Application of the Impedance Method of Modeling Active Materials to Plate Structures

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Abstract

The increased use of induced strain actuators, in everything from skis to speakers, requires more accurate and usable modeling methods. These methods must retain their accuracy while being easy to use and retaining a physical insight into the system. Until recently, available methods have not achieved these three requirements.

The impedance method comes close to meeting these requirements. However the need to find the impedances analytically has limited its use to simple structures. Fairweather (1998) changed this by using FEA analysis to calculate the impedances. He experimentally verified this approach in beams but for plates only compared his results to previously published experimental data. This work starts with Fairweather's work and move the method forward to the point where it can be used under a variety of conditions and has been experimentally verified as such.

To this end, the equations of Fairweather (1998) are used as the starting point. The underlying assumptions are clarified and some modifications have been made. Whereas the original equations were for an actuator aligned with the plate's axis, now any orientation is acceptable. Originally the eigenvectors at the center points of the actuator's sides were used for calculating the impedances. This may not be an accurate representation of the actuator side though so the use of shape functions to represent the eigenvectors along the patch side is investigated.

Three different sets of boundary conditions for a plate were studied: bolted at four corners, four corners plus two interior bolts, and four corners plus two interior holes. Both isotropic (aluminum) and orthotropic (graphite composite) material symmetries were considered. As well as two different actuator orientations: 90-degrees and 45-degrees. These 12 configurations were modeled by FEA and tested experimentally. Additionally, both fine and coarse mesh versions of the FEA models were made. This was to see if mesh density or configuration had any effect on the results.
Comparison of the experimentally recovered and predicted plate responses showed acceptable agreement. Shape function calculations were found to be superior to center point calculations. A finer mesh was found to improve results, though at increased computational cost.
1. Introduction

1.1. Background

Active materials and smart structures are becoming increasingly prevalent in today’s world. They are showing up in everything from skis to speakers to eyeglasses. With this increasing use comes the demand for more accurate and usable mathematical models of the physical system. The models must be easy to use, retain a physical insight into the system and still be accurate. Until recently all of the models for induced strain actuators, such as piezoelectric ceramics (PZT), have failed to meet some or all three of these requirements.

A mathematical model is necessary to predict the dynamic response of the system, develop control strategies, and investigate issues of actuator placement. It is common practice to divide a complex system into several smaller sub-systems and then to make simplifying assumptions on the nature of these sub-systems. These assumptions vary from one type of model to another and while they can make the models easy to use, they tend to limit their accuracy.

At present, there are several different methods for generating dynamic models. These methods will be briefly described here and in more detail in Chapter 3.

- Static Equivalent Force (SEF) Methods
- Finite Element Methods (FEM)
- Models Derived from First Principles
- Impedance Methods

SEF models are the most widely used although they have been shown to significantly miscalculate the systems resonant frequencies, and the actuator force, stress, and strain (Liang et al., 1993b; Zhou et al., 1996; Fairweather 1998). The reason for their
widespread acceptance is their easy formulation and retention of the physical insight into the system. No attempt is made to include the stiffness, mass or frequency response of the actuator in the model.

PZT excitation has been modeled in finite element programs by treating the excitation as a local thermal expansion. The physical properties of the PZT are included as part of the mesh. The drawbacks to this method include the necessity for a very fine mesh around the actuator to properly represent the strain transfer to the host structure, and the lack of commercial FEM packages that provide harmonic thermal expansion analysis (Liang and Rogers, 1989; Hagood et al., 1990; Ha et al. 1992). Physical insight into the interaction between the actuator and host structure is lost in this solution process. In addition, any changes to actuator placement require another FEM solution, which can be expensive.

Some FEM codes, such as ABAQUS, Ansys, and ATILA, have built in capability to handle PZT actuators. The capabilities of ABAQUS and ANSYS were reviewed by Lin, Abatan, and Rogers (1994). Others allow custom elements to be made, thus allowing a PZT element to be tailored to the application. Use of a PZT element allows for better physical insight than the thermal analogy, but it can still be hard to get a good grasp on what is happening. Again, a new solution is required every time the actuator is moved. Commercial FEM codes with built in PZT elements are expensive to buy. Creating custom elements can allow for use of lower cost or in-house designed FEM codes, but the coding can be very complex and much time is wasted getting it work with the analysis and pre- and post- processor packages.

