Student Number:

St Catherine's School
Waverley

August 2011

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 2

General Instructions

- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question
- Complete each question in a separate booklet

- Attempt Questions 1 – 8
- All questions are of equal value
- **Total Marks – 120**

Total marks -120
Attempt Questions 1-8
All questions are of equal value

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

Question 1 (15 marks) Use the Question 1 Writing Booklet. **Marks**

(a) Find $\int \frac{dx}{x^2-2x+5}$ **2**

(b) Use integration by parts to evaluate $\int_0^1 \tan^{-1}x\,dx$ **3**

(c) Use the substitution $t=\tan\frac{x}{2}$ to evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x+\cos x}$ **3**

(d) (i) Find the values of a, b and c such that; **2**

$$\frac{x^2-x-21}{(2x-1)(x^2+4)}=\frac{a}{2x-1}+\frac{bx+c}{x^2+4}$$

(ii) Hence evaluate $\int \frac{x^2-x-21}{(2x-1)(x^2+4)}\,dx$ **2**

(e) Use the substitution $x=\frac{\pi}{2}-u$ to show that **3**

$$\int_0^{\frac{\pi}{2}} \frac{\cos x-\sin x}{1+\sin 2x}\,dx=0$$

Question 2 (15 marks) Use the Question 2 Writing Booklet. **Marks**

(a) (i) Express $z=\sqrt{3}+i$ in modulus-argument form. 1

(ii) Hence show that $z^7+64z=0$ 2

(b) On an argand diagram the point P representing the complex number z moves such that $|z-(1+i)|=1$

(i) Sketch the locus of P 1

(ii) Find the greatest value of $|z|$ 2

(iii) Shade the region common to $|z-(1+i)|\leq 1$ and $0<\arg(z-1)<\frac{\pi}{4}$ 2

(iv) Find the area of the region in part (iii) above 2

(c) If w is one of the complex roots of $z^3=1$

(i) Show that w^2 is also a root. 1

(ii) Show that $1+w+w^2=0$ 1

(iii) Evaluate $(1-w)(1-w^2)(1-w^4)(1-w^8)$ 3

Question 3 (15 marks) Use the Question 3 Writing Booklet. **Marks**

(a) Given that $(x+i)$ is a factor of $P(x)=x^4+3x^3+6x^2+3x+5$ factorise $P(x)$ over the complex field. 4

(b) Given that the equation $x^4-5x^3-9x^2+81x-108=0$ has a root of multiplicity 3, find all the roots of this equation. 3

(c) If α, β, γ are the roots of $x^3-3x^2+2x-1=0$ find the equation whose roots are

(i) $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$ 2

(ii) $\alpha^2, \beta^2, \gamma^2$ 3

(d)

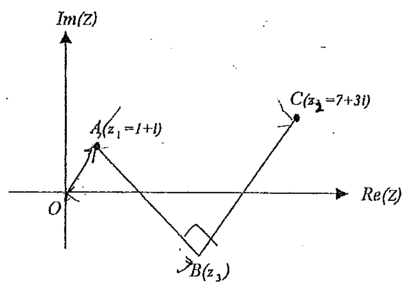


Diagram not to scale

The points A and C represent the complex numbers $z_1=1+i$ and $z_2=7+3i$.

Find the complex number z_3, represented by the point B such that ΔABC is isosceles and right angled at B. 3

Question 4 (15 marks) Use the Question 4 Writing Booklet. **Marks**

The equation of an ellipse E is given by $\dfrac{x^2}{9}+\dfrac{y^2}{5}=1$

(i) Find the eccentricity of E — 1

(ii) Write down

(a) The coordinates of the foci — 1

(b) The equations of the directrices — 1

(c) The equation of the major auxiliary circle A — 1

(iii) Draw a *neat* sketch of E *and* A showing clearly the features in part (b) above (at least one third of a page) — 2

