Sydney Girls High School

2009

TRIAL HIGHER SCHOOL

CERTIFICATE EXAMINATION

Mathematics

Extension Two

his is a trial paper ONLY. It
ɔes not necessarily reflect the
ɔrmat or the contents of the
009 HSC Examination Paper
ı this subject.

Candidate Number

General Instructions

- Reading Time - 5 minutes
- Working time - 3 hours
- Attempt ALL questions
- ALL questions are of equal value
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are supplied
- Board-approved calculators may be used.
- Diagrams are not to scale
- Each question attempted should be started on a new sheet. Write on one side of the paper only.

Use $g = 10ms^{-2}$ where required

Question One (15 marks) Marks

a) Find $\int 4x^3\sqrt{2+x^4}\,dx$ 2

b) Find $\int \frac{dx}{\sqrt{9x^2-1}}$ 2

c) Evaluate $\int_1^2 \log_e x\,dx$ 3

d) Evaluate $\int_1^3 \frac{dx}{x^2-2x+5}$ 3

e) i) Find constants A, B and C such that: 5

$$\frac{3x^2+1}{(x+1)(x^2+1)} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2+1)}$$

ii) Hence, find $\int \frac{3x^2+1}{(x+1)(x^2+1)}dx$

Question Two (15 marks)

Marks

a) Given $z_1 = (2-i)$ and $z_2 = (3+4i)$ write each of the following in the form $x+iy$:

i) $z_1 + z_2$ 1

ii) $\frac{z_2}{z_1}$ 1

iii) $z_1\overline{z_2}$ 1

iv) $\sqrt{z_2}$ 3

b) Evaluate $\left[\sqrt{3}(1+i)\right]^6$ giving your answer in the form $a+ib$ 2

c) Sketch the region given by the intersection of:

$|z-2-2i| \le 2$ and $0 \le \arg(z-2) \le \frac{\pi}{4}$ 3

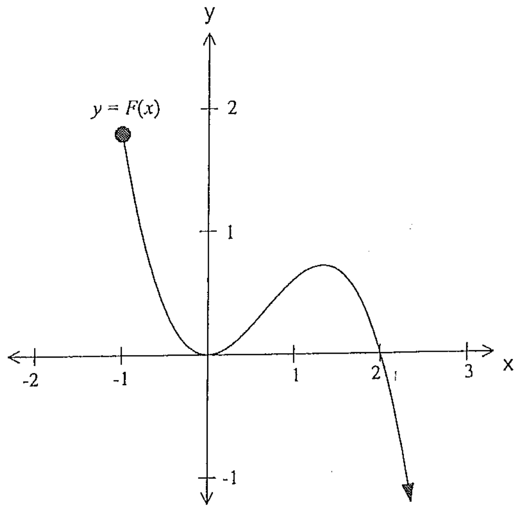
d) If $z = x+iy$ where x and y are real:

i) Find the Cartesian equation of the locus of $\text{Re}\left(z-\frac{1}{z}\right)=0$ 2

ii) Sketch the locus of $\text{Re}\left(z-\frac{1}{z}\right)=0$ 2

Question Three (15 marks)

Marks



a) Given the curve $y=F(x)$ above draw separate ¼ page sketches of

i) $y=|F(x)|$ 1

ii) $y=F(-x)$ 1

iii) $y=F(x-2)$ 1

iv) $y=\frac{1}{F(x)}$ 2

v) $y=e^{F(x)}$ 2

vi) $y=\log_e(F(x))$ 2

vii) $y=F'(x)$ 2

b) By referring to the graph of $y=F(x)$ in part a) above solve: 1

$$\frac{10}{F(x)}>0$$

c) Sketch the graph of $y=2e^{-x^2}$ showing all essential features 3

Question Four (15 marks)

Marks

a) Given the curve $4x^2 + 9y^2 = 36$:

i) Find the eccentricity 1

ii) Find the coordinates of the foci 1

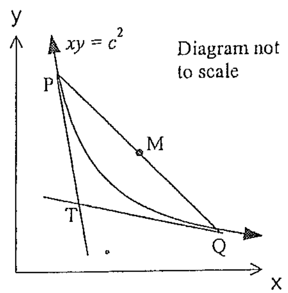
iii) Find the equations of the directrices 1

iv) Sketch the curve 1

b) $P\left(cp, \frac{c}{p}\right)$ and $Q\left(cq, \frac{c}{q}\right)$ are points on the rectangular hyperbola $xy = c^2$.

The tangents from P and Q meet at the point T.

