

# Non-investigative additions to Cambridge Year 11 Extension 1 Solutions

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Since 2019 the solutions [1] to the Year 11 Cambridge Mathematics Extension 1 textbook [2] have gradually been released until 2022 when it seemingly stopped. This left many questions with incorrect solutions and several others without solutions. The additions for the missing non-investigative ones are provided here. Corrections were addressed in [3]. The investigative ones will be addressed in a separate document.

The non-investigative solutions are to

1D Q8e, 8f, 9b, 9f, 10b, 10e

1G Q6g, 6h, 7

2C Q6j, 6k, 6l

Chapter 2 review exercise Q16, 17, 18, 19

3D Q4

3E Q16c

4C Q8e, 8f

7D Q14e, 14g

9D Q10, 11

9E Q4

9F Q5, 11

9H Q14

9I Q16ai-vi, 16bi-bii

9K Q9b

9L Q2a, 2b, 2d

Chapter 9 review exercise Q2a, 2b, 2f, 2g, 2h, 7a, 7b

11E Q2, 3

11H Q3, 4, 6c, 7a, 7b, 7e, 7f, 7g, 7h

Chapter 11 Review exercise Q9i, 15, 23, 31

13D Q1a, 1b

14E Q18c

15A Q17

$$\begin{aligned} \mathbf{1D Q8e} \frac{ax+bx-2a-2b}{3x^2-5x-2} \times \frac{9x^2-1}{a^2+2ab+b^2} &= \frac{(a+b)x-2(a+b)}{(3x+1)(x-2)} \times \frac{(3x-1)(3x+1)}{(a+b)^2} \\ &= \frac{(a+b)(x-2)(3x-1)(3x+1)}{(3x+1)(x-2)(a+b)^2} \\ &= \frac{3x-1}{a+b} \end{aligned}$$

$$\begin{aligned} \mathbf{1D Q8f} \frac{2x^2+x-15}{x^2+3x-28} \div \frac{x^2+6x+9}{x^2-4x} \div \frac{6x^2-15x}{x^2-49} &= \frac{(2x-5)(x+3)}{(x+7)(x-4)} \div \frac{(x+3)^2}{x(x-4)} \div \frac{3x(2x-5)}{(x-7)(x+7)} \\ &= \frac{(2x-5)(x+3)x(x-4)(x-7)(x+7)}{(x+7)(x-4)(x+3)^2 3x(2x-5)} \\ &= \frac{x-7}{3(x+3)} \end{aligned}$$

**1D Q9** There are 6 questions. However the Cambridge solution mixed the order and skipped some.

9a. The Cambridge solutions did 9a

9b. The Cambridge solutions did not do 9b

9c. The Cambridge solutions did 9c but they incorrectly labelled it 9d

9d. The Cambridge solutions did 9d but they incorrectly labelled it 9b.

9e. The Cambridge solutions did 9e but they incorrectly labelled it 9c.

9f. The Cambridge solutions did not do 9f

Hence that leaves 9b and 9f yet to be done

$$\begin{aligned} \mathbf{1D Q9b} \frac{v^2-u^2}{u-v} &= \frac{-(u-v)(u+v)}{u-v} \\ &= -u - v \end{aligned}$$

$$\begin{aligned} \mathbf{1D Q9f} \frac{x-y}{y^2+xy-2x^2} &= \frac{x-y}{(y-x)(2x+y)} \\ &= \frac{-(x-y)}{(x-y)(2x+y)} \\ &= \frac{-1}{2x+y} \end{aligned}$$

**1D Q10** Again there are 6 questions and the Cambridge solution mixed the order and skipped some.

10a. The Cambridge solutions did 10a.

10b. The Cambridge solutions did not do 10b.

10c. The Cambridge solutions did 10c but incorrectly labelled it 10b.

10d. The Cambridge solutions did 10d but incorrectly labelled it 10c.

10e. The Cambridge solutions did not do 10e.

10f. The Cambridge solutions did Q10f but incorrectly labelled it 10d.

So that leaves 10b and 10e yet to be done.

$$\begin{aligned} \mathbf{1D Q10b} \frac{1}{x^2-4} + \frac{1}{x^2-4x+4} &= \frac{1}{(x-2)(x+2)} + \frac{1}{(x-2)^2} \\ &= \frac{x-2+x+2}{(x-2)^2(x+2)} \\ &= \frac{2x}{(x-2)^2(x+2)} \end{aligned}$$

$$\begin{aligned} \mathbf{1D Q10e} \frac{x}{a^2-b^2} - \frac{x}{a^2+ab} &= \frac{x}{(a-b)(a+b)} - \frac{x}{a(a+b)} \\ &= \frac{ax-(a-b)x}{a(a-b)(a+b)} \\ &= \frac{bx}{a(a-b)(a+b)} \end{aligned}$$

**1G Q6g** If  $x$  = number of \$20 notes and  $y$  = number of \$10 notes, we have

$$x + y = 23 \quad \dots (1)$$

$$20x + 10y = 320 \quad \dots (2)$$

$$\therefore (1) \times 10 : 10x + 10y = 230 \quad \dots (3)$$

$$\text{Now } (2) - (3) : 10x = 90$$

$$\text{So } x = 9 \text{ and } y = 23 - 9 = 14.$$

**1G Q6h** Let their displacements be  $x$  km and  $y$  km and  $x_1$  km and  $y_1$  be their initial displacements and w.l.o.g.,  $y_1 > x_1$ . Then  $y_1 - x_1 = 16$ .

