CHAPTER 9

LOGIC'S TOWER OF BABEL

> …today we know that it is possible, logically speaking, to derive almost all of present-day mathematics from a single source, the Theory of Sets. It will suffice therefore to set forth the principles of one single formalized language, to indicate how one can write in this language the Theory of Sets, then to show how from this theory grow one by one the different branches of mathematics as our attention turns to them…[1]

Sparked by Cantor's transfinite set theory, a new mathematics developed at much the same time that psychology was itself emerging as a science. Both were undoubtedly attempts at scratching the same inner itch. The Renaissance ideal of humanity as observers had led to the split of mind and body so necessary to the objective, disinterested stance of science. Obviously such a split is inherently impossible, since mind and body are merely artificial separations of an inherently unified organism. Nor is it possible for a human being to exist except as part of a complex unity which includes all the supposed objects of observation. Eventually any study that goes deep enough finds itself staring at its own reflection.

By the 2nd half of the 19th century, this need to restore unity was the driving, though unacknowledged force in all the arts and sciences. In particular, it was no longer possible for either philosophy or science to ignore the mind of the observer doing the observing. Similarly,

---

[1] quotation by Nicolas Bourbaki in Lucienne Félix, *The Modern Aspect of Mathematics*, p. 58-9. Nicolas Bourbaki is a fictional mathematician created by a group of French mathematicians in about 1930, to express their joint views about modern mathematics.

mathematics was no longer able to ignore the unexamined, implicit assumptions out of which it had emerged. Cantor's careful attempt to describe the nature of number, led to the first rigorous mathematical attempt to deal with infinity. That in turn revealed previously unimaginable paradoxes hidden at the core of infinity. Two major attempts were made to solve those paradoxes: (1) that of the formalists, led by David Hilbert, and (2) the logicians, led by Bertrand Russell.

In both cases they hoped to avoid the problems of infinity by reducing mathematics to finite axiomatic systems. Any mathematical system would consist solely of a finite number of definitions of relevant terms, a finite number of axioms, and a finite number of defined operations. The definitions, axioms, and operations were all to be totally empty of content! Mathematics was assumed to concern itself only with the formal manipulation of empty symbols. Perhaps from all that has already been discussed in this book it will be apparent that this was an impossible goal. Symbols are not empty and, in fact, it is precisely because symbols produce deep feeling responses that mathematics is able to develop.

> A symbolism not interpreted is only a game of signs; it is a language only if it takes on meaning and if one has the use of an interpretation of the symbols and of the symbolic game;…the problem of the role of intuition cannot be sidestepped.[2]

Though both the formalists and the logicians wanted to create such formal axiomatic systems, beyond that they diverged. The formalists were only interested in developing such axiomatic systems for mathematics. Russell wanted to develop a formal axiomatic system for a symbolic logic, that included both traditional logic and mathematics under a single roof.

---

[2] quotation by Jörgensen (?) in Lucienne Félix, *The Modern Aspect of Mathematics*, p. 58-9.

> Mathematics and logic, historically speaking, have been entirely distinct studies. Mathematics has been connected with science, logic with Greek. But both have developed in modern times: logic has become more mathematical and mathematics has become more logical. The consequence is that is has now become wholly impossible to draw a line between the two; in fact, the two are one. They differ as boy and man: logic is the youth of mathematics and mathematics is the manhood of logic.[3]

Both formalists and logicians were themselves under the grip of a symbol, a symbol of unity. This desire for unity was clearly the *living symbol*[4] for mathematics in Hilbert's era. Nearly all of the great mathematicians were striving toward this formal unity of mathematics, either through a formal axiomatic system, or through the reduction of mathematics to symbolic logic. They were not able to stand outside their field and see that they were under a spell. And, though their efforts were fore-doomed, the development of mathematical rigor was a necessary step in mathematics.

---

[3] Bertrand Russell, *Introduction to Mathematical Philosophy*, reprint of 2nd edition of 1920 (New York: Dover Publications, 1933), p. 194.

[4] A term used by Jung to mean a symbol that still has deep meaning for a culture. This will be discussed at some length in chapter 15, in our discussion of Jung's concept of the Self.