M is the midpoint of the chord PQ.



i) Show that the equation of the tangent at the point P is
$x + yp^2 - 2cp = 0$ 2

ii) Find the coordinates of T 3

iii) Find the coordinates of M 2

iv) If $AMBT$ is a rectangle whose sides are parallel to the coordinate axes, show that the points A and B lie on the hyperbola $xy = c^2$ 4

Question Five (15 marks)

Marks

a) A polynomial $P(x)$ has a double root at $x = \alpha$.

i) Prove that $P'(x)$ also has a root at $x = \alpha$ 1

ii) The polynomial $Q(x) = x^4 - 6x^3 + ax^2 + bx + 36$ has a double root at $x = 3$. Find the values of a and b 3

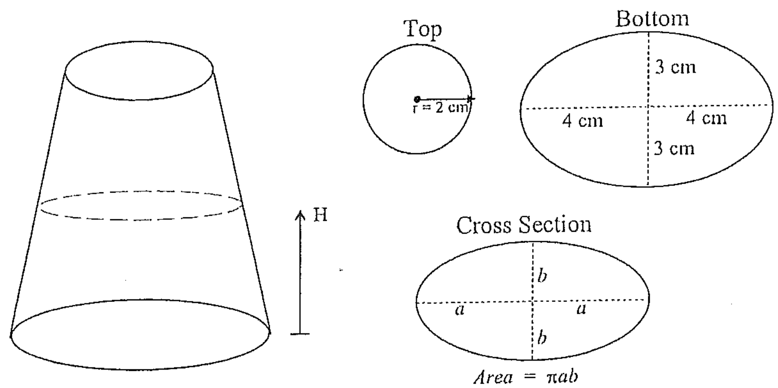
iii) Factorise $Q(x)$ over the complex field 2

b) The polynomial $x^3 - 4x + 10$ has roots α, β and γ

i) Find the polynomial equation with roots α^2, β^2 and γ^2 2

ii) Find the value of $\alpha^2 + \beta^2 + \gamma^2$ 1

c) A solid has an elliptical base and circular top as shown below. The height of the solid is 20cm. All other dimensions are shown below on the right.



Each cross section (slice) parallel to the base is an ellipse. A slice is taken H cm from the base

i) Find an expression for the area of each slice in terms of H 4

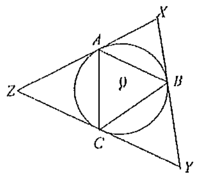
ii) Hence find the exact volume of the solid 2

Question Six (15 marks)

Marks

a) An odd monic polynomial of least degree has a double root at $x=-1$ and a triple root at $x=4$. Write down the equation of the polynomial **1**

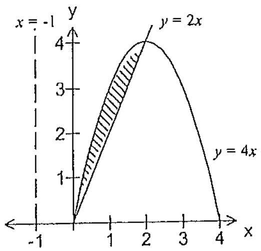
b) XYZ is a triangle whose inscribed circle (centre O) touches the sides at A, B and C. **2**



Show that triangle ABC is always acute angled

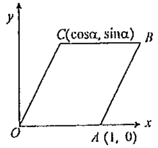
c) Prove by mathematical induction that $x^{2n}-y^{2n}$ is always divisible by $(x+y)$ for $n\geq 1$ where n is an integer. **3**

d) The area shown in the diagram between the curve $y=4x-x^2$ and $y=2x$ is rotated around the line $x=-1$ **4**



<u>Use the method of cylindrical shells</u> to find the volume of the solid formed.

e) OABC is a rhombus. The point O represents the complex number $(0+0i)$, the point A represents the complex number $(1+0i)$ and the point C represents the complex number $(\cos\alpha+i\sin\alpha)$. Copy the diagram onto your paper



i) Find the complex number represented by the point B **1**

ii) <u>Use your diagram</u> to prove that $\tan\dfrac{\alpha}{2}=\dfrac{\sin\alpha}{1+\cos\alpha}$ **1**

iii) <u>Hence(but not otherwise)</u> prove that $1+\cos\alpha=2\cos^2\dfrac{\alpha}{2}$ **3**

Question Seven (15 marks)

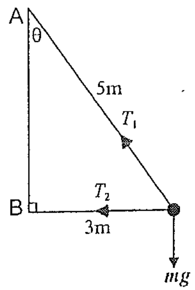
Marks

a) Given that the roots of $(1+i)z^2-(1+\sqrt{3}i)z+(-2\sqrt{3}-2i)=0$ are z_1 and z_2, find

i) $|z_1+z_2|$ **2**

ii) $\arg(z_1)+\arg(z_2)$ **2**

b) A mass of 4kg is attached to two strings of lengths 5 m and 3m. The other ends are attached to two points A and B, such that AB is vertical and the mass is both perpendicular to AB and level with B.



i) Find the minimum angular speed so that the bottom string is just taut. **4**

ii) If the angular speed is changed to $2 rad.s^{-1}$ find T_2 **2**

c) An integral is defined as $I_n=\int_0^1 \dfrac{x^n}{1+x^2}dx$ for $n\geq 0$

i) Evaluate I_0 **1**

ii) Show that $I_n+I_{n+2}=\dfrac{1}{n+1}$ **2**

iii) Evaluate I_2 **2**

uestion Eight (15 marks) **Marks**

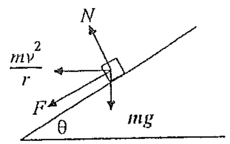
a) A particle moving in a straight line has acceleration given by $a = 4te^{-t^2}\ ms^{-2}$.

i) Find an expression for its velocity at any time t 2

ii) Given that the particle has an initial velocity of $4ms^{-1}$, find its velocity after 2 second of motion. 2

iii) What will its terminal velocity be? 1

b) A train line is banked at an angle θ as shown below.



