Student Number:	
Class:	

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION 2013

MATHEMATICS EXTENSION 2

General Instructions:

- Reading Time: 5 minutes.
- Working Time: 3 hours.
- Write in black pen.
- Board approved calculators & templates may be used
- A Standard Integral Sheet is provided.
- In Question 11 - 16, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged working.

Total Marks 100

Section I: 10 marks

- Attempt Question 1 – 10.
- Answer on the Multiple Choice answer sheet provided.
- Allow about 15 minutes for this section.

Section II: 90 Marks

- Attempt Question 11 - 16
- Answer on lined paper provided. Start a new page for each new question.
- Allow about 2 hours & 45 minutes for this section.

The answers to all questions are to be returned in separate *stapled* bundles clearly labelled Question 11, Question 12, etc. Each question must show your Candidate Number.

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

1 Let $z = 1 + i$. What is the value of z^{12}?

(A) 64

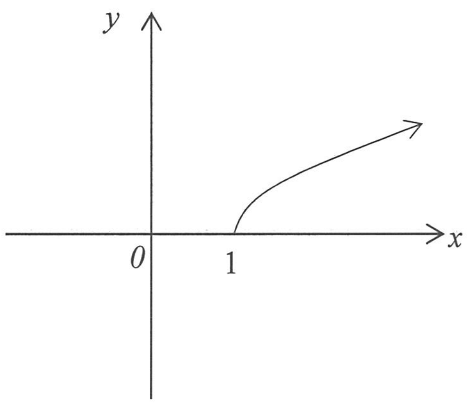
(B) -64

(C) $64i$

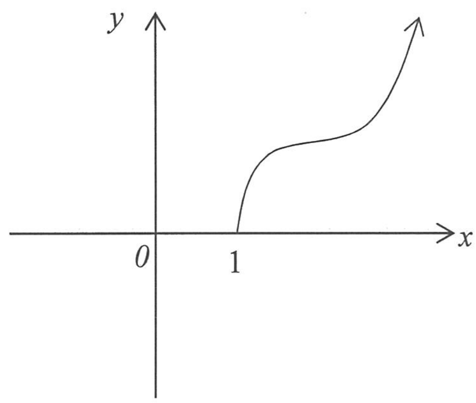
(D) $-64i$

2 Given $f(x) = x^2(x-1)$. Which of the following best represents the graph of $y = \sqrt{f(x)}$?

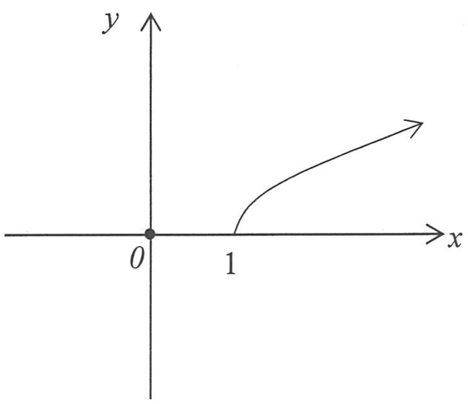
(A)



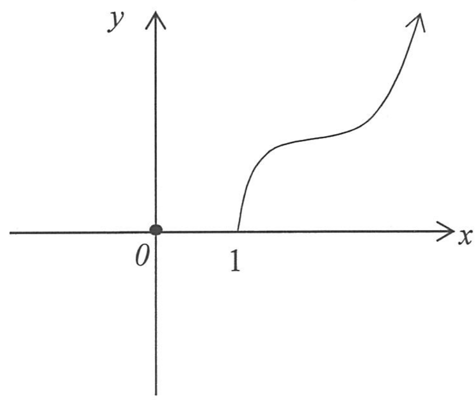
(B)



(C)



(D)



3 Given $2x^2 + xy + 2y^2 = 30$, what are the coordinates of one of the vertical tangents?

(A) (-1, 4)

(B) (4, -1)

(C) (-1, -4)

(D) (1, -4)

4 What is the equation of the chord of contact of tangents from (2, 1) to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$?

(A) $\frac{2x}{9} - \frac{y}{4} = 1$

(B) $\frac{2x}{9} + \frac{y}{4} = 1$

(C) $\frac{x}{9} - \frac{y}{2} = 1$

(D) $\frac{x}{9} + \frac{y}{4} = 1$

5 Given $3x^3 - 2x + 5 = 0$ has roots α, β and γ, what is the equation with roots $\alpha + 1$, $\beta + 1$ and $\gamma + 1$?