(iv) A line parallel to the y-axis meets the x-axis at N and the curves E and A at P and Q respectively. If N has coordinates $(3\cos\theta, 0)$ and given that P and Q are in the first quadrant, show that the coordinates of P are $\left(3\cos\theta, \sqrt{5}\sin\theta\right)$ and the coordinates of Q are $(3\cos\theta, 3\sin\theta)$. — 2

(v) Show that the equations of the tangents at P and Q are $\sqrt{5}\cos\theta\ x+3\sin\theta\ y=3\sqrt{5}$ and $x\cos\theta+y\sin\theta=3$ respectively. — 4

(vi) Show that the point of intersection R of these tangents lies on the major axis of E produced. — 1

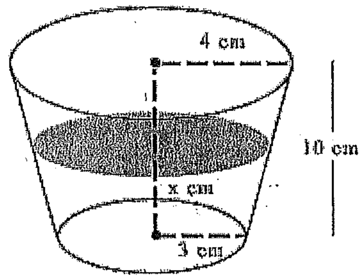
(vii) Prove that $ON.\ OR$ is independent of the position of P and Q on the curves. — 2

Question 5 (15 marks) Use the Question 5 Writing Booklet. **Marks**

(a) Prove that the area bounded by the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ is πab square units — 3

(b) Use the method of **cylindrical shells**, find the volume of the solid formed when the area in (a) is rotated through one complete revolution about the line $y=b$ — 3

(c) A drinking glass is in the shape of a truncated cone, in which the internal diameter of the top and the bottom are 8cm and 6 cm respectively.



(i) If the internal height of the glass, MN, is 10cm, show that the area of the cross-section at a height of x cm above the base is — 3

$$\pi\left(3+\frac{x}{10}\right)^2 \text{ cm}^2$$

(ii) Hence find by integration, the volume of the glass. — 2

(c) The points $P(a\sec\theta, b\tan\theta)$ and $Q(a\sec\phi, b\tan\phi)$ lie on the same branch of the hyperbola $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ and PQ is a focal chord, passing through $S(ae, 0)$. — 4

Use the gradients of PS and QS to show that $e=\dfrac{\sin\theta-\sin\phi}{\sin(\theta-\phi)}$

Question 6 (15 marks) Use the Question 6 Writing Booklet. **Marks**

(a) (i) Show that the equation of the normal to the hyperbola $xy = c^2$ at $P\left(cp, \dfrac{c}{p}\right)$ is $p^3x - py = c(p^4 - 1)$ **2**

(ii) The normal at $P\left(cp, \dfrac{c}{p}\right)$ meets the x-axis at Q. Find the coordinates of Q. **1**

(iii) Find the coordinates of the mid point, R, of PQ. **1**

(iv) Hence find the equation of the locus of R. **2**

(b) Use the compound angle formula for $\cos(x+y)$ and $\cos(x-y)$ to prove the result $\cos S - \cos T = -2\sin\left(\dfrac{S+T}{2}\right)\sin\left(\dfrac{S-T}{2}\right)$ **2**

(c) If I_n is defined such that $I_n = \displaystyle\int_0^{\frac{\pi}{4}} \frac{1-\cos 2nx}{\sin 2x}\,dx$ for $n = 0, 1, 2, 3, \ldots$

(i) Show that $I_1 = \dfrac{1}{2}\ln 2$ **2**

(ii) Using the result proven in part (b) above, show that for $r \geq 1$; **3**

$$I_{2r+1} - I_{2r-1} = \frac{1-(-1)^r}{2r}$$

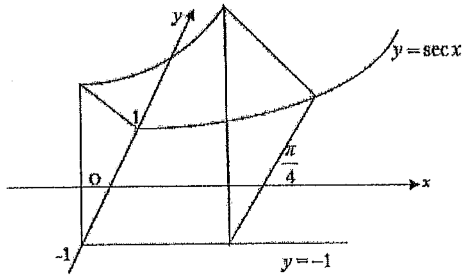
(iii) Hence evaluate I_5 **2**

Question 7 (15 marks) Use the Question 7 Writing Booklet. **Marks**

(a) A string 50 *cm* in length can just sustain a weight of mass 20 *kg* without breaking. A mass of 4 *kg* is attached to one end of the string and revolves *uniformly* on a smooth horizontal table. The other end is fixed to a point on the table. **3**

