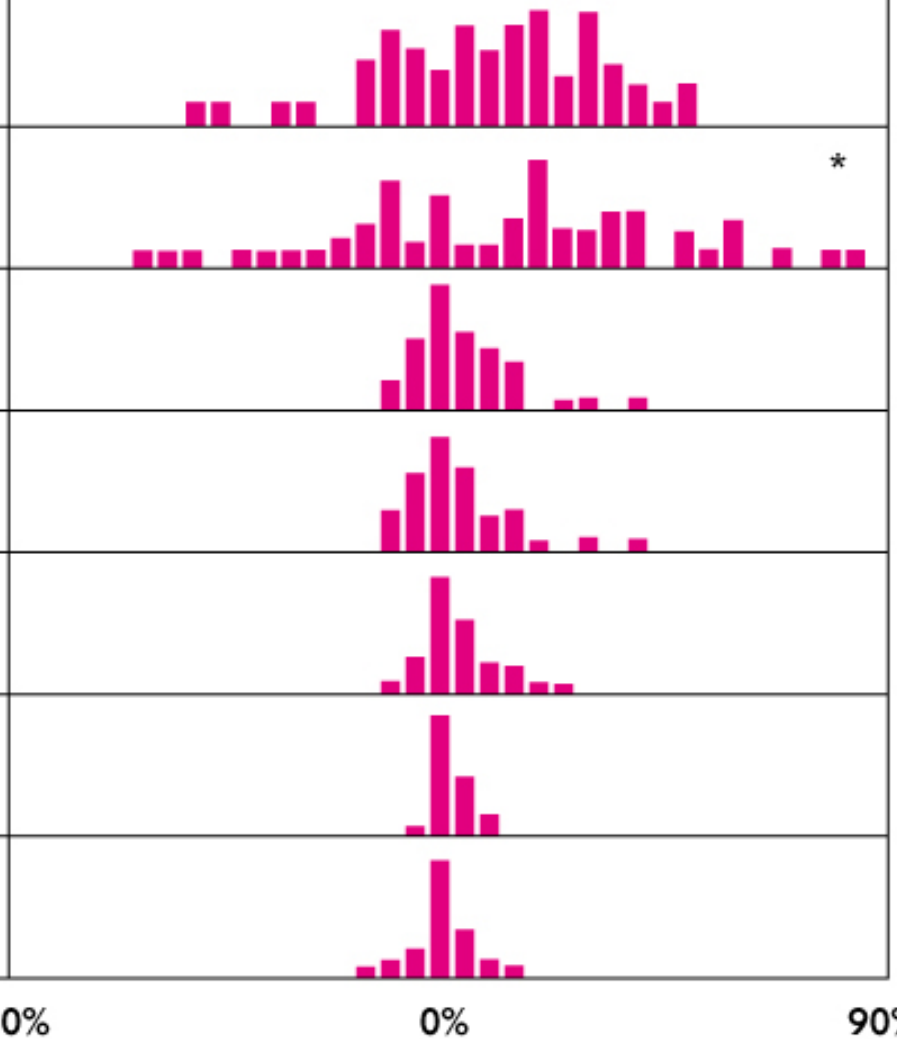
where N is the number of returns

Standard deviation of return

$$SD(R) = \sigma = \sqrt{Var(R)}$$

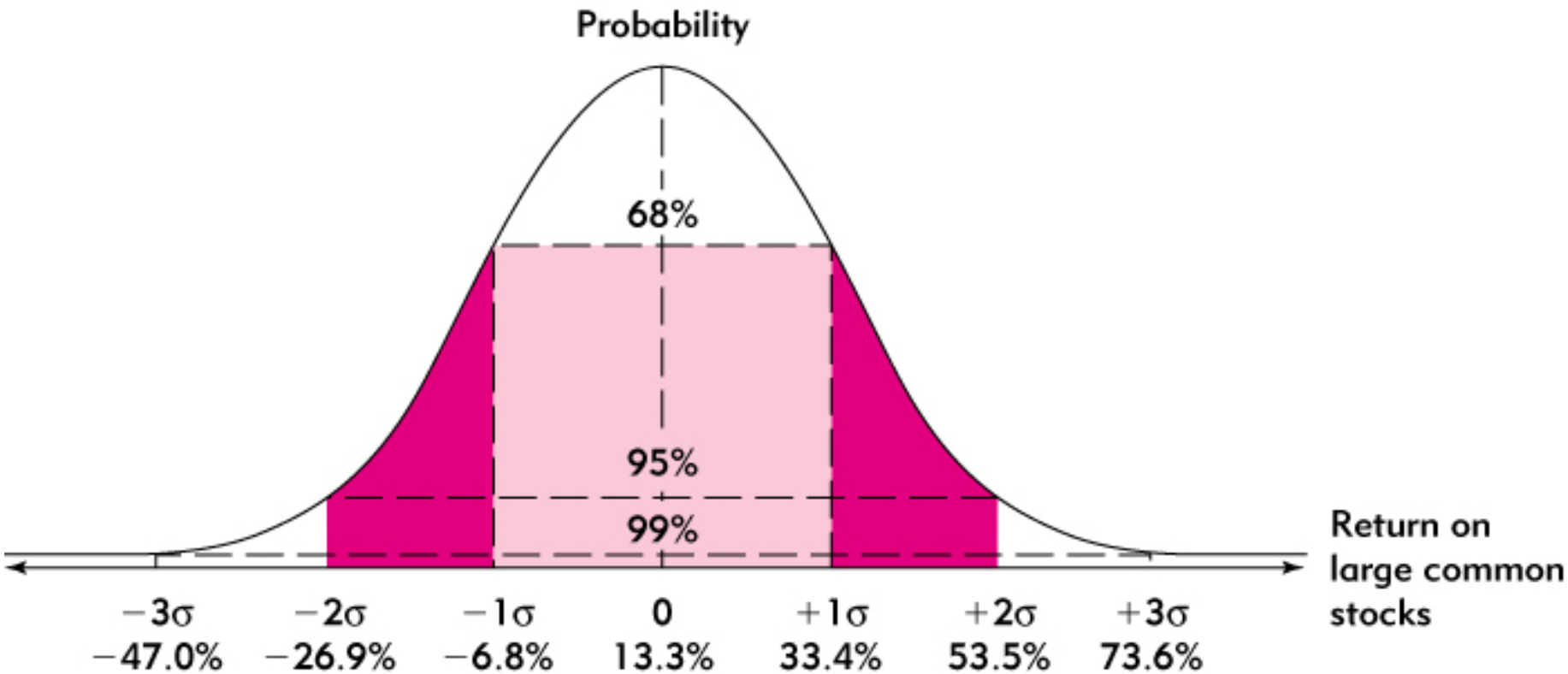
Return Variability

Series	Average Return	Standard Deviation	Distribution
Large-company stocks	13.3%	20.1%	
Small-company stocks	17.6	33.6	*
Long-term corporate bonds	5.9	8.7	
Long-term government	5.5	9.3	
Intermediate-term government	5.4	5.8	
U.S. Treasury bills	3.8	3.2	
Inflation	3.2	4.5	



*The 1933 small-company stock total return was 142.9 percent.

Return Variability



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Return Variability

- The greater the potential reward, the greater the risk.

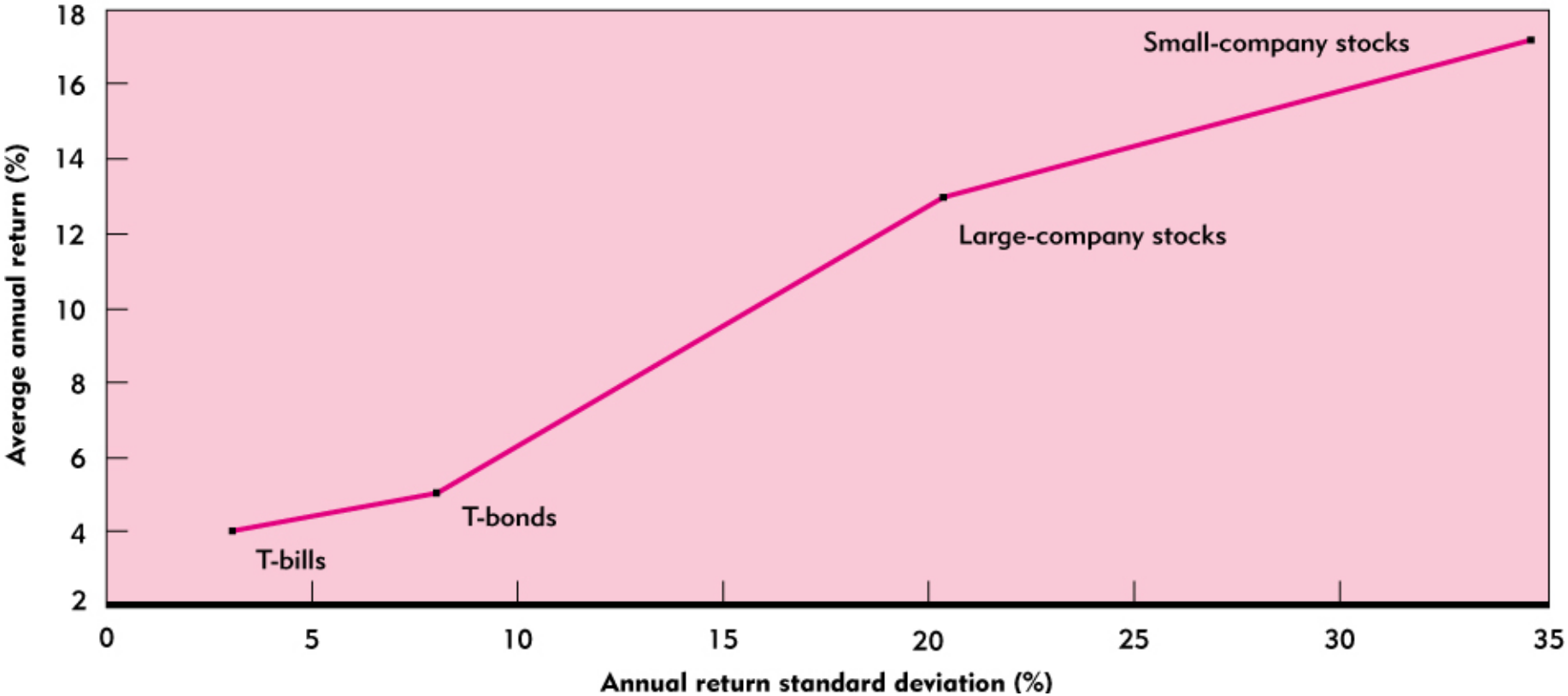
Return Variability

Top 12 One-Day Percentage Changes in the Dow Jones Industrial Average

October 19, 1987	**- 22.6 %**	**March 14, 1907**	**- 8.3 %**
October 28, 1929	**- 12.8**	**October 26, 1987**	**- 8.0**
October 29, 1929	**- 11.7**	**July 21, 1933**	**- 7.8**
November 6, 1929	**- 9.9**	**October 18, 1937**	**- 7.7**
December 18, 1899	**- 8.7**	**February 1, 1917**	**- 7.2**
August 12, 1932	**- 8.4**	**October 27, 1997**	**- 7.2**

Source: Dow Jones

Risk and Return



Risk and Return

- The risk-free rate represents compensation for just waiting. So, it is often called the *time value of money*.
- If we are willing to bear risk, then we can expect to earn a risk premium, at least on average.
- Further, the more risk we are willing to bear, the greater is that risk premium.

Summary:

- We know how to measure risk, using standard deviation for overall risk and beta for market risk.
- We know how to reduce overall risk to only market risk through diversification.
- We need to know how to price risk so we will know how much extra return we should require for accepting extra risk.

