Alternative solutions to 2023 HSC Extension 1 Q14ci

by Derek Buchanan

There have been several inefficient methods going around for this question but here are 2 somewhat more efficient methods:

Method 1

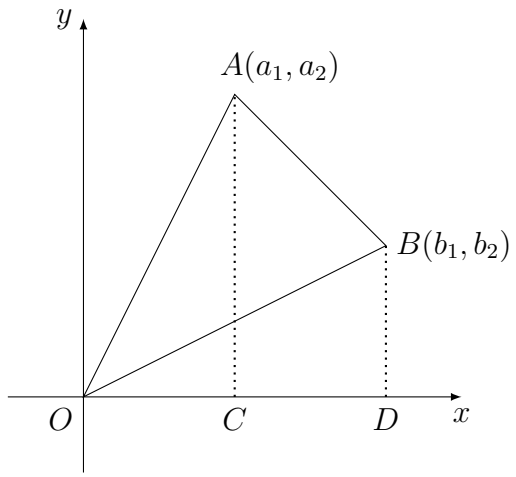
Area of triangle spanned by $\overrightarrow{OA}, \overrightarrow{OB}$ is

$$\tfrac{1}{2}|\overrightarrow{OA} \times \overrightarrow{OB}| = \tfrac{1}{2}\left|\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & 0 \\ b_1 & b_2 & 0 \end{vmatrix}\right| = \tfrac{1}{2}|(a_1b_2 - a_2b_1)\mathbf{k}| = \tfrac{1}{2}|a_1b_2 - a_2b_1|$$

Although there have been comments going around that it would not be accepted by the markers, on HSC Feedback Day 2024, it was confirmed by the markers that it would be accepted, as the question said "or otherwise".

Method 2

This is slightly less efficient than Method 1, but still more efficient than the other solutions that have so far been published.



$$\begin{aligned}
\text{Area}(\triangle OAB) &= |\text{Area}(\triangle OAC) + \text{Area}(\text{trapezium } ACDB) - \text{Area}(\triangle OBD)| \\
&= |\tfrac{1}{2}a_1a_2 + \tfrac{1}{2}(a_2 + b_2)(b_1 - a_1) - \tfrac{1}{2}b_1b_2| \\
&= \tfrac{1}{2}|a_1a_2 + a_2b_1 + b_2b_1 - a_2a_1 - b_2a_1 - b_1b_2| \\
&= \tfrac{1}{2}|a_2b_1 - b_2a_1| \\
&= \tfrac{1}{2}|a_1b_2 - a_2b_1|
\end{aligned}$$