In this chapter we will largely discuss Russell's efforts, culminating in the *Principia Mathematica*. We'll return to Hilbert when we discuss the background for Gödel's ideas.[5] First we begin with *Peano's Postulates*[6], which were to influence both Hilbert and Russell.

PEANO'S POSTULATES

> Having reduced all traditional pure mathematics to the theory of natural numbers, the next step in logical analysis was to reduce this theory itself to the smallest set of premisses and undefined terms from which it could be derived. This work was accomplished by Peano.[7]

The first important product of this new formalism was Peano's Postulates: five axioms from which the full arithmetic of the natural numbers or integers (i.e., 0, 1, 2, 3, …) can be derived. Giuseppe Peano [1858–1932] was an Italian mathematician and logician whose postulates have a simplicity and an elegance that strongly influenced his contemporaries in mathematical logic. Here is a summary of the five postulates, as we think of them today:

(1) Zero is a number.

(2) The successor of any number is a number.

(3) No two numbers have the same successor.

(4) Zero is not the successor of any number.

(5) If a set of numbers contains both zero and also the successor of every number in itself, then it contains all numbers.

---

[5] See chapter 12.

[6] Giuseppe Peano, *Arithmetices Principia Nova Methodo Exposita* (1889).

[7] Bertrand Russell, *Introduction to Mathematical Philosophy*, p. 5.

With the exception of the last axiom, which is what mathematicians refer to as *mathematical induction*, the purpose of the axioms can be readily understood. For example, let's combine the first two. Since "zero is a number", we have at least one number (whatever a number is). Since "the successor of any number is a number", zero's successor is a number. We'll call it "one" as we normally do. Then "one" has a successor, which we'll call "two", and so forth. Clearly we can arrive at all the other positive numbers by taking successors one-at-a-time.

Peano made sure that "no two numbers have the same successor" so he didn't end up in strange paradoxical number systems. After all, we wouldn't have to have "three" be the successor for both "one" and "two".

By saying that "zero is not the successor of any number", he created a first number. There is no number smaller than zero in his system. (Note that once Peano developed zero and the positive integers, it was easy enough to extend this process to negative numbers, fractions, and so forth.)

The fifth axiom, which mathematicians call *induction* is something different, reminiscent of the parallel axiom in geometry.[8] The scientific method is sometimes known as *induction*. It is more properly termed *incomplete induction*, in contrast with *mathematical induction* or *complete induction*. A scientific theorem is provisional; it is always possible that further experience might contradict the theorem. In contrast with the incomplete induction of science, mathematical induction implies that something can be proved once and for all.

Mathematical induction says that if you want to prove some mathematical statement is true for all numbers, you only have to prove it is true in two cases. First, prove it's true for zero,

[8] See chapter 4.

since zero is the first number. Then prove that if it's true for any arbitrary number, then it's true for that number's successor as well. The trick here is that zero is a number, so it must then also be true for "one", then for "two", and on indefinitely for all numbers.

This is obviously a very powerful method of proof since it's quite a bit easier to prove an assertion is true in two circumstances than to prove it's true in any of an infinite number of circumstances (which is why natural science is provisional). But notice that induction involves infinite processes; i.e., there is no way to demonstrate the truth of induction in a finite number of steps. That is both the strength of mathematical induction and its Achilles heel. For, as we will see later, infinite processes tend to involve us in self-referential systems, and self-referential systems cannot always be reduced to logical analysis.

Just as calculus introduced the infinite concepts of infinitesimals and limits, then only later realized their potential problems, mathematical induction was used for hundreds of years before anyone realized that the method presented deep philosophical issues because of its implicit acceptance of infinite processes. By the mid-17th century, French mathematicians Pierre Fermat [1601–1665 and Blaise Pascal [1623–1662] had independently begun using versions of mathematical induction in their work. It was two hundred years, however, before August De Morgan [1806–1871] gave the process the explicit name of mathematical induction (in 1838). Another fifty years passed before Peano codified it fully and included it as the fifth and final postulate necessary for developing arithmetic.