Determining the Required Return

- The required rate of return for a particular investment depends on several factors, each of which depends on several other factors (i.e., it is pretty complex!):
- The two main factors for any investment are:
  - The perceived riskiness of the investment
  - The required returns on alternative investments
- An alternative way to look at this is that the required return is the sum of the RFR and a risk premium:

$$E(R_i) = \text{RFR} + \text{Risk Premium}$$

The Risk-free Rate of Return

- The risk-free rate is the rate of interest that is earned for simply delaying consumption
- It is also referred to as the *pure time value of money*
- The risk-free rate is determined by:
  - The time preferences of individuals for consumption
    - Relative ease or tightness in money market (supply & demand)
    - Expected inflation
  - The long-run growth rate of the economy
    - Long-run growth of labor force
    - Long-run growth of hours worked
    - Long-run growth of productivity

The Risk Premium

- The risk premium is the return required in excess of the risk-free rate
- Theoretically, a risk premium could be assigned to every risk factor, but in practice this is impossible
- Therefore, we can say that the risk premium is a function of several major sources of risk:
  - Business risk
  - Financial leverage
  - Liquidity risk
  - Exchange rate risk

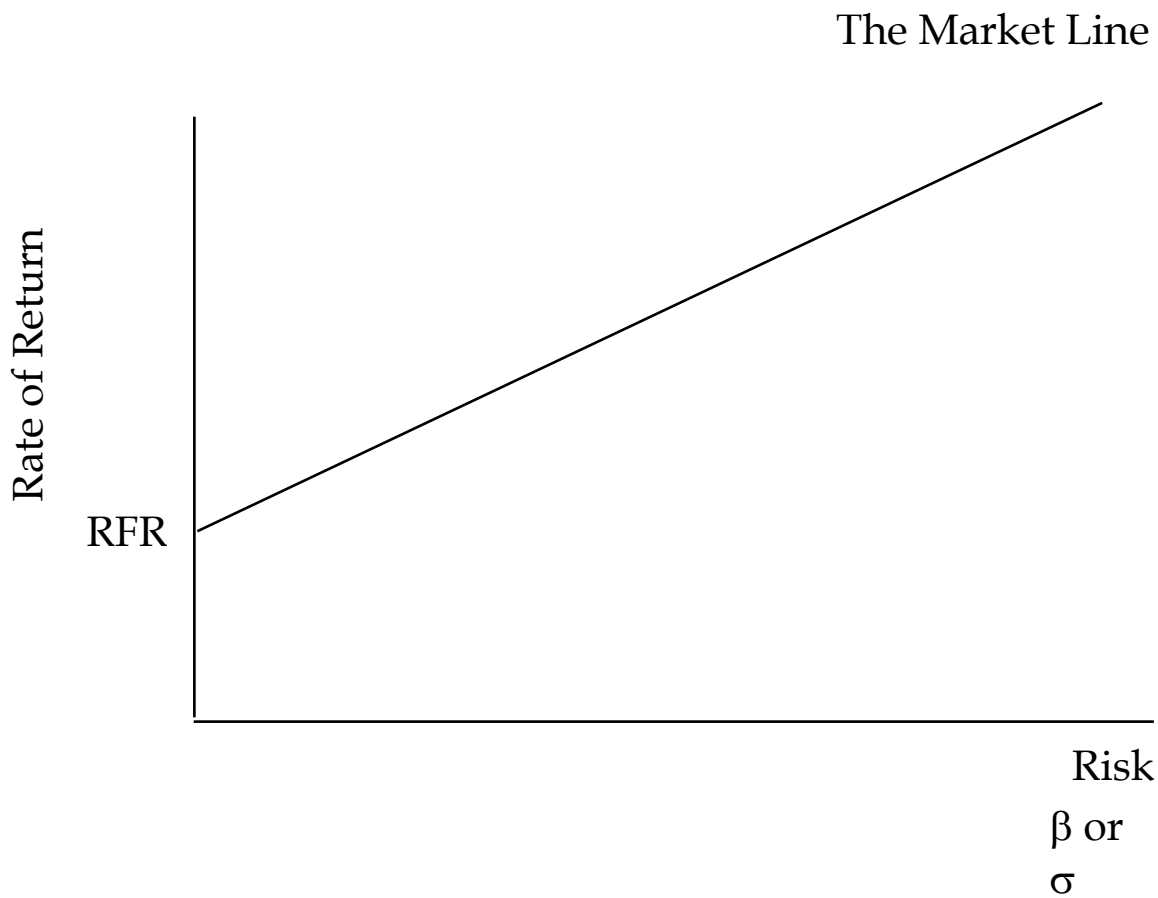
The MPT View of Required Returns

- Modern portfolio theory assumes that the required return is a function of the RFR, the market risk premium, and an index of systematic risk:

$$E(R_i) = R_f + \beta_i\left(E(R_M) - R_f\right)$$

This model is known as the Capital Asset Pricing Model (CAPM).
It is also the equation for the Security Market Line (SML)

Risk and Return Graphically



Portfolio Risk and Return

- A portfolio is a collection of assets (stocks, bonds, cars, houses, diamonds, etc)
- It is often convenient to think of a person owning several "portfolios," but in reality you have only one portfolio (the one that comprises everything you own)

Expected Return of a Portfolio

- The expected return of a portfolio is a weighted average of the expected returns of its components:

$$E\left(R_P\right) = \sum_{i=1}^{N} w_i R_i$$

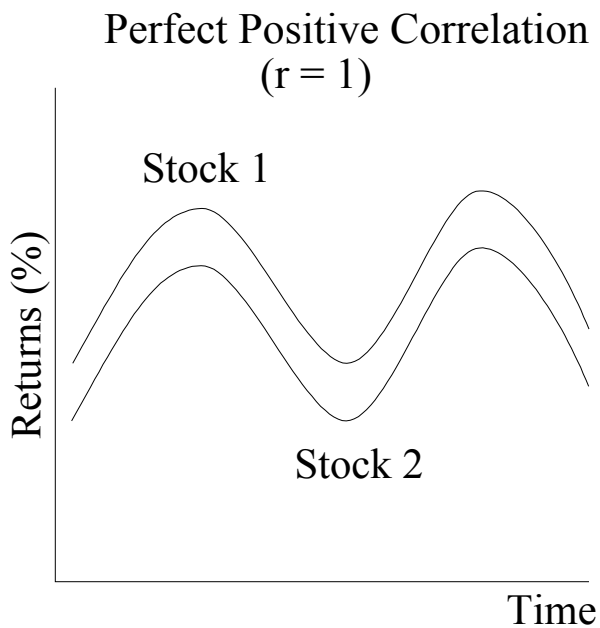
  - Note: w_i is the proportion of the portfolio that is invested in security I, and R_i is the expected return for security I.

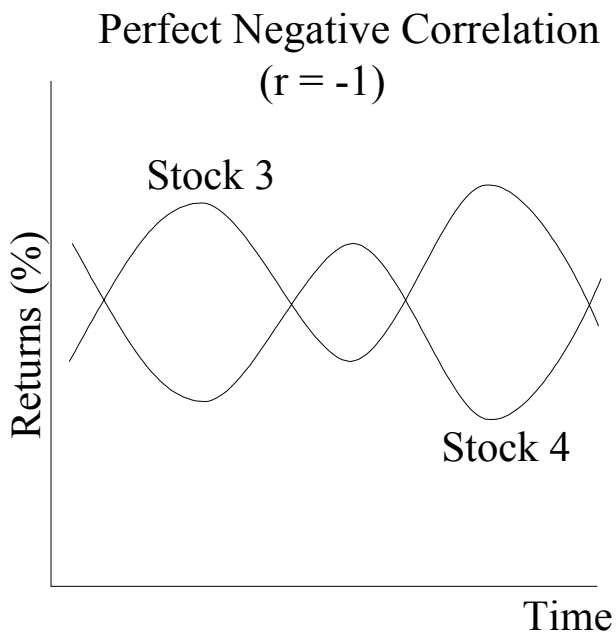
Portfolio Risk

- The standard deviation of a portfolio is not a weighted average of the standard deviations of the individual securities.
- The riskiness of a portfolio depends on both the riskiness of the securities, and the way that they move together over time (correlation)
- This is because the riskiness of one asset may tend to be canceled by that of another asset

The Correlation Coefficient

- The correlation coefficient can range from -1.00 to +1.00 and describes how the returns move together through time.





The Portfolio Standard Deviation

- The portfolio standard deviation can be thought of as a weighted average of the individual standard deviations plus terms that account for the co-movement of returns
- For a two-security portfolio:

$$\sigma_P = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2r_{1,2}\sigma_1\sigma_2 w_1 w_2}$$

An Example: Perfect Pos. Correlation

		Potential Returns		
State of Economy	**Probability**	**ABC**	**XYZ**	**50/50 Portfolio**
Recession	25%	2%	2%	2%
Moderate Growth	50%	8%	8%	8%
Boom	25%	14%	14%	14%
Expected Return		8%	8%	8%
Standard Deviation		4.24%	4.24%	4.24%
Correlation		1.00		

$$\sigma_P = \sqrt{.5^2(0.0424)^2 + .5^2(0.0424)^2 + 2(1.00)(0.0424)(0.0424)(0.5)(0.5)} = 0.0424$$

An Example: Perfect Neg. Correlation

		Potential Returns		
State of Economy	**Probability**	**ABC**	**XYZ**	**50/50 Portfolio**
Recession	25%	2%	14%	8%
Moderate Growth	50%	8%	8%	8%
Boom	25%	14%	2%	8%
Expected Return		8%	8%	8%
Standard Deviation		4.24%	4.24%	0.00%
Correlation		-1.00		

$$\sigma_P = \sqrt{.5^2(0.0424)^2 + .5^2(0.0424)^2 + 2(-1.00)(0.0424)(0.0424)(0.5)(0.5)} = 0.00$$

An Example: Zero Correlation

		Potential Returns		
State of Economy	**Probability**	**ABC**	**XYZ**	**50/50 Portfolio**
Recession	25%	2%	2%	2%
Moderate Growth	50%	8%	2%	5%
Boom	25%	14%	2%	8%
Expected Return		8%	2%	5%
Standard Deviation		4.24%	0.00%	2.12%
Correlation		0.00		

$$\sigma_P = \sqrt{.5^2(0.0424)^2 + .5^2(0.0424)^2 + 2(0)(0.0424)(0.0424)(0.5)(0.5)} = 0.0212$$

Interpreting the Examples

- In the three previous examples, we calculated the portfolio standard deviation under three alternative correlations.
- Here's the moral: The lower the correlation, the more risk reduction (diversification) you will achieve.

Correlation	Risk Reduction
+1.00	None
-1.00	Major (to risk-free in this example)
0.00	Lots (cut risk in half in this example)

CAPM – Required rate of return

Required rate of return on market portfolio consists of risk-free rate and market risk premium.

$$R_M = R_f + RP_M$$

Market risk premium: The additional return over the risk-free rate needed to compensate investors for assuming an systematic risk.

$$RP_M = R_M - R_f$$

Required rate of return on individual stock, *j*, also consists of risk-free rate and risk premium of *j*.

$$R_j = R_f + RP_j$$

If we know the market risk premium, RP_M, and beta for the individual stock, β_j, then risk premium for stock *j*,

$$RP_j = RP_M \times \beta_j$$

Therefore, required rate of return on the individual stock, *j*, can be represented by

$$R_j = R_f + RP_j = R_f + RP_M \times \beta_j = R_f + (R_M - R_f) \times \beta_j$$

$$R_j = R_f + (R_M - R_f) \times \beta_j$$

Equation for Security Market Line (SML)

Examples

Assume: the historic market risk premium has been about 8.5%. The risk-free rate is currently 5%. GTX Corp. has a beta of .85. What return should you expect from an investment in GTX?

$$E(R_i) = R_f + [E(R_M) - R_f] \times \beta_i$$

$$E(R_{GTX}) = 5\% + 8.5\% \times .85 = 12.225\%$$

The Security Market Line (SML): Calculating required rates of return

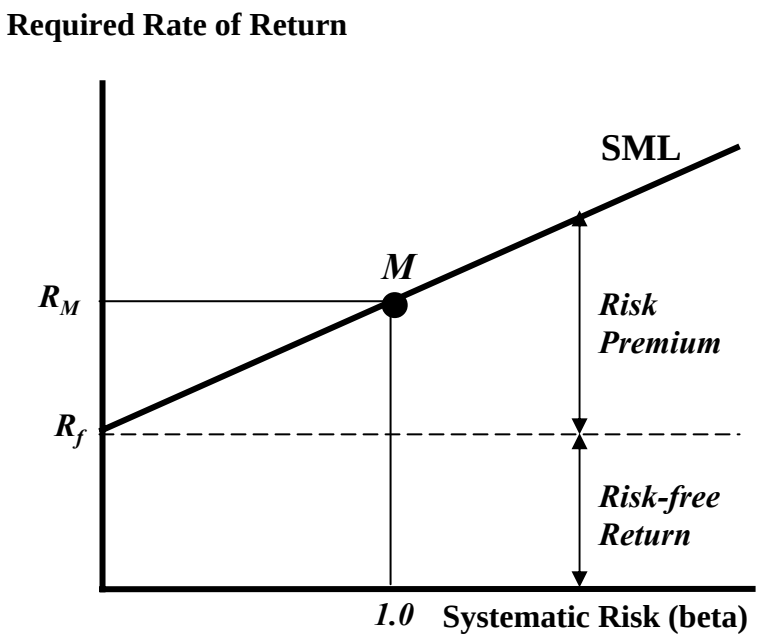
$$\text{SML: } k_i = k_{RF} + (k_M - k_{RF})\,\beta_i$$

- Assume $k_{RF} = 8\%$ and $k_M = 15\%$.
- The market (or equity) risk premium is $RP_M = k_M - k_{RF} = 15\% - 8\% = 7\%$.

CAPM - Beta

Security Market Line (SML)

A line that describes the linear relationship between required rates of return for individual securities (and portfolios) and systematic risk, as measured by beta.



What is the market risk premium?

- Additional return over the risk-free rate needed to compensate investors for assuming an average amount of risk.
- Its size depends on the perceived risk of the stock market and investors' degree of risk aversion.
- Varies from year to year, but most estimates suggest that it ranges between 4% and 8% per year.