(A) $3x^3 - 9x^2 + 7x + 6 = 0$

(B) $3x^3 + 9x^2 + 7x + 6 = 0$

(C) $3x^3 - 9x^2 + 7x + 4 = 0$

(D) $3x^3 + 9x^2 + 7x + 4 = 0$

6 Which of the following is the correct expression for the integral $\int \frac{dx}{4+\sin^2 x}$?

(A) $\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{5}{4}\tan x\right)+C$

(B) $2\sqrt{5}\tan^{-1}\left(\frac{5}{4}\tan x\right)+C$

(C) $\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{\sqrt{5}}{2}\tan x\right)+C$

(D) $2\sqrt{5}\tan^{-1}\left(\frac{\sqrt{5}}{2}\tan x\right)+C$

7 Given $3x^3+6x-5=0$ has roots $\alpha,\ \beta$ and γ, what is the value of $\alpha^3+\beta^3+\gamma^3$?

(A) 5

(B) 9

(C) 15

(D) -1

8 The equation of motion of a particle falling with velocity v m/s is given by $\ddot{x}=10-\frac{v}{2}$. Which of the following is the value of the terminal velocity?

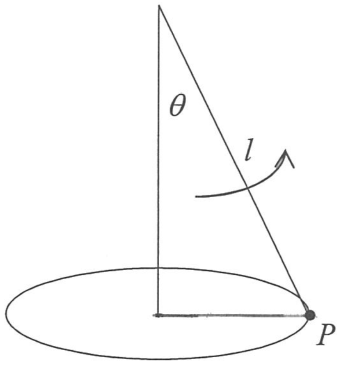
(A) 5

(B) 15

(C) 20

(D) $\sqrt{20}$

9 A bob P of mass m kg is suspended from a fixed point A by a string of length l metres, and acceleration due to gravity g. P describes a horizontal circle with uniform angular velocity ω rad/s.



Which of the following expressions represents the tension in the string?

(A) $ml\omega$

(B) $ml\omega^2$

(C) $mgl\omega$

(D) $mgl\omega^2$

10 Which of the following is the correct expression for the integral $\int e^{\alpha x} \sin \beta x \, dx$?

(A) $\dfrac{e^{\alpha x}}{\alpha^2 + \beta^2}[\beta \sin \beta x + \alpha \cos \beta x] + C$

(B) $\dfrac{e^{\alpha x}}{\alpha^2 + \beta^2}[\beta \sin \beta x - \alpha \cos \beta x] + C$

(C) $\dfrac{e^{\alpha x}}{\alpha^2 + \beta^2}[\alpha \sin \beta x + \beta \cos \beta x] + C$

(D) $\dfrac{e^{\alpha x}}{\alpha^2 + \beta^2}[\alpha \sin \beta x - \beta \cos \beta x] + C$

Section II

90 marks
Attempt Questions 11–16.
Allow about 2 hours and 45 minutes for this section.
In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a NEW page.

(a) $|z|<1$ and $z=\cos\theta+i\sin\theta$, where $-\pi<\theta\le\pi$..

(i) Show $1+z=2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)$. 2

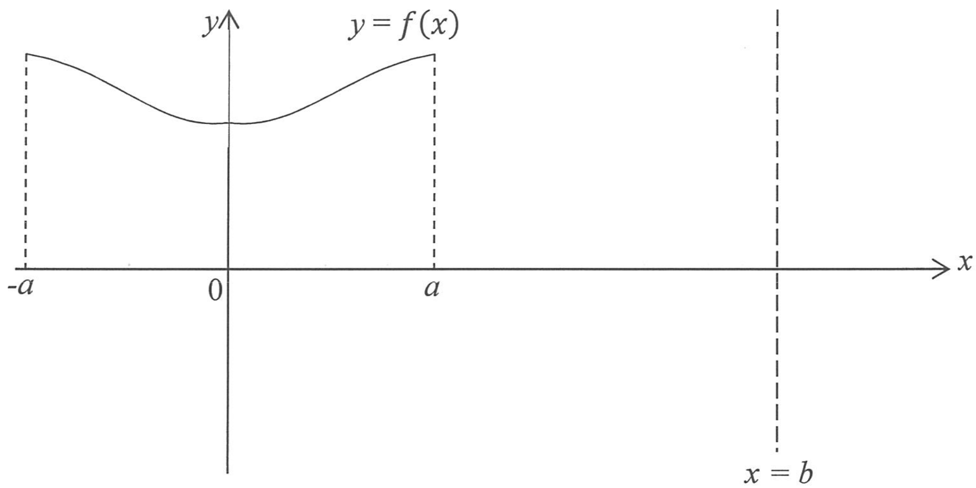
(ii) z_1 and z_2 are complex numbers such that $|z_1|=|z_2|=1$. If z_1 and z_2 have arguments α and β respectively, where $-\pi<\alpha\le\pi$ and $-\pi<\beta\le\pi$, show that $\frac{z_1+z_1z_2}{z_1+1}$ has modulus $\frac{\cos\frac{\beta}{2}}{\cos\frac{\alpha}{2}}$ and Argument $\frac{\alpha+\beta}{2}$. 2