Peano's Postulates were a stunning example of the possibilities of the new formalism. Using these five axioms, it was then at least theoretically possible to derive all the properties of arithmetic. Most colleges gave a first-year mathematics course (for mathematics majors only), where the students derive the major properties of arithmetic using Peano's Postulates, or some similar system of axioms. Note that there is absolutely nothing contained in the five axioms that points to the nature of those properties. It is undoubtedly true that there are properties of the integers that have never yet been discovered. Peano's Postulates only provide the starting point: an elegant axiomatic method for producing arithmetic.[9]

In 1892, Peano announced a project "for a compendium of all the known theorems in mathematics to be stated and proved using [his] logic.[10] Perhaps it was too early, and his methods were still too little known, for only his fellow Italians picked up on his program. It was David Hilbert's similar program of 1900 which was to inspire mathematicians in general.[11]

BERTRAND RUSSELL DISCOVERS PEANO'S LOGIC

In 1854, the same year that Riemann published his strange new geometry[12], George Boole 1815–1864] published his *Laws of Thought*, which presented a symbolic representational system for logical *syllogisms*.[13] These symbols could then be manipulated using a sort of algebra to determine the truth or falsity of complex logical relationships. This was a start at fulfilling Leibniz' goal of a "general method in which all truths of reason would be reduced to a kind of

---

[9] It should be mentioned that Peano Postulates were equally influential for the system of mathematical notation in which they were presented as for the logic itself. The technical notation was to serve as the starting point for Bertrand Russell's development of his own extensive system of mathematical logic.

[10] Nicholas Griffin, (ed.), *The Selected Letters of Bertrand Russell, Vol 1: the Private Years (1884–1914)* (Boston: Houghton Mifflin, 1992), p. 204.

[11] As we will see in chapter 12, when we discuss Gödel's response to Hilbert's program.

[12] See chapter 4.

[13] "From the Greek *syn* ("with" or "together") and *logizesthai* ("to reckon," "to conclude by reasoning"). A form of reasoning whereby, given two sentences or propositions, a third follows necessarily from them" (W. L. Reese, *Dictionary of Philosophy and Religion*, p. 559-60.) E.g., if A implies B, and B implies C, then A implies C.

calculation."[14]. Most contemporary philosophy students take a one or two semester course in which they learn Boole's method, and use it to determine the truth or falsity of progressively more complex logical statements. His system was a significant contribution toward the development of the computer; computer programmer/analysts still use "truth tables" based on Boole's system in summarizing all the possible logical paths a computer program has to consider.

Boole's system was to be combined with Peano's notation, then extended to almost unimaginable levels of complexity by philosopher and mathematician Bertrand Russell [1872–1970]. In the late days of the 19th century, Russell was a bright young man with a gift for describing difficult philosophical matters in elegant prose. He had already been studying mathematical philosophy for some years, and had achieved some notoriety for his work on geometry. But something was still lacking. When he attended the International Congress of Philosophy in Paris in 1900, he found what he had been looking for when he heard Peano speak.

Russell found Peano far and away the most impressive figure at the Congress, a man of unsurpassed logic and precision. He grew convinced that this must be due to the system of mathematical logic Peano had perfected. Within little more than a month following the Congress, Russell had devoured everything written by Peano and his Italian followers, and began first extending their technique, then using it to solve the problems with which he had been struggling.[15] It was an exhilarating time for Russell:

> …The time was one of intellectual intoxication. My sensations resembled those one has after climbing a mountain in a mist, when, on reaching the summit, the mist suddenly

---

[14] See chapter 3.

[15] See Bertrand Russell, *The Autobiography of Bertrand Russell, the Early Years: 1872–World War 1* (New York: Bantam Books, 1968), p. 191-2.

> clears, and the country becomes visible for forty miles in every direction.…Intellectually the month of September 1900 was the highest point of my life.[16]

Peano was satisfied with the sufficiently difficult project of using his technique to "state and prove" all mathematical theorems. Russell had even more exalted ambitions for these new techniques than did Peano: he planned to extend Peano's logic beyond mathematics to philosophy—beginning with the philosophy of mathematics—then beyond that the sky was the limit. Russell thought that ultimately he could reduce all mathematics, all science and all philosophy to this new symbolic logic. A grandiose plan indeed, and one which would never be achieved. Russell quickly convinced his older friend—philosopher/mathematician Alfred North Whitehead [1861–1947]—to join him in his effort. This project would occupy both men for the next ten years, and would eventually culminate in the three volumes, 4,500 pages of their *Principia Mathemetica*.