CAPM

$$R_j = R_f + (R_M - R_f) * \beta_j$$

R_j : market required rate of return on stock j
R_M : required rate of return on market portfolio
$R_M - R_f$: Market Risk Premium

In equilibrium, expected return is the same as the required rate of return, so

$$E(R_j) = R_f + (E(R_M) - R_f) * \beta_j$$

$E(R_j)$: Expected return or market required rate of return
R_f : Risk-free rate
$E(R_M)$: Expected return on market portfolio
$E(R_M) - R_f$: Market Risk Premium
β_j : Index of systematic risk for stock, j

R_f : Risk-free rate

The rate on short- to intermediate-term Treasury securities is used as a proxy for the risk-free rate.

Market Portfolio

As a proxy for the market portfolio, most people use the **Standard & Poor's 500 Stock Price Index (S&P 500 Index)*.**

$E(R_M) - R_f$: Expected Market Risk Premium

Usually the historical average of risk premium of S&P 500 Index is used as the proxy for the expected market risk premium. The expected risk premium for the S&P 500 Index has generally ranged from 5 to 8 percent.

β_j : Index of systematic risk for stock, *j*

Financial economists estimate the beta for each individual stock. The slope coefficient of historical relationship between returns on the market portfolio and an individual stock's returns.

*** S&P 500 Index** : A market-value-weighted index of 500 large-capitalization common stocks selected from a broad cross section of industry group. It is used as a measure of overall market performance.

Calculating required rates of return

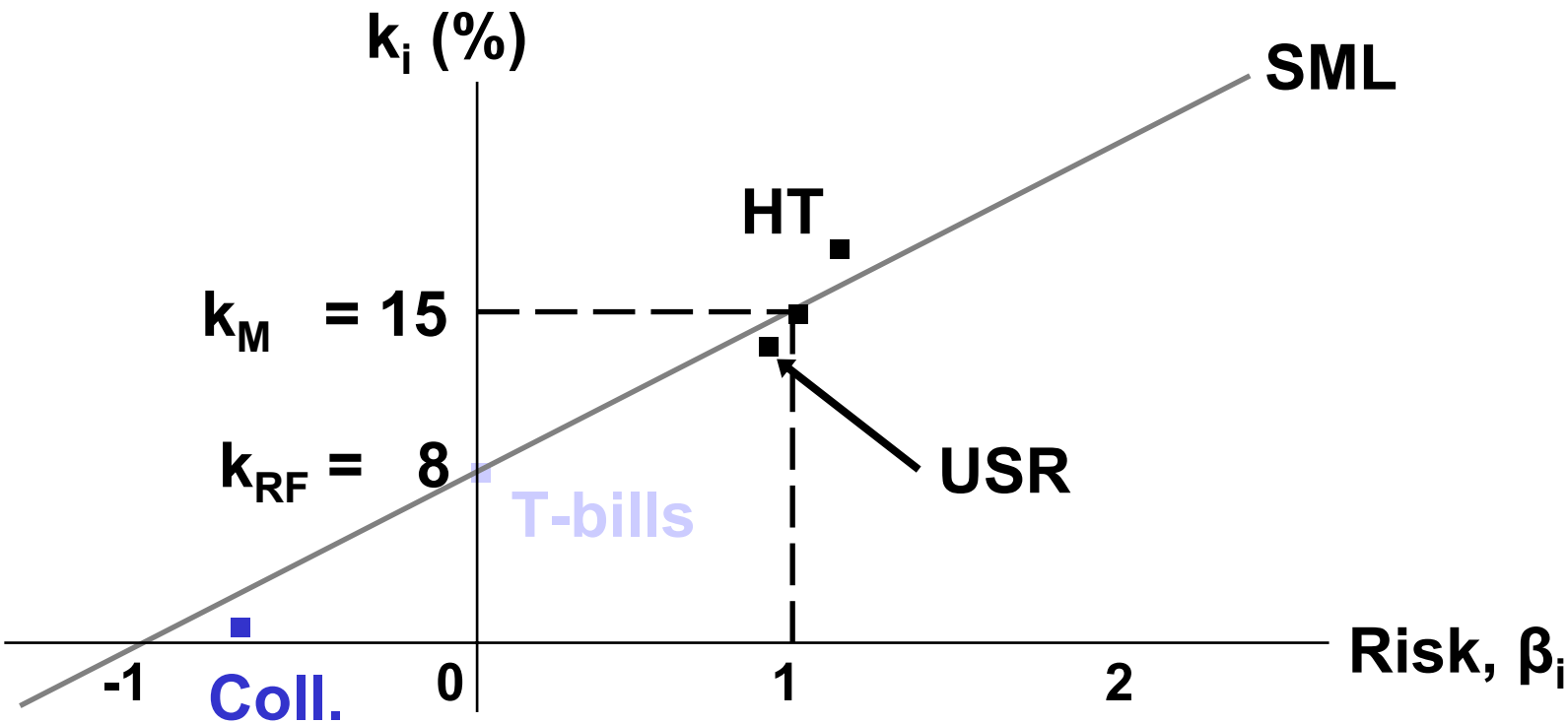
- $k_{HT} = 8.0\% + (15.0\% - 8.0\%)(1.30)$
  $= 8.0\% + (7.0\%)(1.30)$
  $= 8.0\% + 9.1\% = 17.10\%$
- $k_{M} = 8.0\% + (7.0\%)(1.00) = 15.00\%$
- $k_{USR} = 8.0\% + (7.0\%)(0.89) = 14.23\%$
- $k_{T\text{-}bill} = 8.0\% + (7.0\%)(0.00) = 8.00\%$
- $k_{Coll} = 8.0\% + (7.0\%)(-0.87) = 1.91\%$

Expected vs. Required returns

	$\hat{k}$	k	
HT	17.4%	17.1%	Undervalued ($\hat{k} > k$)
Market	15.0	15.0	Fairly valued ($\hat{k} = k$)
USR	13.8	14.2	Overvalued ($\hat{k} < k$)
T - bills	8.0	8.0	Fairly valued ($\hat{k} = k$)
Coll.	1.7	1.9	Overvalued ($\hat{k} < k$)

Illustrating the Security Market Line

SML: $k_i = 8\% + (15\% - 8\%)\ \beta_i$



An example: Equally-weighted two-stock portfolio

- Create a portfolio with 50% invested in HT and 50% invested in Collections.
- The beta of a portfolio is the weighted average of each of the stock's betas.

$$\beta_P = w_{HT}\,\beta_{HT} + w_{Coll}\,\beta_{Coll}$$
$$\beta_P = 0.5\,(1.30) + 0.5\,(-0.87)$$
$$\beta_P = 0.215$$

Calculating portfolio required returns

- The required return of a portfolio is the weighted average of each of the stock's required returns.

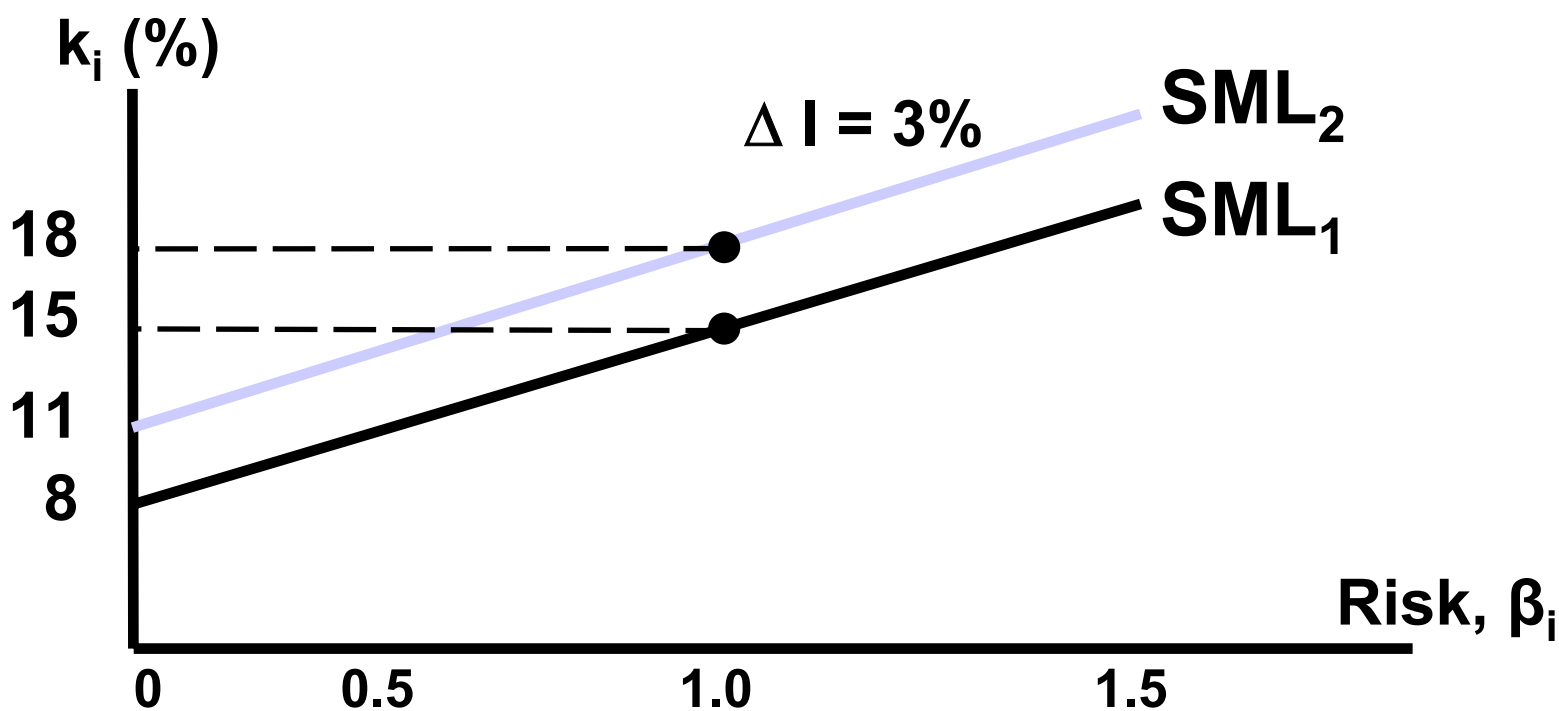
$$k_P = w_{HT}\, k_{HT} + w_{Coll}\, k_{Coll}$$
$$k_P = 0.5\ (17.1\%) + 0.5\ (1.9\%)$$
$$k_P = 9.5\%$$

- Or, using the portfolio's beta, CAPM can be used to solve for expected return.

$$k_P = k_{RF} + (k_M - k_{RF})\ \beta_P$$
$$k_P = 8.0\% + (15.0\% - 8.0\%)\ (0.215)$$
$$k_P = 9.5\%$$

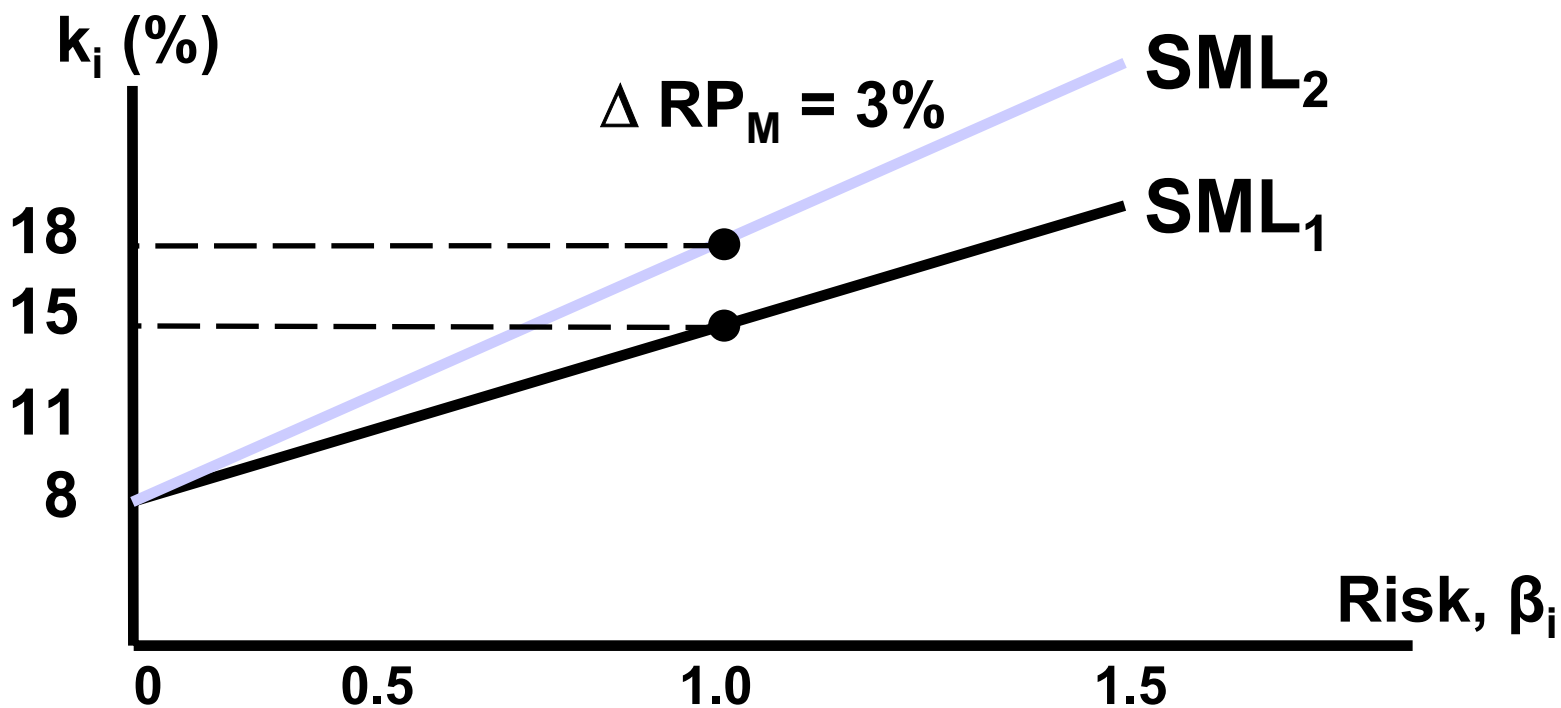
Factors that change the SML

- What if investors raise inflation expectations by 3%, what would happen to the SML?