### Walking in the opposite direction

If  $t$  is in hours then  $\frac{dx}{dt} = m \Rightarrow x = mt + x_1$  and  $-\frac{dy}{dt} = n \Rightarrow y = -nt + y_1$ . If they meet at time  $t = 2$  then  $2m + x_1 = -2n + y_1$  and so  $2(m + n) = y_1 - x_1 = 16$ . Hence

the first equation is

$$m + n = 8 \dots (1)$$

### Walking in the same direction

$\frac{dy}{dt} = n \Rightarrow y = nt + y_1$  and so if they meet when  $t = 8$  then  $8m + x_1 = 8n + y_1$  and so  $8(m - n) = y_1 - x_1 = 16$ . Hence the second equation is

$$m - n = 2 \dots (2)$$

Now (1) + (2) :  $2m = 10$  and so  $m = 5$  km/h and  $n = 8 - 5 = 3$  km/h.

$$\mathbf{1G Q7a} \frac{y}{4} - \frac{x}{3} = 1 \dots (1)$$

$$\frac{x}{2} + \frac{y}{5} = 10 \dots (2)$$

$$-24 \times (1) : 8x - 6y = -24 \dots (3)$$

$$30 \times (2) : 15x + 6y = 300 \dots (4)$$

$$(3) + (4) : 23x = 276$$

$$\therefore x = \frac{276}{23} = 12$$

$$\text{and } y = 4\left(1 + \frac{12}{3}\right) = 20$$

$$\mathbf{1G Q7b} 4x + \frac{y-2}{3} = 12 \dots (1)$$

$$3y - \frac{x-3}{5} = 6 \dots (2)$$

$$(1) \times 3 : 12x + y - 2 = 36$$

$$\therefore y = 38 - 12x \dots (3)$$

$$5 \times (2) : 15y - x + 3 = 30$$

$$\therefore x - 15y = -27 \dots (4)$$

$$\therefore x - 15(38 - 12x) = -27$$

$$\therefore 181x = 543$$

$$\therefore x = 3 \text{ and } y = 38 - 12 \times 3 = 2$$

$$\mathbf{2C Q6j} \sqrt[3]{125} = 5$$

$$\mathbf{2C Q6k} \sqrt[4]{81} = 3$$

$$\mathbf{2C Q6l} \sqrt[5]{32} = 2$$

### Chapter 2 review exercise Q16

$$\begin{aligned} \mathbf{16a} \quad x + \frac{1}{x} &= 3 + \sqrt{10} + \frac{1}{3+\sqrt{10}} \cdot \frac{\sqrt{10}-3}{\sqrt{10}-3} \\ &= 3 + \sqrt{10} + \frac{\sqrt{10}-3}{10-9} \\ &= 2\sqrt{10} \end{aligned}$$

$$\begin{aligned} \mathbf{16b} \quad x^2 + \frac{1}{x^2} &= (x + \frac{1}{x})^2 - 2 \\ &= (2\sqrt{10})^2 - 2 \\ &= 38 \end{aligned}$$

$$\begin{aligned} \mathbf{17} \quad \frac{1}{\sqrt{6}-\sqrt{5}} &= \frac{\sqrt{6}+\sqrt{5}}{(\sqrt{6}-\sqrt{5})(\sqrt{6}+\sqrt{5})} \\ &= \frac{\sqrt{6}+\sqrt{5}}{6-5} \\ &= \sqrt{6} + \sqrt{5} \end{aligned}$$

$$\text{and } \frac{1}{\sqrt{5}-2} = \frac{\sqrt{5}+2}{(\sqrt{5}-2)(\sqrt{5}+2)} = \frac{\sqrt{5}+2}{5-4} = \sqrt{5} + 2$$

But  $\sqrt{6} + \sqrt{5} > \sqrt{4} + \sqrt{5} = \sqrt{5} + 2 \therefore \frac{1}{\sqrt{6}-\sqrt{5}} > \frac{1}{\sqrt{5}-2} \therefore \sqrt{6} - \sqrt{5} < \sqrt{5} - 2$ .

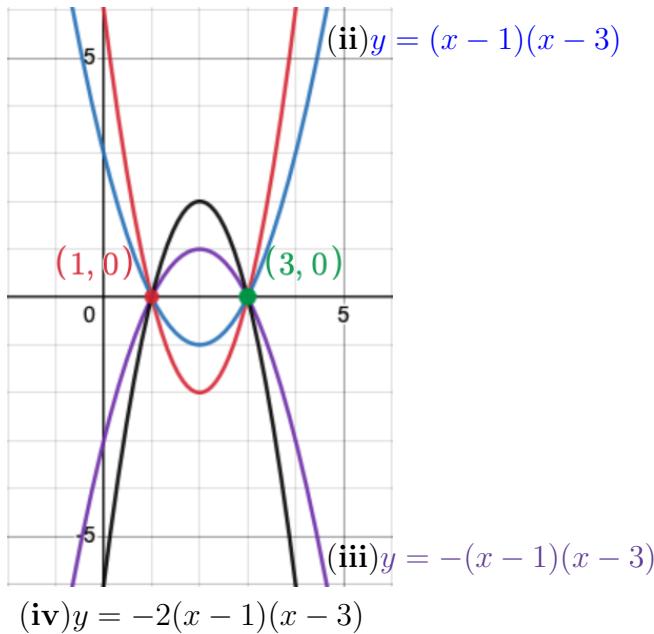
$$\begin{aligned} \mathbf{18} \quad &\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots + \frac{1}{\sqrt{15}+\sqrt{16}} \\ &= \frac{1}{\sqrt{1}+\sqrt{2}} \cdot \frac{\sqrt{2}-\sqrt{1}}{\sqrt{2}-\sqrt{1}} + \frac{1}{\sqrt{2}+\sqrt{3}} \cdot \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{1}{\sqrt{3}+\sqrt{4}} \cdot \frac{\sqrt{4}-\sqrt{3}}{\sqrt{4}-\sqrt{3}} + \cdots + \frac{1}{\sqrt{15}+\sqrt{16}} \cdot \frac{\sqrt{16}-\sqrt{15}}{\sqrt{16}-\sqrt{15}} \\ &= \frac{\sqrt{2}-1}{2-1} + \frac{\sqrt{3}-\sqrt{2}}{3-2} + \frac{\sqrt{4}-\sqrt{3}}{4-3} + \cdots + \frac{\sqrt{16}-\sqrt{15}}{16-15} \\ &= \sqrt{16} - 1 \\ &= 3 \end{aligned}$$