i) If the force of circular motion is given as $\frac{mv^2}{r}$, and the force due to gravity as mg, express $\frac{mv^2}{r}$ and mg in terms of F and N 2

ii) Hence show that $F = \frac{mv^2}{r}\cos\theta - mg\sin\theta$ 1

iii) A railway line is constructed around a curve of radius 500 m to allow an optimum speed of 90km/h. Calculate the angle θ at which the track should be banked. 1

c) i) Show that $\sin\left[(2r+1)\theta\right] - \sin\left[(2r-1)\theta\right] = 2\sin\theta\cos 2r\theta$ 1

ii) Hence show that $\sum_{r=1}^{n}\cos 2r\theta = \frac{1}{2\sin\theta}\left[\sin(2n+1)\theta - \sin\theta\right]$ 2

iii) Hence evaluate $\sum_{r=1}^{100}\cos^2\left(\frac{r\pi}{100}\right)$ 3

STION ONE

let $u = 2 + x^4$

$\frac{du}{dx} = 4x^3$

$I = \int \sqrt{u}\, du$

$= \frac{2u^{\frac{3}{2}}}{3} + c$

$= \frac{2}{3}\sqrt{(2+x^4)^3} + c$

let $u = 3x \quad \frac{du}{dx} = 3$

$= \frac{1}{3}\int \frac{du}{\sqrt{u^2-1}}$

$\frac{1}{3} \ln\left(u + \sqrt{u^2-1}\right) + c$

$\frac{1}{3} \ln\left(3x + \sqrt{9x^2-1}\right) + c$

$u = \log_e x \qquad v' = 1$

$u' = \frac{1}{x} \qquad v = x$

$I = x\log_e x - \int 1\, dx$

$= \left[x\log_e x - x\right]_1^2$

$= 2\log_e 2 - 2 - (\log_e 1 - 1)$

$= \log_e 4 - 1$

$I = \int_1^3 \frac{dx}{(x-1)^2+4}$

$= \frac{1}{2}\left[\tan^{-1}\left(\frac{x-1}{2}\right)\right]_1^3$

$= \frac{1}{2}\left[\tan^{-1} 1 - \tan^{-1} 0\right]$

$= \frac{1}{2}\left(\frac{\pi}{4} - 0\right)$

$= \pi$

(e). (i).