(iii) If $|z_1|=|z_2|=1$ and $\frac{z_1+z_1z_2}{z_1+1}=2i$ find z_1 and z_2 in the form $x+iy$ where x and y are real rational numbers. 4

(b) Shade the region $-\frac{\pi}{4}\le \text{Arg}\, z\le\frac{\pi}{4}$ and $|z|\le 3$. 2

Question 11 (c) is continued over the page.

(c)

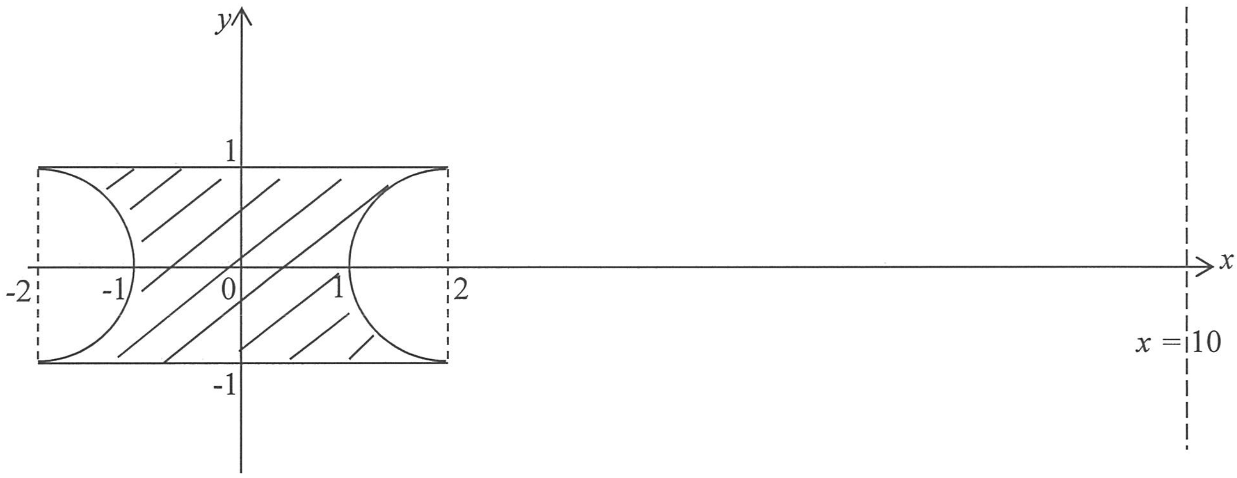


$f(x)$ is an even function such that $f(x) \geq 0$ for $-a \leq x \leq a$.

The region bounded by $y = f(x)$, the x-axis, and the ordinates $x = -a$ and $x = a$ has area A. The region is rotated about the line $x = b$ where $b > a > 0$.

(i) Using the method of cylindrical shells show that the volume V of rotation is $2\pi bA$. 3

(ii)



The region shown with circular ends is rotated about $x = 10$ to form a circular sealing ring. Find the volume of revolution. 2

End of Question 11.

Question 12 (15 marks) Start a NEW page.

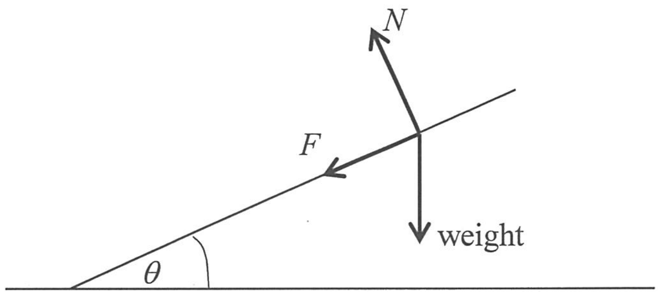
(a) Graph $y = \dfrac{x}{(x+4)(x+2)}$ showing all intercepts with the coordinate axes and all asymptotes. 3

(b) The region bounded by $y = \dfrac{x}{(x+4)(x+2)}$, the x-axis and $x = 1$ is rotated around the y-axis.

