The optimum control of thruster system for dynamically positioned vessels

C.C. Liang \(^a\),* W.H. Cheng \(^b\)

\(^a\) Department of Mechanical Engineering, Da-Yeh University, Changhua, Taiwan, ROC
\(^b\) Department of Electronics Engineering, Nan-Jeon Institute of Technology, Tainan, Taiwan, ROC

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Abstract

An offshore vessel with a dynamic positioning system (DP system) needs fast response to produce thrust to counteract the environmental forces acting on it for the purpose of maintaining its position and heading as close as possible to the working position. Therefore, quick and effective modulation of the thrust is the problem to determine the thrust and the rotation angle of the thruster devices of the ship. This paper presents an effective optimum control for a thruster system, using the sequential quadratic method to achieve economical and effective modulation of the thrust and the direction of the thruster. An optimum control study of a 2280 tons DP coring vessel with five rotary azimuth thruster marine positioning is studied in detail, which can quickly and exactly estimate the thrusts and angles of direction of all the thrusters. The results can provide a valuable thruster system for a dynamically positioned vessel.

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1. Introduction

Modern underwater survey techniques, that is, coring, drilling, pipelaying and ocean observation, have been making increasing use of dynamic positioning (DP) systems for vessels in wave, wind and current environments (Milne, 1983). As a result, the role of the DP system has become more important. The DP system gener-
ally includes an acoustic sensor system, control system, thruster system and power system (Fig. 1).

Generally, dynamically positioned vessels use thrusters and main propellers to produce thrust by means of azimuth thrusters as well as to control manoeuvring (Fig. 2). Thrust can counteract environmental forces acting on the vessel in order to maintain its position and heading as closely as required to some desired position in the horizontal plane (Morgan, 1978). Therefore, each thruster needs to be considered as having two independent variables, magnitude and direction. However, the DP system has three equations that include a longitudinal force, a lateral force, and a moment requirement from the control system to be satisfied. When the numbers of variable are greater than three, it is possible to get multi solutions but a unique solution can be usually obtained. Therefore, Morgan (1978) assumed that some of the azimuth thrusters were fixed, or all of the thrust contributed equally to delivering the required force in one direction (Wichers, 1998). The common way of solving the thrust allocation program problem is to solve a static quadratic minimization program problem.

Fig. 1. Dynamic positioning system of the drill vessel.
with constraints on the thruster system. Jensen (1980) studied estimation and control thrust using static quadratic minimization programming for dynamic positioning of vessels. Lindfors (1993) also presented a thrust allocation method with static quadratic minimization programming in a dynamic positioning system. Sørdalen (1996) proposed a modification of single-value decomposition to control the azimuth directions to reduce wear and tear, and to minimize the required thrust and power in a general manner, thus allowing the enabling and disabling of thrusters. Fossen et al. (1996) discussed an off-line parallel extended Kalman filter algorithm utilizing two measurement series in parallel to estimate the parameters in the DP ship model. Wichers (1998) described a procedure for investigating thruster–hull and thruster–thruster interaction, used with the DPSIM computer program (Pinkster, 1980; Nienhuis, 1986, BMT (British Marine Technology), 1996) and in model tests. Although the above articles have provided much valuable information on the modulation of thrust, how to obtain an effective optimum thrust has rarely been explored. Therefore, this article focuses on the need for a DP vessel that can rapidly and exactly distribute thrust and rotate the angle of thruster, and presents an effective optimum control of thruster system, using the sequential quadratic method (SQM) with constraints on the azimuth thrusters for a thruster system.

2. Design of optimum control of the thruster system

When the control system of a dynamically positioned vessel orders the total thrust and moment commands to the azimuth thruster system as shown in Fig. 2, the thruster system will apply an effective optimum control using the sequential quadratic method (Arora and Jasbir, 1989) to control the azimuth thrusters. The behaviour of vessel thrusters in the sea is described in detail in the following.
2.1. Behaviour of thrusters

In the sea, when the vessel is moved by environmental forces and moments, the control system of the DP vessel computes the total desired thrust and moment commands, \( \Sigma F_{tx}, \Sigma F_{ty} \) and \( \Sigma M_{tz} \), sufficient to move the vessel back to the reference position. If the vessel is equipped with a total of \( n \) thrusters, the thruster system commands the thrusters to produce the thrust \( F_{ti}, i = 1,2,\ldots,n \), and to rotate the direction of the thrusters \( \alpha_{ti} \), as shown in Fig. 3. Moreover, we need to determine the thrust in the surge (X-axis) and sway (Y-axis) directions, that are \( F_{ti}\cos\alpha_{ti} \) and \( F_{ti}\sin\alpha_{ti} \), and the moment in the yaw direction, that are \( F_{ti}\sin\alpha_{ti}\cdot l_{xi} \) and \( F_{ti}\cos\alpha_{ti}\cdot l_{yi} \).