$3x^2+1 = A(x^2+1) + (Bx+c)(x+1)$

$= (A+B)x^2 + (B+c)x + (A+c)$

$A + B = 3$
$B + c = 0 \implies B = -c$
$A + c = 1$

$A - c = 3$, $A + c = 1$ $\quad 2A = 4$, $A = 2$ *

$2c = -2$

$c = -1$ *

$B = 1$ *

(ii). $I = \int \frac{2}{x+1} + \frac{x-1}{x^2+1}$

$= \int \frac{2}{x+1} + \frac{x}{x^2-1} - \frac{1}{x^2+1}$

$= 2\ln(x+1) + \frac{1}{2}\ln(x^2-1)$

$- \tan^{-1}x + c$

(a) (i). $z_1 + z_2 = 5 + 3i$

(ii). $\frac{z_2}{z_1} = \frac{3+4i}{2-i} \times \frac{2+i}{2+i}$

$= \frac{6 + 8i + 3i + 4i^2}{4 - i^2}$

$= \frac{2}{5} + \frac{11}{5}i$

(iii). $z_1 \bar{z}_2 = (2-i)(3-4i)$

$= 6 - 3i - 8i + 4i^2$

$= 2 - 11i$

(iv). $\sqrt{3+4i} = a + ib$

$3+4i = a^2 + 2abi - b^2$

$ab = 2 \implies b = \frac{2}{a}$

$a^2 - \frac{4}{a^2} = 3$

$a^4 - 4 = 3a^2$

$(a^2-4)(a^2+1) = 0$

$a = \pm 2 \qquad b = \frac{2}{a} = \frac{2}{\pm 2}$

$\therefore b = \pm 1$

$\therefore \sqrt{3+4i} = \pm(2+i)$

(b). $\left[\sqrt{3}(1+i)\right]^6$

$= (\sqrt{3})^6\left[(1+i)^3\right]^2$

$= 27\left[1 + 3i + 3i^2 + i^3\right]^2$

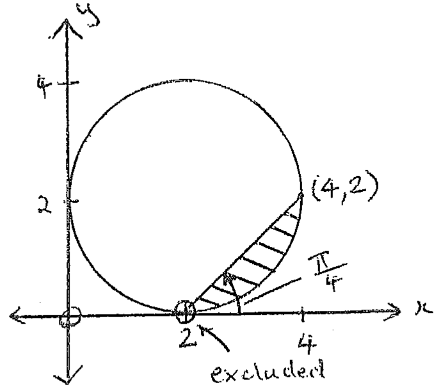
$= 27\left[2i - 2\right]^2$

$= 27\left[4i^2 - 8i + 4\right]$

$= 27(-8i)$

$= -216i$

(c). $\left|z - (2+2i)\right| \leq 2$



(d). (i). $Re\left(z - \frac{1}{z}\right)$

$= Re\left(x+iy - \frac{1}{x+iy}\right)$

$= Re\left(x+iy - \frac{1}{x+iy} \times \frac{x-iy}{x-iy}\right)$

$= Re\left(x+iy - \frac{x}{x^2+y^2} + \frac{iy}{x^2+y^2}\right)$

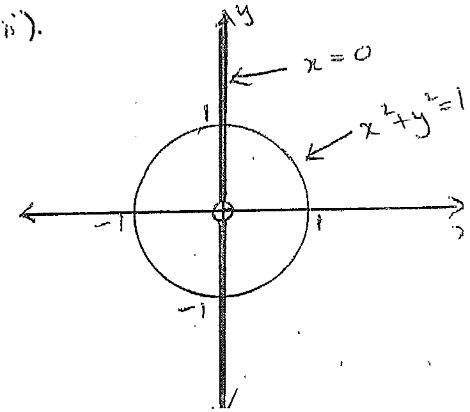
$= x - \frac{x}{x^2+y^2}$

$\therefore x - \frac{x}{x^2+y^2} = 0$

$x\left(1 - \frac{1}{x^2+y^2}\right) = 0$

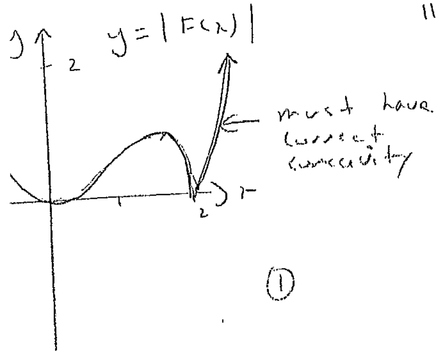
$\therefore x = 0$ (except (0,0)) or $x^2+y^2 = 1$

(ii).