Find the greatest number of complete revolutions the mass can make in a minute without breaking the string. [Use acceleration due to gravity as 10 ms^{-2}]

(b) The base of a solid is the region in the xy plane enclosed by the curve $y = \sec x$, $y = -1$, $x = 0$ and $x = \dfrac{\pi}{4}$. Each cross-section perpendicular to the x-axis is an equilateral triangle.



(i) Show that the area of the triangular cross-section x units from the origin is given by **1**

$$A = \frac{\sqrt{3}}{4}(\sec x + 1)^2$$

(ii) Show by differentiation that $\displaystyle\int \sec x\, dx = \ln(\sec x + \tan x) + c$ **1**

(iii) Hence, show that the volume of the solid is given by; **3**

$$\frac{\sqrt{3}}{4}\left[1 + 2\ln(\sqrt{2}+1) + \frac{\pi}{4}\right] \text{ units}^3$$

(c) If $U_1 = 1$, $U_2 = 5$ and $U_n = 5U_{n-1} - 6U_{n-2}$ for $n \geq 3$, prove by mathematical induction that $U_n = 3^n - 2^n$ for $n \geq 1$ **4**

(d) The complex number z moves so that the sum of its distances from 3 ($|z-3|$) and -3 ($|z+3|$) is 10 units. Find the Cartesian equation of the ellipse described by the locus of z **3**

Question 8 (15 marks) Use the Question 8 Writing Booklet. **Marks**

(a) (i) Given that a, b, and c are three non negative numbers, show that the arithmetic mean $\geq$ geometric mean. **3**

i.e. show that $\frac{a+b+c}{3} \geq \sqrt[3]{abc}$

(You may assume that $a^2 + b^2 \geq 2ab$)

(ii) Given that $a + \frac{1}{a} \geq 2$ show that $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geq 9$ **2**

(b) A body of unit mass falls under gravity through a resisting medium. The body falls from rest from a height of 50 metres above the ground. The resistance to its motion is $\frac{1}{100}v^2$ where v metres per second is the speed of the body when it has fallen a distanc of x metres. The acceleration due to gravity is $g\ ms^{-2}$

(i) Show that the equation of motion of the body is : **2**

$$\ddot{x} = g - \frac{1}{100}v^2$$

(ii) Show that the terminal velocity V of the body is given by **1**

$$V = \sqrt{100g}$$

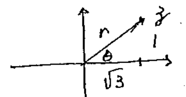
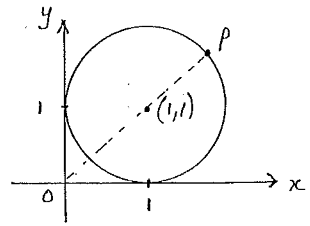
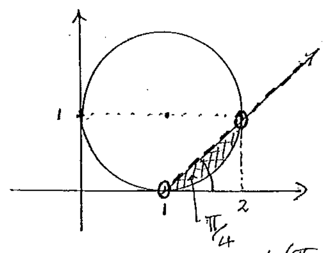
(iii) Hence show that $v^2 = V^2(1 - e^{-\frac{x}{50}})$ **3**

(iv) Find the distance fallen in metres until the body reaches a velocity equal to 50% of the terminal velocity. **2**

(v) Find the velocity reached as a percentage of terminal velocity when the body hits the ground. **2**

