Modeling of elevated water tanks under seismic excitations considering interaction of water and structure

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1) Introduction:
The elevated liquid tanks include not only the water tanks but also the tanks in refineries and the chemical factories with different shapes such as cylindrical and spherical and containing the poisonous, chemical or explosive materials. Considering the condition of this type of structures in civil services and industrial networks shows the importance of the assured action of them during and after earthquake to answer the civil water requirements and avoid of great explosions and environmental damages. The guaranteed work of these structures needs to more studies due to the complication of them. This complication and understanding of interaction of tank and water during the analyze and design under dynamic excitation duplicates the need of presentation of simplified models in codes. Housner [1] in 1957, calculated the hydrodynamic pressure of water on tank wall by an analytical method then he replaced the water pressure by a mass spring model [2]. These models were based on the nature of the wall of the tank. Then, Sheferd developed the two mass model for the elevated water tanks. Fisher in 1979 solved the hydrodynamic pressure equation considering wall flexibility and without surface sloshing. [3] In 1981 Haroun showed that the flexibility of wall of the tank doesn’t have an important effect on sloshing response of the surface water. This was because of unimportant involvement of sloshing modes with structural modes. In 1985, Haroun presented a more complete sloshing mass – spring and impulsive mass in which the flexibility and mass of the structure were under consideration by an additional mass and spring. He also included the rotational excitation of the base in the model. In all mentioned models, only the first sloshing mode has been considered. It should be noticed that in all of these models, only the first asymmetric sloshing mode has been the base of simulation and the symmetric mode is not important. Although this mode is activated during the vertical excitation of the tank but it is not able to cause any base shear and overturning moment. In later works researchers concentrated mainly on nonlinear phenomena such as great sloshing, up lift or different tank geometries. The simplified water tank models are used in different codes. "AWWA", "API" and "UBC" codes, have used the models with individual mass or housner two mass or the related models. The AWWA code have provided the individual mass model for base shear calculation and the housner two mass model for over turning moment calculation. API code has the same methodologies as the “AWWA” with some little changes and in “UBC” code base shear is determined by a factor multiplied to the mass of the tank and water. In first version of Iranian earthquake code only a mass without sloshing has been considered [5]. And the second version of this code has used the two mass housner method based on plan and budget organization code [6].

In next reviews of this code this subject is needed to be studies more after some sever damages to the elevated water tanks in 1991 Manjil earthquake.
In this research, a simplified model containing the first and second sloshing modes and the first structural mass is suggested. Comparing the treatment of this model with Housner model and Iranian code, shows the importance of sloshing modes and even the second mode in some special cases.

2- The fundamental of hydrodynamics in liquid tanks

This study is based on the following assumptions:
- The fluid is incompressible and inviscid.
- The displacements of surface water are small.
- The elevated tank is cylindrical and the radius is “R” with rigid walls and flexible tower (columns).
- The base excitation is horizontal only.
- The materials are linear and the soil foundation interaction is ignored.
- The base of the structure is fixed , p-Δ effects and the rotational degrees of freedom around the axes θ=0 is ignored.

Based on these assumptions laplace equation will be the predominant equation in fluid media:

\[ \nabla^2 \phi = 0 \]  

(1)

The boundary conditions in this case will be as following:

\[ \frac{\partial \phi}{\partial Z} = 0 \quad \text{On the bottom} \]  

(2)

\[ \frac{\partial \phi}{\partial r} = 0 \quad \text{On the wall} \]  

(3)

\[ \frac{\partial \phi}{\partial r^2} + g \frac{\partial \phi}{\partial z} + r \cdot \cos(t) = 0 \quad \text{On the surface of water} \]  

(4)

In which \((r,z,\theta)\) are the cylindrical dimensions parameters, \(a(t)\) horizontal acceleration in base across \(\theta=0\) and \(t\) is time.

If \(\lambda_n\) be the nth root of derivation of Bessel function and with the mentioned boundary conditions and by separation of parameters method, we will have:

\[ \phi(r,\theta,z,t) = \cos(\theta) \sum_{n=1}^{\infty} F_n(t) j_1(\frac{\lambda_n r}{R}) \frac{\cosh(\frac{\lambda_n z}{R})}{j_1(\lambda_n)} \]  

(5)

By (4) and (5) the frequency of sloshing in nth mode will be:

\[ \beta_n = \sqrt{\frac{g \lambda_n}{R}} \cdot \tanh\left(\frac{\lambda_n H}{R}\right) \]  

(6)

And because:

\[ P_d(r,\theta,z,t) = -\rho \left( \frac{\partial \phi}{\partial t} + r \cdot \cos(\theta) a(t) \right) \]  

(7)

in which \(P_d\) is the dynamic pressure. Then:
The simplified dynamic model of fluid.