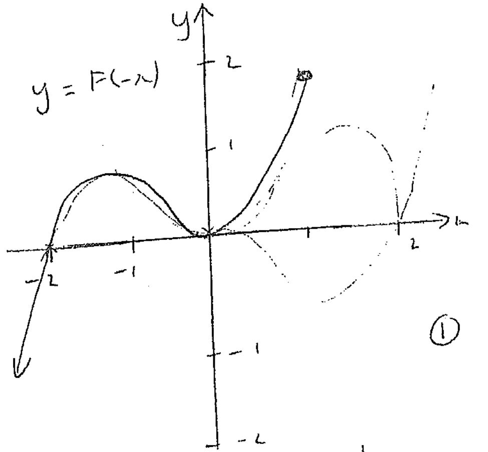


...ion Three

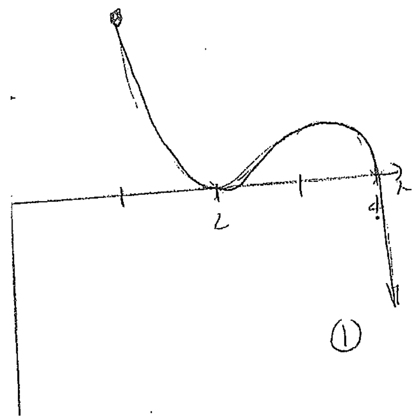
i) $y = |F(x)|$



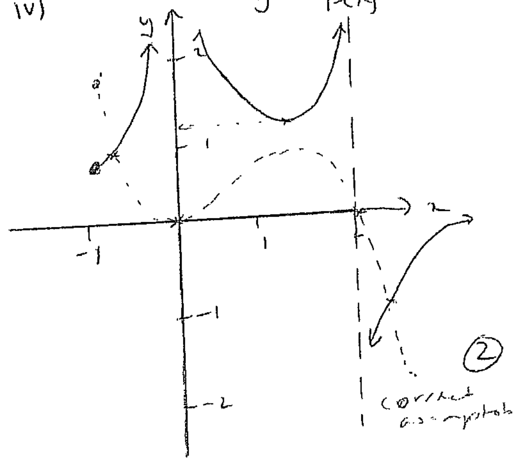
ii) $y = F(-x)$



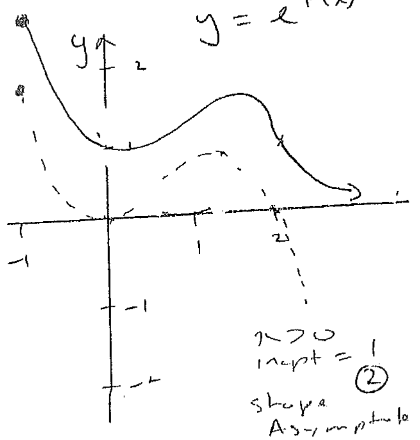
iii) $y = F(x-2)$



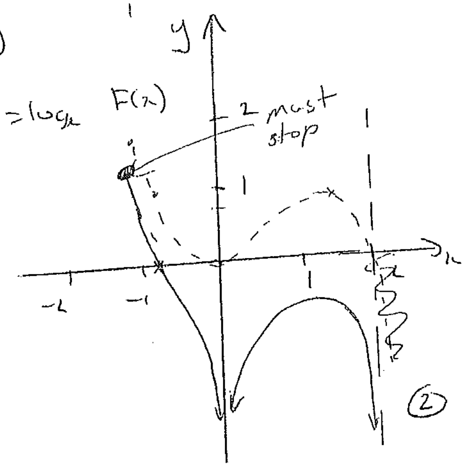
iv) $y = \frac{1}{F(x)}$



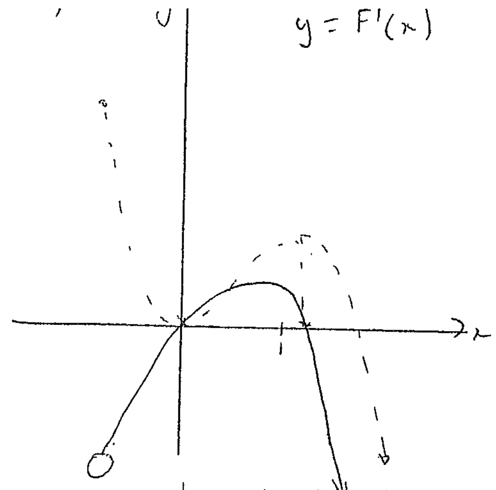
v) $y = e^{F(x)}$



vi) $y = \log_e F(x)$



$y = F'(x)$



b) $\frac{10}{F(x)} > 0$

$\frac{10\,(F(x))^2}{F(x)} > 0 \quad F(x)^2 \Rightarrow \quad F(x) > 0$ for $-1 \le x < 2$ for $x \neq 0$ ①

recognise $-1, 2$ as critical pts

c) $y = 2e^{-x^2}$

when $x = 0,\ y = 2$

y always > 0 (x axis asymptote)

$\frac{dy}{dx} = -4xe^{-x^2}$

for a stny pt $\frac{dy}{dx} = 0$

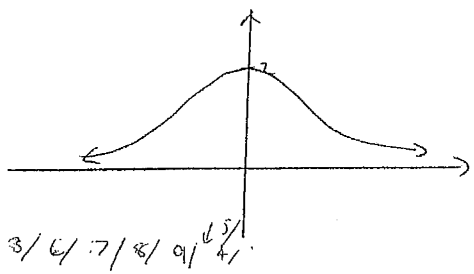
$-4xe^{-x^2} = 0$

$x = 0$

$y = 2$

x	0^-	0	0^+
$\frac{dy}{dx}$	$-$	0	$+$

$\therefore$ max T.pt at $(0, 2)$



Even Function ③

1 y incpt at 2 / T.pt

1 Symmetrical about y axis

1 asymptote

1/ 2/ 3/ 6/ 7/ 8/ 9/ 5/ 4/

$4x^2 + 9y^2 = 36$

a) $\frac{x^2}{9} + \frac{y^2}{4} = 1$ $\quad a^2 = 9, \; b^2 = 4$

i. $b^2 = a^2(1-e^2)$

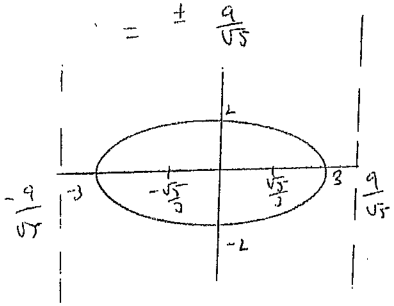
$4 = 9(1-e^2)$ $\quad e = \frac{\sqrt{5}}{3}$ ✓ ①

ii) $x = (\pm ae, 0)$

$= (\pm\sqrt{5}, 0)$ ①

iii) $x = \pm \frac{a}{e}$

$= \pm \frac{9}{\sqrt{5}}$ ①



must show intercepts ①

④

b) i) $y = c^2 x^{-1}$

$\frac{dy}{dx} = -\frac{c^2}{x^2}$ at $x = cp$

$= -\frac{c^2}{c^2p^2}$

$m = -\frac{1}{p^2}$ ①

$y - y_1 = m(x - x_1)$

$y - \frac{c}{p} = -\frac{1}{p^2}(x - cp)$

$yp^2 - cp = -x + cp$

$x + yp^2 - 2cp = 0$ ① —

②

ii) Eqn QT $x + yq^2 - 2cq = 0$

$x = 2cq - yq^2$ ①

$x = 2cp - yp^2$

$\therefore 2cq - yq^2 = 2cp - yp^2$

$yp^2 - yq^2 = 2cp - 2cq$

$y(p+q)(p-q) = 2c(p-q)$ ✓

$y = \frac{2c}{p+q}$ subst in ①

$x = 2cq - \frac{2cq^2}{p+q}$

$x = \frac{2cq(p+q) - 2cq^2}{p+q}$

$= \frac{2cpq}{p+q}$ ✓

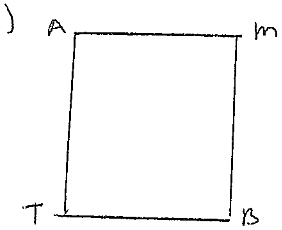
ie $T\left(\frac{2cpq}{p+q}, \frac{2c}{p+q}\right)$

③

iii) $x = \frac{cp + cq}{2}$ $\quad y = \left(\frac{c}{p} + \frac{c}{q}\right) \div 2$