Factors that change the SML

- What if investors' risk aversion increased, causing the market risk premium to increase by 3%, what would happen to the SML?



Verifying the CAPM empirically

- The CAPM has not been verified completely.
- Statistical tests have problems that make verification almost impossible.
- Some argue that there are additional risk factors, other than the market risk premium, that must be considered.

More thoughts on the CAPM

- Investors seem to be concerned with both market risk and total risk. Therefore, the SML may not produce a correct estimate of k_i.

$$k_i = k_{RF} + (k_M - k_{RF})\,\beta_i + ???$$

- CAPM/SML concepts are based upon expectations, but betas are calculated using historical data. A company's historical data may not reflect investors' expectations about future riskiness.

What is the Required Rate of Return?

- **The return on an investment required by an investor given market interest rates and the investment's risk.**

Required rate of return = **Risk-free rate of return** + **Risk premium**

market risk

company-unique risk

Required rate of return = **Risk-free rate of return** + **Risk premium**

market risk

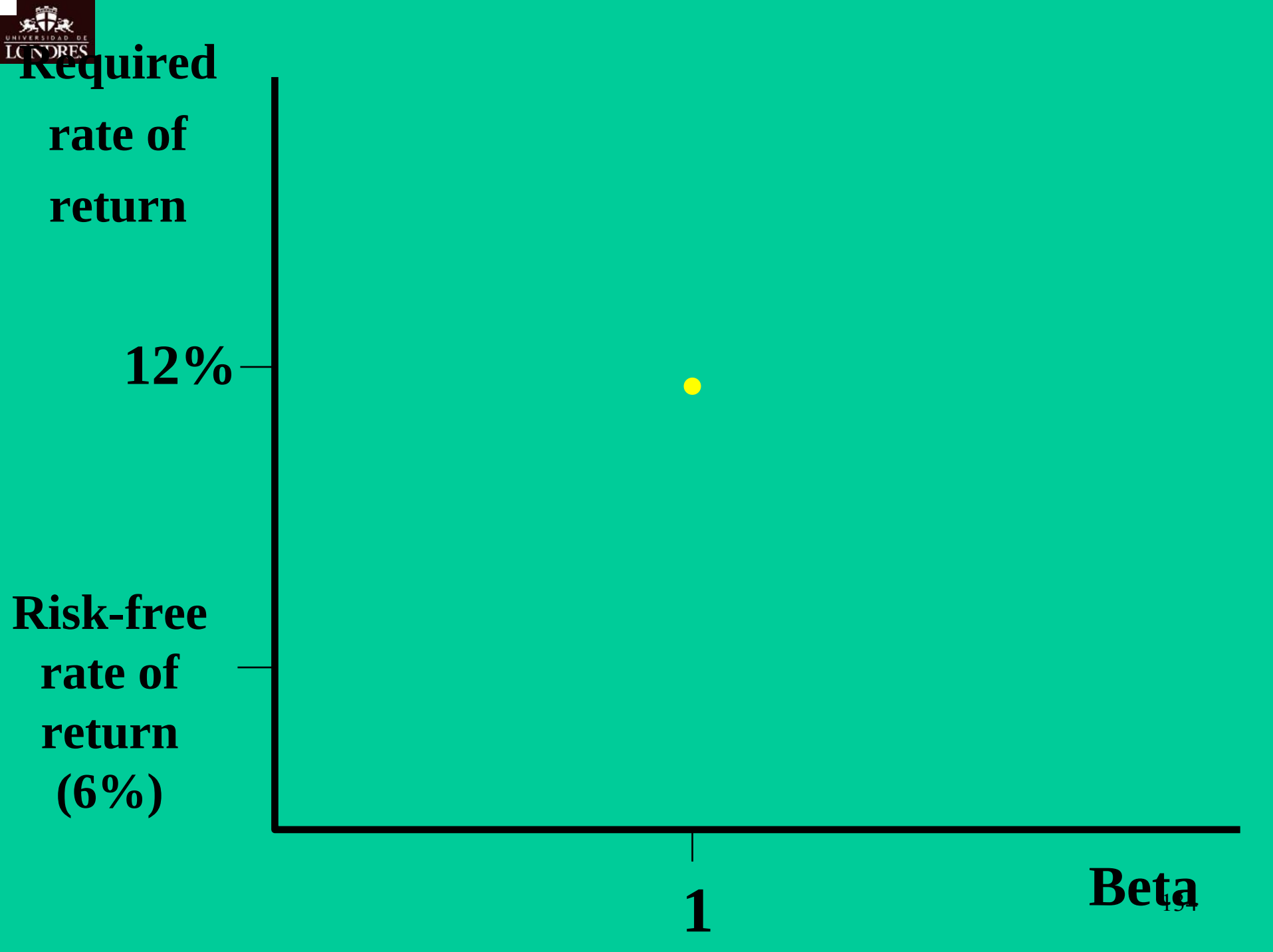
~~**company-unique risk**~~

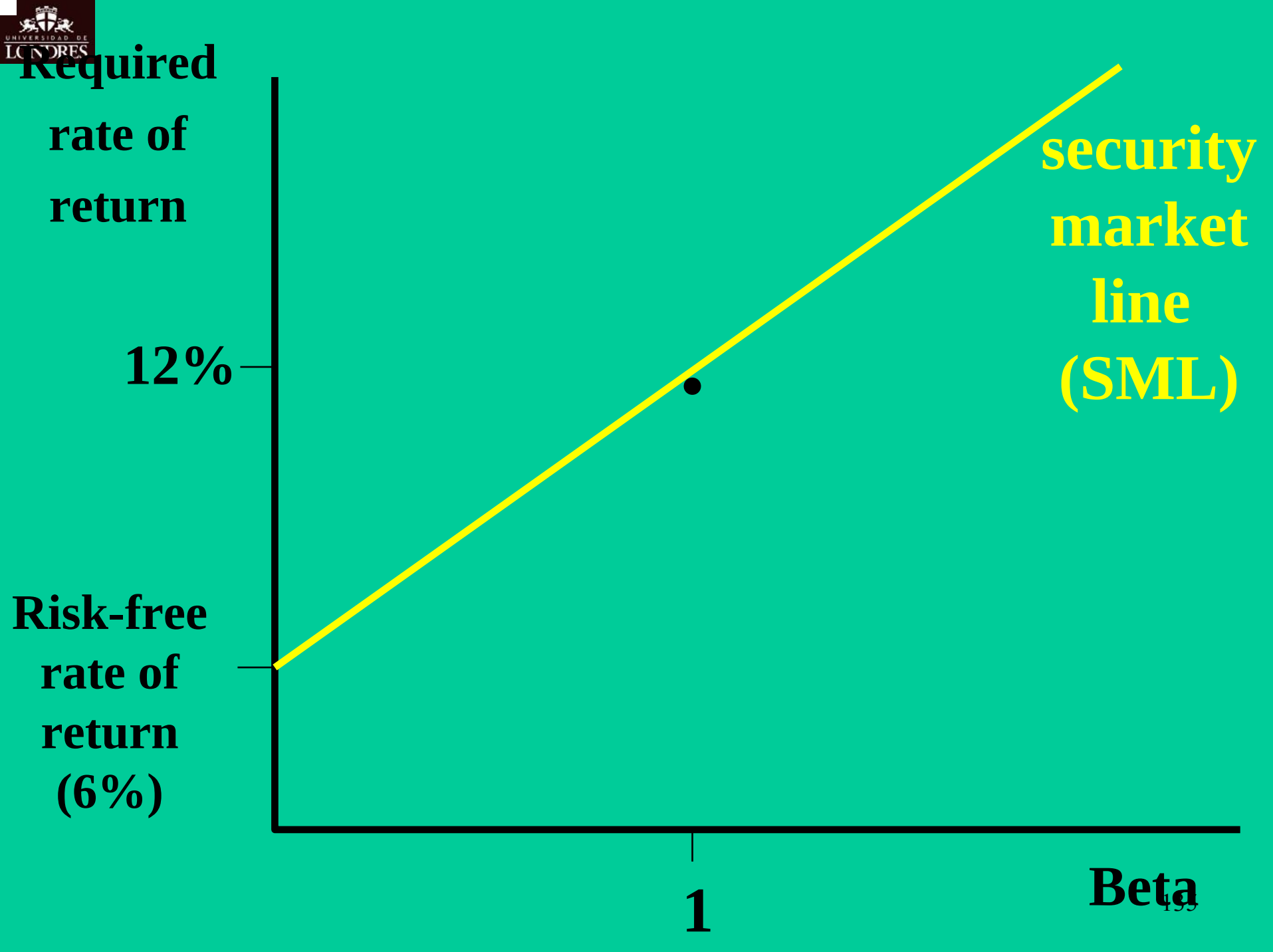
can be diversified away

Required rate of return

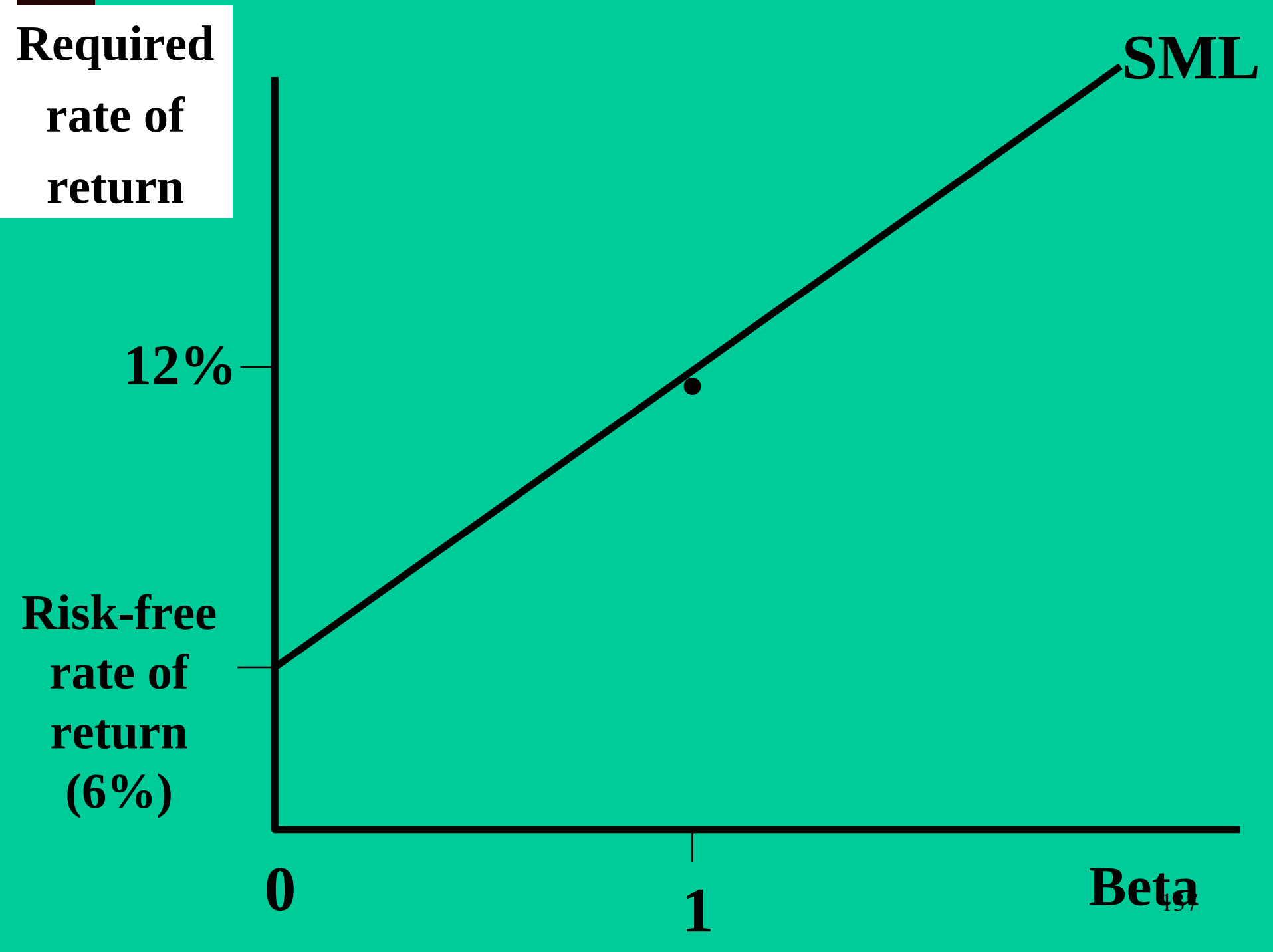
Let’s try to graph this relationship!