$$\mathbf{19a} \quad (1+\sqrt{3})^2 = 1 + 2\sqrt{3} + 3 = 4 + 2\sqrt{3} \therefore \sqrt{4 + 2\sqrt{3}} = 1 + \sqrt{3}$$

$$\begin{aligned} \mathbf{19b} \quad \frac{1+\sqrt{1+x}}{\sqrt{1+x}} &= \frac{1+\sqrt{1+\frac{\sqrt{3}}{2}}}{\sqrt{1+\frac{\sqrt{3}}{2}}} \cdot \frac{\sqrt{4}}{\sqrt{4}} \\ &= \frac{\sqrt{4}+\sqrt{4+2\sqrt{3}}}{\sqrt{4+2\sqrt{3}}} \\ &= \frac{2+1+\sqrt{3}}{1+\sqrt{3}} \cdot \frac{\sqrt{3}-1}{\sqrt{3}-1} \text{ from 19a} \\ &= \frac{3\sqrt{3}+3-3-\sqrt{3}}{3-1} \\ &= \sqrt{3} \end{aligned}$$

### 3D Q4a

$$(\text{i}) \quad y = 2(x-1)(x-3)$$



### 3D Q4b

$(1, 0)$  and  $(3, 0)$ .

**3E Q16c** In the Cambridge solutions, 16b was answered as part of 16a. Part of 16c was answered in 16b except the geometric description for the family of quadratics with this property which is that the vertex is on the line  $y = -1$ .

**4C Q8e and f cf.** Answers would suffice.

**7D Q14e Isosceles**

**7D Q14g** Let  $E$  be  $(x, y)$ . Then  $D$  is the midpoint of  $BE$  and so  $\frac{x+0}{2} = 4$  and  $\frac{y+8}{2} = 2$ . Hence  $E = (8, -4)$ .

**9D Q10a**  $y' = 2x \therefore y'(p) = 2p$ . So the tangent at  $(p, p^2 + 9)$  is  $y - (p^2 + 9) = 2p(x - p)$ .

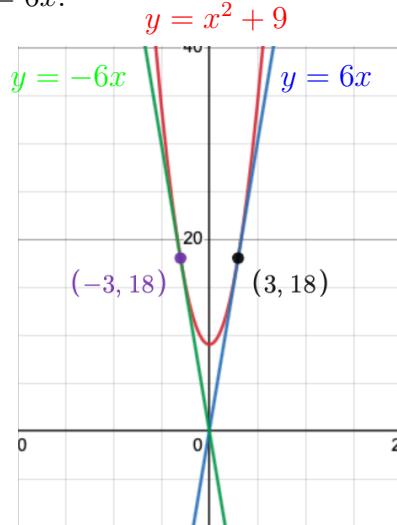
$$\therefore y = 2px + 9 - p^2$$

**9D Q10b** Tangents pass through the origin  $\Rightarrow$  if  $(x, y) = (0, 0)$  then  $0 = 2p(0) + 9 - p^2$  and so  $p = \pm 3$  and so  $P$  is  $(-3, 18)$  or  $(3, 18)$ .

Tangents:

At  $(-3, 18)$  the tangent is  $y = -6x$

At  $(3, 18)$  the tangent is  $y = 6x$ .



**9D Q11a**  $y = x^2 - 10x + 9 \Rightarrow y' = 2x - 10$  and so  $y'(t) = 2t - 10$  and the tangent at  $(t, t^2 - 10t + 9)$  is  $y - (t^2 - 10t + 9) = (2t - 10)(x - t) \therefore y = (2t - 10)x - t^2 + 9$

Passing through  $(0, 0)$  gives  $0 = (2t - 10)(0) - t^2 + 9$  and so  $t = \pm 3$  and so the tangents are  $y = -4x$ ,  $y = -16x$ .

**9D Q11b**  $y = x^2 + 15x + 36 \Rightarrow y' = 2x + 15$  and so  $y'(t) = 2t + 15$  and the tangent at  $(t, t^2 + 15t + 36)$  is  $y - (t^2 + 15t + 36) = (2t + 15)(x - t) \therefore y = (2t + 15)x - t^2 + 36$

Passing through  $(0, 0)$  gives  $0 = (2t + 15)(0) - t^2 + 36$  and so  $t = \pm 6$  and so the tangents are  $y = 27x$ ,  $y = 3x$ .

**9E Q4** The Cambridge solution which they call 4a, 4b, 4c, 4d, 4e, 4f are actually 3a, 3b, 3c, 3d, 3e, 3f and there is no solution given for Q4.

$$y' = 4(5x - 2)^3 \times 5 = 20(5x - 2)^3$$

$$y'' = 3 \times 20(5x - 2)^2 \times 5 = 300(5x - 2)^2$$

$$y''' = 2 \times 300(5x - 2) \times 5 = 3000(5x - 2)$$

$$y'''' = 3000 \times 5 = 15000$$

$$y''''' = 0$$

$$y'''''' = 0$$

**9F Q5** The Cambridge solutions say cf. Answers but more working is required.

**5a**  $y = x^{-1} \Rightarrow y' = -x^{-2} = -1 \Rightarrow x = \pm 1$  and the points are  $(-1, -1)$  and  $(1, 1)$ .

