

Work and Energy

Work is a measure of energy transfer. In the absence of friction, when positive work is done on an object, there will be an increase in its kinetic or potential energy. In order to do work on an object, it is necessary to apply a force along or against the direction of the object's motion. If the force is constant and parallel to the object's path, work can be calculated using

$$W = F \cdot s$$

where F is the constant force and s the displacement of the object. If the force is not constant, we can still calculate the work using a graphical technique. If we divide the overall displacement into short segments, Δs , the force is nearly constant during each segment. The work done during that segment can be calculated using the previous expression. The total work for the overall displacement is the sum of the work done over each individual segment:

$$W = \sum F(s)\Delta s$$

This sum can be determined graphically as the area under the plot of force vs. distance.¹

These equations for work can be easily evaluated using a force sensor and a Motion Detector. In either case, the work-energy theorem relates the work done to the change in energy as

$$W = \Delta PE + \Delta KE$$

where W is the work done, ΔPE is the change in potential energy, and ΔKE the change in kinetic energy.

In this experiment you will investigate the relationship between work, potential energy, and kinetic energy.

OBJECTIVES

- Use a Motion Detector and a force sensor to measure the position and force on a hanging mass, a spring, and a dynamics cart.
- Determine the work done on an object using a force vs. distance graph.
- Use the Motion Detector to measure velocity and calculate kinetic energy.
- Compare the work done on a cart to its change of mechanical energy.

MATERIALS

Power Macintosh or Windows PC
Vernier Motion Detector
Vernier Force Sensor
Universal Lab Interface
Logger *Pro*

dynamics cart
masses (200 g and 500 g)
spring with a low spring constant (~10 N/m)
tape
wire basket (to protect Motion Detector)
rubber band

¹ If you know calculus, you may recognize this sum as leading to the integral $W = \int_{s_{initial}}^{s_{final}} F(s)ds$.

PRELIMINARY QUESTIONS

1. Lift a book from the floor to the table. Did you do work? To answer this question, consider whether you applied a force parallel to the displacement of the book.
2. What was the average force acting on the book as it was lifted? Could you lift the book with a constant force? Ignore the very beginning and end of the motion in answering the question.
3. Holding one end still, stretch a rubber band. Did you do work on the rubber band? To answer this question, consider whether you applied a force parallel to the displacement of the moving end of the rubber band.
4. Is the force you apply constant when you stretch the rubber band? If not, at what point in the stretch is the force the least. At what point is the force the greatest?



PROCEDURE

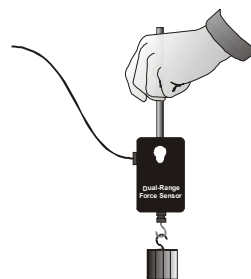
Part I Work When The Force Is Constant

In this part you will measure the work needed to lift an object straight upward at constant speed. The force you apply will balance the weight of the object, and so is constant. The work can be calculated using the displacement and the average force, and also by finding the area under the force *vs.* distance graph.

1. Connect the Vernier Motion Detector to PORT 2 and the Vernier Force Sensor to DIN 1 (PORT 1 if using ULI Force Probe) of the Universal Lab Interface.
2. If you are using a Vernier Dual-Range Force Sensor², open “Exp18DRA” from the *Physics with Computers* experiment files of Logger Pro. Set the range switch on the sensor to 10 N. If you are using the ULI Force Probe, open “Exp18FPA.” For the Student Force Sensor, open “Exp18SFA.” Three graphs will appear on the screen: distance *vs.* time, force *vs.* time, and force *vs.* distance. Data will be collected for 5 s.
3. If you are using the ULI Force Probe, it is necessary to calibrate the sensor. Other Force Sensors can optionally be calibrated using the same procedure, or you can skip this step.
 - Choose Calibrate from the Experiment menu. Click on the PORT 1 icon (DIN 1 for Force Sensors other than the ULI Force Probe) so the port is highlighted. Click .
 - Remove all force from the Force Sensor. Enter a **0** (zero) in the Value 1 field. Hold the sensor vertically with the hook downward and wait for the reading shown for Input 1 to stabilize. Click . This defines the zero force condition.
 - Hang the 500-g mass from the Force Sensor. This applies a force of 4.9 N. Enter **4.9** in the Value 2 field, and after the reading shown for Input 1 is stable, click . Click to close the calibration dialog.
4. Hold the Force Sensor with the hook pointing downward, but with no mass hanging from it. Click and then to set the Force Sensor to zero.