END of PAPER

Q	Solutions	Marks	Comments
Q1a)	$\int \frac{dx}{x^2-2x+5}$		
	$= \int \frac{dx}{(x^2-2x+1)+4}$	1	
	$= \int \frac{dx}{(x-1)^2+2^2}$		
	$= \frac{1}{2}\tan^{-1}\left(\frac{x-1}{2}\right) + C$	1	
b)	$\int_0^1 \tan^{-1}x\,dx$ $u = \tan^{-1}x$ $\quad v = x$ $u' = \frac{1}{1+x^2}$ $\quad v' = 1$	1	
	$\int_0^1 \tan^{-1}x\,dx = \left[x\tan^{-1}x\right]_0^1 - \int_0^1 \frac{x}{1+x^2}\,dx$	1	
	$= \frac{\pi}{4} - \left[\frac{1}{2}\ln(1+x^2)\right]_0^1$		
	$= \frac{\pi}{4} - \frac{1}{2}\ln 2$	1	
c)	$\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x+\cos x}$ $t = \tan\frac{x}{2}$ $\quad \sin x = \frac{2t}{1+t^2}$ $dx = \frac{2dt}{1+t^2}$ $\quad \cos x = \frac{1-t^2}{1+t^2}$		
	$= \int_0^1 \frac{\frac{2dt}{1+t^2}}{1+\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}}$ $x = 0$ $\quad t = 0$ $x = \frac{\pi}{2}$ $\quad t = 1$	1	
	$= \int_0^1 \frac{2dt}{1+t^2+2t+1-t^2}$		
	$= 2\int_0^1 \frac{dt}{2t+2}$		
	$= \int_0^1 \frac{dt}{t+1}$	1	
	$= \left[\ln(t+1)\right]_0^1$		
	$= \ln 2$	1	

Q	Solutions	Marks	Comments
2a)	(i) $z = \sqrt{3} + i$ 		
	$\therefore z = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ $\quad r = \sqrt{3+1} = 2$ $= 2\,\text{cis}\frac{\pi}{6}$ $\quad \theta = \tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6}$	1	
	(ii) $z^7 = \left(2\,\text{cis}\frac{\pi}{6}\right)^7 = 2^7\,\text{cis}\frac{7\pi}{6}$		
	$64z = 128\,\text{cis}\frac{\pi}{6}$		
	$\therefore z^7 + 64z = 128\,\text{cis}\frac{7\pi}{6} + 128\,\text{cis}\frac{\pi}{6}$	1	
	$= 128\left[\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6} + \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right]$		
	$= 128\left[-\cancel{\cos\frac{\pi}{6}} - \cancel{i\sin\frac{\pi}{6}} + \cancel{\cos\frac{\pi}{6}} + \cancel{i\sin\frac{\pi}{6}}\right]$		
	$= 0$	1	
b)	(i) $\left\lvert z - (1+i)\right\rvert = 1$ 	2	
	(ii) $\lvert z\rvert$ is maximum when z is at P	1	
	$\therefore$ Now $OP = \sqrt{2}+1$	1	
	$\therefore$ max value of $\lvert z\rvert = \sqrt{2}+1$	1	
	(iii) 	2	
	(iv) Area shaded region = area of segment $= \frac{1}{2}\left(\frac{\pi}{2} - \sin\frac{\pi}{2}\right)$		
	$= \left(\frac{\pi}{4} - \frac{1}{2}\right)u^2$	1	