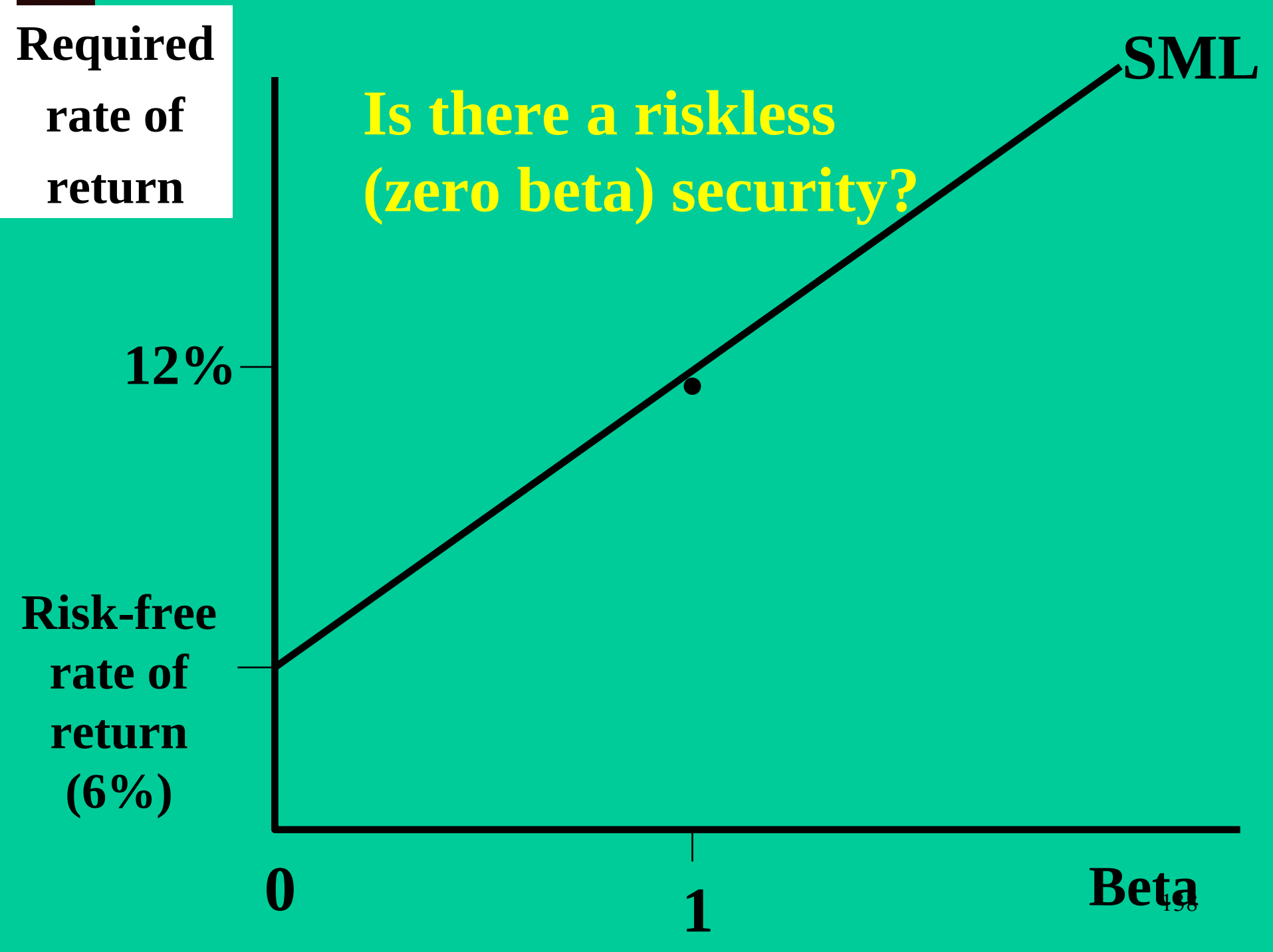
Beta

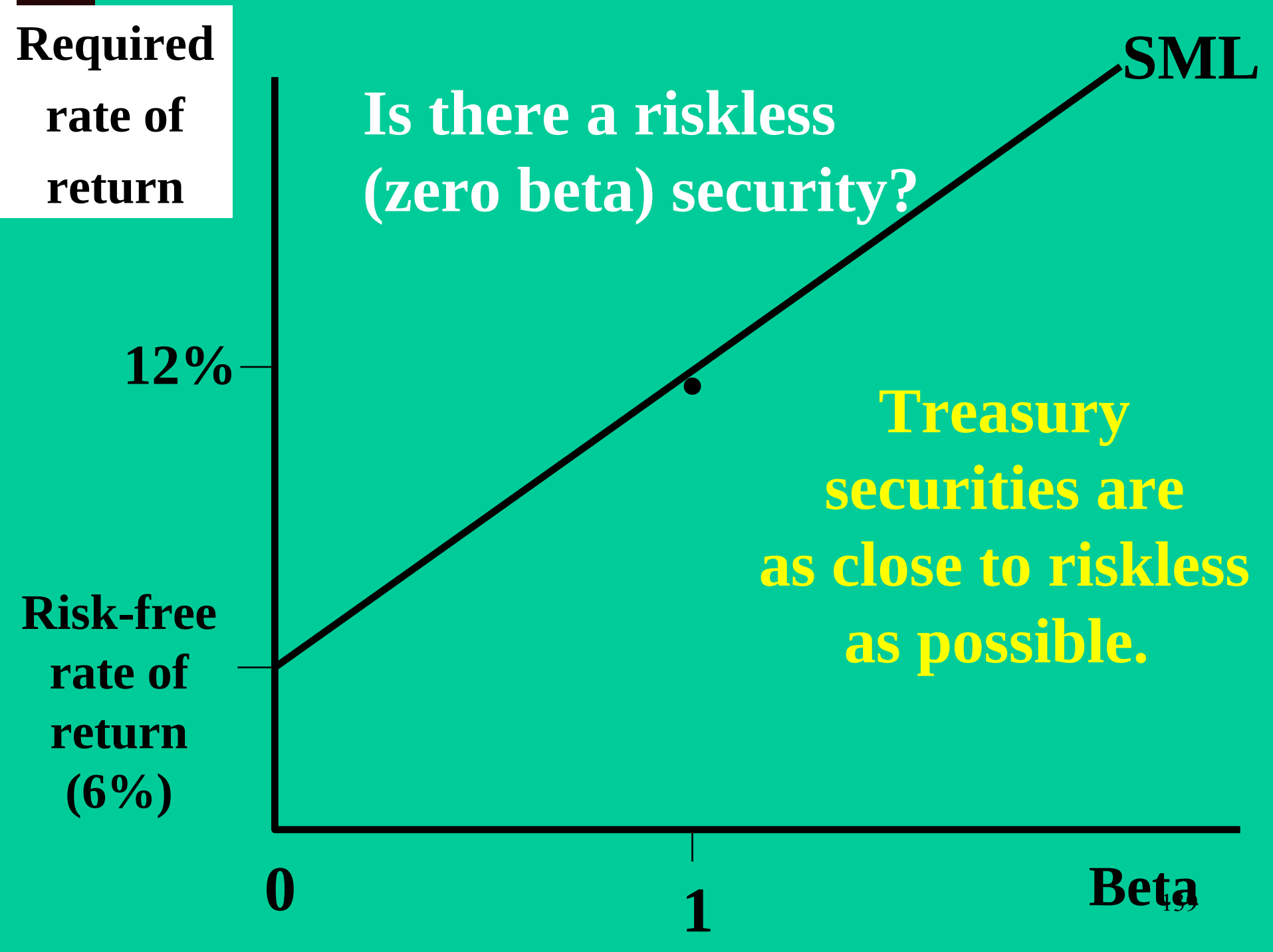

Required rate of return
12%
Risk-free rate of return (6%)
1
Beta

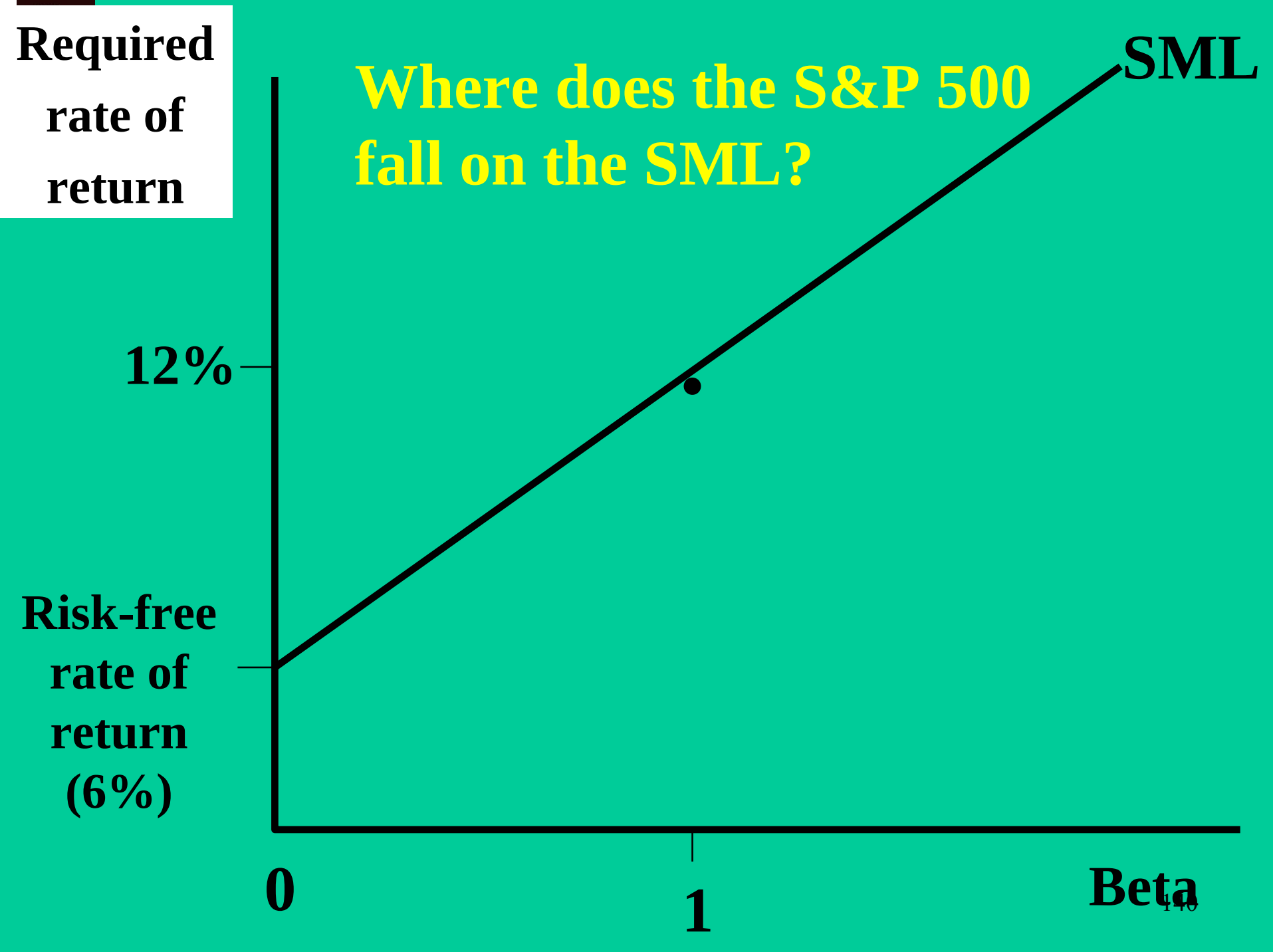

Required rate of return
12%
Risk-free rate of return (6%)
security market line (SML)
1
Beta