From the viewpoint of the DP system, several basic requirements must be established for the thruster system with respect to holding the vessel in position under a given environment while performing a given task. This is to ensure that there will be unknowns to satisfy these equations as shown in the following equations:

(a) The thrust has to satisfy the desired resulting surge forces as follows:

\[
\sum F_{tx} \geq F_{1x}\cos\alpha_{1} + F_{2x}\cos\alpha_{2} + \ldots + F_{nx}\cos\alpha_{n}
\]

(b) The thrust has to satisfy the desired resulting sway forces as follows:

\[
\sum F_{ty} \geq F_{1y}\sin\alpha_{1} + F_{2y}\sin\alpha_{2} + \ldots + F_{ny}\sin\alpha_{n}
\]

Fig. 3. Thrust allocation scheme for the thruster system. (a) General arrangement of azimuth thrust. (b) The forces in surge and sway and moment in the yaw direction.
\[
\sum_{i} F_{ti} \geq F_{t1} \sin \alpha_{t1} + F_{t2} \sin \alpha_{t2} + \ldots + F_{tm} \sin \alpha_{tm}. \tag{2}
\]

(c) The moment has to satisfy the desired resulting yaw moment as follows:
\[
\sum_{i} M_{ti} \geq F_{t1} \cos \alpha_{t1} \cdot ly_{1} + F_{t2} \cos \alpha_{t2} \cdot ly_{2} + \ldots + F_{tm} \cos \alpha_{tm} \cdot ly_{n}
+ F_{t1} \sin \alpha_{t1} \cdot lx_{1} + F_{t2} \sin \alpha_{t2} \cdot lx_{2} + \ldots + F_{tm} \sin \alpha_{tm} \cdot lx_{n}, \tag{3}
\]

where \( F_{t1}, F_{t2}, F_{t3}, \ldots, F_{m} \) is the unknown thrusts from the first to \( n \) th thruster, respectively; \( \alpha_{t1}, \alpha_{t2}, \alpha_{t3}, \ldots, \alpha_{m} \) is the unknown directions of the thrust from the first to \( n \) th thruster, respectively; \( lx_{1}, lx_{2}, \ldots, lx_{n}, ly_{1}, ly_{2}, \ldots, ly_{n} \) is the known distance from the center of rotation of the vessel to the thrust, respectively

However, a general thruster system should be able to provide control under different directions of azimuth thrusters. The structure of azimuth thruster for deriving the low-frequency, desired azimuth angle \( \alpha_{ti} \) of a given thruster is shown in Fig. 3(b), where \( F_{ti} \sin \alpha_{ti}, F_{ti} \cos \alpha_{ti} \) are the two-dimensional vector forces \( F_{ti} \) corresponding to the actual azimuth thruster, which belongs to a quadratic function. According to Eqs. (1) and (3), the thrusts from the thrusters must satisfy the desired thrust and moment commands subject to nonlinear equality and inequality constraints. Consequently, the optimal control problem is best dealt with using the sequential quadratic method.

3. Optimization method SQM

The sequential quadratic method (SQM) is an important method used to minimize an arbitrary objective function subject to nonlinear equality and inequality constraints. The principal of SQM is the formulation of a quadratic approximation of Lagrangian function (Arora and Jasbir, 1989; Schittowski, 1985).

\[
L(X, \lambda) = f(X) + \sum_{i=1}^{m} \lambda_{i} g_{i}(X) \quad i = 1, 2, \ldots, n \tag{4}
\]

where \( f(X) \) are convex functions, \( \lambda_{i} \) are Lagrange Multipliers and \( g_{i}(X) \) are convex functions.

Fig. 4 shows the SQM path to the solution point \( X = \{x_{m1}, x_{m2}, x_{m3}, \ldots, x_{mn}\} \), starting at \( X = \{x_{01}, x_{02}, x_{03}, \ldots, x_{0n}\} \) in \( m \) iterations. The standard form of the general nonlinear constrained optimization problem is as follows: assume the following set of \( n \) design variables contained in the vector \( X \) which are real values in a specified range.