Within a year of Russell's initial intoxication with the possibilities of Peano's logic, he received three blows, each of which seemed to be telling him that there was more to life than logic, each of which he determinedly ignored. First, in February 1901, came the most profound mystical experience of his life. He was already in some emotional distress over a painful illness suffered by Whitehead's wife Evelyn, with whom Russell had an ambiguous relationship, composed about equally of admiration and sexual attraction.[17] In that mood, he heard a new translation of Euripedes' tragedy *Hippolytus*, read by its translator, his friend Gilbert Murray. This translation drew on the then new Nietzchean idea that underneath the calm, rational,

---

[16] Bertrand Russell, *The Autobiography of Bertrand Russell, the Early Years*, p. 192.

[17] Though Russell probably exaggerated the extent of her illness (something he was wont to do); it is likely that her condition was in considerable part hypochondria. See Nicholas Griffin, *The Selected Letters of Bertrand Russell, Vol 1*, p. 215-218.

structured *Appollonian* aspect that we present to the world, lies the wild, irrational, emotional *Dionysian* source of life. In other words, Murray was presenting one of the prefigurations of the *unconscious*.

> Suddenly the ground seemed to give way beneath me, and I found myself in quite another region. Within five minutes I went through some such reflections as the following: the loneliness of the human soul is unendurable; nothing can penetrate it except the highest intensity of the sort of love that religious teachers have preached; whatever does not spring from this motive is harmful, or at best useless.…At the end of those five minutes, I had become a completely different person. For a time, a sort of mystic illumination possessed me.[18]

Despite the intensity of this vision, which seemed clearly to be pushing Russell to acknowledge the importance of instinct and emotion, his logical project was too important for him to question. Eventually "*the mystic insight which I then imagined myself to possess* [my emphasis] has largely faded and the habit of analysis has reasserted itself."[19]

The second blow came three months later, when he discovered a deeply disturbing paradox inherent in his new logic. In effect, he had discovered that Cantor's paradox of the "set of all sets" was not restricted to infinite sets, but applied to all logic.[20] Again, rather than take this as a sign that his project was doomed to failure, Russell refused to take the hint.

---

[18] Bertrand Russell, *The Autobiography of Bertrand Russell, the Early Years*, p. 193-4.

[19] Bertrand Russell, *The Autobiography of Bertrand Russell, the Early Years*, p. 194.

[20] This will be discussed at some length later in this chapter.

> …At first, I supposed I should be able to overcome the contradiction quite easily, and that probably there was some trivial error in the reasoning. Gradually, however, it became clear that this was not the case.[21]

As he continued to work on the paradox to no avail, he confessed: "it seemed unworthy of a grown man to spend his time on such trivialities, but what was I to do?"[22]

Finally, the most devastating emotional blow hit. While bicycling one day, he suddenly realized that he was no longer in love with his first wife Alys. Initially he tried to talk himself out of this mood, and behave normally, but Alys could sense the difference. Within a short period of time, it was clear their marriage was at an end as an emotional partnership. Again Russell refused to confront the situation, and either seek a divorce, or resurrect their marriage on a stronger foundation. He instead bottled up his feelings, and continued to live with Alys in a miserable state that satisfied neither for the next decade, the same period over which he struggled equally on equally miserably with the *Principia Mathematica*.

SYMBOLIC LOGIC MEETS THE BARBER PARADOX

> The question "What is a number?" is one which has been often asked, but has only been correctly answered in our own time. The answer was given by Frege in 1884, in his *Grundlagen der Arithmetik.*[23]

Despite these wake-up calls from the unconscious, Russell continued on with his project. By 1902, he had been able to finish his first assault on the mountain: a book called *The Principles of*

---

[21] Bertrand Russell, *The Autobiography of Bertrand Russell, the Early Years*, p. 195.

[22] Bertrand Russell, *The Autobiography of Bertrand Russell, the Early Years*, p. 195.

[23] Bertrand Russell, *Introduction to Mathematical Philosophy*, p. 11.