(i) Find the values A, B and C such that $\dfrac{x^2}{(x+4)(x+2)} = A + \dfrac{B}{x+2} + \dfrac{C}{x+4}$. 4

(ii) Using the method of cylindrical shells show that the volume V of revolution is given by $V = 2\pi\int_0^1 \dfrac{x^2\,dx}{(x+4)(x+2)}$, hence find the exact value of the volume of revolution. 4

(c)



A car of mass 2000 kg travels around a curve of radius 150 m at a speed of 110km/h. The car experiences a lateral resistance force F of 0.22 × normal force, N, as shown. 4

By resolving the forces vertically and horizontally find the ~~minimum~~ angle θ ~~(to the nearest minute)~~ for the car to negotiate the curve. (Assume acceleration due to gravity of 10 m/s^2).

End of Question 12.

Question 13 (15 marks) Start a NEW page.

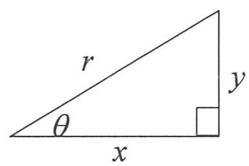
(a) (i) Show $\int_{-a}^{0} f(x)dx = \int_{0}^{a} f(-x)dx$ 1

(ii) Deduce $\int_{-a}^{a} f(x)dx = \int_{0}^{a} \left[f(x)+f(-x)\right] dx$ 1

(iii) Hence evaluate $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{(1+\sin x)^2}$ 4

(b) A shape is defined as $r = \frac{9}{5+4\cos\theta}$ where r is the distance from origin and θ is the angle anticlockwise from the positive x-axis.

(i) Using the notation 3



find the equivalent Cartesian equation and show that the shape is an ellipse translated.

(ii) State the minor axis, major axis and location of the foci. 4

(iii) The area A enclosed by the shape is given by $A = \int_{0}^{2\pi} \frac{1}{2} r^2 d\theta$. 2

Using (b)(i) and(b)(ii) evaluate $\int_{0}^{2\pi} \frac{d\theta}{(5+4\cos\theta)^2}$.

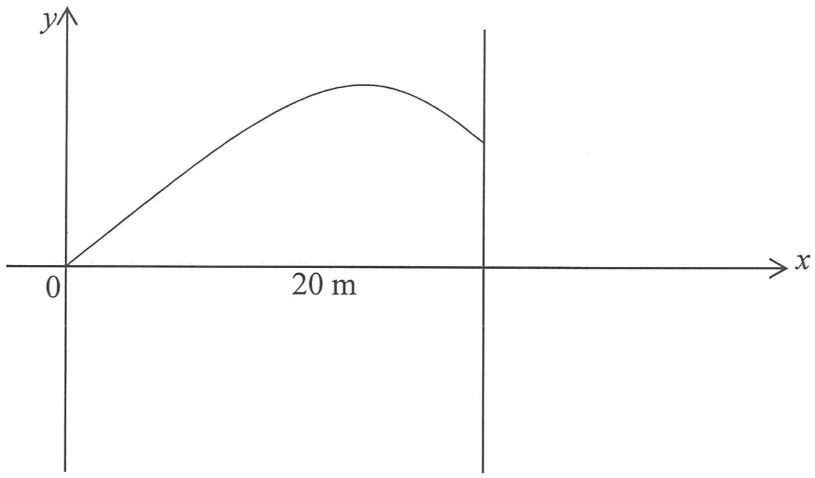
End of Question 13.

Question 14 (15 marks) Start a NEW page.

(a) (i) Find the coordinates of the intersection of the curves $y^2 = 8x$ and $x^2 = 8y$. 1

(ii) The base of a solid is in the region bounded by the curves $y^2 = 8x$ and $x^2 = 8y$, and its cross sections by planes perpendicular to the x-axis are semicircles. Find the volume of the solid. 3

(b)



A liquid particle of mass m kg is projected from the ground and hits a vertical wall 20m from the point of projection as shown.

(i) The equations of motion before the particle hits the wall are 3

$x = 4t$ and $y = 30t - 5t^2$

where t is time in seconds. Show that the particle hits the wall 25 m above the ground with a downwards velocity of 20 m/s.

(ii) After hitting the wall the particle slides down the wall with a resistance force equal to $0.04mv^2$.

(α) If acceleration due to gravity is 10 m/s^2 show that the velocity on return to the ground is approximately 16.44 m/s. 4

(β) Find the total time for the particle to return to the ground. Give your answer to two decimal places. 4

End of Question 14.

