3-Dimensional Eye Movement Monitor
Model EM7
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1. Introduction

The electromagnetic eye movement monitor was invented by David A. Robinson. This method offers high resolution, linearity, low drift, and low noise when compared to methods which record the electro-oculogram, or measure the light reflectance from parts of the eye. Both rotation and torsion of the eye can be measured. With appropriate geometrical calculations, rotation and torsion of both eyes and head can be measured in the head-free situation; such measurements are difficult or impossible with pupil trackers or body-trackers which use charge-coupled-device (CCD) cameras.

An eye-movement monitor can also track arm and body movements.

A coil of wire is either implanted surgically around one eye for animals, or embedded in a contact lens for humans, so that the coil rotates with the eye. A second coil can be added to measure torsion.

Robinson used two pairs of circular Helmholtz coils to generate horizontal and vertical alternating magnetic fields—the 2-dimensional method. The left and right coils produced the horizontal field, while the top and bottom coils produced the vertical field. The subject’s eyes were placed near the center of both coils. These Helmholtz coils produced a nearly uniform field near the center.

For horizontal rotation of the eye coil, a voltage is induced proportional to the magnetic flux traversing the coil. When the coil rotates, more or less flux traverses the coil, which is proportional to the cosine of the angle $\beta$ between the eye-coil axis and the field-coil axis. If and only if the eye movements are confined to the horizontal plane, then the eye-coil voltage is proportional to the sine of the angle $\alpha$ (angles in radians) between the axis of the eye coil and the straight-ahead direction, because $\beta = \pi/2 - \alpha$.

\[
\text{output proportional to } \cos(\beta) = \cos(\pi/2 - \alpha) = \sin(\alpha) \approx \alpha \text{ for small angles}
\]

Remmel simplified Robinson’s system by replacing the 90 deg phase-shifted signals with 2 separate phase-locked frequencies at 50 and 75 KHz. The high-fidelity amplifier was replaced with switching transistors, while the commercial lock-in amplifier was replaced with an electronic switch and several operational amplifiers.

For $\alpha < 30$ deg, the eye-coil voltage is linear to 5%—the linear part of the sine function. Although for larger angles a correction can be applied, the measurement is ill-defined near 90 deg. Worse yet, a horizontal eye rotation to +45 deg gives the same reading as a rotation to +135 deg (assuming the head turns too)!

To eliminate such ambiguities, Adrian Lasker and David Zee added a third field perpendicular to the other two, so that angles in all directions—front, back, right, left, up, down—could be accurately measured—the 3-dimensional method. This method permits measuring eye and head rotation and torsion in the head-free situation.

In Robinson’s system, the horizontal and vertical fields were at the same frequency, but 90 deg out of phase. This will not work for 3-dimensional monitors. Instead, our monitor generates the 3 fields at 48, 60, and 80 KHz, and thus separates the X, Y, and Z components by frequency. A master oscillator at 480 KHz is frequency-divided by 10, 8, and 6 to produce square waves at the 3 frequencies, all phase-locked together. In the eye-coil amplifier, the 3 frequencies are separated

\[\text{References:}\]
through use of lock-in amplifiers (also called phase-sensitive detectors). The signal is then low-pass filtered to remove the carrier frequencies to give analog voltages $V_x$, $V_y$, and $V_z$, which are proportional to the 3 direction cosines.

2. Specifications and Warranty

The instrument consists of four parts: power box, electronics box, field box and coils, and preamplifiers (one for each amplifier). The power box supplies +/−25 V DC, filtered but not regulated, by a cable to the electronics box; the voltage regulators are inside the electronics box. Because the power box is separate, the user has the option to place the electronics box on a vestibular turntable without having to send 120/240 V AC over the slip rings, and without having the mass of the power supply on the turntable. The slip rings can carry the eye-movement output signals (+/−10 V range) over the slip rings, thus eliminating the need to send the tiny eye-coil signals over slip rings.

**Power:** 100-120 V AC or 220-240 V AC at 50 or 60 Hz; the 120/240 voltage is selected by changing the transformer primary connections inside the power box. Current is 280/140 mA, with a 500 mA fuse. The instrument regulates down to 80/160 V AC.

**Voltage regulators in electronics box:** +12 and -12 V DC. Accuracy ±0.1 V. Ripple and noise are < 100 µV peak-peak.

**Field coils and field box:** The cable from the electronics box to the field box sends +12 and -12 V DC and three TTL square-wave voltages at 48, 60, and 80 KHz. Inside the field box, a pair of MOSFET switching transistors generates a 12 V peak-peak square wave; this wave goes to a transformer which steps down the voltage to about 4 V at 6 A for the field coils. Because a capacitor has been placed across each coil to attenuate radio-frequency noise (not a resonant circuit), the waveform to the coils is a square wave with a superimposed damped sine wave (ringing).

The field-coil currents pass through the aluminum bars, which are uninsulated and are safe to touch, having only about 4 volts. The weak magnetic fields have negligible effect on body tissues and functioning.

**Eye coils:** Remmel Labs does not manufacture eye coils. See section 3 for sources and methods.

**Eye-coil amplifier and demodulator:** The first amplifier stage—the preamplifier with about 40X gain—is inside of a copper pipe, which should be placed next to the subject. The copper shielding greatly reduces noise caused by induced voltage in the eye-coil cables caused by the magnetic field. In early-model monitors, this unwanted noise caused the need for excessive adjustment of the OFFSET potentiometer, and caused annoying changes in the signal when the eye-coil cables were moved. The RG58/U cable supplied with the monitor should be used to connect the preamplifier to the electronics box, because other cables will give inadequate shielding.

Amplifier output impedance is 500 Ω; output range is +/−10 V, suitable for most computer A/D converters.

Gain is defined as DC output divided by zero-to-peak square-wave input. With the GAIN potentiometer at maximum (1000), the gain is about 78,000 for channel X, 68,000 for channel Y, and 52,000 for channel Z.

The OFFSET potentiometer adds a DC voltage to the output, like the position knob on an oscilloscope. The output can be adjusted over +/−6 V.

The low-pass filter after the demodulator has a transfer function $1/(1 + \tau s)^3$, where $s = j\omega = 2\pijf$, and $j = \sqrt{-1}$. Because $\tau = 0.5\text{ms}$, the cutoff frequency $f_c = 1/(2\pi\tau) = 320\text{ Hz}$. This filter virtually eliminates the 48, 60, and 80 KHz carrier frequencies from the output. This filter has
negligible effect on eye movement signals, because they have little energy above 30 Hz.\textsuperscript{4}

**Noise voltage:** The monitor noise has been discussed previously.\textsuperscript{5} The noise voltage $V_n$ is the RMS output voltage divided by the amplifier gain, and equals 18 nV RMS. The noise is measured with the preamplifier grounded, because an eye coil has essentially zero resistance. There is negligible noise from the field power supply. Essentially all noise is generated in the transistor inside the preamplifier. The noise spectrum (see figure) is constant at low frequencies, and drops off sharply at higher frequencies because of the 320 Hz low-pass filter.

The noise spectrum was measured using an A/D converter with a computer. The computer sampled the signal output every 1.042 ms for a total of 256 points. These points are random with presumably a Gaussian, normal distribution, except that the successive points are correlated because of the finite frequency response. Then the fast Fourier transform (FFT) of these points was computed; the transformed points are the voltages at each frequency, which may be positive or negative numbers, and presumably also have normal distributions. Then 100 such spectra were measured and computed. At each frequency, the root-mean-square (standard deviation) of the 100 points was computed, and plotted on the graph as a function of frequency. The points are still somewhat scattered because of random fluctuations. Note that there are no peaks at 60 or 120 Hz, implying that no noise from the mains supply has gotten into the signal. The noise drops off according to the third-order low-pass filter with the 320 Hz cutoff frequency.

The *typical system* for describing performance was chosen to be a Skalar or Chronos-Vision contact lens with radius $R = 9$ mm and $N_2 = 9$ turns, and a field coil with $D = 60$ cm, which is large enough for a man’s head and shoulders, or for a cat or monkey.

*Noise angle* $A_n$ is that angle, which produces an output, which equals the RMS noise voltage $V_n$ of the amplifier. The noise angle depends on $D$, $R$, and $N_2$; for a typical system $A_n = 0.98$ arc seconds RMS. The monitor will easily measure microsaccades!