Mathematics.[24] Though he had been working on this book since 1898, it was Peano's logic that enabled Russell to finally finish it. Now that he was deeply committed to the goal of making logic supreme, he sat down and seriously studied a book he had had for several years, but never previously been able to understand: *Grundlagen der Arithmetik, Vol. 1* (i.e., *The Foundation of Arithmetic*), by German mathematician and logician Gottlob Frege [1848–1925], which covered much the same territory as Russell's *Principles of Mathematics*. Russell was amazed to find that Frege had anticipated much of Russell's own new ideas; in fact, in many cases Frege had gone beyond Russell in the rigor of his presentation. However, Frege had not yet come upon the paradox we mentioned as the second of Russell's blows, and with which Russell had since been struggling.

On June 16, 1902, Russell wrote to Frege praising his work, and closing with a description of the paradox. The second and concluding volume of Frege's work was at the printer's when he received Russell's letter. Poor Frege was stunned to find his life's work destroyed by a single puzzle. However, he was too honest a man to deny the issue. He immediately replied to Russell that his "conundrum makes not only his Arithmetic, but all possible Arithmetics totter,"[25] then added a brief appendix to the book at the last minute, in which he said that:

---

[24] Published the following year.

[25] Nicholas Griffin, *The Selected Letters of Bertrand Russell, Vol 1*, p. 245.

> A scientist can hardly encounter anything more undesirable than to have the foundation collapse just as the work is finished. I was put in this position by a letter from Mr. Bertrand Russell.[26]

Russell appreciated the innate decency of Frege's act. As he told a friend in a letter many years later:

> As I think about acts of integrity and grace, I realize that there is nothing in my experience to compare with Frege's dedication to truth. His entire life's work was on the verge of completion,…his second volume was about to be published, and upon finding that his fundamental assumption was in error, he responded with intellectual pleasure clearly submerging any feelings of disappointment.[27]

How could a paradox be so powerful that it could destroy a life's work? Most of us regard a paradox as a type of puzzle to amuse or annoy, but hardly a cause for deep concern. The word paradox has many meanings; for example, "an assertion that seems false but actually is true" or that "seems true but actually is false." Or "a line of reasoning that seems impeccable but which leads to a logical contradiction" (hence a fallacy). However, the word paradox will be used in this book to mean a statement that, if assumed to be true, leads to the conclusion that the statement is false. If assumed to be false, it implies that it is true. Hence, "an assertion whose truth or falsity is undecidable."[28]

---

[26] quotation from Martin Gardner, *Gotcha: Paradoxes to Puzzle and Delight* (San Francisco: W. H. Freeman and Company, 1982), p. 16

[27] Nicholas Griffin, *The Selected Letters of Bertrand Russell, Vol 1*, p. 245.

[28] all quotations from Martin Gardner, *Gotcha: Paradoxes to Puzzle and Delight*, p. vii.

Russell coined a popular version of the paradox which had bedeviled Frege and called it *The Barber Paradox*. Consider a village where the barber shaves every male villager if and only if the villager does not shave himself. That's clear enough—some of the villagers shave themselves and some let the barber do the shaving, but everyone get shaved. The paradox arises when we ask whether the barber (who incidentally is also a male; if not, the paradox obviously disappears) shaves himself. If we assume that he does, then, since the barber only shaves those who don't shave themselves, the barber cannot shave himself. Hence, if he does shave himself, he doesn't shave himself. If we assume that he doesn't shave himself, we get stuck in the circle again, since the barber shaves everyone who doesn't shave himself. Hence, if he doesn't shave himself, he necessarily does shave himself.

> By analyzing the paradoxes to which Cantor's set theory had led, [Russell] freed them from all mathematical technicalities, thus bringing to light the amazing fact that our logical intuitions (i.e., intuitions concerning such notions as: truth, concept, being, class, etc.) are self-contradictory.[29]

The version of the paradox which Russell described to Frege concerned Cantor's theory of sets, which formed a central part of both Russell's and Frege's methods. You will recall that Cantor[30] had discovered a paradox concerning the set of all sets. Since it was necessarily the largest set imaginable, how could we reconcile the fact that its power set had to be still larger, yet by definition must be included in the set of all sets? Russell discovered that there was a still more pervasive version of this paradox which extended to logic itself. Remember that "a set is a

---

[29] Kurt Gödel, "Russell's Mathematical Logic", 1944, in Paul Benacerraf & Hilary Putnam, *Philosophy of Mathematics: Selected Readings*, p. 452.