$= \frac{c(p+q)}{2}$ $\quad = \frac{c(p+q)}{2pq}$ ✓ ②

iv)



m $\left(\frac{c(p+q)}{2}, \frac{c(p+q)}{2pq}\right)$

$\frac{2cpq}{p+q}, \frac{2c}{p+q}$

From diagram

A $\left(\frac{2cpq}{p+q}, \frac{c(p+q)}{2pq}\right)$ ✓ ②

sub in $xy = c^2$

LHS $= \frac{2cpq}{p+q} \times \frac{c(p+q)}{2pq}$

$= c^2$

$=$ RHS ①

From diagram

B $\left(\frac{c(p+q)}{2}, \frac{2c}{p+q}\right)$ ✓

subst in

$xy = c^2$

LHS $= \frac{c(p+q)}{2} \times \frac{2c}{p+q}$

$= c^2$ ①

$=$ RHS $\therefore$ A and B lie on the hyperbola

④

Question 5:

a) (i) $P(x)=(x-\alpha)^2 Q(x)$

$P'(x)=2(x-\alpha)Q(x)+(x-\alpha)^2 Q'(x)$

$=(x-\alpha)\left[2Q(x)+(x-\alpha)Q'(x)\right]$

$\therefore P'(x)$ has a factor of $(x-\alpha)$ and so has a root at $x=\alpha$.

(ii) $Q(x)=x^4-6x^3+ax^2+bx+36$

$Q(3)=0 \text{ and } Q'(3)=0$

$Q(3)=3^4-6(3)^3+a(3)^2+3b+36$

$45=9a+3b$

$15=3a+b \quad ---(1)---$

$Q'(x)=4x^3-18x^2+2ax+b$

$Q'(3)=4(3)^3-18(3)^2+2a(3)+b$

$54=6a+b \quad ---(2)---$

$(2)-(1)$:

$39=3a$

$a=13$

Sub $a=13$ into (1):

$15=3(13)+b$

$b=-24$

(iii) $x^4-6x^3+13x^2-24x+36=(x-3)^2(ax^2+bx+c)$

Equating coefficients:

$a=1$

$9c=36$

$c=4$

$9b-6c=-24$

$9b-6(4)=-24$

$b=0$

$Q(x)=(x-3)^2(x^2+4)$

$=(x-3)^2(x^2-4i^2)$

$=(x-3)^2(x-2i)(x+2i)$

b) (i) $x^3-4x+10=0$

$x=\alpha,\beta,\gamma$

If $x=\alpha^2,\beta^2,\gamma^2$

$\sqrt{x}=\alpha,\beta,\gamma$

$(\sqrt{x})^3-4\sqrt{x}+10=0$

$x\sqrt{x}-4\sqrt{x}+10=0$

$\sqrt{x}(x-4)=-10$

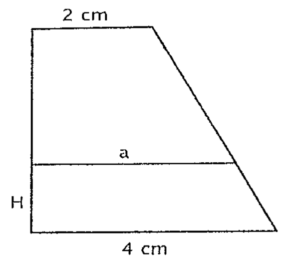
$x(x-4)^2=100$

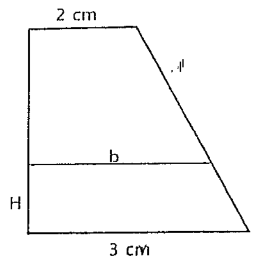
$x(x^2-8x+16)=100$

$x^3-8x^2+16x-100=0$

(ii) $\alpha^2+\beta^2+\gamma^2=8$

c) (i)





$a=mH+k$

When $H=0, a=4$

$k=4$

When $H=20, a=2$

$2=20m+4$

$m=-\dfrac{1}{10}$

$a=-\dfrac{1}{10}H+4$

$b=mH+l$

When $H=0, b=3$

$k=3$

When $H=20, a=2$

$2=20m+3$

$m=-\dfrac{1}{20}$

$b=-\dfrac{1}{20}H+3$

(ii) *Area slice* $= \pi ab$

$$= \pi\left(-\frac{1}{10}H+4\right)\left(-\frac{1}{20}H+3\right)$$

$$= \pi\left(\frac{1}{200}H^2-\frac{1}{2}H+12\right)$$

$$\text{Volume slice} = \pi\left(\frac{1}{200}H^2-\frac{1}{2}H+12\right)\delta H$$

$$\text{Volume solid} = \sum_{H=0}^{20}\pi\left(\frac{1}{200}H^2-\frac{1}{2}H+12\right)\delta H$$

$$= \pi\int_0^{20}\left(\frac{1}{200}H^2-\frac{1}{2}H+12\right)dH$$

$$= \pi\left[\frac{1}{600}H^3-\frac{1}{4}H^2+12H\right]_0^{20}$$

$$= \pi\left(\frac{1}{600}\{20\}^3-\frac{1}{4}\{20\}^2+12\{20\}\right)-0$$

$$= \pi\left(\frac{40}{3}-100+240\right)$$

$$= \frac{460\pi}{3}\ \text{units}^3$$

Question 6:

a) $P(x)=x(x-1)^2(x+1)^2(x-4)^3(x+4)^3$

b) (Suggested solution)

Join *OA, OB, OC*.

Let $\angle CAB = x$

$\therefore \angle BCY = x$ (angle between a chord and tangent at the point of contact equal to angle in the alternate segment).