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Q	Solutions	Marks	Comments
3a)	If $(x+i)$ is a factor of $P(x) = x^4+3x^3+6x^2+3x+5$ then $(x-i)$ is also a factor of $P(x)$ ($P(x)$ real coefficients)	1	
	$\therefore (x+i)(x-i) = (x^2+1)$ is also a factor		
	Now $x^4+3x^3+6x^2+3x+5 = (x^2+1)(x^2+3x+5)$	1	
	$\therefore$ the zeros of $P(x)$ are $i, -i, \frac{-3\pm\sqrt{11}i}{2}$	1	
	$\therefore P(x) = (x-i)(x+i)\left(x+\frac{3-\sqrt{11}i}{2}\right)\left(x+\frac{3+\sqrt{11}i}{2}\right)$	1	
b)	$P(x) = x^4-5x^3-9x^2+81x-108 = 0$		
	$P'(x) = 4x^3-15x^2-18x+81$		
	$P''(x) = 12x^2-30x-18 = 0$ for possible roots of multiplicity 3.		
	$\therefore 2x^2-5x-3 = 0$		
	$(2x+1)(x-3) = 0$		
	$\therefore x = -\frac{1}{2}, 3$	1	
	now $P'(3) = 108-135-54+81 = 0$		
	and $P(3) = 81-135-81+243-108 = 0$	1	
	$\therefore x = 3$ is a root of multiplicity 3.		
	$\therefore (x-3)^3$ is a factor of $P(x)$		
	also, roots are $3, 3, 3, \alpha$		
	Now product of roots $= 27\alpha = -108$		
	$\therefore \alpha = -4$		
	$\therefore$ the roots of $P(x) = 0$ are $x = 3, 3, 3, -4$	1	
c)	$x^3-3x^2+2x-1 = 0$ α, β, γ are roots		
	(i) let $y = \frac{1}{\alpha}$ $\therefore \alpha = \frac{1}{y}$		
	$\therefore \left(\frac{1}{y}\right)^3 - 3\left(\frac{1}{y}\right)^2 + 2\left(\frac{1}{y}\right) - 1 = 0$	1	
	$\frac{1}{y^3} - \frac{3}{y^2} + \frac{2}{y} - 1 = 0$		
	$\therefore 1 - 3y + 2y^2 - y^3 = 0$		
	which equates to the polynomial in x of $x^3-2x^2+3x-1 = 0$	1	

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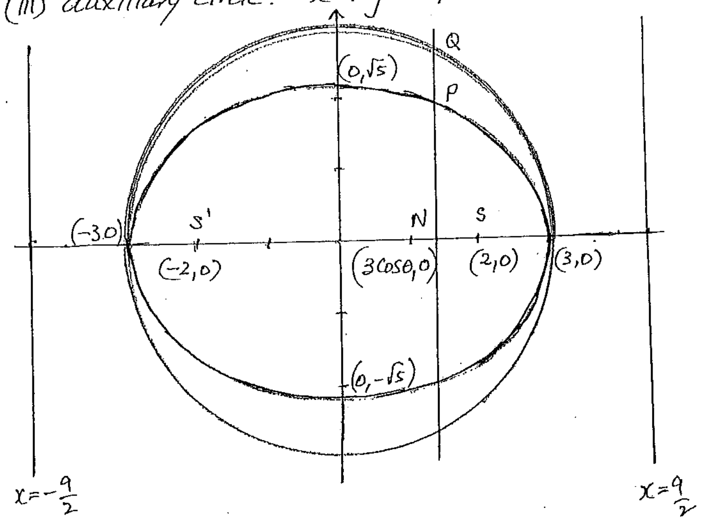
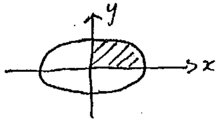
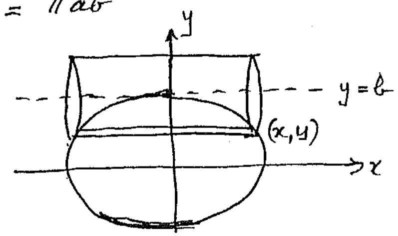
Q	Solutions	Marks	Comments
Q4a)	$\frac{x^2}{9} + \frac{y^2}{5} = 1$		
	for ellipse $b^2 = a^2(1-e^2)$		
	$\therefore 5 = 9(1-e^2)$		
	$\therefore e^2 = \frac{4}{9}$		
	$e = \frac{2}{3}$	1	
b)	(i) foci $(\pm ae, 0)$ ie. $(\pm 2, 0)$	1	
	(ii) directrices: $x = \pm\frac{a}{e}$ ie. $x = \pm\frac{9}{2}$	1	
	(iii) auxiliary circle: $x^2+y^2 = 9$	1	
c)		2	
d)	Coordinates of Q $(3\cos\theta, 3\sin\theta)$	1	
	Coordinates of P $(3\cos\theta, \sqrt{5}\sin\theta)$	1	
e)	$\therefore$ at P $x = 3\cos\theta$ $y = \sqrt{5}\sin\theta$		
	$\frac{dx}{d\theta} = -3\sin\theta$ $\frac{dy}{d\theta} = \sqrt{5}\cos\theta$		
	now $\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$		
	$= -\frac{\sqrt{5}\cos\theta}{3\sin\theta}$	1	