² If you have an older Dual-Range Force sensor with a 5-N range instead of a 10-N range, use the 5-N setting. Instead of the experiment file listed above, open the similarly named file with a “5” in the name.

5. Hang a 200-g mass from the Force Sensor.
6. Place the Motion Detector on the floor, away from table legs and other obstacles. Place a wire basket over it as protection from falling weights.
7. Hold the Force Sensor and mass about 0.5 m above the Motion Detector. Click  to begin data collection. Wait about 1.0 s after the clicking sound starts, and then slowly raise the Force Sensor and mass about 0.5 m straight upward. Then hold the sensor and mass still until the data collection stops at 5 s.
8. Examine the distance vs. time and force vs. time graphs by clicking the Examine button, . Identify when the weight started to move upward at a constant speed. Record this starting time and height in the data table.



9. Examine the distance vs. time and force vs. time graphs and identify when the weight stopped moving upward. Record this stopping time and height in the data table.

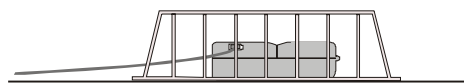




Figure 1

10. Determine the average force exerted while you were lifting the mass. Do this by selecting the portion of the force vs. time graph corresponding to the time you were lifting (refer to the position graph to determine this time interval). Do not include the brief periods when the up motion was starting and stopping. Click the Statistics button, , to calculate the average force. Record the value in your data table.
11. On the force vs. distance graph select the region corresponding to the upward motion of the weight. (Click and hold the mouse button at the starting distance, then drag the mouse to the stopping distance and release the button.) Click the Integrate button, , to determine the area under the force vs. distance curve during the lift. Record this area in the data table.
12. Print the graphs.

Part II Work Done To Stretch A Spring

In Part II you will measure the work needed to stretch a spring. Unlike the work needed to lift a mass, the work done in stretching a spring is not a constant. The work can still be calculated using the area under the force vs. distance graph.

13. If you are using a Vernier Dual Range Force Sensor, open “Exp18DRB” from the Physics with Computers experiment files of Logger Pro. Set the range switch to 10 N. If you are using the ULI Force Probe, open “Exp18FPB.” For the Student Force Sensor, open “Exp18SFB.” Three graphs will appear on the screen: distance vs. time, force vs. time, and force vs. distance. Data will be collected for 5 seconds.

Experiment 18

14. Attach one end of the spring to a rigid support. Attach the Force Sensor hook to the other end. Rest the Force Sensor on the table with the spring extended but relaxed, so that the spring applies no force to the Force Sensor.
15. Place the Motion Detector about one meter from the Force Sensor, along the line of the spring. Be sure there are no nearby objects to interfere with the distance measurement.