**Amplifier drift:** For testing purposes, a TEST square wave of 60 $\mu$V zero-to-peak amplitude at 48, 60, or 80 KHz is provided from the electronics box. For the drift measurements, this test

\textsuperscript{4}Zuber et al., Frequency characteristics of the saccadic eye movement, Biophysical J. 8 (1968) 1288
square wave was used, instead of a field coil or eye coil. The GAIN potentiometer was maximum.
The OFFSET potentiometer was set so that there was zero output with TEST set to off. Results:

<table>
<thead>
<tr>
<th></th>
<th>X output</th>
<th>Y output</th>
<th>Z output</th>
</tr>
</thead>
<tbody>
<tr>
<td>cold start at t = 0</td>
<td>5.55 V</td>
<td>5.16 V</td>
<td>4.25 V</td>
</tr>
<tr>
<td>t = 90 min</td>
<td>5.35</td>
<td>4.96</td>
<td>4.02</td>
</tr>
<tr>
<td>t = 880 min</td>
<td>5.33</td>
<td>4.92</td>
<td>3.98</td>
</tr>
</tbody>
</table>

During 90 min warmup, the outputs all decreased by 4-5%, and by only 1% thereafter. (Turn the
monitor on 90 min before starting your experiment for best results.) For the typical system, an
X output of 5.55 V corresponds to an angle of 1.47 deg. Drift of 1% thus corresponds to an error of
0.84 minutes of arc. The monitor thus will stably measure 1 arc minute eye movements over times
of 10 minutes.

Drift was also measured with input shorted, which is equivalent to an eye coil pointed straight
ahead. GAIN was maximum and the output was initially zeroed using OFFSET. The drifts in the
outputs are similar to those above; results:

<table>
<thead>
<tr>
<th></th>
<th>X output</th>
<th>Y output</th>
<th>Z output</th>
</tr>
</thead>
<tbody>
<tr>
<td>t = 0</td>
<td>0 V</td>
<td>0 V</td>
<td>0 V</td>
</tr>
<tr>
<td>t = 90 min</td>
<td>-0.07</td>
<td>-0.05</td>
<td>0.03</td>
</tr>
<tr>
<td>t = 880 min</td>
<td>-0.09</td>
<td>-0.07</td>
<td>-0.06</td>
</tr>
</tbody>
</table>

Cross talk occurs when an X output signal appears at the Y output, or vice versa. Cross talk
was measured with the 60 µV zero-to-peak signal going into the preamp. GAIN was maximum, and
the outputs were zeroed with no signal, using OFFSET. All combinations:

<table>
<thead>
<tr>
<th></th>
<th>X output</th>
<th>Y output</th>
<th>Z output</th>
</tr>
</thead>
<tbody>
<tr>
<td>input=X</td>
<td>5.43 V</td>
<td>0.03 V</td>
<td>-0.03 V</td>
</tr>
<tr>
<td>input=Y</td>
<td>-0.04</td>
<td>5.02</td>
<td>-0.07</td>
</tr>
<tr>
<td>input=Z</td>
<td>-0.04</td>
<td>0.04</td>
<td>4.10</td>
</tr>
</tbody>
</table>

The cross talk is thus < 2 %.

**Field coils:** Remmel Labs makes field coils from aluminum bars, which are self-supporting, and
also carry the currents. We will make coils of any length D up to 1.8 m, which are large enough
for people to sit inside comfortably. Coils larger than D = 60 cm are shipped disassembled. 60 cm
coils are suitable for monkeys and cats, with enough room for the animal’s chair or box. For a more
uniform field and more roominess, use a bigger coil. Coils with D = 36 cm can be mounted on a
head helmet. We will make special coils for special needs.

Each square coil is one turn consisting of 4 straight sections, each of length D. Two coils make
up a pair called **dual square coils**, which are designed to make a nearly uniform field near the center.
One pair is for the X field, one pair for Y, and one for Z. There thus are 6 coils all together, consisting
of 24 aluminum bars (see figure). Coils with $D < 1$ meter are made from square bars 1.27 cm (0.5
inch) width; larger coils are made from 2.54 cm (1 inch) bars. Polycarbonate plastic brackets (not
shown in figure) hold the six coils together. The coils have perfect cubic symmetry.

The next figure shows the details of the coils and plastic brackets, with dimension distorted to
show details at corners.

**Field-coil mass** in MKS units:
D = bar length in meters
\(d = \text{density of aluminum} = 2702 \, Kg/m^3\)
W = width of bar in meters; there are 24 bars
\(m = 24dW^2D\), neglecting plastic, screws and field power box.

**Moment of inertia I** about vertical axis of cube: This number is needed to evaluate the loading of a motor-driven turntable for vestibular experiments. As a slight overestimate, concentrate all the mass of all the coils (see above equation) at one corner of a square at a distance \(D/\sqrt{2}\) from the axis. Neglect masses of screws and brackets.

\[
I = m(D/\sqrt{2})^2 = 12dW^2D^3
\]

The coils require no shielding because, by Faraday’s law of induction, the changing magnetic field produces an electric field in the air, which equals the electric field within the adjoining aluminum.

**Warranty:** If within 3 months you are dissatisfied with any Remmel product, you may return it for a full refund.

If at any time the instrument needs repairs, parts and labor will be provided free of charge. The customer is responsible for shipping.

The warranty is void, if modifications are made without permission.

3. Getting Started

**Assembling big coils** (small coils are shipped assembled): Parts list:
12 short aluminum bars with 5 mm threaded holes in ends
12 slightly longer bars
8 polycarbonate plastic pieces with 5 holes–brackets
8 polycarbonate pieces with 4 holes–braces
12 very short screws–5x10 mm
74 short screws–5x16 mm
18 long screws–5x35 mm
6 very long screws–5x40 mm
12 nylon washers
Drawings are not to scale (for clarity).
White bars and squares: coils.
Gray polygons: plastic brackets holding coils together (one for each corner).
Part of the bottom, top and side bars can be seen through the translucent brackets.
Black rectangles: side view of plastic braces fastened onto right and left coils only.

D = outside length of square aluminum coil made from 4 bars. The locations where bar are fastened together are not shown.
W = width of bar (1.27 cm for smaller coils, or 2.54 cm for larger coils).
S = spacing between bars = thickness of plastic brackets = 0.635 cm

The usable internal volume is slightly larger than DxDxD.
3 mm hex wrench

There are black letters and numbers written on the bars, braces, and brackets, such as "I3". The 6 sets of 4 bars are labeled I (inferior), S (superior), R (right), L (left), A (anterior), and P (posterior). Each set of 4 bars has ends labeled: WIRE, 2, 3, or 4.

Find the 2 long and 2 short bars labeled I. From these, take the 2 bars with a WIRE label (one bar is shorter than the other). Match up the WIRE ends—this is the insulated corner where the wires will be attached. Take note of the fact that in the longer bar there is a plastic tube force-fit into one of the holes for electrical insulation. Use a very long screw and two nylon washers to fasten this corner together. One washer should be between the two bars; the other washer goes between the screw’s head and the bar to insulate the bars. Thus an electric current cannot flow between these two bars.

Use long screws to fasten the other three corners together by matching the I2, I3, and I4 labels. Current can flow around these corners.

Assemble the other 5 squares in the same manner. Notice that each square has a big label, such as INFERIOR (BOTTOM), SUBJECTS RIGHT, SUBJECTS LEFT, SUPERIOR (TOP), ANTERIOR (FRONT), and POSTERIOR (BACK). These large labels will be visible from the front when the coils are assembled.

For the R (right) and L (left) squares only, braces are provided. Fasten the braces using 4 short screws so that, for instance, the "R2" labels on the bar and the brace are together. You may have to rotate a bar in order to get the labels to match.

Place the I (inferior) square on the floor. Fasten the 4 brackets with the labels I-WIRE, I2, I3, and I4 to the corresponding corners, using short screws.

Place the R (right) square vertically and fasten it to the I square’s bracket so that the labels on the brackets match up, i.e., the R-WIRE label on a bar matches the R-WIRE label on a bracket.

Do the same for the L (left) square.