Question 15 (15 marks) Start a NEW page.

The hyperbola $xy = c^2$ meets the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $P\left(ct_1, \frac{c}{t_1}\right)$ and $Q\left(ct_2, \frac{c}{t_2}\right)$ where $t_1 > t_2 > 0$. Tangents to the hyperbola at P and Q meet at T, while tangents to the ellipse at P and Q meet at V.

(i) Show the above information on a sketch. 1

(ii) Show that the parameter of point $\left(ct, \frac{c}{t}\right)$ which lies on the intersection of $xy = c^2$ and $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ satisfies the equation $b^2c^2t^4 - a^2b^2t^2 + a^2c^2 = 0$. 2

(iii) Given the equation of the tangent to the hyperbola at $\left(ct, \frac{c}{t}\right)$ is $x + t^2y = 2ct$, show that the coordinates of T are $\left(\frac{2ct_1t_2}{t_1+t_2}, \frac{2c}{t_1+t_2}\right)$. 2

(iv) Given that the equation of the tangent to the ellipse at (x_1, y_1) is $b^2x_1x + a^2y_1y = a^2b^2$, show that the coordinates of V are $\left(\frac{a^2}{c(t_1+t_2)}, \frac{b^2t_1t_2}{c(t_1+t_2)}\right)$. 2

(v) Show that the line TV passes through the origin. 3

(vi) Point V lies at a focus of the hyperbola.

(α) Show that the ellipse is a circle. 2

(β) Find the radius of the circle in terms of c. 3

End of Question 15.

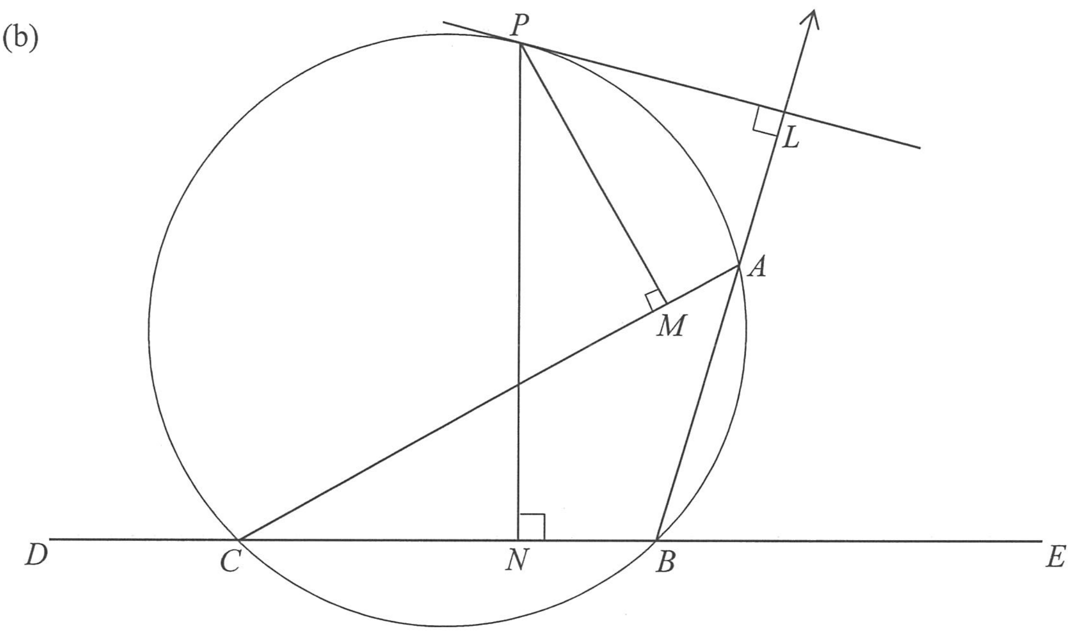
Question 16 (15 marks) Start a NEW page.

(a) $I_n = \int_0^{2\pi} (1+\cos\theta)^n \, d\theta$ for n $\geq$ 0.

(i) Show $I_{n+1} = \dfrac{2n+1}{n+1} I_n$. 4

(ii) Find I_3. 1

(b)



ABC is a triangle inscribed in a circle. *L*, *M* and *N* are the feet of the perpendiculars from *P* to *AB*, *AC* and *BC* respectively.

(i) Copy the diagram. 1

(ii) Show *P*, *M*, *A* and *L* are concyclic points. 2

(iii) Show *P*, *C*, *N* and *M* are concyclic points. 2

(iv) Show that *L*, *M* and *N* are collinear. 5

End of paper.