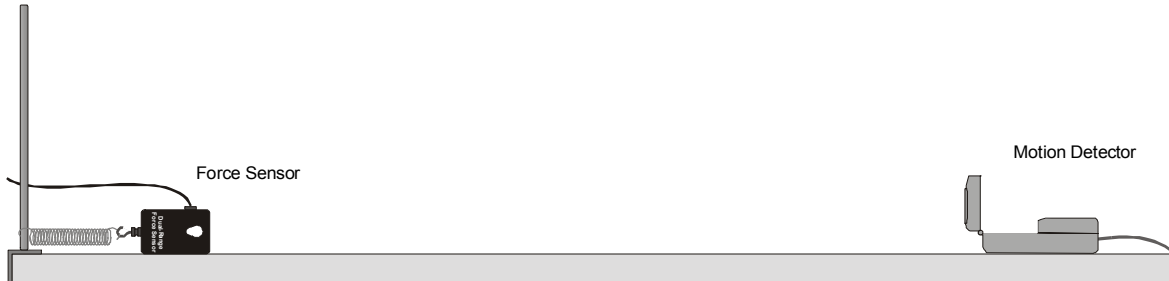


Figure 2

16. Using tape, mark the position of the leading edge of the Force Sensor on the table. The starting point is when the spring is in a relaxed state. Hold the end of the Force Sensor that is nearest the Motion Detector as shown in Figure 3. The Motion Detector will measure the distance to your hand, not the Force Sensor. With the rest of your arm out of the way of the Motion Detector beam, click . On the dialog box that appears, click . Logger Pro will now use a coordinate system which is positive *towards* the Motion Detector with the origin at the Force Sensor.

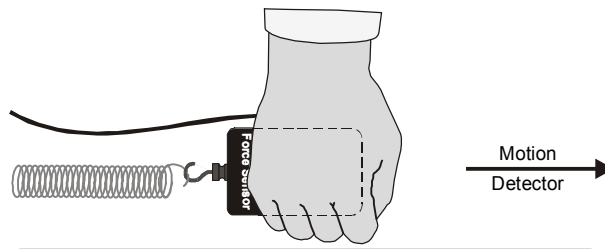






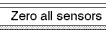



Figure 3

17. Click to begin data collection. Within the limits of the spring, move the Force Sensor and slowly stretch the spring about 50 cm over several seconds. Hold the sensor still until data collection stops. Do not get any closer than 40 cm to the Motion Detector.
18. Examine the distance vs. time and force vs. time graphs by clicking the Examine button, . Identify the time when you started to pull on the spring. Record this starting time and distance in the data table.
19. Examine the distance vs. time and force vs. time graphs and identify the time when you stopped pulling on the spring. Record this stopping time and distance in the data table.
20. Click the force vs. distance graph, then click the Linear Regression button, , to determine the slope of the force vs. distance graph. The slope is the spring constant, k . Record the slope and intercept in the data table.
21. The area under the force vs. distance graph is the work done to stretch the spring. How does the work depend on the amount of stretch? On the force vs. distance graph select the region corresponding to the first 10 cm stretch of the spring. (Click and hold the mouse button at the starting distance, then drag the mouse to 10 cm and release the button.) Click the Integrate

- button, , to determine the area under the force vs. distance curve during the stretch. Record this area in the data table.
22. Now select the portion of the graph corresponding to the first 20 cm of stretch (twice the stretch). Find the work done to stretch the spring 20 cm. Record the value in the data table.
 23. Select the portion of the graph corresponding to the maximum stretch you achieved. Find the work done to stretch the spring this far. Record the value in the data table.
 24. Print the graphs (optional).

Part III Work Done To Accelerate A Cart

In Part III you will push on the cart with the Force Sensor, causing the cart to accelerate. The Motion Detector allows you to measure the initial and final velocities; along with the Force Sensor, you can measure the work you do on the cart to accelerate it.

25. If you are using a Vernier Dual Range Force Sensor, open “Exp18DRC” from the Physics with Computers experiment files of Logger Pro. Set the range switch to 10 N. If you are using the ULI Force Probe, open “Exp18FPC.” For the Student Force Sensor, open “Exp18SFC.” Three graphs will appear on the screen: distance vs. time, force vs. time, and force vs. distance. Data will be collected for 5 seconds.
26. Remove the spring and support. Determine the mass of the cart. Record in the data table.
27. Place the cart at rest about 1.5 m from the Motion Detector, ready to roll toward the detector.
28. Click . On the dialog box that appears, click . Logger Pro will now use a coordinate system which is positive towards the Motion Detector with the origin at the cart.
29. Prepare to gently push the cart toward the Motion Detector using the Force Sensor. Hold the Force Sensor so the force it applies to the cart is parallel to the sensitive axis of the sensor.
30. Click  to begin data collection. When you hear the Motion Detector begin clicking, gently push the cart toward the detector using only the hook of the Force Sensor. The push should last about half a second. Let the cart roll toward the Motion Detector, but catch it before it strikes the detector.
31. Examine the distance vs. time and force vs. time graphs by clicking the Examine button, . Identify when you started to push the cart. Record this time and distance in the data table.
32. Examine the distance vs. time and force vs. time graphs and identify when you stopped pushing the cart. Record this time and distance in the data table.
33. Determine the velocity of the cart after the push. Use the slope of the distance vs. time graph, which should be a straight line after the push is complete. Record the slope in the data table.
34. From the force vs. distance graph, determine the work you did to accelerate the cart. To do this, select the region corresponding to the push (but no more). Click the Integrate button, , to measure the area under the curve. Record the value in the data table.
35. Print the graphs (optional).