The use of first principles can allow for both the mechanical and electrical aspects of the actuator to be taken into account. Which aspects and to what degree, depend upon the principle applied. The drawback to this approach is that for anything more complex than a simple beam these models become very high order. This makes them not well suited for typical design studies, such as optimal placement of actuators.

Impedance modeling is a new approach to modeling PZT actuators developed by Liang et
al. (1993, 1994, 1995) at Virginia Polytechnic Institute. The mechanical impedance of the
host structure is used to determine the load at the actuator boundary. This is then used to
determine the dynamic response of the system and actuator. These models have been
shown to have very good agreement with experimental measurements as opposed to the
simpler SEF predictions (Fairweather, 1998; Zhou et al., 1996; Sermoneta et al. 1995,
Rossi et al. 1993).

This technique requires analytical solution of the host structure’s impedance, which is
derived from solution of the structure dynamics. This severely limits its application, since
there are few geometries of interest for which a closed form expression of their mechanical
impedance exists. The complexity of generating these impedances and, somewhat, the
novelty of the approach has kept this technique from gaining widespread use.

Fairweather (1998) used FEM to generate the impedances instead of solving for them
analytically. This allows any structure geometry to be used and makes it very easy for
studies, such as optimal placement of actuators, to be done. He showed that the FEM
impedance model of a beam with a PZT actuator agrees with both the analytical
impedance model and with experimental data. For plates he showed agreement with the
analytical impedance model of Zhou et al. (1996) for simply supported plates, but did not
take any experimental measurements. No further boundary conditions or experimental
measurements were taken.

1.2. Approach

The method as put forth by Fairweather (1998) shows promise but has not been
experimentally verified. Also there were issues of different boundary conditions, material
symmetries, methods of computing the applied moment, and off axis actuator placement
left open. This work will address these issues and show that the method works well when
the underlying FEA model is accurate.

The first step is to lay out the necessary equations to calculate the response of the plate.
These are basically the equations from Fairweather (1998), cleaned up, streamlined and with the underlying assumptions clarified. These equations were originally developed to handle patches aligned with the plate's axis and to use the eigenvectors at the center of the patch sides to determine the applied moment. Adjustments will be made to allow the patch to be at any angle and to use shape functions, representing the eigenvectors along the entire side of the patch, to determine the applied moment.

Finite element models will be constructed for both aluminum and composite plates, so as to test different material symmetries. These models will be made with a coarse and fine mesh, so as to see if the mesh density has any effect on the method's performance. Different boundary conditions will also be included in these models. The basic model will have the four corners constrained by bolts. Then two interior bolts will be added to make a boundary condition that would be near impossible to solve for analytically. Then finally the constraints will be removed from the interior bolt holes.

After creating the FEA models they will be experimentally verified. The plates described by the FEA models will be built and have piezoceramic patches attached to them. The patches will be actuated and the plate's response will be recorded. The response will then be compared to the predicted response to gauge how well the method performed.

1.3. Thesis Arrangement

Chapter 2 gives an overview of the area of active materials and smart structures. It compares some of the available sensors and actuators. The advantages and drawbacks of the different material types are presented along with some representative properties. It then goes on to go through some of the current applications and research in the area and where the future of the field is headed.

Chapter 3 goes into more detail on piezoelectric ceramics since, that is the actuator type being used for the impedance model. A brief history of piezoelectrics and their different actuator configurations is presented. The different modeling methods mentioned above
are presented in more detail with their features and limitations clearly listed.

Chapter 4 goes through the development of the impedance model for plates. It follows the steps from Fairweather (1998) to determine the plate's response to an actuator. The underlying assumptions will be clarified some from Fairweather's work. The necessary steps to use shape functions instead of center point calculations and to include off axis actuators will be presented.

Chapter 5 presents the FEA models and how they were developed. The process for verifying the models by modal testing is gone through. The types of elements and the various models produced are also presented.

Chapter 6 goes through the actual experiments and their results. First the method for creating the plates is presented. Then the experimental setup and procedure is gone through. Then the method for taking the FEA results, using them to predict the plate's response and comparing them to the experimental data is presented. The experimental and predicted responses are then compared for the different plate and FEA configurations. Finally some observations as to the performance of them method, the effect of mesh density, and the approach to calculating applied moments are made.