Q	Solutions	Marks	Comments
5a)	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $y^2=b^2\left(1-\frac{x^2}{a^2}\right)$ $y^2=\frac{b^2}{a^2}(a^2-x^2)$ $y=\pm\frac{b}{a}\sqrt{a^2-x^2}$	1	
	Shaded area is given by  $A=\int_0^a \frac{b}{a}\sqrt{a^2-x^2}\,dx$ $=\frac{b}{a}\int_0^a\sqrt{a^2-x^2}\,dx$	1	could also be done by substitution let $u=a\sin\theta$
	now $\int_0^a\sqrt{a^2-x^2}\,dx$ is a quadrant of a circle radius a $=\frac{1}{4}\pi a^2$ $\therefore A=\frac{b}{a}\times\frac{1}{4}\pi a^2$ $=\frac{1}{4}\pi ab$	1	
	$\therefore$ area of ellipse $=\pi ab$		
b)	radius of shell is $(b-y)$ height of shell is $2x$ 	1	
	$\therefore \delta V=2\pi(b-y).2x$ $\therefore V=4\pi\int_{-b}^{b}(b-y).\frac{a}{b}\sqrt{b^2-y^2}\,dy$ $=\frac{4\pi a}{b}\int_{-b}^{b}\left(b\sqrt{b^2-y^2}-y\sqrt{b^2-y^2}\right)dy$	1	

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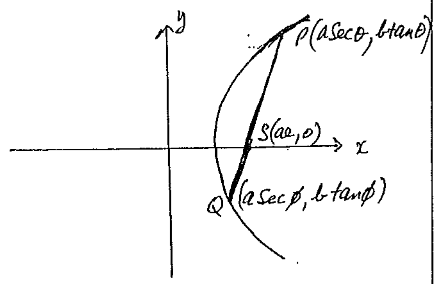
Q	Solutions	Marks	Comments
Q5c)	Gradient of PS: $\frac{b\tan\theta-0}{a\sec\theta-ae}$ 	1	
	gradient of SQ: $\frac{b\tan\phi-0}{a\sec\phi-ae}$. $\therefore \frac{b\tan\theta}{a\sec\theta-ae}=\frac{b\tan\phi}{a\sec\phi-ae}$	1	
	$ab\tan\theta\sec\phi-abe\tan\theta=ab\tan\phi\sec\theta-abe\tan\phi$ $\therefore abe\tan\phi-abe\tan\theta=ab\tan\phi\sec\theta-ab\tan\theta\sec\phi$ $\therefore\quad e=\frac{ab(\tan\phi\sec\theta-\tan\theta\sec\phi)}{ab(\tan\phi-\tan\theta)}$	1	
	$=\dfrac{\frac{\sin\phi}{\cos\phi}.\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}.\frac{1}{\cos\phi}}{\frac{\sin\phi}{\cos\phi}-\frac{\sin\theta}{\cos\theta}}$ $=\frac{\sin\phi-\sin\theta}{\sin\phi\cos\theta-\sin\theta\cos\phi}$ $=\frac{\sin\phi-\sin\theta}{\sin(\phi-\theta)}$ $=\frac{\sin\theta-\sin\phi}{\sin(\theta-\phi)}$	1	

