QUESTION 1

Marks

(a) Find $\int \frac{e^x}{(e^x+1)^2}.dx$ **2**

(b) Find $\int \frac{x^2}{(1-x^2)^{\frac{3}{2}}}.dx$ **3**

(c) Evaluate $I = \int_0^{\frac{\pi}{3}} \sec^4\theta.\tan\theta.d\theta$ **3**

(d) Evaluate $I = \int_0^{\frac{\pi}{2}} \frac{1}{1+\sin\theta+\cos\theta}.d\theta$ **3**

(e) (i) Express $\frac{3x+7}{(x+1)(x+2)(x+3)}$ in partial fractions **4**

(ii) Hence, evaluate $\int_0^1 \frac{(3x+7).dx}{(x+1)(x+2)(x+3)}$

Question 2.

Marks

(a) Given $z = 1 + i\sqrt{3}$ and $w = 1 + i$ **6**

(i) Evaluate: $\dfrac{z}{w}$ in Cartesian Form

(ii) Plot z, w and $\dfrac{z}{w}$ on the Argand Diagram

(iii) Express z and w in modulus – argument form.

(iv) Show $\dfrac{z}{w} = \sqrt{2}cis(\dfrac{\pi}{12})$ and hence find the exact value of $\cos\dfrac{\pi}{12}$

(b) (i) Find both square roots of $8 + 6i$, in the form $x = iy$ **4**

(ii) Solve the quadratic equation $z^2 + (2 + 4i)z - 11 - 2i = 0$

(c) Given the locus of z is $|z - 2 - 2i| = 1$ **5**

(i) Sketch the locus of z on the Argand Diagram.

(ii) Find the maximum value of $\arg z$

(iii) Find the maximum value of mod z

3+7

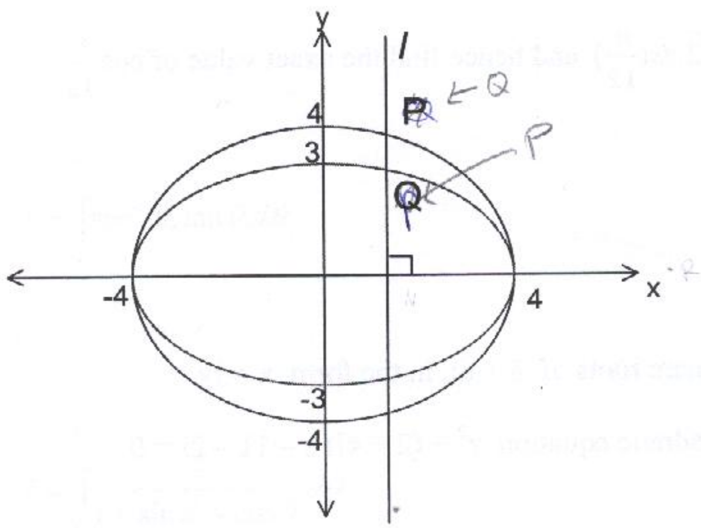
Question 3.

Marks

(a) Given $\frac{x^2}{25}-\frac{y^2}{9}=1$ **6**

Find: (a) eccentricity
(b) foci
(c) directrices
(d) asymptotes
(e) the equation of the tangent at $P(5\sec\theta, 3\tan\theta)$

(b) **9**



$x = 4\cos\theta; y = 4\sin\theta$

The diagram shows the ellipse, E, with equation $\frac{x^2}{16}+\frac{y^2}{9}=1$ and its auxiliary circle C
The coordinates of a point P on E are $(4\cos\theta, 3\sin\theta)$.

A straight line, l, parallel to the y axis intersects the x axis at N and the curves E and C at the points P and Q respectively.

(i) Find the eccentricity of E,

(ii) Write down the coordinate of N and Q,

(iii) Find the equations of the tangents at P and Q to the curves E and C respectively,

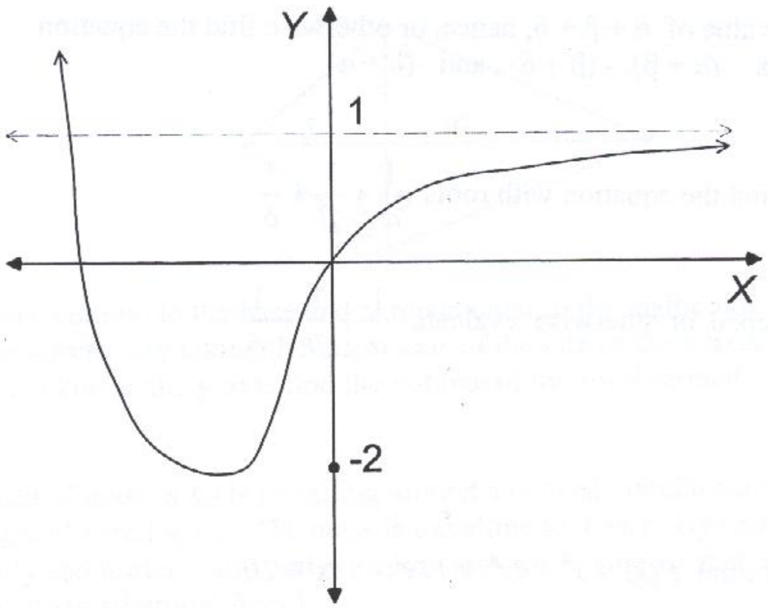
(iv) The tangents at P and Q intersect at a point R. Show that R lies on the x axis,

(v) Prove that $ON.OR$ is independent of the positions P and Q.