The concentrated masses and related stiffness are calculated in each mode to establish the simplified model. The equality between the base shear and overturning moment in the model and the analytical solutions will be the base to find the concentrated masses and spring properties.

The model base shear in each mode is determined by the Duhamel integral. The base shear and overturning moment are calculated by the following equations:

\[
Q(t) = \int_0^H \int_0^{2\pi} P_d(r, \theta, z, t) R \cos \theta \, dr \, dz \\
M(t) = \int_0^H \int_0^{2\pi} P_d(R, \theta, z, t) R z \cos \theta \, d\theta \, dz + \int_0^R \int_0^{2\pi} P_d(r, \theta, 0, t) r^2 \cos \theta \, d\theta \, dr
\]

On the right side of equation 10, the first part relates to the pressure on the wall and the second part relates to pressure on the bottom of the tank.

If \( \bar{M} = \rho R^2 H \) and \( C_n = \frac{\lambda_n}{R} \), then

the impulsive mass of the water and its position are calculated by frequency independent part of the base shear and over turning moment:

\[
m_{sw} = \bar{M} \left(1 + \sum_{n=1}^{\infty} \frac{2}{C_n (1 - \lambda_n^2)} \tanh(C_n) \right)
\]

\[
h_{sw} = H \left[ \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{C_n (1 - \lambda_n^2)} \left( \tanh(C_n) - \frac{1}{C_n} + \frac{2}{C_n \cosh(C_n)} \right) + \frac{1}{4 \left( \frac{H}{R} \right)^2} \right]
\]

And finally processing the frequency dependent part of \( Q(t) \) and \( M(t) \) will result the \( m_n \)-the sloshing mass in nth mode – and the related height \(-h_n\)-.
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\[ \beta_n^2 = \frac{\lambda_n g}{R} \tanh\left( \lambda_n \frac{H}{R} \right) \]  

(13)

\[ m_n = \frac{k_n}{\beta_n^2} \frac{2}{C_n(1 - \lambda_n^2)} \tanh(C_n) \]  

(14)

\[ k_n = H \left[ 1 - \frac{1}{C_n \tanh(C_n)} + \frac{2}{C_n \sinh(C_n)} \right] \]  

(15)

The stiffness of the simulated spring in the model in nth mode is then defined from equality of base shear calculated in analytical method and duhamel equation result:

\[ k_n = -\frac{2 \beta_n^2}{C_n(1 - \lambda_n^2)} \tanh(C_n) \]  

(16)

In elevated water tanks the flexible tower is modeled by a spring \((k_s)\) and also the impulsive water mass, \((m_{sw})\) and the wall and bottom mass of the tank are brought together.

And finally the simplified dynamic model in Fig(1) is suggested, in which \(\ddot{\theta}_x(t)\) is the horizontal acceleration across \(\theta=0\), and \(m_i, k_i, \xi_i\) are the mass, stiffness and damping in each ith sloshing mode and the impulsive mass is: \(m_i = m_{sw} + m_{w}\), and \(k_s\) and \(\xi_s\) are the stiffness and damping of the structure of tower. \(\xi_s, \xi_i\) are acquired from last experiments.

After calculating \(x_1, x_2\) and \(x_s\), it is needed to control the small displacement assumption.

### 4) Numerical analyses:

To consider the performance of the model a computer program is written to analyze the model by modal analysis and time history method.

Then an example is considered by the program with the following properties:

- four columns by height =25m from concrete
- column section : square

The research is continued through three conditions and the applied earthquake records mentioned in table 1
- first sloshing mode only (2DOF)
- first and second sloshing mode (3DOF)
- housner method: HS
- besides for regulation of the research and adoption with Iranian earthquake code ,we have normalized the records base on the PGA=0.35g
The response of the structure is calculated in seven ratios of $\frac{H}{R}$ from 0.5 to 2.0 and four amounts of $k_s$ (10%, 30%, 50% and 100%) of the main $k_s$ (Table 2). In this table $T_1$ and $T_2$ are the period of sloshing modes and $T_3$ is the period of the first mode of the structure, based on [4], the effects of the nonlinear waves in base shear in $\frac{H}{R} > 0.5$ are neglctable.

Calculating the maximum base shear in each condition some comparison between the different models and conditions were provided. During analysis based on the Iranian code the property factor “$R=3$” is included.

There is a good adoption between the suggested method and Housner method up to $\frac{H}{R} < 1.5$ in ELCENTRO earthquake but upper than this limit it is broken. In this case, the effect of second sloshing mode is neglectable. Also in ABBAR earthquake, neglecting the water level, the effect of second sloshing mode is considerable only in lower percentages of column stiffness and in normal conditions there is not any considerable difference with “2Dof” model.