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Q	Solutions	Marks	Comments
8b)	$\cos(x+y) = \cos x\cos y - \sin x\sin y$ —— ①		
	$\cos(x-y) = \cos x\cos y + \sin x\sin y$ —— ②		
	let $S = x+y$ and $T = x-y$		
	$S+T = 2x \quad \therefore x = \frac{S+T}{2}$	1	
	$S-T = 2y \quad \therefore y = \frac{S-T}{2}$		
	$\therefore ①-②$ $\cos S - \cos T = -2\sin x\sin y$	1	
	ie. $\cos S - \cos T = -2\sin\frac{S+T}{2}\sin\frac{S-T}{2}$		
c)	$I_n = \int_0^{\pi/4} \frac{1-\cos 2nx}{\sin 2x}\,dx \quad n = 0,1,2,3,\dots$		
	(i) $I_1 = \int_0^{\pi/4} \frac{1-\cos 2x}{\sin 2x}\,dx$		
	$= \int_0^{\pi/4} \frac{2\sin^2 x}{2\sin x\cos x}\,dx$	1	
	$= \int_0^{\pi/4} \frac{\sin x}{\cos x}\,dx$		
	$= -\Big[\ln\lvert\cos x\rvert\Big]_0^{\pi/4}$		
	$= -\ln\frac{1}{\sqrt2}$	1	
	$= \frac{1}{2}\ln 2$		
	(ii) $I_{2r+1} - I_{2r-1} = \int_0^{\pi/4} \frac{1-\cos(4xr+2x)}{\sin 2x}\,dx - \int_0^{\pi/4} \frac{1-\cos(4xr-2x)}{\sin 2x}\,dx$	1	
	$= \int_0^{\pi/4} \frac{\cos(4xr-2x) - \cos(4xr+2x)}{\sin 2x}\,dx$		
	$= \int_0^{\pi/4} \frac{-2\sin 4xr\sin(-2x)}{\sin 2x}\,dx$ from (b)	1	
	$= 2\int_0^{\pi/4} \sin 4xr\,dx$		
	$= -\frac{1}{2r}\Big[\cos 4xr\Big]_0^{\pi/4} = -\frac{1}{2r}\Big[\cos\pi r - 1\Big]$		
	$= -\frac{1}{2r}\Big[(-1)^r - 1\Big] = \frac{1-(-1)^r}{2r}$	1	

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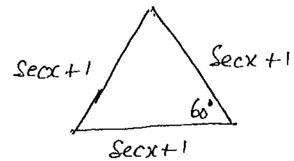
Q	Solutions	Marks	Comments
7b)	(i) 		
	Area $\Delta = \frac{1}{2}(\sec x+1)^2\sin 60^\circ$	1	
	$= \frac{1}{2}(\sec x+1)^2\frac{\sqrt3}{2}$		
	$= \frac{\sqrt3}{4}(\sec x+1)^2$		
	(ii) $\frac{d}{dx}\ln(\sec x+\tan x) = \frac{1}{\sec x+\tan x}\cdot(\sec x\tan x + \sec^2 x)$		
	$= \frac{\sec x\cancel{(\tan x+\sec x)}}{\cancel{(\sec x+\tan x)}}$		
	$= \sec x$	1	
	(iii) Volume of typical slice is given by		
	$\delta V = \frac{\sqrt3}{4}(\sec x+1)^2\delta x$	1	
	$\therefore V = \frac{\sqrt3}{4}\int_0^{\pi/4}(\sec x+1)^2\,dx$		
	$= \frac{\sqrt3}{4}\int_0^{\pi/4}(\sec^2 x + 2\sec x + 1)\,dx$	1	
	$= \frac{\sqrt3}{4}\Big[\tan x + 2\ln(\sec x+\tan x) + x\Big]_0^{\pi/4}$		
	$= \frac{\sqrt3}{4}\Big[1 + 2\ln(\sqrt2+1) + \frac{\pi}{4} - 0\Big]$		
	$\therefore V = \frac{\sqrt3}{4}\Big[1 + 2\ln(\sqrt2+1) + \frac{\pi}{4}\Big]$ units^3	1	