$\angle OCY = 90^0$ (radius $\perp$ tangent at point of contact)

$\angle OCB + \angle BCY = 90^0$ (adjacent complementary $\angle$s)

So $\angle BCY < 90^0$

$\therefore \angle CAB < 90^0$ ($\angle CAB = \angle BCZ$)

Similarly $\angle ABC < 90^0$ and $\angle BCA < 90^0$

$\therefore \Delta ABC$ is always acute-angled.

c) Step 1: *Prove true when* $n=1$

$x^2-y^2=(x+y)(x-y)$

which is divisible by $(x+y)$.

Proven true for $n=1$.

Step 2: *Assume true for* $n=k$.

$x^{2k}-y^{2k}=M(x+y)$

Step 3: *Prove true for* $n=k+1$

$$LHS = x^{2(k+1)}-y^{2(k+1)}$$
$$= x^{2k}\cdot x^2 - y^{2k}\cdot y^2$$
$$= \left[M(x+y)+y^{2k}\right]x^2 - y^{2k}\cdot y^2$$
$$= Mx^2(x+y)+y^{2k}\cdot x^2 - y^{2k}\cdot y^2$$
$$= Mx^2(x+y)+y^{2k}(x^2-y^2)$$
$$= Mx^2(x+y)+y^{2k}(x+y)(x-y)$$
$$= (x+y)\left[Mx^2+y^{2k}(x-y)\right]$$

which is divisible by $(x+y)$.

Step 4: If true for $n=k$, then proven true for $n=k+1$.

Proven true for $n=1$, so proven true for $n=2$. If true for $n=2$, then proven true for $n=3$ and so on. Hence, by mathematical induction, true for all positive integers n.

d)

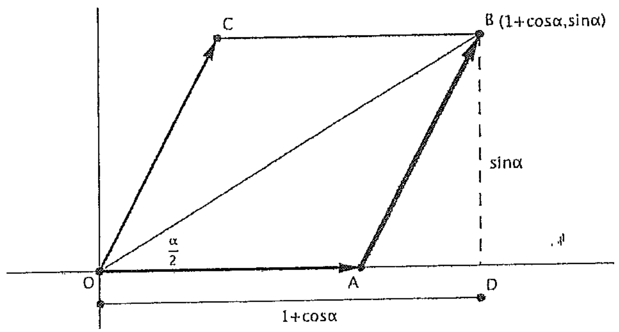
$$Vol.\ shell = \pi r^2 h$$
$$= \pi\left(\{1+x+\delta x\}^2 - \{1+x\}^2\right)(y_2 - y_1)$$
$$= \pi\left(\{1+x+\delta x\} + \{1+x\}\right)\left(\{1+x+\delta x\} - \{1+x\}\right)\left(\{4x-x^2\}-2x\right)$$
$$= \pi(2+2x+\delta x)(\delta x)(2x-x^2)$$
$$= \pi\left(2\delta x + 2x\delta x + \{\delta x\}^2\right)(2x-x^2)$$
$$= \pi(2+2x)(2x-x^2)\delta x \quad as\ (\delta x)^2 \cong 0$$
$$= \pi(-2x^3+2x^2+4x)\delta x$$

$$V = 2\pi\int_0^2 (-x^3+x^2+2x)dx$$
$$= 2\pi\left[-\frac{1}{4}x^4 + \frac{1}{3}x^3 + x^2\right]_0^2$$
$$= 2\pi\left(-\frac{1}{4}\times(2)^4 + \frac{1}{3}\times(2)^3 + (2)^2\right) - 0$$
$$= \frac{16\pi}{3}\ units^3$$

e) (i) $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{OC}$

$$= (1+0i) + (\cos\alpha + i\sin\alpha)$$
$$\therefore B(1+\cos\alpha, \sin\alpha)$$

(ii)



D is a point on the x axis such that $BD \perp x$ axis

Using ΔOBD:

$$\tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha}$$

(iii)

$$\tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha}$$
$$1+\cos\alpha = \sin\alpha \times \frac{\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}}$$
$$= 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} \times \frac{\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}}$$
$$= 2\cos^2\frac{\alpha}{2}$$

$$= \left| \frac{1+\sqrt{3}i}{1+i} \times \frac{1-i}{1-i} \right|$$

$$= \left| \frac{1-i+\sqrt{3}i+\sqrt{3}}{1+1} \right|$$

$$= \sqrt{\left(\frac{1+\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}-1}{2}\right)^2} \checkmark$$

$$= \frac{\sqrt{1+2\sqrt{3}+3+3-2\sqrt{3}+1}}{2}$$

$$= \frac{\sqrt{8}}{2}$$

$$= \sqrt{2}$$

$\arg(z_1)+\arg(z_2)$

$\arg(z_1 z_2)$ ✓

$$\arg\left\{ \frac{-2\sqrt{3}-2i}{1+i} \times \frac{1-i}{1-i} \right\}$$

$$\arg\left\{ \frac{-2\sqrt{3}+2\sqrt{3}i-2i-2}{2} \right\}$$

$$\arg\left\{ -1-\sqrt{3}+i(\sqrt{3}-1) \right\}$$

$= \theta$

$$\tan\theta = \frac{\sqrt{3}-1}{-1-\sqrt{3}} \times \frac{-1+\sqrt{3}}{-1+\sqrt{3}} \checkmark$$