[30] See chapter 7.

collection of things of the same kind", and that "the set is not the same as the members—the set is the collection, the assemblage, not the things assembled."[31] For that reason, most sets are not members of themselves. For example, the set of all even numbers is not an even number; the set of left-handed tennis players is not a left-handed tennis player. Let's refer to such sets as *normal*. But, contrary to expectations, there are sets which are members of themselves. For example, the set of all concepts which can be imagined is itself a concept which can be imagined. Therefore, it is a member of itself. We'll call such sets *abnormal*.

Russell mentally constructed a higher level set, the set of all normal sets. In other words, the members of this set were themselves sets, sets which did not contain themselves. The paradox arose, just as it did in the "Barber Paradox", when Russell asked whether this set was normal. If he assumed that it was normal, then since it was defined as containing all normal sets, it must contain itself. However, by the definition of normal set, a set which contains itself is not normal. Hence, if it's normal, then it's abnormal. Try assuming the opposite, that it's abnormal. Following a similar line of logic, you arrive at the conclusion that it must be normal. So, if it's normal, it's abnormal and, if it's abnormal, then it's normal.

In March of 1985, the public television science program "Nova" had an episode on the history of mathematics. In that episode they gave a superbly simple illustration of Russell's paradox. They asked the viewer to imagine that there was a central librarian for a state. She requested all the regional librarians to compile a directory volume which listed all the books in their particular library. When she received these directories, she found that some of the librarians had included their directory as one of the books in the library, and some had excluded it.

[31] From chapter 7.

She separated the directories into two different groups on that basis, then began to compile a master directory which listed all the directories in the second group; i.e., those directories which did not include themselves. However, when she had finished listing all the directories, she hit a snag. Should she list the master directory itself or not. And, of course, she was stuck just as Frege was stuck.

SELF-REFERENTIAL SYSTEMS

Why does such a paradox arise? The problem is because with both the "Barber Paradox" and in the paradox of the "set of all sets which do not contain themselves", we are dealing with *self-referential* systems. Self-referential systems are systems which refer to themselves. For example: consider the following sentence: “This sentence has five words.” The sentence does, in fact, have five words, so the statement, though self-referential, is true. There is also no problem with false self-referential sentences, such as “This sentence has four words,” which is false because it actually has five words. However, some self-referential statements present a more paradoxical situation: "This sentence is false." It's clearly self-referential, but is it true or false? When we try to decide, we find ourselves stuck in a vicious cycle. The truth or falsity of the statement is indeterminable.

As this simple example demonstrates, self-referential systems quickly become too complex for logic. The self-referential system par excellence is the mind. Since the mind is the only tool that we have for thinking about anything, that means that the mind is also the only tool which we can use to think about the nature of the mind. Jung was fond of emphasizing “the fundamental fact that in psychology the object of knowledge is at the same time the organ of knowledge, which is true of no other science."[32]

However, Jung was wrong in assuming that the problem is confined to psychology; if we think deeply enough about anything, we are led ineluctably to think about the process of thought. As an example which we have already discussed at some length, Kant ran up against the same issue in philosophy. Once the Renaissance ideal led men and women to separate themselves from nature in order to observe nature objectively, it was inevitable that they would eventually turn that observation upon themselves. Berkeley and Hume were merely the first in the Western World to demonstrate the nasty little paradoxes that appear when the mind thinks about itself.

Anthropologist Gregory Bateson stressed that all living creatures are complex feed-back systems. A feed-back system provides information to itself about its performance, which in turn is used to change that performance. That change leads to further feed-back, thus forming a continuous loop. The information, which the system feeds back to itself so that it can better adapt to reality, is of a higher order of reality than the behavior that it comments on. Hence living creatures are complex self-referential systems.[33]

As physicist Werner Heisenberg [1901–1976] discovered, even confining our observation to non-living systems is not sufficient to protect us from the problem. At the sub-atomic level, our observation changes the object under observation, which is another way of saying that the subject, the object, and the act of observation are actually a single system—the split between

---

[32] C. G. Jung, *Collected Works, Vol. 10: Civilization in Transition* (Princeton: Bollingen Series, Princeton University Press, 1964), par. 1025