Fasten the remaining 4 brackets to the tops of the right and left squares, matching labels. Fasten the S (superior) coil to the top, matching labels on the brackets.

The A (anterior) and P (posterior) squares are fastened to the brackets with only one screw at each corner. Check that the big labels SUPERIOR (TOP) etc. are visible from the front in their proper orientations.

Near the center of the lower bar of the POSTERIOR square are two small holes threaded for 5 mm screws; the field-coil power box is mounted here. This box has 6 black wires connected to a screw-terminal connector. Using two short screws (5x16 mm), fasten the plastic piece to the bottom bar so that the box is toward the inside of the field coils. The wires should be on top and projecting inward. The plastic insulates the box from the field coil.

Use very short screws and the 3 mm hex wrench to fasten color-coded wires to bars at the threaded holes marked by colored rectangles. Connect three more wires to connect opposite squares, such as the right and left squares, which function as a pair (the fields from these two squares add together). Use electrical or package-wrapping tape or cable ties to fasten the loose wires to the bars.

Verify that nylon washers have been inserted for electrical insulation at all 6 corners where wires have been connected. Retighten all screws to ensure good electrical contact.

Setup: Place the field coils at least 2 m from the electronics box to avoid inducing noise into that box. The field coil may be placed directly on the floor, on a nonmetallic table, or supported by nonmetallic legs fastened to the aluminum bars.

Place the power supply box on the floor or far from the electronics box to avoid inducing 60 Hz noise into the circuits.
If large pieces of metal are in the floor, walls, or ceiling of your lab, you may have to place the field coils 2 m away from there. Small metal objects such as screws may be placed inside the coils with little effect on the fields. However, a large aluminum plate or iron pipe will distort the field. An amplifier inside the field can be shielded with a copper or iron pipe or aluminum box. If in doubt, insert the object and measure the result. Often the effect of a piece of metal can be automatically corrected during calibration.

Parts:
- mains cord and power box
- electronics box
- cable connecting power to electronics box (3-pin and 5-pin connectors)
- preamplifiers (inside copper pipes) and 7 meter long RG58/U cables
- field-coil box and black connecting wires
- cable connecting electronics to field-coil box (7-pin connectors)
- field coils

Connect the power box to the electronics box, and the electronics box to the field box.

There is always the possibility that the magnetic field will induce an unwanted voltage into your eye-coil cable. In previous models of Remmel eye-movement monitors, a long cable ran from the eye coil to the electronics box, which often resulted in such unwanted voltages. To reduce this possibility, a preamplifier was built with 40X gain and shielded inside of a copper pipe. You must place this preamplifier as close to the subject’s eye coil as possible. Then the amplified signal is sent through the long RG58/U cable, which is well shielded, thus virtually eliminating unwanted voltages. Do NOT substitute a thinner cable for the RG58/U.

Either use your eye coil or make a test eye coil (called a search coil) of about 2 cm diameter and one turn. Connect the test signal output called ”400X” to your oscilloscope to see the eye-coil voltage amplified 400 times. Put the eye coil in the center of the field coils to see the 48, 60, and 80 KHz signals as you rotate the eye coil in various directions. For clarity, unscrew one wire from each of the X and Z coils, so that the Y signal can be viewed alone; the signal should be a square wave at about 60 KHz with superimposed ringing. Rotate the coil to verify the cosine dependency.

**Eye coils: Human subjects:** Contact lenses require one coil of wire for horizontal/vertical measurements, and a second coil for torsion measurements. Eye coils are manufactured by:

Chronos Vision
Wiesenweg 9, 12247 Berlin, Germany
phone +49(0)30 / 7694 25-25; fax +49(0)30 / 7694 25-26
www.chronos-vision.de
info@chronos-vision.de or andrew-clarke@charite.de

The ”lens” is made of silicone rubber with a hole in the center for light to pass. Contact Chronos Vision for instructions for safe use; Remmel Labs is not responsible for eye injuries.

**Eye coils: Animals:** Cooner Sales makes multistranded stainless-steel wire coated with Teflon, type AS632 or thicker:

Cooner Sales
9265 Owensmouth Av., Chatsworth, CA 91311 USA
phone: 818-882-8311 fax: 818-709-8281
www.coonerwire.com

The eye-coil resistance should be about 25 Ω.

A good way to implant the wire is to pass it underneath the tendons of the four recti muscles, then twist the leads tightly together, and push a loop of wire toward the back of the orbit so that the
eye can easily rotate without pulling on the wire. Then pass the wire under the skin to a connector on top of the head. (For an experiment which will use the animal for only one day, an eye coil can be preformed and then either sutured or glued with superglue onto the eyeball.)

Make sure to eliminate voltage induced into your eye-coil wire by a small loop of the wire. All eye-coil wires must remain stationary during the experiment. Shielding any exposed wire with at least 1 mm of copper or 10-20 layers of aluminum foil should help reduce stray pickup. Remember that magnetic fields at about 50 KHz can penetrate a short distance into copper and aluminum (skin depth about 0.4 mm). Please e-mail Remmel Labs if you need help (remmellabs@juno.com).

Check eye-coil insulation before each experiment. For animals with implanted coils, measure the resistance between the eye coil and a piece of metal in contact with body fluid. For contact lenses, put one ohmmeter lead in a beaker of saline, connect the other lead to the eye coil, and immerse the coil. The resistance should be greater than 20 MΩ. If the resistance is finite, the coil must be discarded, or for animals, a new coil must be implanted. A leaky coil will give spurious results!

**Data taking:** Use a computer with a 16-bit A/D converter with a +-10 volt range and a 32-channel multiplexing switch (MUX). Digitize each voltage at 2 ms intervals, or longer intervals if you do not need such good time resolution. (The monitor has a 3-pole low-pass filter with a 300 Hz cutoff frequency.) Purchase enough RAM to store the 16-bit integers into a continuous array for the total time of one data-taking segment. Then write the whole array to a disk file. Analyze this data later.

There are many data-acquisition programs available for PCs.

You also have the option to digitally filter your data, e.g., to remove the high-frequency noise. Remmel Labs can suggest digital filtering algorithms, some implemented in the C++ language.

**Angles and coordinates:** Physics: The X axis is to the subject’s right. The Y axis is up. The Z axis is in front of the subject (a left-handed coordinate system), although you can change the direction of the monitor output signals by flipping the "INVERT" switches. The three magnetic fields are phase-locked at about 48, 60, and 80 KHz in the X, Y, and Z directions, respectively. The X, Y, and Z signals are completely separate because they are at different frequencies, and are calibrated independently.

According to Faraday’s law of induction, the eye-coil voltage is:

$$V = N A \cos(\beta) dB/dt$$

N is the number of turns in the eye coil, and A is its area. B is the magnetic field. The derivative dB/dt means that the coil does not respond to DC (constant) fields, but only to changing (alternating) fields. \(\beta\) is the angle in radians between the magnetic field direction and the coil axis. The cosine occurs because as the coil rotates, less "magnetic flux" passes through the coil.

When the eye-coil axis is aligned with the field \((\beta = 0)\), maximum flux traverses the coil, and V is maximum.

When the eye-coil axis is perpendicular to the field \((\beta = \pi/2)\), no field passes through the coil, and V = 0.

When \(\beta\) becomes still larger, V becomes negative.

Thus V is proportional to the "direction cosine".

The eye-coil voltage is amplified, rectified (phase-sensitive demodulated), and low-pass filtered. \(V_x\) is the DC voltage at the X output:

$$V_x = K C_x$$
where \( C_x = \cos(\beta_x) \), where \( \beta_x \) is the angle between the eye-coil axis and the X axis, and where \( K \) is a constant which depends on the number of turns \( N \), the coil area \( A \), the magnetic field \( B \), and the amplifier gain.

The OFFSET potentiometer is adjusted, so that \( V_x = 0 \) when the eye is looking straight ahead (\( \beta_x = \pi/2 \)) or the coil is outside the field.

The GAIN potentiometer is adjusted so that \( V_x \) is some convenient value, say 8 volts, when \( \beta_x = 0 \), so that the full range of the A/D converter is used; call this voltage \( V_{\text{max}} \). Then

\[
C_x = V_x/V_{\text{max}} = \text{direction cosine}
\]

\( C_x, C_y, \) and \( C_z \) uniquely specify the eye direction forwards, sideways, up, down, or backwards, even in the completely head-free situation.