**5b**  $y = \frac{1}{2}x^{-2} \Rightarrow y' = -x^{-3} = -1 \Rightarrow x = 1$  and the point is  $(1, \frac{1}{2})$ .

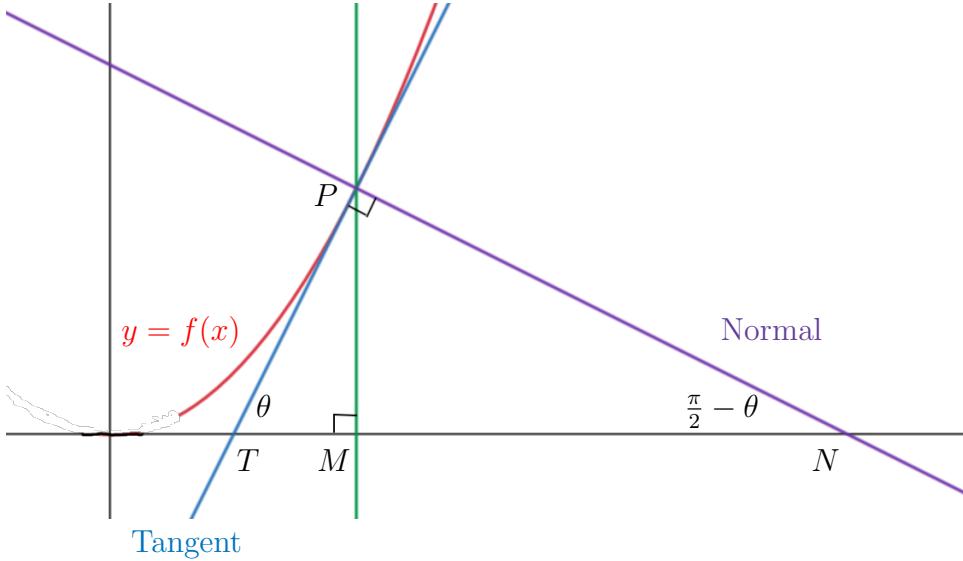
**9F Q11** The Cambridge solutions say cf. Answers but more working is required.

$y = (x + a)^{-1} \Rightarrow y' = -(x + a)^{-2} \Rightarrow y'(6) = -(6 + a)^{-2} = -1 \Rightarrow (6 + a)^2 = 1$  and so  $6 + a = \pm 1$  and  $a = -6 \pm 1 = -5$  or  $-7$ .

$$\begin{aligned}\mathbf{9H Q14a} \quad y' &= 5x^4(x - 1)^4(x - 2)^3 + x^5 \times 4(x - 1)^3 \times 1(x - 2)^3 + x^5(x - 1)^4 \times 3(x - 2)^2 \times 1 \\ &= x^4(x - 1)^3(x - 2)^2(5(x - 1)(x - 2) + 4x(x - 2) + 3x(x - 1)) \\ &= x^4(x - 1)^3(x - 2)^2(12x^2 - 26x + 10) \\ &= 2x^4(x - 1)^3(x - 2)^2(6x^2 - 13x + 5) \\ &= 2x^4(x - 1)^3(x - 2)^2(2x - 1)(3x - 5) \text{ with zeroes } 0, 1, 2, \frac{1}{2}, \frac{5}{3}\end{aligned}$$

$$\begin{aligned}\mathbf{9H Q14b} \quad y' &= 1 \times (x - 2)^4(2x + 1)^{\frac{1}{2}} + x \times 4(x - 2)^3 \times 1(2x + 1)^{\frac{1}{2}} + x(x - 2)^4 \times \frac{1}{2}(2x + 1)^{-\frac{1}{2}} \times 2 \\ &= (x - 2)^3(2x + 1)^{-\frac{1}{2}}((x - 2)(2x + 1) + 4x(2x + 1) + x(x - 2)) \\ &= (x - 2)^3(2x + 1)^{-\frac{1}{2}}(11x^2 - x - 2) \\ \text{with zeroes } 2 \text{ and } \frac{1 \pm \sqrt{1^2 - 4 \times 11 \times (-2)}}{2 \times 11} &= 2 \text{ and } \frac{1 \pm \sqrt{89}}{22}.\end{aligned}$$

**9I Q16**



$$\mathbf{Q16a \ i} \tan\left(\frac{\pi}{2} - \theta\right) = \frac{PM}{MN} = \cot\theta = \frac{1}{\tan\theta} \therefore \frac{y}{MN} = \frac{1}{y'} \therefore MN = yy'$$

$$\mathbf{Q16a \ ii} \tan\theta = \frac{PM}{TM} \therefore \frac{y}{TM} = y' \therefore TM = \frac{y}{y'}$$

$$\mathbf{Q16a \ iii} \sec^2\theta = 1 + \tan^2\theta = 1 + (y')^2 \therefore \sec\theta = \sqrt{1 + (y')^2}$$

$$\mathbf{Q16a \ iv} \cosec\theta = \frac{1}{\sin\theta} = \frac{\cos\theta}{\cos\theta\sin\theta} = \sec\theta \cot\theta = \frac{\sec\theta}{\tan\theta} = \frac{\sqrt{1+(y')^2}}{y'}$$

$$\mathbf{16a \ v} \sin\left(\frac{\pi}{2} - \theta\right) = \frac{PM}{PN} = \cos\theta = \frac{1}{\sec\theta} \therefore \frac{y}{PN} = \frac{1}{\sqrt{1+(y')^2}} \therefore PN = y\sqrt{1 + (y')^2}$$

$$\mathbf{16a \ vi} \sin\theta = \frac{PM}{PT} = \frac{1}{\cosec\theta} \therefore \frac{y}{PT} = \frac{y'}{\sqrt{1+(y')^2}} \therefore PT = \frac{y\sqrt{1+(y')^2}}{y'}$$

**16b i**  $y = x^2 \Rightarrow y' = 2x$  and so if  $x = 3$ ,  $y = 3^2 = 9$  and  $y' = 2 \times 3 = 6$ . Hence

$$MN = yy' = 9 \times 6 = 54$$

$$TM = \frac{y}{y'} = \frac{9}{6} = \frac{3}{2}$$

$$PN = y\sqrt{1 + (y')^2} = 9\sqrt{1 + 6^2} = 9\sqrt{37}$$

$$PT = \frac{y\sqrt{1+(y')^2}}{y'} = \frac{9\sqrt{37}}{6} = \frac{3\sqrt{37}}{2}$$