[33] see Gregory Bateson, *Steps to an Ecology of Mind* (New York: Ballantine Books, 1972) and *Mind and Nature: a Necessary Unity* (New York: E. P. Dutton, 1979).

subject and object is artificial. Thus, even in physical experiments, the subject is once again involved in a self-referential system.[34]

There is no necessary paradox in self-referential systems as long as the systems are finite. Mathematicians have been able to successfully prove the consistency and completeness of many finite mathematical systems. If the system is small enough and simple enough, they simply develop every implication of the system for every member of the system. Thus, if human beings can be viewed independently from the world around them, and if they are nothing more than the finite amount of "things" (such as proteins and nucleic acid, etc.) that make up their physical composition, there is a way out of the problem of self-referencing. The mind is then reducible to the brain, and the problem of the brain thinking about itself eventually winds down since it is at most a finite process.

That is still the most popular scientific position. The point of this book is that such a position seems increasingly untenable as we dig deeper into any of the systems that explore the universe, from philosophy to mathematics to physics to psychology; in fact, to any field at all. All fields of thought eventually involve themselves in infinite self-referential systems.[35] And the only way out of that dilemma lies in an archetypal hypothesis, such as developed by Jung and Gödel in their separate fields. But we are getting ahead of ourselves and need to get back to Russell's project.

---

[34] Werner Heisenberg, *The Physical Principles of the Quantum Theory*, reprint of 1930 edition (New York: Dover Publications, 1949).

[35] and if not literally infinite, of such a large size as to be forever beyond the ability to calculate. Allan Turing was the first to realize that there are such practical limits to what can be achievable by science.

PRINCIPIA MATHEMATICA: LOGIC'S TOWER OF BABEL

Though Russell was initially buoyed in reading Frege, by the discovery that he was not alone in his quest, Frege was no more able to solve Russell's paradox than Russell himself. Russell found himself in a strange position. Except for the paradox, his grandiose goal of subsuming all nature within logic seemed achievable. For example, in 1902, he was teaching a course at Cambridge where:

> …I began with 22 Pp's [i.e., primitive proposition, or axioms] of general logic…and I deduced from them all of pure mathematics, including Cantor[36] and geometry, without any new Pp's or primitive concepts. All this will appear in the book that I plan to publish with Whitehead. We could even deduce rational mechanics [i.e., Newtonian physics]….[With some alternative definitions, we could do] the same for non-Newtonian mechanics. [37]

On the other hand, if he tried to resolve the paradox—and resolve it he must, or give up on his goal—the task became Herculean in size and scope. Russell struggled on, never dealing fully with any of his three calls from the unconscious. He suppressed his mystical experience, and tried to compensate by writing a never-completed book, aptly titled *Prisons*, which contained what might be thought of as a logician's religion; i.e., Russell accepted that humans have a need for the religious feeling, but he also acknowledged that there is no actual basis for religion. In other words, live a lie because it's a necessary lie. This was pretty pallid stuff that satisfied no one, including himself.

---

[36] I.e., Cantor's transfinite set theory.

[37] Nicholas Griffin, *The Selected Letters of Bertrand Russell, Vol 1*, p. 227.

Russell seemed to be a man torn between his mind and his heart.[38] In 1903, his mood might be summed up in a letter he wrote to a female friend:

> But as for me, I have felt no emotions of any kind, except on rare occasions, for some time now; and this is a state of things most convenient for work, though very dull.[39]

His work consisted of an endless, mind-deadening struggle to somehow resolve the paradox. Russell confessed that, during 1903 and 1904, he used to sit all morning staring at a blank piece of paper, break for lunch, then stare again all afternoon. At day's end he sat with nothing more than when the day began.[40] Things began to improve slightly in 1905, when he discovered what has come to be called the *Theory of Descriptions*, then improved more in 1906, when he discovered the *Theory of Types*. However, these discoveries were stopgaps, not intellectual breakthroughs like his discovery of Peano in 1900. Though they enabled him to once more continue his project, even Russell recognized the limitation of his solution.

In effect he solved the paradox by defining it away. He began with classes, which were much the same as sets except that they were defined such that they could neither be members of themselves, nor have any actual existence. A class was defined to be on a higher order than its members. Classes of classes were on a higher order than classes, and so forth. Hence there could be no such thing as the set of sets which were not members of themselves, since by definition his

---

[38] This state of affairs wasn't to be broken until 1911, when the *Principia* was completed, and he began a passionate affair with Lady Ottoline Morrell.