By the Pythagorean theorem: \( C_x^2 + C_y^2 + C_z^2 = 1 \).

To summarize, the Z signal is needed, because \( V_x = 0 \) and \( V_y = 0 \) when the eye is straight forward or straight backward—an ambiguity!

**Spherical coordinates:** Define \( \theta \) as the angle between the Z axis and the eye-coil axis. Thus \( \theta = 0 \) is straight ahead. Project the vector of the eye-coil axis onto the XY plane. Define \( \phi \) as the angle clockwise from the X axis as viewed from the front (sorry, left-handed coordinate system). Then by trigonometry the direction cosines are:

\[
C_x = \sin(\theta)\cos(\phi); \quad C_y = \sin(\theta)\sin(\phi); \quad C_z = \cos(\theta)
\]

Thus inverting: \( \theta = \arccos(c_z) \) and \( \phi = \arctan(C_y/C_x) \) (arctan going over four quadrants).

These angles are good for all directions front and back. The correct eye angles re the X, Y, and Z axes are measured in the head-free system, even when the subject rotates his head and trunk!

The rotations of the head and body parts can be measured by fixing coils to the body.

**2-dimensional calibration:** For pure horizontal eye movements within 30 deg of straight ahead, we define the horizontal angle re straight ahead: \( \alpha = \pi/2 - \beta_x \). We use the trigonometric identity: \( \cos(u) = \sin(\pi/2 - u) \). We further use the linear approximation for the sine of small angles:

\[
V_x/V_{\text{max}} = C_x = \cos(\beta_x) = \sin(\pi/2 - \beta_x) \approx \pi/2 - \beta_x = \alpha_x
\]

The same small-angle approximation, accurate to 5% over 30 deg, can be used for vertical eye movements.

Calibration is done by adjusting OFFSET so that \( V_x = 0 \) for straight ahead, and by adjusting GAIN so that the output voltage is some convenient value for, say, 10 deg to the right.

**4. Filtering and reducing noise**

The monitor produces alternating magnetic fields, which will induce voltages into nearby loops of wire or other conductors. Other electronics, including the monitor box itself, should be placed > 2 meters from the coils.

For some experiments, single-unit recording or electro-encephalogram (EEG) measurements must be made. The signal cables used for such measurements must be heavily shielded, because magnetic fields at 60 KHz will penetrate some distance into copper or aluminum shielding (skin depth 0.266 mm for copper). The frequency components in action potentials and in EEG waves are all below 10 KHz, so low-pass filtering at \( f_c = 10 \) KHz will remove the induced noise.

A Butterworth filter of order \( N \) is designed to have a flat response at low frequencies and a *nice* steep fall-off at high frequencies. It has a transfer function:
\[ |T(f)|^2 = 1/(1 + (f/f_c)^{2N}) \]

The following N=4 Butterworth filter has a \( f_c = 10 \) KHz cutoff frequency:

- **C1-4**: 2.2 nF
- **Q1**: LF347N or similar operational amplifier
- **R0**: 750 ohm
- **R1**: 500 ohm
- **R2,3,8,9**: 7.5 Kohm
- **R4,6**: 10 Kohm
- **R5**: 12 Kohm
- **R7**: 1.5 Kohm

Power should come from a regulated +5 to 15 volt supply or two 9 V batteries.

## 5. Troubleshooting

1. The power box produces about 25 V DC between pins A and C, and -25 V between pins B and C. Measure these voltages. Check whether the fuse is blown; replace with a 500 mA slow-blow fuse, or if this still blows, try a 1A fuse.

2. Connect the cable between the power box and the electronics box. The electronics box has +12 and -12 V regulators. At the 7-pin connector, you should measure +12 V between pin A and ground (the aluminum box), and -12 V between pin B and ground.

3. Pins C, D, and E of that connector carry the square-wave sync signals, which are TTL voltages (0 to 3 V DC) at 48, 60, and 80 KHz, respectively. Verify these voltages and frequencies with an oscilloscope, accurate to about 10%.

4. Connect the 7-pin cable between the electronics and field boxes. Measure the ground resistance between the mains plug, the power box metal, the electronics box metal, and the field box metal. The resistance should be < 1Ω.

5. With no eye-coil input, verify that the output voltage at each BNC connector varies between about -6 V and +6 V as the OFFSET potentiometer is rotated.
6. Test signals: Turn all GAIN potentiometers to maximum. Connect a DC voltmeter to the X output of the top amplifier.

Connect a BNC cable from the TEST SIGNAL output on the electronics box to a preamplifier input (preamplifiers are built inside copper pipes). This TEST SIGNAL is a 60 $\mu$V zero-to-peak square wave at 48, 60, or 80 KHz, depending on whether the rotary switch is set to X, Y, or Z.

Connect a BNC cable from the preamplifier output to the top eye-coil input on the electronics box.

Move the rotary switch to OFF. Adjust the X OFFSET potentiometer for zero output.

Turn the rotary switch to X (48 KHz). A 60 $\mu$V signal, locked in phase to the master oscillator, now goes into the preamplifier input. The output should now be about 4.7 V within 10%, because the gain is 78,000. Use the INVERT switch to invert the signal.

Repeat the test for Y. You should get about 4.1 V.

Repeat the test for Z. You should get about 3.1 V.

7. Noise: Ground the preamplifier input. Turn GAIN to maximum. Turn X OFFSET to 500. Observe the X output voltage on an oscilloscope set to AC input. The voltage should fluctuate with white noise with frequency components mostly below 320 Hz, and with a root-mean-square amplitude of about 1.4 mV. Repeat for all channels.

TIGHTEN ALL SCREWS ON THE FIELD COILS FOR GOOD ELECTRICAL CONNECTIONS.

8. Field coils: Use an oscilloscope to measure the voltages at the X, Y, and Z outputs of the field box at the screw terminals (most AC voltmeters will not work at these high frequencies). The signals should look approximately like 4 V square waves with ringing. Also measure the voltages across the corners of the field coils, to test for short circuits.

9. Make an artificial eye coil appropriate for your experiment, and use it to test the monitor under simulated conditions.

6. Safety, compatibility, immunity, and quality

The CE mark imprinted on the monitor indicates, that we have self-certified, that it has passed tests for sale in the Common Market. Our testing company (notified body) is Intertek Testing Services. The monitor is classified as a laboratory instrument, not a medical device, but must meet the medical standards for safety.

Safety: The USA electrical safety standards (UL-2601-1) for medical electronics have been harmonized, i.e., brought into agreement with the Common Market standards:

IEC 60601-1 Medical electrical equipment, Part 1: General requirements for safety

IEC stands for International Electrotechnical Commission (Europe), and UL stands for Underwriters Laboratory (USA). Remmel Labs performed the safety tests according to Common Market standards. There are requirements for fuses, labeling, wire insulation, mechanical construction, etc. The following three tests are done on every monitor:

1. The mains circuit (transformer, switch, fuse, connector, and cord) has insulation which withstands 1500 V AC between the mains wires and ground. This type B equipment is symbolized by the man symbol. The power box puts out +/-25 V DC (extra-low voltage) to the rest of the system.
2. Class 1 equipment is grounded if basic insulation fails. The ground wire from the mains all the way to the electronics box must be $< 0.1\Omega$; to achieve this, 10 gauge (5.3 mm$^2$) wire is used between the power and electronics boxes.

3. If the ground is disconnected, leakage current can flow from the monitor out the eye-coil cable to the subject and then to ground. The eye coil has such thin insulation, that it is considered directly connected to the subject. If the mains ground is disconnected, the leakage must be $< 0.5$ mA. Typically we measure only 0.05 mA. Leakage current is primarily from capacitative coupling between high-voltage components (transformer etc.) to ground.

**Mandatory safety inspections:** Ask an electrician to test the ground connection of your mains outlet. In the USA the white (neutral) wire comes into the building and is connected to 3 things:

- A wire or water pipe going into the earth (hence the name ground).
- All the ground wires of the electrical outlets (green/yellow stripe or bare).
- Neutral wires of all outlets (white in USA and blue elsewhere).

