The following was posted to sci.stat.edu by Don Burrill. I have taken the liberty of making the forumlae a bit more readable (by using an equation editor), and I added the table summarizing T and T′ for the 4 types of means.

B. Weaver (13-Apr-2004)

From: "Donald Burrill" <dfb@mv.mv.com>
Subject: Re: "Flavours" of mean
Date: Thursday, July 11, 2002 10:12 PM

---- snip the original post asking about various means ----

You appear to have understood the basic concept, although I think your way of expressing it may be somewhat misleading. For what it may be worth, I would describe the situations thus:

Given a set of numbers X_i (i=1,...n), one calculates a mean by

(1) applying a transformation T to each of the numbers;
(2) summing all the transformed numbers;
(3) dividing that sum by n;
(4) applying the inverse transformation T' to the quotient of (3).

In the case of the arithmetic mean, T = T' = the identity transform;
for the geometric mean, T = the logarithm (to any convenient base b) and T' = the antilogarithm (to the same base),
so that if T = *ln*, T' = exp (i.e., e^X);
for the harmonic mean, T = T' = the reciprocal;
for the RMS (root mean square), T = square and T' = square root.

Type of Mean	**T**	**T′**
Arithmetic mean	Identity transform	Identity transform
Geometric mean	Logarithm (any base)	Antilogarithm (same base)
Harmonic mean	Reciprocal	Reciprocal
Root mean square (RMS)	Square	Square root

This schema has the advantage that the arithmetic mean is more clearly seen to behave just like the others, with a suitable choice of transformation (namely, the identity).

In language similar to the notation you used above, we might write

$$Mean = T'\left[\frac{\sum T(X_i)}{n}\right]$$

and the particular mean one gets depends on one's choice of T.

Cheers! -- Don.

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