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Q	Solutions	Marks	Comments

Q7d)

$|z-3| + |z+3| = 10$

let $z = x+iy$

$|(x-3)+iy| + |(x+3)+iy| = 10$ — 1

$\sqrt{(x-3)^2+y^2} + \sqrt{(x+3)^2+y^2} = 10$

$\therefore \sqrt{(x-3)^2+y^2} = 10 - \sqrt{(x+3)^2+y^2}$

$\therefore (x-3)^2 + \cancel{y^2} = 100 - 20\sqrt{(x+3)^2+y^2} + (x+3)^2 + \cancel{y^2}$

$\therefore 20\sqrt{(x+3)^2+y^2} = 100 + 12x$

$\sqrt{(x+3)^2+y^2} = 5 + \frac{3x}{5}$ — 1

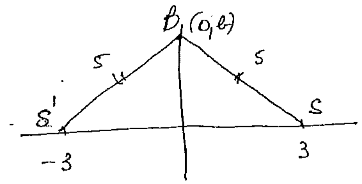
Squaring both sides $x^2 + \cancel{6x} + 9 + y^2 = 25 + \cancel{6x} + \frac{9x^2}{25}$

$25x^2 + 225 + 25y^2 = 625 + 9x^2$

$16x^2 + 25y^2 = 400$

$\frac{x^2}{25} + \frac{y^2}{16} = 1$ — 1

OR



for sum of distances from foci to be 10 (= 2a) ellipse must travel through (5,0) $\therefore a = 5$ — 1

also $S'B + SB = 10$ $\therefore S'B = SB = 5$ — 1

$\therefore b = 4$ (Pythagoras)

$\therefore$ equation of ellipse is

$\frac{x^2}{25} + \frac{y^2}{16} = 1$ — 1

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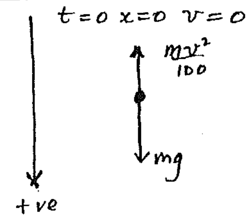
Q	Solutions	Marks	Comments

8b) (i)

$ma = mg - \frac{mv^2}{100}$



$\therefore a = g - \frac{v^2}{100}$ — 1

$\therefore \ddot{x} = g - \frac{v^2}{100}$ — 1

is the equation of motion

(ii) terminal velocity when $a = 0$ $v = V$

$\therefore g - \frac{V^2}{100} = 0$

$\therefore V^2 = 100g$

$\therefore V = \sqrt{100g}$ also $g = \frac{V^2}{100}$ — 1

(iii) $\ddot{x} = v\frac{dv}{dx} = g - \frac{v^2}{100}$

from (ii) $g = \frac{V^2}{100}$

$\therefore v\frac{dv}{dx} = \frac{V^2}{100} - \frac{v^2}{100}$

$\therefore \frac{dv}{dx} = \frac{V^2 - v^2}{100v}$

$\therefore \frac{dx}{dv} = \frac{100v}{V^2 - v^2}$ — 1

$\therefore \int dx = \int \frac{100v}{V^2 - v^2}\,dv$ (integrating both sides)

$\therefore x + c = -50\ln(V^2 - v^2)$

when $v = 0$ $x = 0$ $\Rightarrow c = -50\ln V^2$

$\therefore x - 50\ln V^2 = -50\ln(V^2 - v^2)$ — 1

$\therefore x = 50\ln V^2 - 50\ln(V^2 - v^2)$

$\therefore \frac{x}{50} = \ln\left(\frac{V^2}{V^2 - v^2}\right)$

$\therefore \frac{V^2}{V^2 - v^2} = e^{\frac{x}{50}}$

$V^2 - v^2 = V^2 e^{-\frac{x}{50}}$

$v^2 = V^2\left(1 - e^{-\frac{x}{50}}\right)$ — 1