3.1. Design variables

The design variables are:

\[
X = \{x_{n}\} = \{x_{1}, x_{2}, x_{3}, \ldots, x_{n}\}. \tag{5}
\]
3.1.1. Objective function
Minimize
\[ f(X) = f(x_1, x_2, \ldots, x_n). \]  
\[ (6) \]

3.1.2. Design constraints
These are the equality constraints
\[ h_i(X) = 0; \ i = 1 \text{ to } p \]  
\[ (7) \]
and the inequality constraints
\[ g_j(X) \leq 0; \ j = 1 \text{ to } m \]  
\[ (8) \]
and the explicit bounds on design variables
\[ X_l \leq x_n \leq X_u, \]  
\[ (9) \]
where \( x_n \) is the vector of the design parameter and the bounds \( X_l \) and \( X_u \) on the design variables are referred to as side constraints. \( f(X) \) is the objective function.

SQM has several important advantages: it exhibits the descent property, is feasible for all iterations and is conceptually simple and computationally efficient. The basic steps in SQM involve solving a linear or nonlinear programming sub-problem to find the direction vector and then finding the step-length along this direction by
performing a constrained line search. After the current point is updated, the above steps are repeated until a termination criterion is satisfied.

3.2. Optimum control using SQM

In the present optimum design study, the thrust $F_{ti}$, $i = 1,2...n$ and the azimuth angles $\alpha_{ti}$ just satisfy to counteract the mean environmental force over a given time period. Therefore, an objective function ensures that the total thrust commands given by each of the $n$ thruster devices under these constraints will be the minimum. Additionally, the thrust and moment behaviour constraints are considered in optimization problems. The mathematical model of optimum control for the thruster system is described in the following.

3.2.1. Mathematical model

The design variables, objective function and design constraints with the behavior of the thrust and moment of the thrusters are described in the following. The definitions of the following notations can be found in the previous sections.

3.2.2. Design variables

In the light of the analysis of thruster system, the design variables are the thrust $F_{ti}$ and the azimuth angles $\alpha_{ti}$, $i = 1,2,3,...n$ (Fig. 3(b)). The design variables are:

$$X = \{x_i\} = \{F_{t1}, \alpha_{t1}, F_{t2}, \alpha_{t2}, ..., F_{tn}, \alpha_{tn}\} \quad (10)$$

3.2.3. Objective function

For economy and the effectiveness of the thruster system, the total thrust commands given by each of the $n$ thruster devices will be the minimum.

$$\text{Min } f(X) = (F_{t1}\cos\alpha_{t1} + F_{t1}\sin\alpha_{t1} + F_{t2}\cos\alpha_{t2} + F_{t2}\sin\alpha_{t2} + ... + F_{tm}\cos\alpha_{tm} + F_{tm}\sin\alpha_{tm}) \quad (11)$$

3.2.4. Design constraints

(a) Constraints on the behavior of the thrust in the X direction: The sum of the minimum thrust of $n$ thrusters in the X direction, $F_{mx}\cos\alpha_{mx}$, must satisfy (nonnegative) the thrust requirement (Fig. 3(b)):

$$g_1(X) = \sum F_{tx} - (F_{t1}\cos\alpha_{t1} + F_{t2}\cos\alpha_{t2} + ... + F_{tm}\cos\alpha_{tm}) \leq 0. \quad (12)$$

(b) Constraints on the behaviour of the thrust in the Y direction: The sum of the minimum thrust of $n$ thrusters in the Y direction, $F_{my}\sin\alpha_{my}$, must satisfy (nonnegative) the thrust requirement (Fig. 4(b)):

$$g_2(X) = \sum F_{ty} - (F_{t1}\sin\alpha_{t1} + F_{t2}\sin\alpha_{t2} + ... + F_{tm}\sin\alpha_{tm}) \leq 0 \quad (13)$$
(c) Constraints on the behavior of the moment in the Z direction: The sum of minimum moment of \( n \) thruster in the Z direction must satisfy (equal) the thrust requirement (Fig. 3(b)):

\[
h_3(X) = \sum M_z - (F_{t1}\cos\alpha_{t1}l_1 + F_{t2}\cos\alpha_{t2}l_2 + \ldots + F_{tn}\cos\alpha_{tn}l_n) + F_{t1}\sin\alpha_{t1}l_1 + F_{t2}\sin\alpha_{t2}l_2 + \ldots + F_{tn}\sin\alpha_{tn}l_n = 0.
\]

(d) Constraints on the behavior of the horsepower per thruster: When determining the number of required thrusters, an inequality can be written based on the maximum available housepower per thruster \( F_{mi}, i = 1,2,\ldots,n \):

\[
g_{2+i} = F_{ti} - F_{mi} \leq 0 \quad i = 1,2,\ldots,n
\]

Consequently, Eqs. (1)–(6) regard an optimum control law of the thruster system, and Eqs. (10)–(15) are used in the present optimum control design for a dynamic positioning system.

