Terry Lee NFM Ch11 corrections

by Derek Buchanan

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In New Fundamental Mathematics (Lee, 2019) hereafter referred to as NFM, there is the problem of adding or subtracting values from tables which have been rounded to 4 decimal places - and the result is not correct to 4 decimal places. There are more accurate methods to add or subtract and then round off at the last step to ensure the result is correct to 4 decimal places. This follows similar articles (Buchanan, D. [1], [2] and [3], 2021) for NSM, Cambridge and MIF.

In this article I provide the following corrections to NFM Chapter 11:

Example 11.10(b), Exercise 11.3 Q6(b) and 11.5 Review Exercise 11 Q8(b), Q8(c) and Q8(d)

We will use the error function, wolframalpha.com and the CASIO fx 100 AU PLUS 2nd edition calculator to do the calculations.

Where $\operatorname{erf} z := \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2}\, dt$ then $P := P(0 < z < z_1) = 0.5 \operatorname{erf}(\frac{z_1}{\sqrt{2}})$.

For the CASIO fx-100AU PLUS 2nd edition calculator in Statistics Mode, Press MODE 3 1 AC to put it into Statistics Mode and exit the Editor Screen.

Then press SHIFT 1 5 2 z_1) = to get $Q(z_1) := P(0 < z < z_1)$ for some positive z-score z_1.

Example 11.10(b)

The problem with adding or subtracting values from the z-score table is that they have already been rounded to 4 decimal places, but the result is not correct to 4 decimal places. So although from the table, $0.3413 + 0.4772 = 0.8185$, nevertheless 0.8185 is not the correct answer.

It is more accurate to only round off at the last step as follows:

$$\begin{aligned}
P(186 < X < 207) &= P\left(\tfrac{186-193}{7} < z < \tfrac{207-193}{7}\right) \\
&= P(-1 < z < 2) \\
&= 0.5\,\text{erf}\left(\tfrac{1}{\sqrt{2}}\right) + 0.5\,\text{erf}(\sqrt{2}) \\
&= 0.81859\ldots \text{ using wolframalpha.com} \\
&\approx \color{red}{0.8186} \text{ not } 0.8185
\end{aligned}$$

Alternatively using the CASIO fx 100 AU PLUS 2nd edition calculator, $Q(1) + Q(2) = 0.81859 \approx \color{red}{0.8186}$ not 0.8185.

Exercise 11.3 Q6(b)

$$\begin{aligned}
100P(250 < x < 298) &= 100P\left(\tfrac{250-266}{16} < z < \tfrac{298-266}{16}\right) \\
&= 100P(-1 < z < 2) \\
&= 50\,\text{erf}\left(\tfrac{1}{\sqrt{2}}\right) + 50\,\text{erf}\left(\sqrt{2}\right) \\
&= 81.859\ldots \\
&\approx \color{red}{81.9\%} \text{ not } 81.5\%
\end{aligned}$$

Alternatively using the calculator, $100Q(1) + 100Q(2) = 81.859 \approx \color{red}{81.9\%}$ not 81.5%.

11.5 Review Exercise 11 Q8(b)

$$\begin{aligned}
P(165 < x < 185) &= P\left(\tfrac{165-178}{7} < z < \tfrac{185-178}{7}\right) \\
&= P(-\tfrac{13}{7} < z < 1) \\
&= 0.5\,\text{erf}\left(\tfrac{13}{7\sqrt{2}}\right) + 0.5\,\text{erf}\left(\tfrac{1}{\sqrt{2}}\right) \\
&= 0.80969\ldots \\
&\approx \color{red}{0.8097} \text{ not } 0.8099
\end{aligned}$$

11.5 Review Exercise 11 Q8(c)

$$\begin{aligned}
P(170 < x < 180) &= P\left(\tfrac{170-178}{7} < z < \tfrac{180-178}{7}\right) \\
&= P(-\tfrac{8}{7} < z < \tfrac{2}{7}) \\
&= 0.5\,\text{erf}\left(\tfrac{8}{7\sqrt{2}}\right) + 0.5\,\text{erf}\left(\tfrac{2}{7\sqrt{2}}\right) \\
&= 0.4859\ldots \\
&\approx \color{red}{0.486} \text{ not } 0.487
\end{aligned}$$

11.5 Review Exercise 11 Q8(d)

$$\begin{aligned} P(180 < x < 190) &= P\left(\tfrac{180-178}{7} < z < \tfrac{190-178}{7}\right) \\ &= P(\tfrac{2}{7} < z < \tfrac{12}{7}) \\ &= 0.5\,\text{erf}\left(\tfrac{12}{7\sqrt{2}}\right) - 0.5\,\text{erf}\left(\tfrac{2}{7\sqrt{2}}\right) \\ &= 0.34431\ldots \\ &\approx 0.3443 \text{ not } 0.3423 \end{aligned}$$

References

Buchanan, D. [1], Corrections to NSM Advanced Exercises 20.4, 20.5 and Review 20 (Additional), 2021
at http://www.angelfire.com/ab7/fourunit/NSM-corrections-20.4-20.5.pdf

Buchanan, D. [2], Corrections to Cambridge Advanced Year 12 Section 10D, 2021
at http://www.angelfire.com/ab7/fourunit/Cambridge-Adv-Y12-10D-cor.pdf

Buchanan, D. [3], Corrections to MIF Advanced Year 12 Chapter 10, 2021
at http://www.angelfire.com/ab7/fourunit/MIF-Advanced-Y12-Ch10-cor.pdf

Lee, T., New Fundamental Mathematics, 2019