**16b ii**  $y = \frac{3x-1}{x+1} \Rightarrow y' = \frac{(x+1)(3)-(3x-1)(1)}{(x+1)^2} = \frac{4}{(x+1)^2}$  and so if  $x = 3$ ,  $y = \frac{3 \times 3 - 1}{3+1} = 2$  and  $y' = \frac{4}{(3+1)^2} = \frac{1}{4}$ . Hence

$$MN = yy' = 2 \times \frac{1}{4} = \frac{1}{2}$$

$$TM = \frac{y}{y'} = \frac{2}{1/4} = 8$$

$$PN = y\sqrt{1 + (y')^2} = 2\sqrt{1 + (\frac{1}{4})^2} = 2\sqrt{\frac{17}{16}} = \frac{1}{2}\sqrt{17}$$

$$PT = \frac{y\sqrt{1+(y')^2}}{y'} = \frac{\sqrt{17}/2}{1/4} = 2\sqrt{17}$$

**9K Q9b** The Cambridge solution to 9c is mislabelled 9b and they did not do 9b.

For 9b zeroes are when  $\cos x^\circ + \sin x^\circ = 0$  and  $\tan x^\circ = -1$  so  $x \in \{135 + 180n : n \in \mathbb{Z}\}$

Discontinuities are for  $\cos x^\circ - \sin x^\circ = 0$  and  $\tan x^\circ = 1$  so  $x \in \{45 + 180n : n \in \mathbb{Z}\}$

**9L Q2a, 2b, 2d** cf. Answers would suffice.

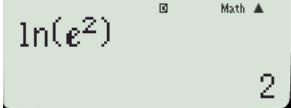
**Chapter 9 review exercise Q2a, 2b, 2f, 2g, 2h** cf. Answers

**Chapter 9 review exercise Q7a**  $3 \times 3(3x + 7)^2 = 9(3x + 7)^2$

**Chapter 9 review exercise Q7b**  $2 \times -2(5 - 2x) = -4(5 - 2x)$

**11E Q2** The Cambridge solutions say "cf. answers" - except there ARE no answers!

11E Q2a  $\ln(e^2)$



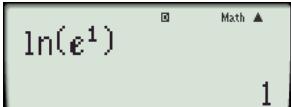
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11E Q2b  $\ln(e^3)$



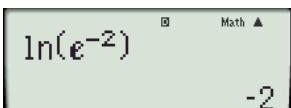
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11E Q2c  $\ln(e^1)$



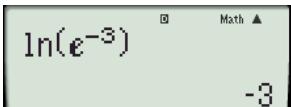
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11E Q2d  $\ln(e^{-2})$



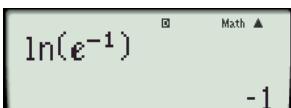
-2

11E Q2e  $\ln(e^{-3})$



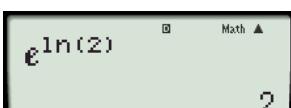
-3

11E Q2f  $\ln(e^{-1})$



-1

11E Q3a  $e^{\ln(2)}$



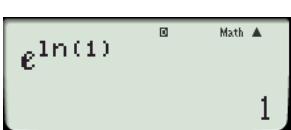
2

11E Q3b  $e^{\ln(3)}$



3

11E Q3c  $e^{\ln(1)}$



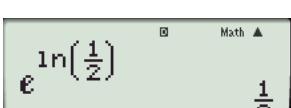
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11E Q3d  $e^{\ln(10)}$



10

11E Q3e  $e^{\ln(\frac{1}{2})}$



$\frac{1}{2}$

11E Q3f

**11H Q3** The Cambridge solutions say cf. Answers, but more working is required.

**3a**  $x = \sin^{-1} \frac{1}{2} = \frac{\pi}{6}$  or  $\pi - \sin^{-1} \frac{1}{2} = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ .

**3b**  $x = \pi \pm \cos^{-1} \frac{1}{2} = \pi \pm \frac{\pi}{3} = \frac{2\pi}{3}$  or  $\frac{4\pi}{3}$

**3c**  $x = \pi - \tan^{-1} 1 = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$  or  $2\pi - \tan^{-1} 1 = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$ .

**3d**  $x = \sin^1 = \frac{\pi}{2}$ .

**3e**  $\cos x = \frac{\sqrt{3}}{2} \therefore x = \cos^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{6}$  or  $2\pi - \cos^{-1} \frac{\sqrt{3}}{2} = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$ .

**3f**  $\tan x = \frac{1}{\sqrt{3}} \therefore x = \tan^{-1} \frac{1}{\sqrt{3}} = \frac{\pi}{6}$  or  $\pi + \tan^{-1} \frac{1}{\sqrt{3}} = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$ .

**3g**  $\cos x = -1 \therefore x = \cos^{-1}(-1) = \pi$

**3h**  $\sin x = -\frac{1}{\sqrt{2}} \therefore x = \pi + \sin^{-1} \frac{1}{\sqrt{2}} = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$  or  $2\pi - \sin^{-1} \frac{1}{\sqrt{2}} = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$ .

**11H Q4** The Cambridge solutions say cf. Answers, but more working is required.

**4a**  $\sin \theta = \pm 1 \therefore \theta = \sin^{-1} 1 = \frac{\pi}{2}$  or  $2\pi - \sin^{-1} 1 = 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$ .

**4b**  $\tan \theta = \pm 1 \therefore \theta = \tan^{-1} 1 = \frac{\pi}{4}$ ,  $\pi \pm \tan^{-1} 1 = \pi \pm \frac{\pi}{4} = \frac{3\pi}{4}, \frac{5\pi}{4}$   
or  $2\pi - \tan^{-1} 1 = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$ .

**4c**  $\cos \theta = \pm \frac{1}{2} \therefore \theta = \cos^{-1} \frac{1}{2} = \frac{\pi}{3}$ ,  $\pi \pm \cos^{-1} \frac{1}{2} = \pi \pm \frac{\pi}{3} = \frac{2\pi}{3}, \frac{4\pi}{3}$   
or  $2\pi - \cos^{-1} \frac{1}{2} = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$ .

**4d**  $\cos \theta = \pm \frac{\sqrt{3}}{2} \therefore \theta = \cos^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{6}$ ,  $\pi \pm \cos^{-1} \frac{\sqrt{3}}{2} = \pi \pm \frac{\pi}{6} = \frac{5\pi}{6}, \frac{7\pi}{6}$   
or  $2\pi - \cos^{-1} \frac{\sqrt{3}}{2} = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$ .