Every device passes current from the live wire (black in USA, brown elsewhere) to the neutral wire (white in USA, blue elsewhere). Almost no current is supposed to flow in the ground wire. Because current flows in the neutral wire, there is always a volt or so between the neutral and ground wires.

In order to safeguard the subject’s eyes, the following tests should be repeated by your safety engineer annually, or when a problem is suspected:

1. Measure the resistance of the ground from the mains plug to the metal of the electronics box; it should be $< 0.1\Omega$.
2. Mains-circuit insulation: Connect the power box to the electronics box. On the mains plug, attach a 1500 V AC source between the ground and one other prong. The test fails if any arcing occurs inside the monitor or cord.
3. Subject leakage test: A special electrical set-up is required to do this test. Disconnect the mains ground, but leave the hot and neutral wires connected. Connect a $1\,K\Omega$ resistor between the aluminum box and mains ground. Increase the mains voltage 10% above normal. Measure the AC voltage across the resistor and divide by 1000 to compute the leakage current; it must not exceed 0.5 mA.

**Compatibility** means that the device does not adversely affect other equipment. The magnetic fields will affect other equipment such as microelectrode amplifiers and oscilloscopes, if they are placed inside or near the fields (although at present there is no legal limit on radiating such fields).

Intertek did the following compatibility tests:

- EN61326: Electrical equipment measurement, control and laboratory use-EMC requirements.
- EN1000-3-2: ...limits on harmonic current emissions.
- EN1000-3-3: ...limitations of voltage fluctuations and flicker...
- EN55011: radiated and conducted emissions [30-1000 MHz].

**Immunity** means that the device continues to function properly (or at least is not damaged) by outside disturbances. Intertek tested:

- EN61000-4-2: ...electrostatic discharge...
- EN61000-4-3: ...radiated radio-frequency electromagnetic field...
- EN61000-4-4: ...electrical fast transient burst...
- EN61000-4-5: ...surge immunity...[lightning]
- EN61000-4-6: ...conducted disturbances, induced by radio-frequency fields
- EN61000-4-8: [60 Hz] power frequency magnetic fields from motors and transformers
EN61000-4-11: ...voltage dips, short interruptions and voltage variations...

**Quality:** The ISO9000 quality standards are not required for the CE mark. Nevertheless, we ourselves are implementing ISO9002: Quality systems–Model for quality assurance in production, installation and servicing. Our quality system has these major parts:

1. Instruction manual (this manual) with circuit diagrams, set-up directions, etc.
2. Check list for safety and proper functioning of every monitor, kept in each customer’s file.
3. Lab notebooks containing test data, design changes, copies of documents, etc.
4. Computer programs for printed circuit design, manual text, etc. preserved on CD-ROM back-up disks.
5. The warranty (section 2) guarantees free repairs and unlimited assistance.
6. Records of all customers in case a safety or quality issue is discovered.

### 7. Electromagnetic theory

The field and eye coils are a transformer. We calculate:

A. Magnetic field
B. Old magnetic field calculations
C. Inductance $L_1$
D. Field-coil current $I_1$
E. Magnetic field at the center
F. Eye-coil voltage $V_2$
G. Noise angle $A_n$
H. Noise from fluctuations in electron flow

**A. Magnetic field** Our original coils were circular Helmholtz coils. Dual square coils are easier to construct, and allow easier access to the animal or subject.

The following calculations have been published. A computer program called ”magnet.c++” was written to calculate the field shape throughout the coils (see figure). We reproduce the calculations below.

A set of coils is needed to produce a nearly uniform field near the eye. If a pair of circular coils spaced about one diameter apart are wired so that the fields add, by symmetry, the field near the center will be very uniform. These coils can be either circular or square; we use ”dual square coils” because they are easily made from four aluminum bars of length $D = 1.8$ m, bolted together at the ends. One corner of each square is insulated and wires (not shown) are attached so that current flows around each square. The fields are produced by three sets of dual square coils, one pair for each of the X, Y and Z fields.

Program *magnet.c++* drew the shape of the field produced by the Y-coils (see figure), using the Biot-Savart law:

$$d\vec{B} = \left(\frac{i\mu_0}{4\pi}\right) d\vec{S} \times \frac{\vec{R}}{r^3}$$

(× denotes cross product)

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8R.S. Remmel, Magnetic field program, http://www.remmellabs.com

\[ d\vec{B} = \text{infinitesimal magnetic field at point } \vec{X} \]
\[ \vec{S} = \text{a point on the wire} \]
\[ d\vec{S} = \text{infinitesimal displacement along wire in direction of current flow} \]
\[ i = \text{current in amperes} \]
\[ \mu_0 = \text{permeability of vacuum } (4\pi \times 10^{-7}, \text{ MKS}) \]
\[ \vec{R} = \vec{X} - \vec{S} = \text{vector from wire } \vec{X}; r = \text{length of } \vec{R} \]

The total \( \vec{B} \) at \( \vec{X} \) is the line integral over \( d\vec{S} \) around the loop of wire, i.e., over all of the four bars of both coils. Current is assumed to flow at the exact center of the bar. With this assumption, the Biot-Savart law can be analytically integrated along one bar, using a table of integrals, and the contribution of the eight bars added. Thus with a simple calculation–simple at least for a computer–the nonuniform field can be calculated everywhere:

\[ \vec{P}_1 = \text{one end of a bar}; \vec{P}_2 = \text{other end}; \text{current flows from } \vec{P}_1 \text{ to } \vec{P}_2. \]

Describe \( \vec{S} = \vec{P}_1 + (\vec{P}_2 - \vec{P}_1)g, \) where \( g \) is a number between 0 and 1. When \( g = 0, \vec{S} = \vec{P}_1; \) when \( g = 1, \vec{S} = \vec{P}_2. \) By changing \( g, \) the wire from \( \vec{P}_1 \) to \( \vec{P}_2 \) is described.

Differentiate: \( d\vec{S} = (\vec{P}_2 - \vec{P}_1)dg. \) Thus:

\[ d\vec{B} = \left( i\mu_0/4\pi \right) (\vec{P}_2 - \vec{P}_1)dg \times (\vec{X} - \vec{P}_1 - (\vec{P}_2 - \vec{P}_1)g)/r^3 \]

Make use of the fact that the cross product of a vector with itself is zero.

\[ d\vec{B} = \left( i\mu_0/4\pi \right) \left( (\vec{P}_2 - \vec{P}_1) \times \vec{X} - \vec{P}_2 \times \vec{P}_1 \right)dg/(cg^2 + bg + a)^{3/2} \]

where \( \bullet \) indicates dot product and:

\[ a = (\vec{X} - \vec{P}_1) \bullet (\vec{X} - \vec{P}_1), \]
\[ b = -2 \bullet (\vec{X} - \vec{P}_1) \bullet (\vec{P}_2 - \vec{P}_1) \text{ and} \]
\[ c = (\vec{P}_2 - \vec{P}_1) \bullet (\vec{P}_2 - \vec{P}_1) \]

Integrate over \( g \) from 0 to 1 to give the total field produced by one bar, using an integral table ((\( \vec{P}_1, \vec{P}_2 \) and \( \vec{X} \) are constants):

\[ \vec{B} = \left( 2i\mu_0/4\pi \right) \left( (\vec{P}_2 - \vec{P}_1) \times \vec{X} - \vec{P}_2 \times \vec{P}_1 \right) \times ((2c + b)/\sqrt{c + b + a} - b/\sqrt{b}) / (4ac - b^2) \]

In the program the function ”segment” calculates the field produced by one bar. The X, Y and Z fields are calculated by the functions ”fieldx”, ”fieldy” and ”fieldz”, which add up the contributions of the eight bars. We thus easily compute the fields everywhere (figure). The 4 tiny squares indicate the field-coil bars.

Note that the field is nearly uniform near the center. For I = 1A current, the program calculates \( B_y = 4.9626 \times 10^{-7} \text{ Tesla at the center for the vertical field.} \)

The problems with the above calculations are (1) the integral might have an error, and (2) high-frequency current flows on the surface of the bars (skin depth = 0.335 mm in aluminum at 60 KHz), not at the center as assumed. To check this, the completely independent program magnet2.c++\textsuperscript{10} did a Monte Carlo integration. A point was chosen at random anywhere on the surface of any coil bar, and an infinitesimal current was assumed to flow there, and the field \( \vec{B} \) produced at point \( \vec{X} \) was calculated by the Biot and Savart law. One million points were chosen randomly and \( \vec{B} \) averaged. When so many points were randomly chosen, the bar surfaces were mostly covered, and the averaged \( \vec{B} \) should approach that found in program ”magnet.c++”.