DATA TABLE

Part I		
	Time (s)	Distance (m)
Start Moving		
Stop Moving		

Average force(N)	
Work done (J)	
Integral (during lift): force vs. distance (N•m)	
ΔPE (J)	

Part II		
	Time (s)	Distance (m)
Start Pulling		
Stop Pulling		

Spring Constant (N/m)	
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	Stretch		
	10 cm	20 cm	Maximum
Integral (during pull) (N•m)			
ΔPE (J)			

Part III		
	Time (s)	Distance (m)
Start Pushing		
Stop Pushing		

Mass (kg)	
Final velocity (m/s)	
Integral – during push (N•m)	
ΔKE of cart (J)	

ANALYSIS

1. In Part I, the work you did lifting the mass did not change its kinetic energy. The work then had to change the potential energy of the mass. Calculate the increase in gravitational potential energy using the following equation. Compare this to the average work for Part I, and to the area under the force vs. distance graph:

$$\Delta PE = mg\Delta h$$

where Δh is the distance the mass was raised. Record your values in the data table. Does the work done on the mass correspond to the change in gravitational potential energy? Should it?

2. In Part II you did work to stretch the spring. The graph of force vs. distance depends on the particular spring you used, but for most springs will be a straight line. This corresponds to Hooke's law, or $F = -kx$, where F is the force applied by the spring when it is stretched a distance x . k is the spring constant, measured in N/m. What is the spring constant of the spring? From your graph, does the spring follow Hooke's law? Do you think that it would always follow Hooke's law, no matter how far you stretched it? Why is the slope of your graph positive, while Hooke's law has a minus sign?
3. The elastic potential energy stored by a spring is given by $\Delta PE = \frac{1}{2} kx^2$, where x is the distance. Compare the work you measured to stretch the spring to 10 cm, 20 cm, and the maximum stretch to the stored potential energy predicted by this expression. Should they be similar? **Note:** Use consistent units. Record your values in the data table.
4. In Part III you did work to accelerate the cart. In this case the work went to changing the kinetic energy. Since no spring was involved and the cart moved along a level surface, there is no change in potential energy. How does the work you did compare to the change in kinetic energy? Here, since the initial velocity is zero, $\Delta KE = \frac{1}{2} m v^2$ where m is the total mass of the cart and any added weights, and v is the final velocity. Record your values in the data table.

EXTENSIONS

1. Show that one N•m is equal to one J.
2. Start with a stretched spring and let the spring do work on the cart by accelerating it toward the fixed point. Use the Motion Detector to determine the speed of the cart when the spring reaches the relaxed position. Calculate the kinetic energy of the cart at this point and compare it to the work measured in Part II. Discuss the results.
3. Repeat Part I, but vary the speed of your hand as you lift the mass. The force vs. time graph should be irregular. Will the force vs. distance graph change? Or will it continue to correspond to $mg\Delta h$?
4. Repeat Part III, but start with the cart moving away from the detector. Pushing only with the tip of the Force Sensor, gently stop the cart and send it back toward the detector. Compare the work done on the cart to the change in kinetic energy, taking into account the initial velocity of the cart.