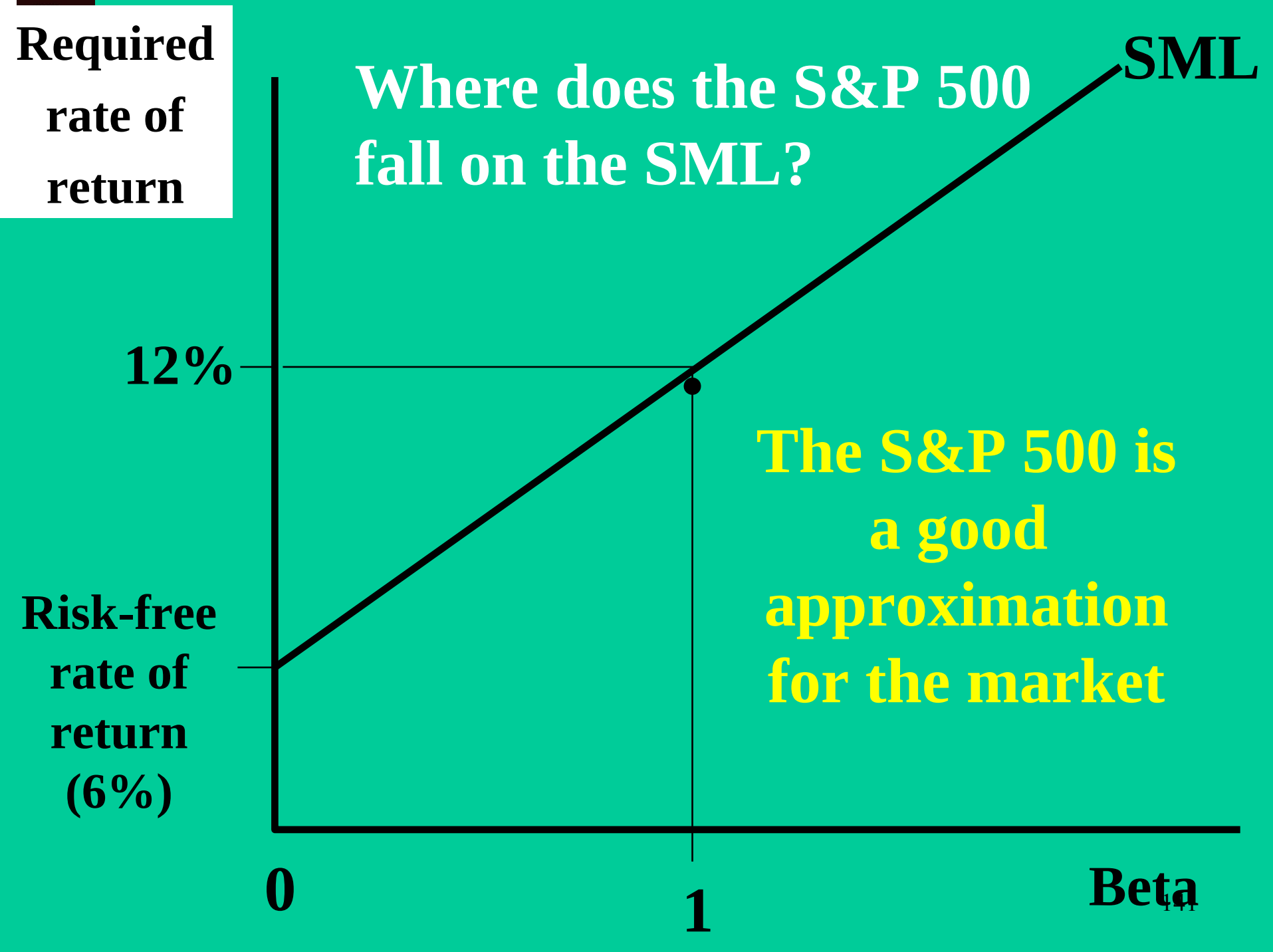
This linear relationship between risk and required return is known as the Capital Asset Pricing Model (CAPM).

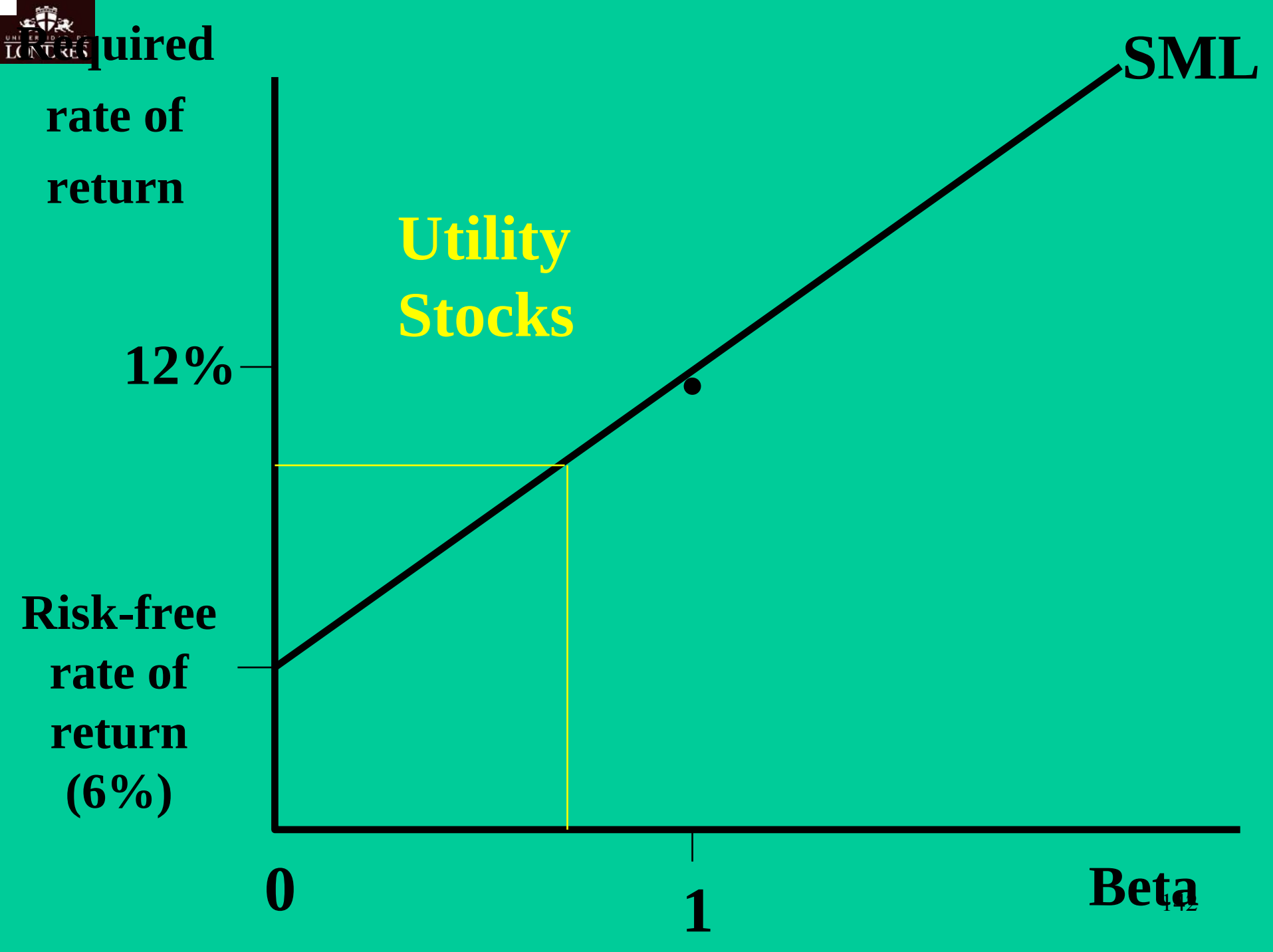

Required rate of return
SML
12%
Risk-free rate of return (6%)
0
1
Beta

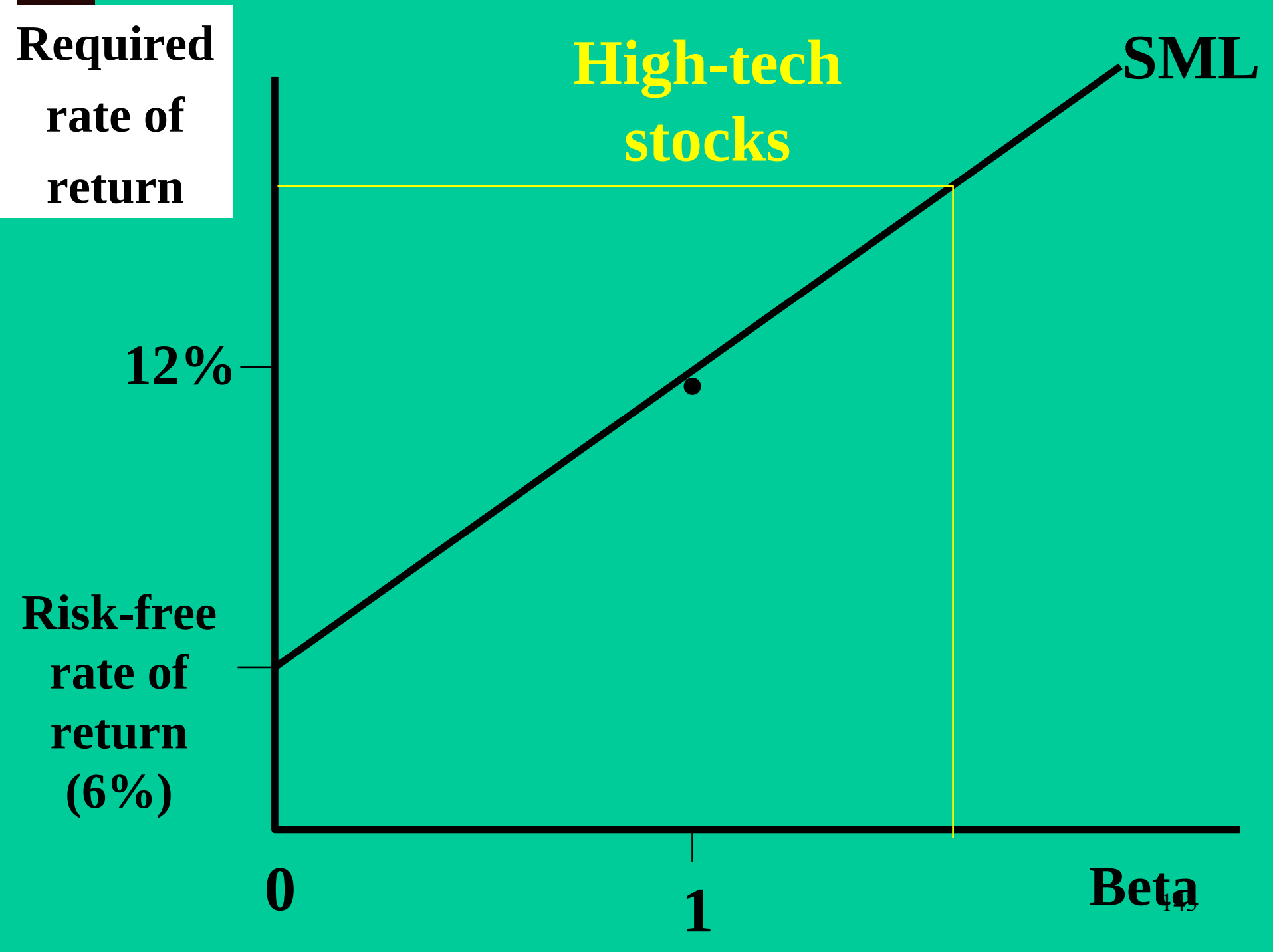

Required rate of return
SML
Is there a riskless (zero beta) security?
12%
Risk-free rate of return (6%)
0
1
Beta


Required rate of return
SML
Is there a riskless (zero beta) security?
12%
Treasury securities are as close to riskless as possible.
Risk-free rate of return (6%)
0
1
Beta


Required rate of return
Where does the S&P 500 fall on the SML?
SML
12%
Risk-free rate of return (6%)
0
1
Beta


Required rate of return
Where does the S&P 500 fall on the SML?
SML
12%
The S&P 500 is a good approximation for the market
Risk-free rate of return (6%)
0
1
Beta


Required rate of return
SML
Utility Stocks
12%
Risk-free rate of return (6%)
0
1
Beta


Required rate of return
High-tech stocks
SML
12%
Risk-free rate of return (6%)
0
1
Beta

The CAPM equation:

$$k_j = k_{rf} + \beta_j (k_m - k_{rf})$$

where:

k_j = the required return on security j,

k_{rf} = the risk-free rate of interest,

β_j = the beta of security j, and

k_m = the return on the market index.