Chapter 7 summarizes what was done in this work. The results of Chapter 6 are revisited and the conclusions are reiterated. A suggested plan for moving the method further along is also presented.
2. Overview of Active Materials and Smart Structures

The following chapter gives an overview of the area of active materials and smart structures. This will give an idea of the materials available for use as both sensors and actuators in smart structures and the problems associated with them.

A thorough review of the state of the art can be achieved by referring to Crawley (1994) and a NASA (1992) technical report. Most of the material below comes from these two sources, with more recent information added. Numerous papers have also been published on the designing of smart structures (deLuis and Crawley, 1990; LaPeter et al., 1991; Hagood et al. 1994; Zhou et al., 1995).

2.1. Definition of a Smart Structure

What is a smart structure? The term smart or intelligent structure is used to encompass a group of structures and systems that all share the same basic property. This is that they are able to sense their environment, decide what type of corrective action is to be taken, and take the necessary action. Ideally, this would be done autonomously.

An everyday example of this would be photogrey lenses for eyeglasses. These lenses sense the ambient light conditions and brighten or darken in a self-acting manner. In this example, all three components, the sensor, actuator and control unit, are all integrated into one material (Neumann, 1996). The system is completely autonomous and requires no additional units. Most systems however do not reach this level of integration and fall under the broader groupings of which smart structures is a subset.

Crawley (1994) presented smart structures as a subset of broader categories. This is shown in Figure 2.1. An adaptive structure is one which has actuators distributed throughout it. These actuators would be used to modify the characteristics of the structure. An example of this would be an aircraft wing with articulated leading and trailing edges.
A structure is considered sensory when it has sensors distributed through it. These sensors could detect strains, temperatures, and displacements among other things. Structures with embedded sensors to detect damage would fall into this category.

The overlap between these two fields contains three types of structures. Any structure that can modify its properties by use of a closed loop control system falls into the category of controlled structures. As you go from controlled to active to smart the degree of integration of the sensors and actuators continues to increase. An active structure would have highly distributed sensors and actuators that are actually part of the load bearing system. To be considered truly smart, a structure should also have the control functions distributed throughout the system.

To date, most of the systems that are often thought of as smart instead fall into the active or controlled areas. Few systems, such as the photogrey glasses, truly qualify as a smart structure. However, the trend is to make more systems smart. The ultimate goal of this work is the mimicking of the human body, with the sensors acting as the nerves, the actuators as the muscles, and the control functions as the brain.

### 2.2. Actuators

Actuators for smart systems must be able to take the control inputs to the system and convert them into physical quantities, which effect changes in the system. These actuators should be capable of being highly distributed throughout the system. Normally the actuator takes an electrical signal and converts it into a strain or displacement. Its maximum achievable strain, bandwidth, and stiffness can describe the performance of such an actuator.
The strain induced by an actuator is called its actuation strain, which is the controllable strain not due to stress. There are many ways in which actuation strains can be caused, such as variations of temperature and exposure to moisture, but only a few these are useful for control. Among these are piezoelectricity, electrostriction, magnetostriction, and the shape memory effect. These four form the basis for most of the actuators used in smart structures, and together with MEMS (microelectromechanical systems) are the ones that will be covered here. The chart below compares the properties of these actuators.

<table>
<thead>
<tr>
<th>Actuation Method</th>
<th>PZT G - 1195</th>
<th>PVDF</th>
<th>PMN - BA</th>
<th>Nitinol</th>
<th>Terfenol - D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max Strain (ppm)</td>
<td>300</td>
<td>300</td>
<td>600</td>
<td>20000</td>
<td>1800</td>
</tr>
<tr>
<td>$E \mu \times 10^6$</td>
<td>9</td>
<td>0.3</td>
<td>17</td>
<td>$4 m^a, 13 a^b$</td>
<td>7</td>
</tr>
<tr>
<td>$T_{max} (°C)$</td>
<td>360</td>
<td>100</td>
<td>high</td>
<td>45</td>
<td>380</td>
</tr>
<tr>
<td>Hysteresis (%)</td>
<td>10</td>
<td>&gt;10</td>
<td>&lt;1</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>Bandwidth (kHz)</td>
<td>kHz</td>
<td>kHz</td>
<td>kHz</td>
<td>1 Hz</td>
<td>100 Hz</td>
</tr>
<tr>
<td>Temp. Sen. (%)</td>
<td>0.05</td>
<td>0.8</td>
<td>0.9</td>
<td>-</td>
<td>0.3</td>
</tr>
</tbody>
</table>