$$= \frac{3-2\sqrt{3}+1}{1-3}$$

$$= \frac{4-2\sqrt{3}}{-2}$$

$$= -2+\sqrt{3}$$

$\theta = -15°$

i) $I_0 = \int_0^1 \frac{1}{1+x^2}\,dx$

$$= \left[\tan^{-1}x\right]_0^1$$

$$= \frac{\pi}{4} - 0$$

$$= \frac{\pi}{4} \checkmark$$

ii) $\int_0^1 \frac{x^n}{1+x^2}\,dx + \int_0^1 \frac{x^{n+2}}{1+x^2}\,dx$

$$= \int_0^1 \frac{x^n(1+x^2)}{1+x^2}\,dx \checkmark$$

$$= \int_0^1 x^n\,dx$$

$$= \left[\frac{x^{n+1}}{n+1}\right]_0^1 \checkmark$$

$$= \frac{1}{n+1}$$

iii) $I_0 + I_2 = \frac{1}{0+1}$ ✓

$$\frac{4T_1}{5} = 4 \times 10$$

$T_1 = 50$ ✓

$T_2 + T_1 \sin\theta = mr\omega^2$ ✓

$$0 + 50 \times \frac{3}{5} = 4 \times 3 \times \omega^2$$

$$30 = 12\omega^2$$

$$\omega^2 = \frac{30}{12} = \frac{5}{2}$$

$$\omega = \sqrt{\frac{5}{2}} = \frac{\sqrt{10}}{2} \checkmark$$

ii) $T_1 = 50$

$$T_2 + 50 \times \frac{3}{5} = 4 \times 3 \times 2^2 \checkmark$$

$$T_2 + 30 = 48$$

$$T_2 = 48 - 30 = 18 \checkmark$$

$$= -2\int -2t e^{-t^2}\,dt$$

$$= -2e^{-t^2} + c \checkmark$$

ii) $4 = -2e^0 + c$

$c = 6$ ✓

$$v = -2e^{-t^2} + 6$$

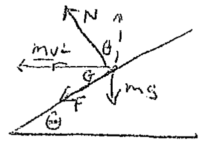
$$= -2e^{-2^2} + 6$$

$$= 6 - 2e^{-4} \checkmark$$

iii) as $t \to \infty$

$v \to 0 + 6 = 6\ \text{m s}^{-1}$ ✓

b) i)



$$\frac{mv^2}{r} = F\cos\theta + N\sin\theta \checkmark$$

$$mg + F\sin\theta = N\cos\theta$$

$$mg = N\cos\theta - F\sin\theta \checkmark$$

ii) $\frac{mv^2}{r}\cos\theta = F\cos^2\theta + N\sin\theta\cos\theta$

$$mg\sin\theta = N\cos\theta\sin\theta - F\sin^2\theta \checkmark$$

$$\frac{mv^2}{r}\cos\theta - mg\sin\theta = F\cos^2\theta + F\sin^2\theta = F$$

iii) $0 = \frac{m \times \left(\frac{90000}{3600}\right)^2 \cos\theta}{500} - m \times 10 \times \sin\theta$ ✓

$$0 = 1.25\cos\theta - 10\sin\theta$$

$$10\sin\theta = 1.25\cos\theta$$

$$\tan\theta = 0.125$$

$$\theta = 7.125016349°$$

$$= \sin 2r\theta\cos\theta + \sin\theta\cos 2r\theta - \sin 2r\theta\cos\theta + \sin\theta\cos 2r\theta$$

$$= 2\sin\theta\cos 2r\theta \checkmark$$

iv) $\cos 2r\theta$

$$= \frac{\sin[(2r+1)\theta] - \sin[(2r-1)\theta]}{2\sin\theta} \checkmark$$

$$\sum_{r=1}^{n} \cos 2r\theta$$

$$= \sum_{r=1}^{n} \frac{\sin[(2r+1)\theta] - \sin[(2r-1)\theta]}{2\sin\theta}$$

$$= \frac{\sin 3\theta - \sin\theta}{2\sin\theta} + \frac{\sin 5\theta - \sin 3\theta}{2\sin\theta} + \frac{\sin 7\theta - \sin 5\theta}{2\sin\theta} + \ldots + \frac{\sin[(2n+1)\theta] - \sin[(2n-1)\theta]}{2\sin\theta} \checkmark$$

$$= \frac{\sin[(2n+1)\theta] - \sin\theta}{2\sin\theta}$$

iii) $\sum_{r=1}^{100} \frac{\cos\frac{2r\pi}{100} + 1}{2}$ ✓

$$= \frac{1}{2}\sum_{r=1}^{100}\left(\cos\frac{2r\pi}{100} + 1\right)$$

$$= \frac{1}{2} \times 100 + \frac{1}{2}\sum_{r=1}^{100}\cos\frac{2r\pi}{100} \checkmark$$

$$= 50 + \frac{1}{2} \times \frac{\sin\frac{201\pi}{100} - \sin\frac{\pi}{100}}{2\sin\frac{\pi}{100}}$$

$$= 50 + \frac{\sin\left(2\pi + \frac{\pi}{100}\right) - \sin\frac{\pi}{100}}{4\sin\frac{\pi}{100}}$$

$$= 50 \checkmark$$