Example:

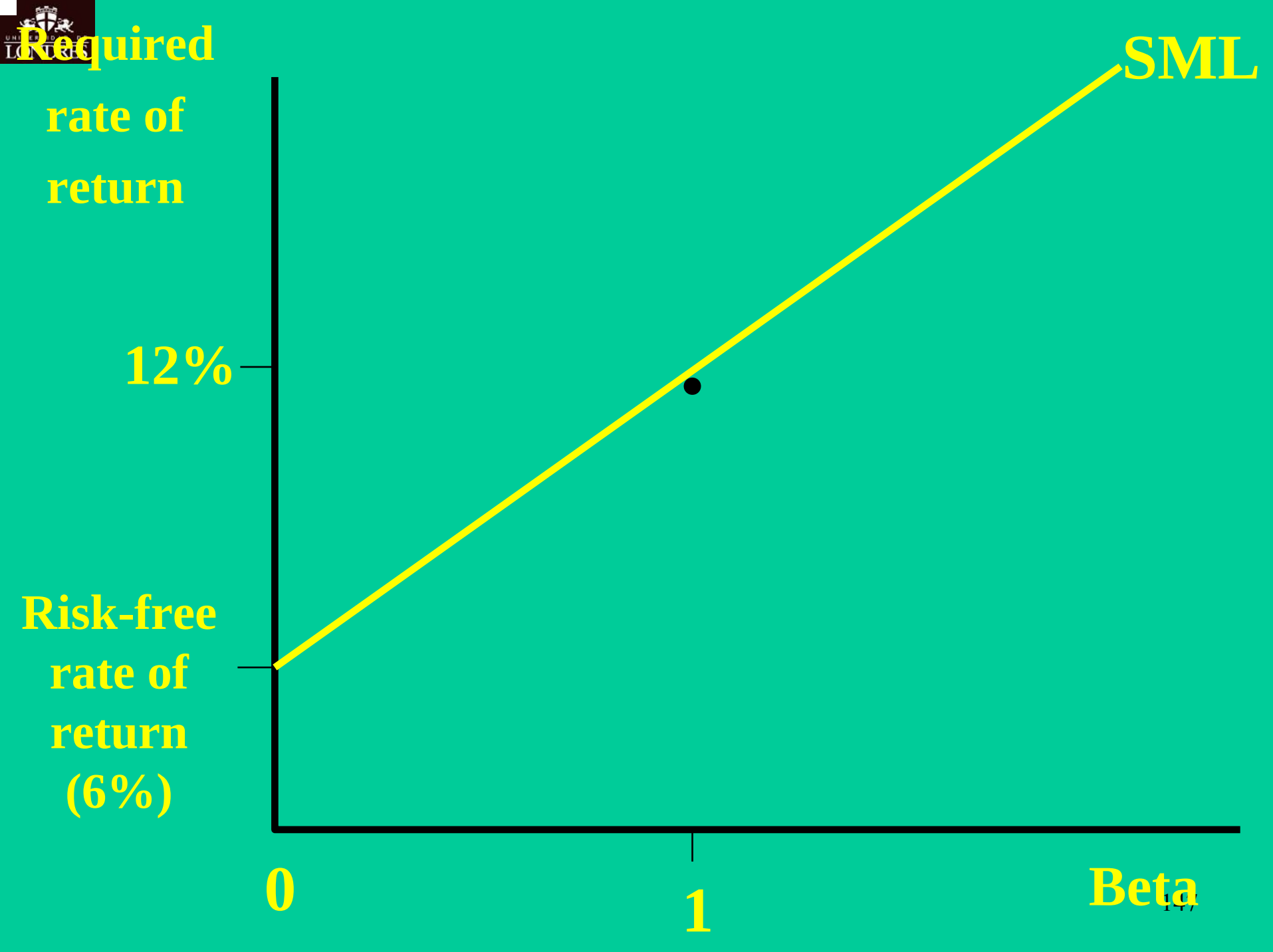
- **Suppose the Treasury bond rate is 6%, the average return on the S&P 500 index is 12%, and Walt Disney has a beta of 1.2.**
- **According to the CAPM, what should be the required rate of return on Disney stock?**