\textsuperscript{10}2nd magnetic field program, http://www.remmellabs.com
The advantage of Monte Carlo integration is that the programmer merely inserts (1) the geometry (integration over all bar surfaces), and (2) the function (Biot and Savart law). No integrals need explicitly be done. But this method is time consuming.

Nine points were chosen along the vertical axis, the horizontal axis and off diagonal; the Monte Carlo points agreed with the analytically integrated points with a std. dev. of 0.21%, consistent with statistical fluctuations.

**B. Old magnetic field calculations:** The following calculations were done for an older, smaller dual square coils with $D = 0.3$ meter made from 1.5875 cm (5/8 inch) aluminum bars, with a gap of 0.00635 cm (1/4 inch). The coils have cubic symmetry. The equations were approximately scaled for coils of any size. The MKS system was used. Axes:
- $+X$ is to the subject’s right (horizontal field-coil axis)
- $+Y$ is up (vertical axis)
- $+Z$ is toward subject’s back (right-handed coordinates)

**Definition of symbols:**
- $A_n =$ noise angle at which $V_2$ equals $V_n$
- $A_x =$ angle between eye-coil axis and X axis
- $A_y =$ angle between eye-coil axis and Y axis
- $A_z =$ angle between eye-coil axis and Z axis
- $\vec{B} =$ magnetic field at point $\vec{X}$
- $D =$ outside length of field coil (0.3 meter for measurements)
- $G = 7.59 \times 10^{-7}$ Tm/A = magnetic field constant
- $g =$ gap between two field coils = 6.35 mm = 1/4 inch of plastic
- $H = \pi G/K = 0.533 =$ transfer-function constant
- $I_1 =$ field-coil current in amperes
- $J =$ current density at point $\vec{X}_c$ in $A/m^2$
- $K =$ $4.47 \mu H/m =$ field-coil inductance constant
- $L_1 =$ inductance of field coil in Henries
- $N_2 =$ turns on eye coil (9 for Skalar or Chronos-Vision lens)
- $N_T =$ turns on secondary of transformer L82 (there are 6 turns on primary)
- $R =$ radius of eye coil (9 mm for Skalar or Chronos-Vision lens)
$V_n = 18 \text{nV RMS} = \text{noise voltage at input of amplifier}$

$V_1 = \text{voltage across field coil}$

$V_2 = \text{induced voltage in eye coil}$

$V_p = 6 \text{volts} = \text{zero-peak square wave from MOSFET transistors in field box}$

$w = \text{width of aluminum bar} = 5/8 \text{inch} = 1.5875 \text{cm}$

$\delta = \text{skin depth of aluminum at 60 KHz} = 0.366 \text{mm}$

$\theta = \pi/2 - A_x \text{ radians} = \text{angle with respect to straight ahead}$

$\mu_0 = \text{permeability of vacuum} = 4\pi \times 10^{-7} \text{H/m}$

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**Magnetic field $\vec{B}$:** Current $\vec{J}$ at point $\vec{X}_c$ produces a field at point $\vec{X}$ according to the Biot-Savart law (referenced previously) written in integral form:

$$\vec{B}(\vec{X}) = \frac{\mu_0}{4\pi} \int \int \frac{\vec{J} \times (\vec{X} - \vec{X}_c)}{|\vec{X} - \vec{X}_c|^3} dX_c dY_c dZ_c$$

The 3 integrals are over the 3-dimensional current distribution, i.e., the integral is evaluated only where current is flowing. The $\times$ denotes cross product. Note that this is an inverse-square relationship, which implies that the magnetic field drops off quickly outside of the field coils. This equation was used to calculate the magnetic field produced by a 60 KHz sinusoidal current of peak amplitude $I_1$ flowing in dual square coils with $D = 0.3$ meter. The coil axis was vertical.

The integral was done by Monte Carlo integration, which is most useful when complicated geometry and boundary conditions are present. The computer picked four numbers at random:

- $N$ went from 1 to 32, the surface number of the 8 bars having 4 surfaces each.
- $P$ went from $-D/2$ to $D/2$, the distance along the length of the bar.
- $S$ went below the surface several skin depths to give current at that depth. Because the random numbers were uniformly distributed, the full range of each integration variable was covered with equal probability. Each point (see figure) was computed with 30,000 random points for smoothing. The remaining statistical fluctuations can be seen in the lower two graphs. For at bar current $I_1$, the magnetic field at the center was:

$$B_y = 2.53 \times 10^{-6} I_1 \text{ Tesla}$$

For scaling to different coil sizes, if all dimensions are doubled, the field will be halved:

$$B_y \text{ at center} = 2.53 \times 10^{-6} \frac{0.3}{D} = GI_1/D$$

where $G = 7.59 \times 10^{-7} \text{ Tm/A}$. The nonuniformity of $B_y$ along the Y axis is shown in the upper left figure. The field has a minimu at $Y = 0$, and two maxima at $Y = \pm 0.15 \text{ m}$, where the center of each field coil is located. The field dropped away rapidly outside of the coils, approximately inversely as the cube of distance (dipole field).

The upper right figure shows $B_y$ as a function of X. The field was strongest at the center. Thus near the center, the field has a saddle point – increasing in the $\pm Y$ directions, but decreasing in the $\pm X$ and $\pm Z$ directions. $B_y$ becomes negative in the X and Z directions outside of the coils, because the field lines wrap around the outside of the coils.

The lower two graphs are expanded to show the nonuniformity; $B_y$ was normalized to unity at the center. The field is uniform to 2% out to 2 cm.

**C. Inductance $L_1$:** It is important to calculate the inductance of the field coil, because the loading to the field-coil power supply is nearly a pure inductive load. Remember that an inductor obeys the differential equation:
AXIS OF COIL PAIR IS ALONG Y-AXIS

COILS ARE D=0.3 = SQUARE
GAP BETWEEN COILS IS g=0.35 mm.
1 = 1 A PEAK SQUARE AT f=90 kHz.
2.036897E-6 = By at center

FIG. 4-1: By AS FUNCTION OF X
By in Tesla
X in meters (Y=0, Z=0)

FIG. 4-2: EXPANDED SCALE OF FIG. 4-1
NORMALIZED TO 1 AT CENTER
By / By at center
X in meters (Y=0, Z=0)

FIG. 4-3: EXPANDED SCALE OF FIG. 4-1
NORMALIZED TO 1 AT CENTER
By / By at center
Y in meters (X=0, Z=0)

FIG. 4-4: EXPANDED SCALE OF FIG. 4-2
NORMALIZED TO 1 AT CENTER
By / By at center
Y in meters (X=0, Z=0)
\[ V_1 = L_1 \frac{dI_1}{dt} \]

Thus if a square wave voltage is applied to an inductor, the current is a triangle wave. Inductance is calculated by this formula in MKS units: \(^{11}\)

\[
L_1 = \frac{\mu_0}{4\pi} \int dX_1 \int dY_1 \int dZ_1 \int dX_2 \int dY_2 \int dZ_2 \frac{\vec{J}(X_1) \cdot \vec{J}(X_2)}{|X_1 - X_2|}
\]

This 6-dimensional integral evaluates the interaction of \( \vec{J} \) at \( \vec{X}_1 \) with \( \vec{J} \) at \( \vec{X}_2 \). This expression has a singularity when \( \vec{X}_1 = \vec{X}_2 \). This singularity has little effect, because wires are not infinitesimally thin. Numerical integration was done by the Monte Carlo method. The inductance was:

\[ L_1 = 1.34 \pm 0.13 \mu H \] by the Monte Carlo method.
\[ L_1 = 1.50 \mu H \] by direct measurement with an LC circuit.

An LC resonance circuit was made by placing a 2.21 \( \mu F \) capacitor across the field coil. The resonance frequency was measured as 87.5 KHz. We used the formula \( L = 1/C/(2\pi f)^2 \). Theory and experiment agreed within errors.