**11H Q6c** The Cambridge solutions say cf. Answers, but more working is required.  
Note also there is an error in the textbook where it says to solve for  $\theta$  where  $0 \leq x \leq 2\pi$  which should be  $0 \leq \theta \leq 2\pi$ .

$$\tan \theta = -1 \Rightarrow \theta = \pi - \tan^{-1} 1 = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \text{ or } 2\pi - \tan^{-1} 1 = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}.$$

$$\tan \theta = 2 \Rightarrow \theta = \tan^{-1} 2 \approx 1.11 \text{ or } \pi + \tan^{-1} 2 \approx 4.25$$

**11H Q7a Q7b, 7e, 7f, 7g, 7h** cf. answers is not a solution

**7a** The Cambridge solutions provided a partial solution leading to  $\tan \theta = 0$  or  $-1$ .

From  $\tan \theta = 0, \theta = 0, \pi, 2\pi$

From  $\tan \theta = -1, \theta = \pi - \tan^{-1}(-1) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$  or  $2\pi - \tan^{-1}(-1) = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$ .

**7b** The Cambridge solutions provided a partial solution leading to  $\sin \theta = 0$  or  $\frac{1}{2}$ .

From  $\sin \theta = 0, \theta = 0, \pi, 2\pi$ .

From  $\sin \theta = \frac{1}{2}$ ,  $\theta = \sin^{-1} \frac{1}{2} = \frac{\pi}{6}$  or  $\pi - \sin^{-1} \frac{1}{2} = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ .

**7e** The Cambridge solutions provided a partial solution leading to  $\cos \theta = -1$  or  $\frac{1}{2}$ .

From  $\cos \theta = -1$ ,  $\theta = \pi$

From  $\cos \theta = \frac{1}{2}$ ,  $\theta = \cos^{-1} \frac{1}{2} = \frac{\pi}{3}$  or  $2\pi - \cos^{-1} \frac{1}{2} = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$ .

**7f** The Cambridge solutions provided a partial solution leading to  $\sin \theta = 1$  or  $-\frac{1}{2}$ .

From  $\sin \theta = 1$ ,  $\theta = \sin^{-1} 1 = \frac{\pi}{2}$ .

From  $\sin \theta = -\frac{1}{2}$ ,  $\theta = \pi + \sin^{-1} \frac{1}{2} = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$  or  $2\pi - \sin^{-1} \frac{1}{2} = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$ .

**7g** The Cambridge solutions provided a partial solution leading to  $\sin \theta = \frac{1}{3}$ .

From this,  $\theta = \sin^{-1} \frac{1}{3} \approx 0.34$  or  $\pi - \sin^{-1} \frac{1}{3} \approx 2.80$ .

**7h** The Cambridge solutions provided a partial solution leading to  $\cos \theta = -\frac{1}{3}$ .

From this,  $\theta = \pi \pm \cos^{-1} \frac{1}{3} \approx 1.91$  or  $4.37$ .

**Chapter 11 Review exercise Q9i** The Cambridge solutions say "cf. answers" which might have been acceptable had the answer been correct. The answer is incorrect.

$\frac{d}{dx}(4e^{\frac{1}{2}x}) = \frac{1}{2} \times 4e^{\frac{1}{2}x} = 2e^{\frac{1}{2}x}$  not  $2e^{\frac{1}{2}x}$  as in the answers in the textbook.

**Chapter 11 Review exercise Q15** cf. Answers will suffice.

**Chapter 11 Review exercise Q23** The Cambridge solutions say "cf. answers" - but more working is required.

**23a**  $x = \cos^{-1} \frac{1}{\sqrt{2}} = \frac{\pi}{4}$  or  $2\pi - \cos^{-1} \frac{1}{\sqrt{2}} = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$ .

**23b**  $x = \pi - \tan^{-1} \sqrt{3} = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$  or  $2\pi - \tan^{-1} \sqrt{3} = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$ .

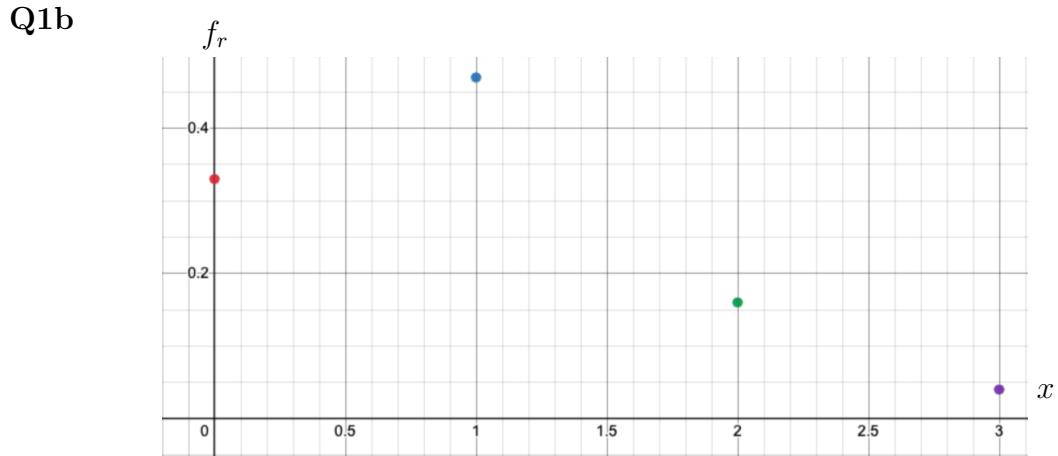
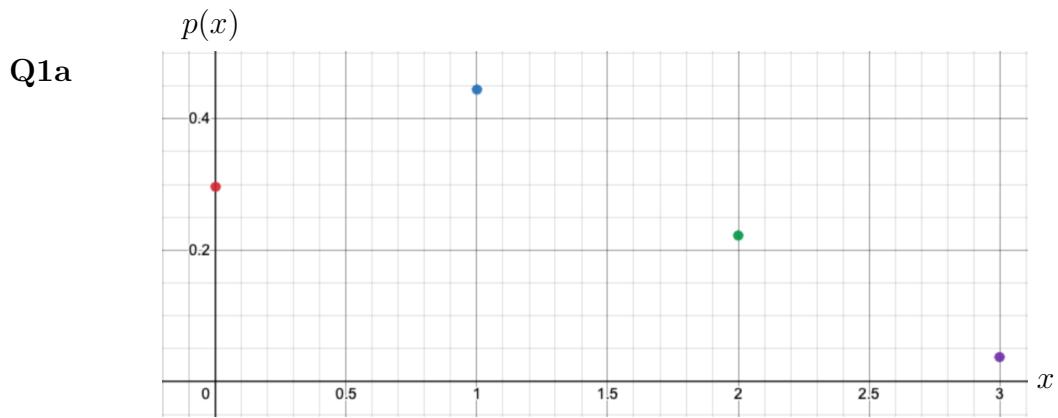
**Chapter 11 Review exercise Q31**