Question 4

Marks

(a) 6



Given the above curve is $y = f(x)$

Sketch (i) $y = f(-x)$

(ii) $y = \dfrac{1}{f(x)}$

(iii) $y = e^{f(x)}$

(iv) $y = \ln f(x)$

(v) $y^2 = f(x)$

(b) (i) By expanding $(\cos\theta + i\sin\theta)^4$ find expressions for $\sin 4\theta$ and $\cos 4\theta$. 9

(ii) Prove $\tan 4\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$

(iii) Solve Solve $\tan 4\theta = 1$ for $0 \le \theta \le \pi$

(iv) By taking $x = \tan\theta$, find the roots of $x^4 + 4x^3 - 6x^2 - 4x + 1 = 0$

(v) Using (iv) find the value of: $\tan^2\dfrac{\pi}{16} + \tan^2\dfrac{5\pi}{16} + \tan^2\dfrac{9\pi}{16} + \tan^2\dfrac{13\pi}{16}$.

Question 5

Marks

(a) Let α, β and δ be the roots of $x^3 - x^2 + 2x - 1 = 0$ **6**

(i) Find the value of $\alpha + \beta + \delta$, hence, or otherwise find the equation with roots $-(\alpha + \beta)$, $-(\beta + \delta)$, and $-(\delta + \alpha)$

(ii) (a) Find the equation with roots $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\delta}$

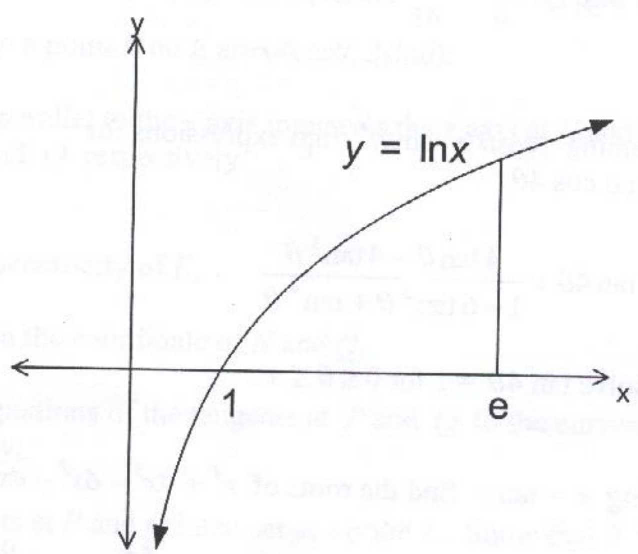
(b) hence, or otherwise, evaluate $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\delta}$

(b) Given the polynomial $P(x) = x^4 - 4x^3 + 11x^2 - 14x + 10$ **5**

(i) If (x) has zeroes $a + bi$, $a - 2bi$, where a and b are real find the values of a and b

(ii) Hence, express $P(x)$ as the product of two quadratic factors with real Coefficients.

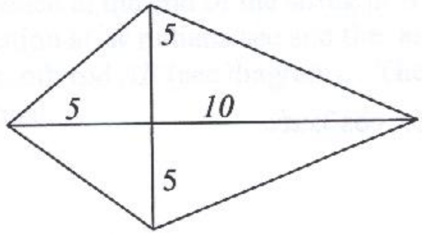
(c) Using the method of cylindrical shells, find the volume generated by revolving the region bounded by $y = \log_e x$ the *x-axis* and $1 \le x \le e$, about the *y-axis*. **4**



Question 6

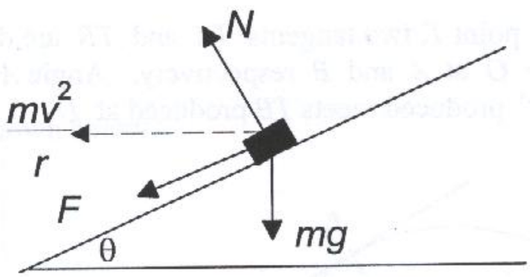
Marks

(a) The base of a solid is the kite shown below, with measurements in cm. **6**



Each slice vertical to the base and perpendicular to the major axis of the kite is a semi-circle. By taking the major axis of the kite as the x-axis and the minor axis of the kite as the y-axis find the volume of the solid formed.