It was shown that although the maximum base shear is about the code results in columns with normal condition of stiffness, but as the column gets more flexible, the suggested model shows less base shear amounts than the code results in all water surface levels.

The results of analysis based on code method and suggested method match only in normal stiffness condition and high rates of $\frac{H}{R}$ in Mexico city earthquake.

But in the tanks with small $\frac{H}{R}$ the code method results are under estimated. It is interesting that in this condition, the base shear in empty tank is more than the full tank. This causes some differences in flexible columns, which states the effect of the frequency containing of the earthquake record.

5) Conclusion

Based on the discussed subjects the good performance of the suggested method is confirmed. Besides, this study expresses that:

1- In some conditions a tank which is not full has been observed with larger responses comparing with a full tank. This point is not relevant to earthquake code of Iran.
2- In some conditions code results are less than the results, which are obtained in models, which, consider the effect of sloshing modes. This subject is more saleintiant while analysis with earthquakes with longer period.
3- There is not any important difference between the results of the models including and excluding the second sloshing mode except when the stiffness of the column reduces.
4- Based on the results mentioned in table 2 when $T_3/T_2$ increases more than %70 the effects of second mode of sloshing will be considerable.
5- Although there is not any problem to design the water towers based on the earthquake code according to ELCENTRO and ABBAR records but this structure is not reliable when facing with earthquakes with longer predominant periods like Mexico city-ABASTOS or Imperial valley record.
<table>
<thead>
<tr>
<th>Record Title</th>
<th>Date</th>
<th>Predominant period</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABBAR</td>
<td>1990</td>
<td>0.3 ~ 0.7</td>
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<tr>
<td>NAGHAN</td>
<td>1977</td>
<td>~0.5</td>
</tr>
<tr>
<td>ELCENTRO</td>
<td>1940</td>
<td>0.2 ~ 0.85</td>
</tr>
<tr>
<td>IMPERIAL VALLEY CA BONDS CORNER</td>
<td>1971</td>
<td>0.5~0.85</td>
</tr>
<tr>
<td>MEXICO CITY SEC.COM</td>
<td>1985</td>
<td>1.8 ~ 2.2</td>
</tr>
<tr>
<td>MEXICO CITY C.DE.ABASTOS</td>
<td>1985</td>
<td>3.2 ~3.8</td>
</tr>
</tbody>
</table>

Table 1- Properties of earthquake records applied to the elevated tank during dynamic analysis
Table 2 - Frequency responses of structure in different $\frac{H}{R}$ ratios

<table>
<thead>
<tr>
<th>$\frac{H}{R}$</th>
<th>2.0</th>
<th>2.0</th>
<th>1.75</th>
<th>1.75</th>
<th>1.5</th>
<th>1.5</th>
<th>1.25</th>
<th>1.25</th>
<th>1.0</th>
<th>1.0</th>
<th>0.75</th>
<th>0.75</th>
<th>0.5</th>
<th>0.5</th>
<th>0.5</th>
<th>$\frac{H}{R}$</th>
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<tbody>
<tr>
<td>$T_3$</td>
<td>1.584</td>
<td>2.273</td>
<td>1.583</td>
<td>2.225</td>
<td>1.582</td>
<td>2.172</td>
<td>1.581</td>
<td>2.12</td>
<td>1.58</td>
<td>2.069</td>
<td>1.579</td>
<td>2.032</td>
<td>1.584</td>
<td>2.026</td>
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<tr>
<td>$T_2$</td>
<td>1.498</td>
<td>1.612</td>
<td>1.453</td>
<td>1.605</td>
<td>1.401</td>
<td>1.601</td>
<td>1.349</td>
<td>1.598</td>
<td>2.798</td>
<td>1.295</td>
<td>2.834</td>
<td>1.243</td>
<td>1.596</td>
<td>2.933</td>
<td>1.204</td>
<td>1.603</td>
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<tr>
<td>$T_1$</td>
<td>1.19</td>
<td>1.593</td>
<td>2.743</td>
<td>1.148</td>
<td>1.592</td>
<td>2.745</td>
<td>1.104</td>
<td>1.592</td>
<td>2.765</td>
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<td>$T_3$</td>
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<td>2.722</td>
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<td>0.636</td>
<td>1.59</td>
<td>2.78</td>
<td>0.609</td>
</tr>
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</table>
References:
1- G.W. Housner , “Dynamic pressure on elevated fluid containers” , Bulletin of the seismological society of America , 1959 , pp 15 ~ 35


5- Code of design of buildings in front of earthquake in Iran. (Standard 2800)

6- Design of water tanks, Plan and budget organization of Iran.