[39] Bertrand Russell, *The Autobiography of Bertrand Russell, the Early Years*, p. 225.

[40] Bertrand Russell, *The Autobiography of Bertrand Russell, the Early Years*, p. 200.

classes did not include themselves. The problem wasn't in any way truly resolved; it was just pushed off to infinity.[41]

This solution of shoving the problem out of sight, like an ostrich sticking its head in the sand, was the same that some of Russell's philosophical allies called the *Vienna Circle*[42] used later in the 1920's in their creation of a philosophy known as *logical positivism*. Logical positivism insisted that since the ultimate questions about reality—metaphysical questions—were not open to either logical or experimental verification, they were not questions at all. Logical positivists confined all questions about the ultimate nature of reality to the dust heap of non-sense; i.e., literally not sensible, since they did not refer directly or indirectly to sensory perception. Their attitude was that if questions are too bothersome to fit into a theory, just define them away as non-questions.

However, Russell at least felt he had a way out, even if it wasn't the triumph he had hoped for.

> …After this it only remained to write the book out. Whitehead's teaching work left him not enough leisure for this mechanical job. I worked at it from ten to twelve hours a day for about eight months in the year, from 1907 to 1910.[43]

And what had they then accomplished? Well, the manuscript was so huge that they had to hire "an old four-wheeler"[44] to cart it off to the publisher. The publisher knew the audience for such a

---

[41] We will discuss this more in chapters 12 and 16, when we consider Gödel's insights, and his famous incompleteness proof.

[42] See chapter 16.

[43] Bertrand Russell, *The Autobiography of Bertrand Russell, the Early Years*, p. 201.

[44] Bertrand Russell, *The Autobiography of Bertrand Russell, the Early Years*, p. 201.

book was so tiny that they would be taking a loss on it. They were willing to absorb part of the loss, but not all. The Royal Society picked up another part, but ultimately Russell had to pay £100 for the privilege of having this product of ten years work published. And, having done that, he said later that he knew of only six people in the entire world who had read the whole book.[45] In fact, it is often said among mathematicians and logicians that no one except Russell and Whitehead has ever read all of the *Principia*, and even they may not have read all of each other's work. Though his life was to take many fascinating turns over another sixty years, he said that "my intellect never quite recovered from the strain."[46]

While it was presented as a joint work, it was clear that it was Russell's project from beginning to end. Whitehead's later work went in a direction opposite to Russell; his philosophical position accords well with the viewpoints presented in this book, while Russell remained confident to his death that logic was sufficient to the solution of all of humanity's problems. Yet it seems clear now that the *Principia* not only didn't come close to accomplishing Russell's full goals, it is questionable whether it accomplished anything of substance. In Gödel's words:

> As to the question of how far mathematics can be built up on this basis [i.e., that of Russell and Whitehead in *Principia Mathematica*]…, it is clear that the theory of real numbers in its present form cannot be obtained…the question whether (or to what extent) the theory of integers can be obtained…must be considered as unsolved at the present time. It is to be noted, however, that, even in case this question should have a positive

[45] Nicholas Griffin, *The Selected Letters of Bertrand Russell, Vol 1*, p. 297.

[46] Bertrand Russell, *The Autobiography of Bertrand Russell, the Early Years*, p. 202.

> answer, this would be of no value for the problem of whether arithmetic follows from logic…[47]

If Russell had been able to develop a consistent and complete arithmetic from the axioms of symbolic logic, then logic would indeed rule the universe. Logic would explain arithmetic, arithmetic would explain geometry and calculus, which would in turn explain physics and astronomy and the other sciences, and the sciences would explain the world. The universe would have been reduced to the manipulations of logical symbols. Like the biblical tower of Babel, Russell and Whitehead were attempting to construct an edifice beyond humankind's capability. Like the biblical tower, it also collapsed because of the limits of language. Kurt Gödel was to destroy what was left of Russell's dream much as Russell destroyed Frege's. As we will see, this was because Gödel's view of reality was broader than Russell's, as Jung's was broader than Freud's.

---

[47] Kurt Gödel, "Russell's Mathematical Logic", p. 463.