Scaling: If dimensions are doubled, inductance doubles:

\[ L_1 = 1.34 \mu H \frac{D}{0.3} = KD, \text{ where } K = 4.47 \mu H/m \]

This inductance is too low to be driven directly by the power supply, which works best when the inductive load is 20 \( \mu H \). Thus transformer L82 (see circuit diagrams) was used with 6 turns on the primary, and \( N_T \) turns on the secondary.

Depending on coil size \( D \), \( N_T \) should be no larger than:

\[ N_T << 6\sqrt{L_1/20\mu H} = 6\sqrt{KD/20\mu H} = 2.8\sqrt{D} \]

If \( N_T \) is a fraction, it is rounded down.

This equation is plotted on a log-log scale on the next graph. For \( D = 0.5 \) meter, for example, \( N_T \) should be 2, whereas for \( D = 1.8 \) m, \( N_T \) should be 3.

**D. Field-coil current \( I_1 \):** The power supply puts out a \( V_P = \pm 6 \) V square wave. The voltage across the secondary of L82 is:

\[ V_1 = V_p N_T / 6 \]

The transformer has 6 turns on the primary and \( N_T \) turns on the secondary. Field-coil current \( I_1 \) is the integral of square-wave \( V_1 \) according to the equation governing inductors, and is a triangle wave. Peak current occurs when \( V_1 \) is applied for 5.2 \( \mu s \) (1/4 cycle at 48 KHz):

\[ \text{peak } I_1 = 5.2 \times 10^{-6} V_1 / L_1 = 5.2 \times 10^{-6} V_p N_T / 6 KD \]

For a \( D = 0.5 \) meter coil with \( N_T = 2 \), peak \( I_1 = 4.7 \) A.

**E. Magnetic field at center:** From previous equations:

\[ \text{peak } B_y \text{ at center} = \frac{5.2 \times 10^{-6} V_p N_T G}{6 KD^2} \]

\(^{11}\) J.D. Jackson, Classical Electrodynamics, Wiley, New York (1962) 199
Fig. 5/5. Performance vs. coil size.

Skalar Eye Coil:
- \( R = 0.009 \, \text{m} \)
- \( N_2 = 9 \, \text{turns} \)

Minimum turns for N1:
- Noise angle \( \text{atan} \, \text{arc sec} \)
- Field at center, 100 Gauss
This is plotted in the figure. For $D = 0.5$ meter, peak $B_y = 7.1 \mu T = 0.071$ gauss, which is $< 10\%$ of the Earth’s field.

**Eye-coil voltage $V_2$:** By Faraday’s law of induction:

$$V_2 = -N_2 d(\text{flux})/dt = -N_2 \pi R^2 \cos(A_y) dB_y/dt,$$

where we multiply eye-coil area $\pi R^2$ by $\cos(A_y)$ to account for the rotated eye. $A_y$ is the angle between the eye-coil axis and the vertical field-coil axis. Substitute into the above equation the value for $B_y$ at the center, which we differentiate with respect to time:

$$V_2 = -N_2 \pi R^2 \cos(A_y) \frac{G}{D} \frac{dI_1}{dt}.$$

Use the inductor equation, $V = LI/dt$ for the derivative:

$$V_2 = -N_2 \pi R^2 \cos(A_y)(G/D)V_1/L_1.$$

Substitute for $L_1$:

$$V_2 = N_2 \pi R^2 \cos(A_y)(G/D)V_1/KD = HV_1 N_2 (R^2/D^2) \cos(A_y),$$

where $H = \pi G/K = 0.533$. Substitute the value for $V_1$:

$$V_2 = H(V_p N_T/6)N_2 (R^2/D^2) \cos(A_y).$$

Instead of $A_y$, use the vertical angle $\theta$ with respect to straight ahead: $\theta = \pi/2 - A_y$ radians for eye movements in the vertical plane. Then $\cos(A_y) = \cos(\pi/2 - \theta) = \sin(\theta) \approx \theta$ for small $\theta$. Thus:

$$V_2 = (HV_p N_T N_2/6) (R^2/D^2) \theta \text{ for small } \theta.$$

**G. Noise angle $A_n$:** The measured RMS noise voltage of the amplifier is 18 nV with respect to the amplifier input. The noise angle $A_n$ is that angle at which $V_n$ equals coil voltage $V_2$. Solve for $A_n$:

$$A_n = 6V_n (D^2/R^2)/(HV_p N_T N_2) \text{ radians}.$$

$A_n$ is plotted in the figure. For a $D = 0.5$ meter field coil, and an $R = 9$ mm eye coil with $N_2 = 9$ turns (Skalar or Chronos-Vision lens), and with $N_T = 2$, the RMS noise angle is $5.79 \times 10^{-6} \text{ radians} = 1.194 \text{ arc sec}$. This high sensitivity can be used to study microsaccades and vernier acuity. For most applications $N_2 = 1$ turn will be adequate.

**H. Noise from fluctuations in electron flow:**

The first stage of the eye-coil amplifier is shown in the Amplifier and demodulator circuit diagram. It consists of a single NPN transistor coupled to the eye coil by a capacitor. Essentially all noise is produced by fluctuations in electron flow through this transistor. The relevant noise is around 50 KHz.

Assumptions: Noise in the field-coil power supply is negligible (verified by measurements). The presence of the cable between J44 and J45 has no effect on frequency response. Subsequent amplifier stages contribute little noise (verified by grounding the input of stage 2). Assume that the eye coil has negligible resistance (20 $\Omega$ is small enough). C45 is a short circuit for 50 KHz signals. R59
provides bias current so that the collector is at about +1.5 V. R40 is nearly zero but is inserted to stabilize the gain. Thus with input shorted, the base-emitter voltage is constant.

The probability of one electron flowing from emitter to collector is determined by this base-emitter voltage. Assume that each electron flows independently of the others—Poisson statistics applies. The collector current nearly equals the emitter current. Because there is about 3 V across R42, which is 100 Ω, the collector current $I_c = 30 \text{ mA}$.

The collector current was deliberately made this high so that the statistical fluctuations in the current are relatively smaller, because if about $N$ electrons flow per second, the fluctuations go as $N^{1/2}$, so that the fluctuations in voltage across R42 go as $N^{-1/2}$. This amplifier has lower noise than any available operational amplifier.

Let us assume that one electron flowing through Q41 acts like a unit impulse $e\delta(t)$, where $e =$ charge of electron $= 1.59 \times 10^{-19}$ Coulomb. The delta function is defined as infinite at zero, zero everywhere else, but with unit area. This unit impulse (actually a very short pulse because of the finite response time of the amplifiers) passes through the demodulator and enters the low-pass filter.

We wish to compute the equivalent noise at the amplifier input. The impulse $e\delta(t)$ passes through R42 = 100 Ω; then we must divide by $g = 40$, the gain of the first stage. Thus when 1 electron flows, the equivalent input voltage $x(t)$ is:

$$x(t) = R_{42} e\delta(t) / g$$

The demodulator consists of an electronic switch MUX connected to the signal from Q43 and the inverted signal from Q44. This lock-in demodulator takes a square wave and full wave rectifies it. If the square wave is not at the lock-in frequency, the waveform is chopped up and subsequently averages to zero. We assume without proof that the demodulator has unity voltage gain converting AC to DC, and thus converts the noise coming through at the lock-in frequency into low-frequency noise.

Next the amplifier has three first-order low-pass filters consisting of capacitors C50, C51, C52, and associated resistors. Each filter has time constant $\tau = 0.5 \text{ ms}$. The transfer function $H(s)$ is:

$$H(s) = \frac{1}{1 + \tau s}$$

and the cutoff frequency is $f_c = 1/2\pi\tau = 320 \text{ Hz}$. Use the theory of Laplace transforms. Input signal $= x(t)$; Laplace transform $= X(s)$.

Output signal $= y(t)$; Laplace transform $= Y(s)$.

Transfer function of filter $= H(s)$. Then:

$$Y(s) = H(s) X(s)$$

Because the Laplace transform of $\delta(t) = 1$, the inverse Laplace transform of the transfer function $H(s)$ is the impulse response $y(t)$. Look up in a table the inverse transform of $H(s)$:

$$y(t) = \text{impulse response for 1 electron (normalized to input)} = eR_{42} t^2 \exp(-t/\tau) / 2\tau^3 g$$

---


\textsuperscript{13} op. cit., p. 574
This is a slowly-rising pulse with an exponential decay.