**31a**  $\sin(x - \frac{3\pi}{2}) = \cos(\frac{\pi}{2} - (x - \frac{3\pi}{2})) = \cos(2\pi - x) = \cos(-x) = \cos x \therefore \theta = \frac{3\pi}{2}$ .

**31b**  $\cos(x - \frac{\pi}{2}) = \cos(\frac{\pi}{2} - x) = \sin x \therefore \theta = \frac{\pi}{2}$ .

**31c**  $\tan x = 1 \therefore x = \tan^{-1} 1 = \frac{\pi}{4}$ .

**13D Q1a, 1b** The Cambridge solutions just say "No working required, cf. answers" - and this mostly suffices except that there are no graphs given in the answers in the textbook. So the graphs are provided here.



**14E Q18c i**  $\frac{12 \times ({}^9C_3 \times 1 + {}^8C_3 \times 2 + {}^7C_3 \times 3 + {}^6C_3 \times 4 + {}^5C_3 \times 5 + {}^4C_3 \times 6 + {}^3C_3 \times 7)}{2} = 2772.$

**14E Q18c ii**  $9240 - 2772 = 6468$

**15A Q17** The Cambridge solutions say “Students check the answer themselves” - but that is not a solution.

**17a**

$$1 = 2^0$$

$$1 + 1 = 2 = 2^1$$

$$1 + 2 + 1 = 4 = 2^2$$

$$1 + 3 + 3 + 1 = 8 = 2^3$$

$$1 + 4 + 6 + 4 + 1 = 16 = 2^4$$

$$1 + 5 + 10 + 10 + 5 + 1 = 32 = 2^5$$

$$1 + 6 + 15 + 20 + 15 + 6 + 1 = 64 = 2^6$$

$$1 + 7 + 21 + 35 + 35 + 21 + 7 + 1 = 128 = 2^7$$

$$1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1 = 256 = 2^8$$

$$1 + 9 + 36 + 84 + 126 + 126 + 84 + 36 + 9 + 1 = 512 = 2^9$$

$$1 + 10 + 45 + 120 + 210 + 252 + 210 + 120 + 45 + 10 + 1 = 1024 = 2^{10}$$

$$1 + 11 + 55 + 165 + 330 + 462 + 462 + 330 + 165 + 55 + 11 + 1 = 2048 = 2^{11}$$

$$1 + 12 + 66 + 220 + 495 + 792 + 924 + 792 + 495 + 220 + 66 + 12 + 1 = 2096 = 2^{12}$$

**17b**

$$2|2$$

$$3|3$$

$$5|5 \text{ and } 5|10 = 2 \times 5$$

$$7|7, 7|21 = 3 \times 7 \text{ and } 7|35 = 5 \times 7$$

$$11|11, 11|55 = 5 \times 11, 11|165 = 15 \times 11, 11|330 = 30 \times 11 \text{ and } 11|462 = 42 \times 11$$

**17c**

$1 + 1 = 2$   
 $1 + 1 + 1 = 3$   
 $1 + 1 + 1 + 1 = 4$   
 $1 + 1 + 1 + 1 + 1 = 5$   
 $1 + 1 + 1 + 1 + 1 + 1 = 6$   
 $1 + 1 + 1 + 1 + 1 + 1 + 1 = 7$   
 $1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 8$   
 $1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 9$   
 $1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 10$   
 $1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 11$   
 $1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 12$   
 $1 + 2 = 3$   
 $1 + 2 + 3 = 6$   
 $1 + 2 + 3 + 4 = 10$   
 $1 + 2 + 3 + 4 + 5 = 15$   
 $1 + 2 + 3 + 4 + 5 + 6 = 21$   
 $1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$   
 $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$   
 $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$   
 $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$   
 $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 = 66$   
 $1 + 3 = 4$   
 $1 + 3 + 6 = 10$   
 $1 + 3 + 6 + 10 = 20$   
 $1 + 3 + 6 + 10 + 15 = 35$   
 $1 + 3 + 6 + 10 + 15 + 21 = 56$   
 $1 + 3 + 6 + 10 + 15 + 21 + 28 = 84$   
 $1 + 3 + 6 + 10 + 15 + 21 + 28 + 36 = 120$   
 $1 + 3 + 6 + 10 + 15 + 21 + 28 + 36 + 45 = 165$   
 $1 + 3 + 6 + 10 + 15 + 21 + 28 + 36 + 45 + 55 = 220$   
 $1 + 4 = 5$   
 $1 + 4 + 10 = 15$   
 $1 + 4 + 10 + 20 = 35$   
 $1 + 4 + 10 + 20 + 35 = 70$   
 $1 + 4 + 10 + 20 + 35 + 56 = 126$   
 $1 + 4 + 10 + 20 + 35 + 56 + 84 = 210$   
 $1 + 4 + 10 + 20 + 35 + 56 + 84 + 120 = 330$   
 $1 + 4 + 10 + 20 + 35 + 56 + 84 + 120 + 165 = 495$   
 $1 + 5 = 6$   
 $1 + 5 + 15 = 21$   
 $1 + 5 + 15 + 35 = 56$   
 $1 + 5 + 15 + 35 + 70 = 126$   
 $1 + 5 + 15 + 35 + 70 + 126 = 252$   
 $1 + 5 + 15 + 35 + 70 + 126 + 210 = 462$   
 $1 + 5 + 15 + 35 + 70 + 126 + 210 + 330 = 792$   
 $1 + 6 = 7$   
 $1 + 6 + 21 = 28$   
 $1 + 6 + 21 + 56 = 84$   
 $1 + 6 + 21 + 56 + 126 = 210$   
 $1 + 6 + 21 + 56 + 126 + 252 = 462$   
 $1 + 6 + 21 + 56 + 126 + 252 + 462 = 924$   
 $1 + 7 = 8$