(b) An object of mass m kg is travelling around a banked circular track of radius r and angle of banking θ. The mass is travelling at v m/s. By resolving forces vertically and horizontally derive expressions for N (the normal force) and F (the sideways frictional force). **5**



Given the radius of the curve is $1km$ and $\tan\theta = \frac{1}{100}$ find the velocity in m/s which will ensure no sideways friction (i.e. $F = 0$). Take $g = 10m/s^2$

(c) If $x^3 + 3kx + l = 0$ has a double root, where k and l are real, prove that $l^2 + 4k^3 = 0$ **4**

Question 7

Marks

(a) (i) Simplify: $\sin(A - B) + \sin(A + B)$ **3**

(ii) Hence find $\int_0^{\frac{\pi}{4}} \sin 5x . \cos 3x . dx.$

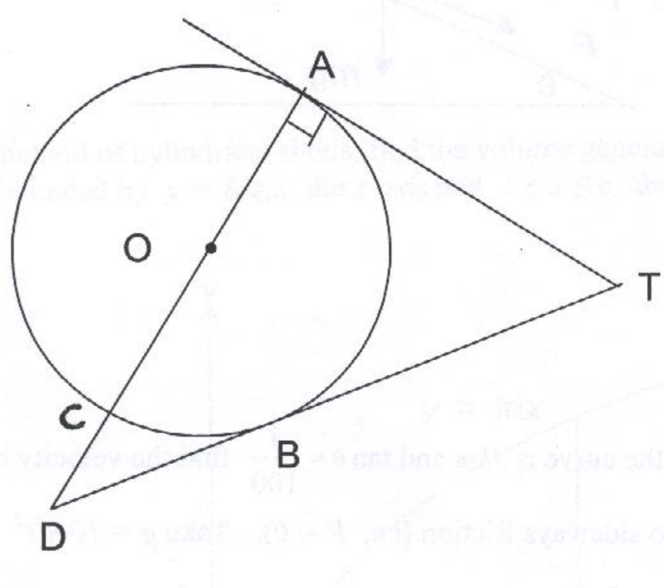
(b) Given $f(x) = \dfrac{\ln x}{x}$ **4**

(i) Find the stationary point and its nature.

(ii) Show $f(e) > f(\pi)$

(iii) Show $e^{\pi} > \pi^{e}$

(c) From an external point T, two tangents TA and TB are drawn to touch a circle with centre O at A and B respectively. Angle ATB is acute. The diameter AC produced meets TB produced at D. **8**



(i) Prove that $< CBD = \dfrac{1}{2} < ATB$

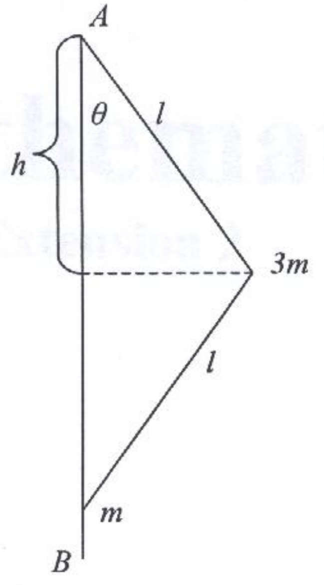
(ii) Prove that ΔABC is similar to ΔTBO

(iii) Deduce that $BC . OT = 2.(OA)^2$

Question 8

Marks

(a) A light string $2l$ metres long is attached to point A. A mass of $3m$ kg is attached to the middle of the string and a second mass of m kg is in the form of a ring and is attached at the end of the string at B. The $3m$ kg mass is rotating in circular motion at w radians/sec and the m kg mass is free to move up or down the smooth rod AB (see diagram). The string makes an angle of θ with the vertical. **7**



(i) Find an expression for h in terms of g and w only.

(ii) If the $3m$ kg and m kg masses are interchanged and the speed of the rotating mass is doubled to $2w$ determine if h is increased or decreased. (note $w > 1$)

(b) A magic square is shown below **2**

4	3	8
9	5	1
2	7	6

Note that the sum of the diagonals, rows and columns is fifteen
Three different numbers are chosen at random from the square.
Find the probability that the sum of the numbers is 15 if:
(i) A five is chosen first
(ii) A two is chosen first

(c) z is a complex number such that: **3**

$$z = k(\cos\theta + i\sin\theta) \quad \text{where } k \text{ is real.}$$

Show $\arg(z + k) = \dfrac{\theta}{2}$

(d) Given $I_n = \dfrac{1}{n!}\displaystyle\int_0^a x^n . e^{-x} . dx.$ **3**

Show $\dfrac{a^n}{n!} = e^a(I_{n-1} - I_n)$

Note $n! = n \times (n-1) \times (n-2) \times \ldots\ldots \times 3 \times 2 \times 1$