4. Simulation

In this section, we present a simulation results for positioning of a coring vessel (Displacement = 2280 tons, LBP = 80 m, Beam = 12 m, Draft = 1 m) in which the present optimum control for the thruster system of a DP system was installed. The coring vessel was simulated with five azimuth thrusters, and the distance between each thruster as shown Fig. 5. The thrusters were bi-directional.

In the simulation a given task was accomplished in an unknown offshore marine environment. The vessel was moved by the various environmental forces. The vessel had to maintain within \( \pm 3.0 \) m of a certain position located in the reference position of the working sets, the motion path of the coring vessel during positioning as shown in Fig. 6. Therefore, the longitudinal resultant thrust, lateral resultant thrust and moment commands were obtained by the control system of the DP system shown in Fig. 7(a),(b),(c). Fig. 7(a),(b),(c) shows that the thrust and moment commands changed values at 25 s interval times in an overall 1000 s. Then the resultant desired thrust and moment commands were distributed among the vessel’s five thrusters by the optimum control SQM of the thruster system.

In the first interval time \( \Delta t_1 = [0 \text{ s}, 25 \text{ s}] \), from the control system we obtained the longitudinal resultant thrust, lateral resultant thrust and moment commands, \([\Sigma F_{tx1}, \Sigma F_{ty1}, \Sigma M_{tz1}] = [0 \text{ KN}, -154 \text{ KN}, 81 \text{ KN} - \text{m}]\), as shown in Fig. 7 and Table 1. The mathematical model of the optimum control SQM for the thruster system is given below.

4.1. Design variables

There were design variables for the thrust and the angle of the five azimuth thrusters. Based on Eq. (10), we can list these variables as follows:
4.1.1. Objective function

The objective function was the minimum total desired thrust output by the five thruster devices under these constraints. Therefore, using Eq. (11), we could determine the objective function as

\[
\text{Min} f(\ldots) = (F_{t1}\cos\alpha_{t1} + F_{t1}\sin\alpha_{t1} + F_{t2}\cos\alpha_{t2} + F_{t2}\sin\alpha_{t2} + \\
+ F_{t3}\cos\alpha_{t3} + F_{t3}\sin\alpha_{t3} + F_{t4}\cos\alpha_{t4} + F_{t4}\sin\alpha_{t4} + F_{t5}\cos\alpha_{t5} \\
+ F_{t5}\sin\alpha_{t5})
\]  

(17)
4.1.2. Design constraints

According to the behaviour of the thruster, the resultant longitudinal thrust, the resultant lateral thrust and the moment commands, \([\Sigma F_{tx}, \Sigma F_{ty}, \Sigma M_{tz}] = [0 \text{ KN}, -154 \text{ KN}, 81 \text{ KN} - \text{ m}]\), from the control system, we could use Eqs. (12)–(15) to obtain 14 constraints:

\[
g_1 = 0KN - (F_{t1}\cos\alpha_{t1} + F_{t2}\cos\alpha_{t2} + F_{t3}\cos\alpha_{t3} + F_{t2}\cos\alpha_{t2}) \leq 0,
\]

\[
g_2 = -154KN - (F_{t1}\sin\alpha_{t1} + F_{t2}\sin\alpha_{t2} + F_{t3}\sin\alpha_{t3} + F_{t2}\sin\alpha_{t4}) \leq 0,
\]

\[
h_1 = 81KNm - (F_{t1}\cos\alpha_{t1}\cdot l_y_1 + F_{t2}\cos\alpha_{t2}\cdot l_y_2 + F_{t3}\cos\alpha_{t3}\cdot l_y_3 + F_{t4}\cos\alpha_{t4}\cdot l_y_4 + F_{t5}\cos\alpha_{t5}\cdot l_y_5) \leq 0,
\]

\[
g_{2+i} = F_{ti} - 600KN \leq 0, \quad i = 1 \text{ to } 5,
\]

\[
g_{7+i} = |\alpha_{ti}| - 180^\circ \leq 0, \quad i = 1 \text{ to } 5.
\]