$$\begin{aligned}
1 + 7 + 28 &= 36 \\
1 + 7 + 28 + 84 &= 120 \\
1 + 7 + 28 + 84 + 210 &= 330 \\
1 + 7 + 28 + 84 + 210 + 462 &= 792 \\
1 + 8 &= 9 \\
1 + 8 + 36 &= 45 \\
1 + 8 + 36 + 120 &= 165 \\
1 + 8 + 36 + 120 + 330 &= 495 \\
1 + 9 &= 10 \\
1 + 9 + 45 &= 55 \\
1 + 9 + 45 + 165 &= 220 \\
1 + 10 &= 11 \\
1 + 10 + 55 &= 66 \\
1 + 11 &= 12
\end{aligned}$$

### 17d

$$\begin{aligned}
1 &= 11^0 \\
11 &= 11^1 \\
121 &= 11^2 \\
1331 &= 11^3 \\
14641 &= 11^4
\end{aligned}$$

$$\begin{aligned}
1 \times 10^5 + 5 \times 10^4 + 10 \times 10^3 + 10 \times 10^2 + 5 \times 10 + 1 \\
= 1 \times 10^5 + 6 \times 10^4 + 1 \times 10^3 + 0 \times 10^2 + 5 \times 10 + 1 \\
= 161051
\end{aligned}$$

Hence 1, 5, 10, 10, 5, 1 becomes 161051 by carrying and  $161051 = 11^5$

Likewise,  $1 \times 10^6 + 6 \times 10^5 + 15 \times 10^4 + 20 \times 10^3 + 15 \times 10^2 + 6 \times 10 + 1 = 1771561$

and so 1, 6, 15, 20, 15, 6, 1 becomes  $1771561 = 11^6$

$1 \times 10^7 + 7 \times 10^6 + 21 \times 10^5 + 35 \times 10^4 + 35 \times 10^3 + 21 \times 10^2 + 7 \times 10 + 1 = 19487171$

1, 7, 21, 35, 35, 21, 7, 1 becomes  $19487171 = 11^7$

$1 \times 10^8 + 8 \times 10^7 + 28 \times 10^6 + 56 \times 10^5 + 70 \times 10^4 + 56 \times 10^3 + 28 \times 10^2 + 8 \times 10 + 1 = 214358881$

1, 8, 28, 56, 70, 56, 28, 8, 1 becomes  $214358881 = 11^8$

$1 \times 10^9 + 9 \times 10^8 + 36 \times 10^7 + 84 \times 10^6 + 126 \times 10^5 + 126 \times 10^4 + 84 \times 10^3 + 36 \times 10^2 + 9 \times 10 + 1 = 2357947691$

1, 9, 36, 84, 126, 126, 84, 36, 9, 1 becomes  $2357947691 = 11^9$

$1 \times 10^{10} + 10 \times 10^9 + 45 \times 10^8 + 120 \times 10^7 + 210 \times 10^6 + 252 \times 10^5 + 210 \times 10^4 + 120 \times 10^3 + 45 \times 10^2 + 10 \times 10 + 1 = 25937424601$

1, 10, 45, 120, 210, 252, 210, 120, 45, 10, 1 becomes  $25937424601 = 11^{10}$

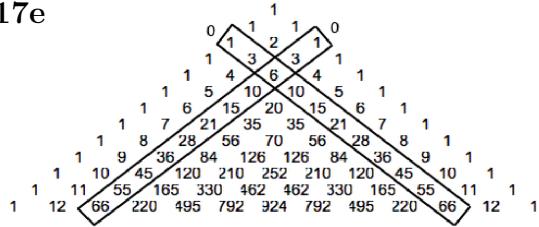
$1 \times 10^{11} + 11 \times 10^{10} + 55 \times 10^9 + 165 \times 10^8 + 330 \times 10^7 + 462 \times 10^6 + 462 \times 10^5 + 330 \times 10^4 + 165 \times 10^3 + 55 \times 10^2 + 11 \times 10 + 1 = 285311670611$

1, 11, 55, 165, 330, 462, 462, 330, 165, 55, 11, 1 becomes  $285311670611 = 11^{11}$ .

$$1 \times 10^{12} + 12 \times 10^{11} + 66 \times 10^{10} + 220 \times 10^9 + 495 \times 10^8 + 792 \times 10^7 + 924 \times 10^6 + 792 \times 10^5 + 495 \times 10^4 + 220 \times 10^3 + 66 \times 10^2 + 12 \times 10 + 1 = 3138428376721$$

1, 12, 66, 220, 495, 924, 792, 495, 220, 66, 12, 1 becomes  $3138428376721 = 11^{12}$ .

**Q17e**



From triangular numbers identified

$$0 + 1 = 1 = 1^2, 1 + 3 = 4 = 2^2$$

$$3 + 6 = 9 = 3^2, 6 + 10 = 16 = 4^2$$

$$10 + 15 = 25 = 5^2, 15 + 21 = 36 = 6^2$$

$$21 + 28 = 49 = 7^2, 28 + 36 = 64 = 8^2$$

$$36 + 45 = 81 = 9^2, 45 + 55 = 100 = 10^2$$

$$55 + 66 = 121 = 11^2$$

## References

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[2] Pender, B., et al. *CambridgeMATHS Stage 6 Mathematics Extension 1 Year 11*, Cambridge University Press, 2019.

[3] Webb, T., Corrections to Cambridge Year 11 Extension 1 Solutions, available at <http://www.angelfire.com/ab7/fourunit/Cambridge-y11-e1-sol-cor.pdf>, 2023.