For $T >> \tau$, calculate the average $\mu$ and standard deviation $\sigma$ of $y(t)$. Because $T >> \tau$, the integrals can be taken to infinity:

$$\mu = \frac{1}{T} \int_{0}^{T} y(t) dt = eR_{42}/Tg$$

variance $= \sigma^2 = \frac{1}{T} \int_{0}^{T} (y(t) - \mu)^2 dt$, giving:

$$\sigma = \frac{eR_{42}}{4g} \sqrt{3/T\tau}$$

This $\sigma$ is for 1 electron. Actually about $N = TI_c/e$ electrons flow in time $T$. When $N$ independent random numbers are added, the standard deviation increases by $N^{1/2}$. Thus the theoretical prediction of RMS noise $V_n$ referred to the input is:

$$V_n = \sigma N^{1/2} = \frac{eR_{42}}{4g} (\sqrt{3eI_c}/\tau) = 3.34 \text{ nV RMS theory.}$$

$T$ has cancelled out.

The noise with input grounded was measured by turning GAIN to maximum, and further amplifying the output signal by 1000X. A true-RMS AC voltmeter measured the output; this was divided by the total gain to give:

measured $V_n = 18nV$

There thus are sources of noise in addition to fluctuations in electron flow.
8. Circuit Diagrams, A. Power supply

Color codes:  black  ground
red  +12 V  yellow  +5 V  orange  +25 V
blue  -12 V  green  -5 V  violet  -25 V

C1-2  10 mF, 50 V electrolytic, Sprague 36DY103F050AB2A
C3-4  2.2 mF, 35 V electrolytic
C5-8  47 uF, 35 V electrolytic
D1-10  1N4007 1 ampere diodes
D11  Hewlett Packard HLMP-3950 green LED
F1  USA:  0.5 A Littlefuse slo blo, type 313-500;  fuseholder: 342014A
      Europe:  0.5 A 5x20 mm Littlefuse 239.500;  fuseholder: 034550LF1H
J1  Beldon 17252A-B1-0  IEC receptacle
Cord  Mains cord for customer’s country
J2  Amphenol 97-3102A-14S-5S 5-pin receptacle
J3  Amphenol 97-3102A-14S-1S 3-pin receptacle
Q1  2N3055 NPN power transistor
Q2  2N5875 PNP power transistor
Q3-5  Fairchild PN3643 NPN transistor
Q6-8  Fairchild PN3645 PNP transistor
Q9  Motorola MC7812CT  +12 V voltage regulator
Q10  Motorola MC7912CT  -12 V voltage regulator
R1  1 Kohm, 2 W, 5%
R2-3  0.68 ohm, 2 W, 5%
R4-11  10 Kohm, 1/4 W, 1%
R12-13  1 ohm, 5 W, 5% wirewound
R14-15  1.5 Kohm, 1/4 W, 1%
R16-17  220 ohm, 2 W, 5%
S1  Eaton Arrow Harty DPST rocker switch, type E120M-11B
T1  Avel D4022 transformer.
      Primaries in series or parallel for 240 V or 120 V.
      Secondaries in series to give 36 V center tapped.
Cable  Amphenol 97-3106A-14S-5P  5-pin plug and 97-3057-1007 cable clamp
       Amphenol 97-3106A-14S-1P  3-pin plug and 97-3057-1007 cable clamp
       7 meters of 10-gauge stranded wire for ground
       7 meters of 18-gauge stranded wire for other wires
8. Circuit diagrams, B. Eye-coil amplifier and demodulator

C41-44,48  47 uF, 35 V electrolytic
C46       1 mF, 16 V electrolytic (was 47 uF)
C47,51     1 nF ceramic
C50       100 nF plastic
C45,52     1 uF plastic
D60,61     1N5338B  5.1 V, 5 W zener diode
J41-45     female BNC connector
L1-2      140 uH, 5 turns on Fair-rite 5975-000501 ferrite core
MUX       CD4053BE triple SPDT electronic switch
Q41       Fairchild PN3643 NPN transistor
Q42-45    Sections of LF347N quad operational amplifier

Resistors are 1/4 W, 1% unless noted otherwise.
R40       0.68 ohm, 2 W
R41       20 ohm
R42       100 ohm
R44,51-54 10 Kohm
R45,49    9.09 Kohm
R46,50    1 Kohm
R47       200 ohm
R48,55    ETI type DC22-10-10K  10 Kohm, 10-turn potentiometer
R56       1 Mohm
R57       499 Kohm
R58       499 ohm
R59       4.99 Kohm
R60,61    22 ohm, 5 W
S2        SPDT miniature toggle switch

Preamplifier cable: 7 meters of RG58/U with Cambridge CPFI-UG88-1 connectors
Q42-44 and MUX are powered by ±5 V.
Q45 is powered by ±12 V.
The TTL signal marked "A" throws the electronic switch BCD in the MUX.

Pin assignments;

<table>
<thead>
<tr>
<th></th>
<th>X module</th>
<th>Y module</th>
<th>Z module</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q42 pins 1-3</td>
<td>pins 12-14</td>
<td>pins 8-10</td>
<td>pins 5-7</td>
</tr>
<tr>
<td>Q43 pins 12-14</td>
<td>pins 8-10</td>
<td>pins 5-7</td>
<td></td>
</tr>
<tr>
<td>Q44 pins 12-14</td>
<td>pins 8-10</td>
<td>pins 5-7</td>
<td></td>
</tr>
<tr>
<td>Q45 pins 8-10</td>
<td>pins 5-7</td>
<td>pins 1-3</td>
<td></td>
</tr>
</tbody>
</table>

MUX pins A,B,C,D pins 9,4,5,3 pins 11,14,13,12 pins 10,15,1,2

Switch S2: pin E: orange green gray
pin F: violet white brown

These colored wires come from the oscillators from X-phase, XX-phase etc.
8. Circuit diagrams, C. Oscillators

Color codes;  orange  X-phase
violet  XX-phase (X inverted)
green  Y-phase
white  YY-phase
gray  Z-phase
brown  ZZ-phase

C20  47 uF, 35 V electrolytic
C21,24-29  1 nF ceramic
C30  1 uF plastic

D20  1N5338B  5.1 V, 5 W zener diode

J21  Amphenol 97-3102A-16S-1S  7-pin female connector
J22  female BNC connector

Q20  Texas Instruments SN74123N dual pulse generator
Q24,26,27,28  Texas Instruments SN74164N  8-bit shift register
Q25,29  Texas Instruments SN74S04N hex inverter
        Q29 is used for the X frequency.
        Q25 is used for the Y and Z frequencies.

Q30  Fairchild PN3643 NPN transistor

R20  22 ohm, 5W
R21  7.5 Kohm in parallel with 49.9 Kohm
R24-29  200 ohm
R30  1 Kohm
R31  100 Kohm
R32  1 ohm

S20  rotary switch
8. Circuit diagrams, D. Field box

C81-83 1 mF, 16 V electrolytic
C84 100 nF plastic

D81-82 1N914B diode

J81 Amphenol 97-3102A-16S-1S  7-pin female connector

L82 6 turns primary on Fair-rite 5975-000501 ferrite core
2 turns on secondary for field coils with D <= 1 meter
3 turns on secondary for larger coils

L83 140 uH, 5 turns on Fair-rite core
L84 22 uH, 2 turns on ferrite core

Q81 Fairchild PN3645 PNP transistor
Q82-83 Fairchild PN3643 NPN transistor
Q84-85 Motorola MTP60N06HD power MOSFET

R81 51.1 ohms, 1/4 W, 1%
R82 100 ohms, 1/4 W, 1%
R83,85 470 ohm, 2 W, 5%
R84 2 Kohm, 1/4 W, 1%

Cable (7 meter long):
Two Amphenol 97-3106A-16S-1P  7-pin male plugs
Two Amphenol 97-3057-1008 cable clamps
Three 18 gauge stranded wire
Three RG174U coax cables for pins C, D, and E (shield grounded)