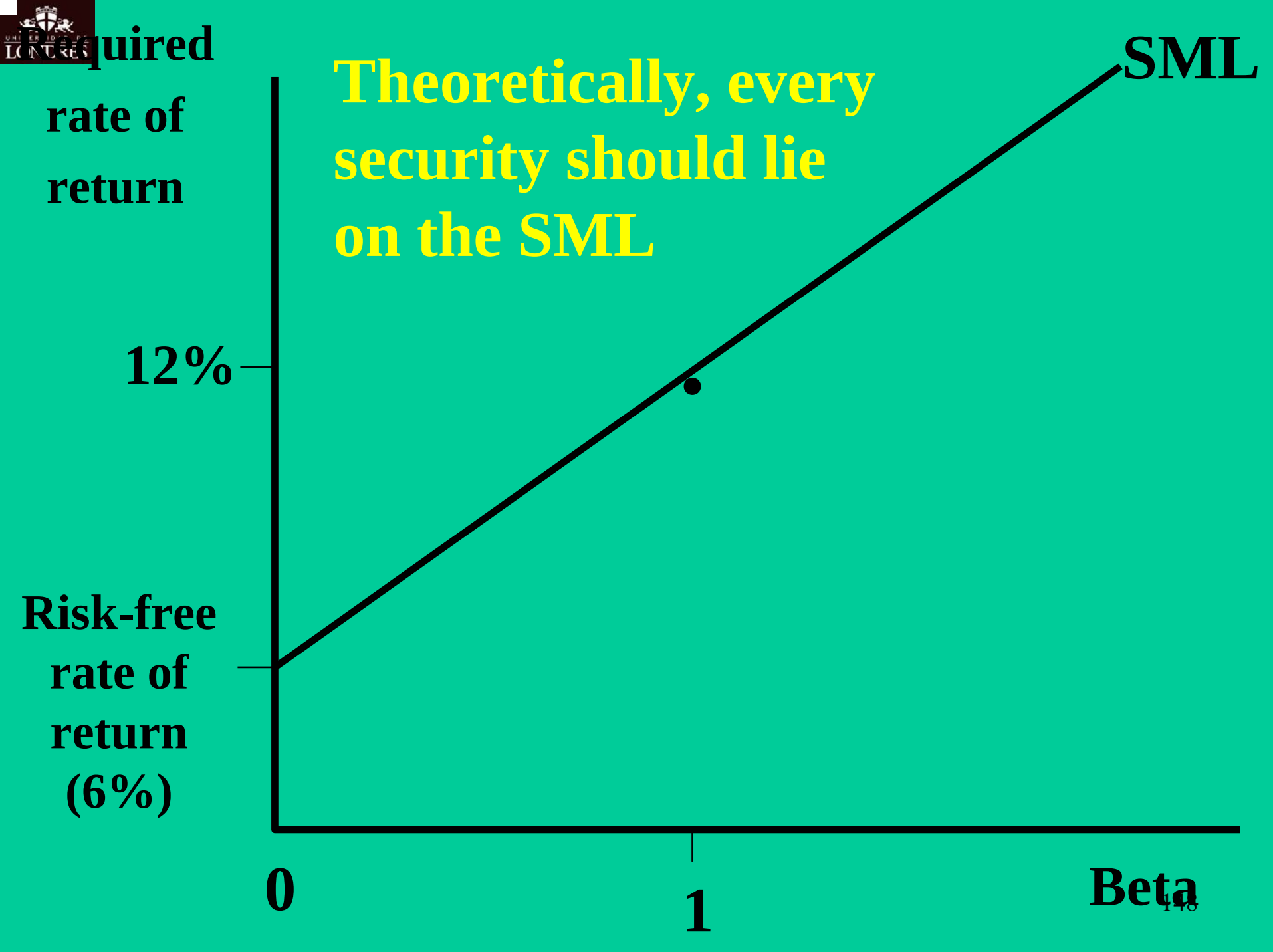
$$k_j = k_{rf} + \beta(k_m - k_{rf})$$

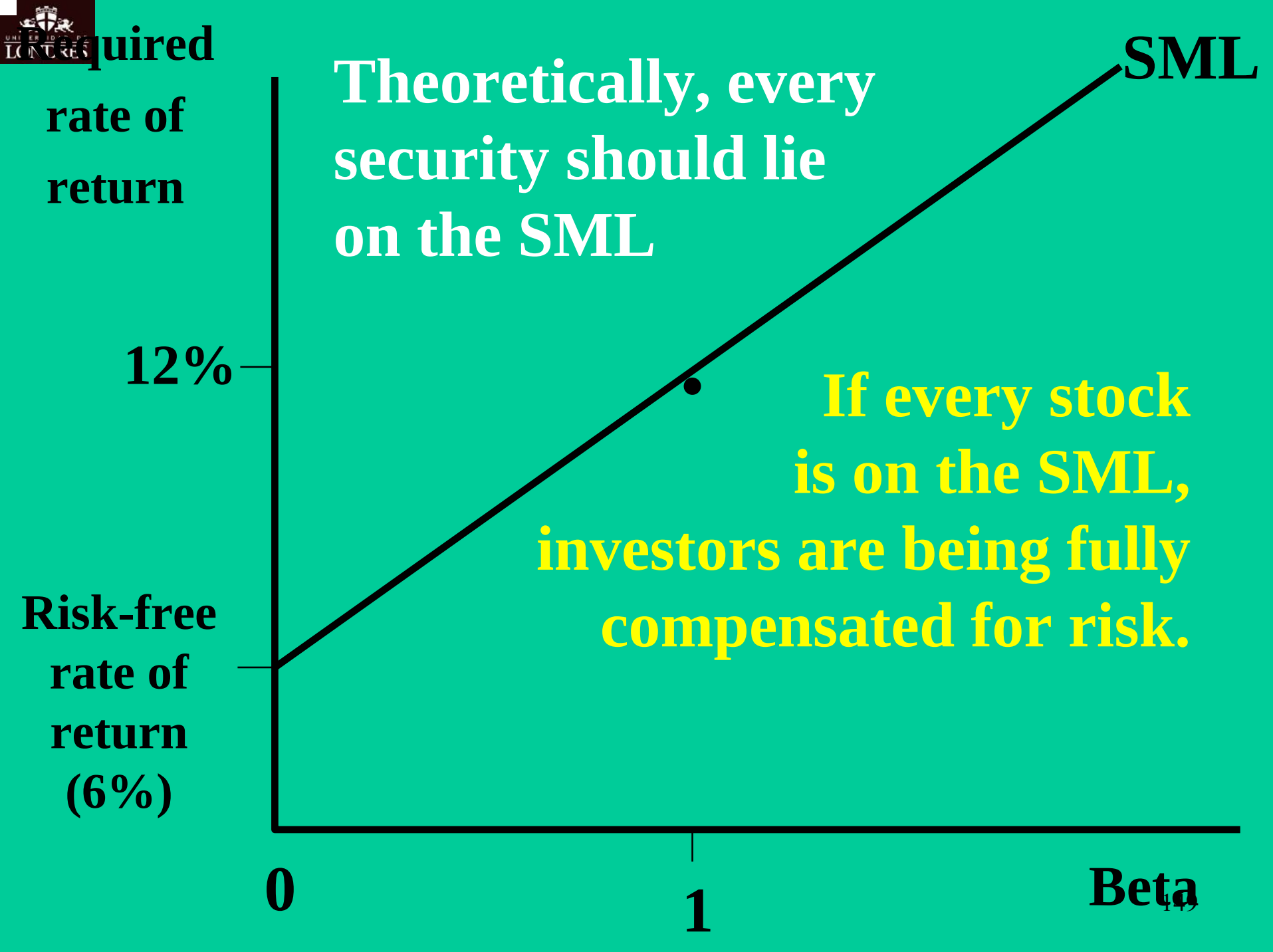
$$k_j = .06 + 1.2\,(.12 - .06)$$

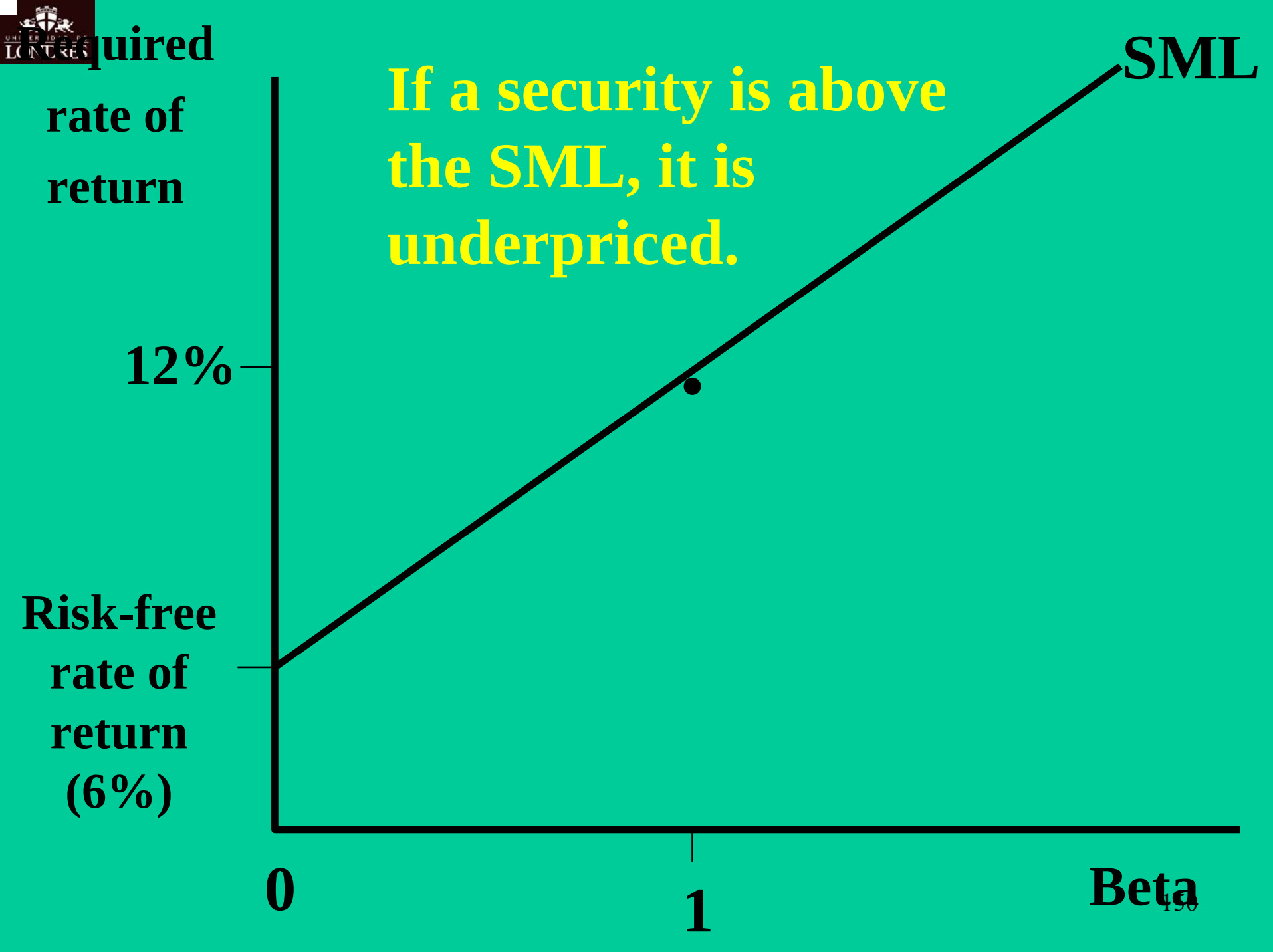
$$k_j = .132 = 13.2\%$$

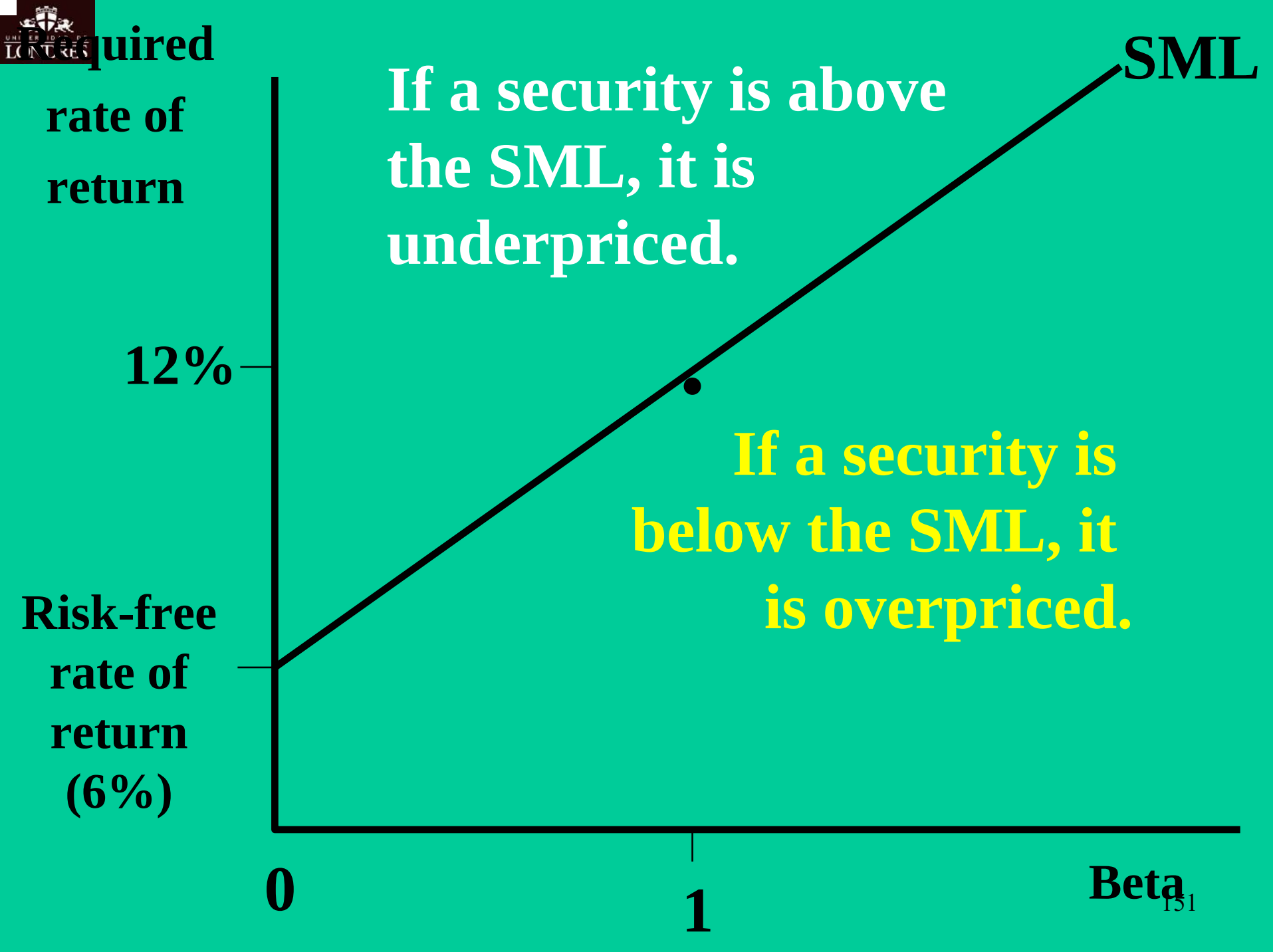
According to the CAPM, Disney stock should be priced to give a 13.2% return.


Required rate of return
SML
12%
Risk-free rate of return (6%)
0
1
Beta


Required rate of return
Theoretically, every security should lie on the SML
SML
12%
Risk-free rate of return (6%)
0
1
Beta


Required rate of return
SML
Theoretically, every security should lie on the SML
12%
If every stock is on the SML, investors are being fully compensated for risk.
Risk-free rate of return (6%)
0
1
Beta


Required rate of return
SML
If a security is above the SML, it is underpriced.
12%
Risk-free rate of return (6%)
0
1
Beta


Required rate of return
SML
If a security is above the SML, it is underpriced.
12%
If a security is below the SML, it is overpriced.
Risk-free rate of return (6%)
0
1
Beta

PROBLEMS

Problems on Portfolio

Problem 1

The yield to maturity (YTM) on a Treasury Bond whose remaining maturity is one year is 5%.

Jackie wants to invest her money for one year into the Treasury Bond or XYZ stock or into both of them.

She believes that expected return and standard deviation of XYZ stock are 14% and 20%, respectively.

Jackie doesn't like either asset because return on the Treasury Bond is too little and standard deviation of XYZ is too large.

If her maximum tolerance towards risk (measured by standard deviation) is 15%.

What is the best portfolio to her satisfying her risk tolerance?

Problems on Portfolio

Problem 2

Harry doesn't like them, either, because he wants higher expected return. Suppose that Harry wants at least 18% of expected return (return objective) and that he can borrow money at the risk-free rate. What is the best portfolio to him satisfying his return objective?

Problems on Portfolio

Problem 3

Suppose that stock I and J are available to you and have the following statistical characteristics;

	I		J
Expected Return	0.25		0.08
Variance	0.04		0.01
Standard Deviation	0.20		0.10
Covariance		-0.001	
Correlation Coefficient		-0.058	

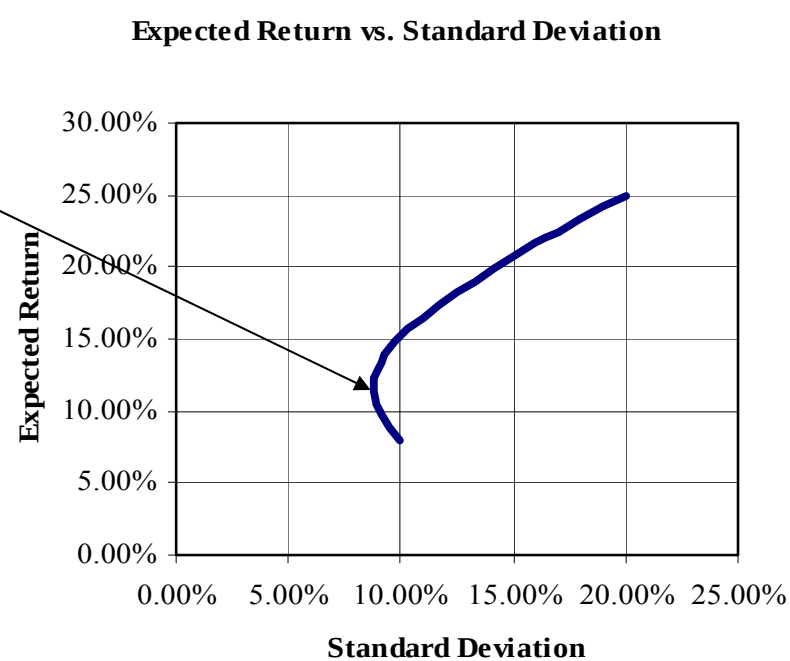
Construct the minimum risk (minimum variance) portfolio.

Problems on Portfolio

Problem 3

Using Excel Spreadsheet,

W_I	W_J	R_P	Sigma P
0.00	1.00	8.00%	10.00%
0.05	0.95	8.85%	9.50%
0.10	0.90	9.70%	9.12%
0.15	0.85	10.55%	8.87%
0.20	**0.80**	**11.40%**	**8.76%**
0.25	0.75	12.25%	8.80%
0.30	0.70	13.10%	8.99%
0.35	0.65	13.95%	9.31%
0.40	0.60	14.80%	9.76%
0.45	0.55	15.65%	10.31%
0.50	0.50	16.50%	10.95%
0.55	0.45	17.35%	11.67%
0.60	0.40	18.20%	12.46%
0.65	0.35	19.05%	13.29%
0.70	0.30	19.90%	14.17%
0.75	0.25	20.75%	15.08%
0.80	0.20	21.60%	16.02%
0.85	0.15	22.45%	16.99%
0.90	0.10	23.30%	17.98%
0.95	0.05	24.15%	18.98%
1.00	0.00	25.00%	20.00%

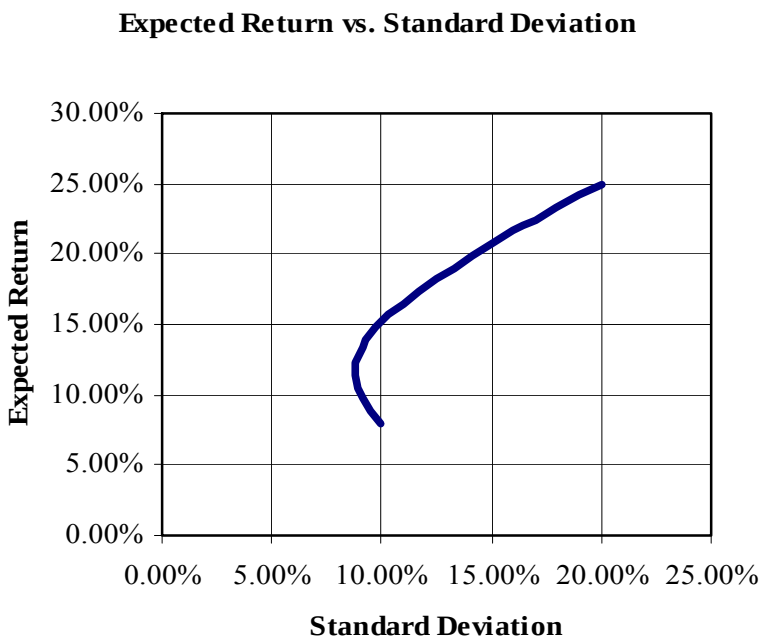


Problems on Portfolio

Problem 4

Suppose that risk-free rate is 5% and that you can invest or borrow any amount of money at the risk-free rate and you can invest any amount of money into stock I and J.

Suppose that your maximum risk tolerance (measured by standard deviation) is 20%. Pinpoint your best portfolio profile.



Problems on Portfolio

Problem 5

Suppose that you want at least 23% of expected annual return.

Pinpoint your best portfolio profile.

Problems on CAPM

Problem 6

Most financial analysts agree that the excess return for the S&P 500 Index will be 7 percent per year. Current YTM on 10-year Treasury Bonds is 6 percent. Beta for Stock XYZ is estimated as 1.3 from an econometric model. What is the market required rate of return on XYZ stock? What is the expected return on this stock if the market is in equilibrium?

Problem 7

Plot SML using information in Problem 6 and pinpoint the XYZ stock on the SML.

Problem 8

The XYZ company has just paid $3 of cash dividends per share and you believe that the cash dividend will grow at 6% per year forever. Suppose that your belief on the dividend growth rate is correct and that current market price of one share of XYZ stock is $32. If CAPM is an appropriate pricing model, is the XYZ stock overvalued or undervalued?