Then, we input the above equations into the thruster system. The optimum control of the thruster system using a sequential quadratic method, we could solve for the thrust and rotated angle of each thruster. From the optimum control of the thruster system, we could obtain the thrusts and the directions of thrust of the five thruster, which were \([F_{t1}, \alpha_{t1}, F_{t2}, \alpha_{t2}, F_{t3}, \alpha_{t3}, F_{t4}, \alpha_{t4}, F_{t5}, \alpha_{t5}] = [50 \text{ KN}, -177 \text{ deg.}, 95 \text{ KN}, -29 \text{ deg.}, -20 \text{ KN}, 90 \text{ deg.}, 82 \text{ KN}, 170, 111 \text{ KN}, -65 \text{ deg.}]\).

At the next interval time \(\Delta t_2 = [25 \text{ s}, 50 \text{ s}]\), the control system outputs the longitudinal resultant thrust, the lateral resultant thrust and the moment commands,
Fig. 7. The result of thrust and moments commands for dynamical positioning of the coring vessel. (a) The longitudinal (X-axis) result thrust commands. (b) The lateral (Y-axis) result thrust commands. (c) The result moment (Z-axis) commands.

\[
\begin{bmatrix}
F_{tx2} \\
F_{ty2} \\
M_{tz2}
\end{bmatrix} = 
\begin{bmatrix}
-68 \text{ KN} \\
-127 \text{ KN} \\
-135 \text{ KN} \cdot \text{m}
\end{bmatrix},
\]
to the thruster system. The solutions of the thrust and the angle of the azimuth thrust were obtained by repeating the above process with Eqs. (10–15). Meanwhile, we can repeat the same process with Eqs. (10–15) for the five thrusters at 25 s interval times in the overall 1000 s.

The results of time intervals for optimum control of the thruster system of the coring vessel are listed in Table 1 during 250 s. In Table 1 shows that the desired thrust and moment commands of each interval time are irregular, because the environment forces are changeable forces. The thrust system has five thrusters, which contain five thrust and five rotation angle. These values were calculated by the results of optimum control. We also found that these values were irregular. But the thrust...
Table 1
The first 10 items of the result of optimum control of thrust system using SQP

<table>
<thead>
<tr>
<th>No.</th>
<th>Control system</th>
<th>Optimum control of thrust system</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time</td>
<td>Five thruster</td>
</tr>
<tr>
<td></td>
<td>Interval time</td>
<td>Required thrust (x)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(sec)</td>
</tr>
<tr>
<td>1</td>
<td>0-25</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>50-75</td>
<td>-210</td>
</tr>
<tr>
<td>4</td>
<td>75-100</td>
<td>-272</td>
</tr>
<tr>
<td>5</td>
<td>100-125</td>
<td>-445</td>
</tr>
<tr>
<td>6</td>
<td>125-150</td>
<td>-272</td>
</tr>
<tr>
<td>7</td>
<td>150-175</td>
<td>-172</td>
</tr>
<tr>
<td>8</td>
<td>175-200</td>
<td>69</td>
</tr>
<tr>
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<td>200-225</td>
<td>81</td>
</tr>
<tr>
<td>10</td>
<td>225-250</td>
<td>90</td>
</tr>
</tbody>
</table>
system can move the vessel to the desired position. Fig. 8(a),(c),(e),(g),(i) are the curves of the output thrust for the thruster in overall 1000 seconds. Fig. 8(b),(d),(f),(h),(j) shows the angle of the thrusters within $\pm 180^\circ$ in an overall 1000 seconds. We found that the three curves of Fig. 8(b),(d),(f), the change angle of the front three thrusters, are similar. And the two curves of Fig. 8(h),(j), the change

![Fig. 8. Result of modulation of the thrust and the angle of the azimuth thruster with optimum control using SQP.](image-url)
The purpose of a thruster system is to maintain a vessel within a given distance from a desired position using a thrust-producing mechanism as quickly and accurately as possible. The paper has presented an optimum control making an effective and reduced lost thrust modulation for a thruster system using the sequential quadratic method. We have shown that the dynamic positioning coring vessel using SQM can easily, quickly and accurately solve a positioning problem with many variables and constraints. Thus, the optimum control of the thruster system using SQM is economical and effective. It can automatically modulate the thrusts and angles of the thrusters, thus reducing wear and tear, and minimizing the required thrust and power.

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