$^a m = \text{martensite}$

$^b a = \text{austentite}$

### 2.2.1. Piezoelectric Materials

Piezoelectric actuators are among the most common of all actuators used in smart materials. One of the reasons for this is that they can serve as both sensor and actuator. Piezoelectricity is the ability of a material to develop an electric charge when subjected to a mechanical strain. The inverse of this effect, the ability to take an electrical charge and convert it to a mechanical strain, is called the inverse piezoelectric effect and is what is used to actuate a structure. Strain is induced locally into the structure by this effect and generates forces and moments.
Piezoelectric materials generally used in smart structures come in one of two forms, piezoceramics and piezoelectric polymers. The piezo’s ability to actuate the structure is a function of its stiffness, electromechanical coupling coefficients, flexibility, and limits on applied voltage. Piezo films, such as PVDF in Table 2.1, have high voltage limits, but low stiffness and electromechanical coupling coefficients. Piezoceramics, such as lead-zirconate-titanate (PZT), are much stiffer and have large mechanical coupling coefficients. For this reason, polymers are usually not chosen as actuators, whereas ceramics lend themselves to this role.

Piezoelectrics have been used as actuators in many different applications. They have been successfully embedded (Hagood et al., 1988) in composites and surface mounted on both composites and other substrates (Crawley et al., 1987). In composites, they have been used for a number of applications including vibration damping (Hagood et al., 1991) and disturbance rejection (Lazarus et al., 1992). The unimorph configuration has been used for speakers in both commercial applications, as tweeters, and in noise cancellation applications. More applications will be given in §2.4.

More details on piezoelectrics as actuators can be found in Chapter 3. There the limitations and advantages of using piezoelectrics as actuators will be covered. Some of these include nonlinear response at high voltage levels, hysteresis, and aging. In addition, problems arising from embedding them in composites will also be covered. Some of these problems can include curing temperature limiting service temperature, discontinuities sin the composite, serviceability, and brittleness.

### 2.2.2. Magnetostrictive Materials

Magnetostrictive materials are able to induce actuation strains in a system through the coupling of an applied magnetic field and the magnetic dipoles in a material. This effect is capable of producing large strain, up to 0.2% for Terfenol-D, but is inherently nonlinear. Large magnetic fields need to be used, which complicates its use as a distributed actuator. Table 2.2 gives a comparison between the properties of some magnetostrictive materials.
Table 2.2 Comparison of Magnetostrictive Materials (NASA, 1992)

<table>
<thead>
<tr>
<th></th>
<th>Nickel</th>
<th>Metglas</th>
<th>Terfenol - D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Strain</td>
<td>50 ppm</td>
<td>50 ppm</td>
<td>1800 ppm</td>
</tr>
<tr>
<td>Field Required</td>
<td>6000 G</td>
<td>1 G</td>
<td>500 G</td>
</tr>
<tr>
<td>Resistivity</td>
<td>$7 \times 10^{-8} , \Omega m$</td>
<td>$1 \times 10^{-6} , \Omega m$</td>
<td>$6 \times 10^{-7} , \Omega m$</td>
</tr>
<tr>
<td>Available in Foil</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

Though Terfenol-D has the largest induced strain its lack of availability in foil form prevents it from being useful as an embedded actuator. Metglas’ applied strain is a significantly lower than Terfenol-D, but its low required field and availability in foil makes it well suited for use as an embedded actuator.

A 1992 NASA study suggested that magnetostrictive foil actuators are superior to piezoceramics in four areas: reliability, property stability, manufacturability, and flexibility. It was considered more reliable because it lacks embedded leads, which may break off, does not require a large wiring harness, and uses low-voltage power supplies. It was found more stable due to its low temperature sensitivity, low creep, and failure to break down under high operating fields. Its lack of electrodes, inherent toughness in ribbon forms and flexibility were reasons for its superior manufacturability. Its superior flexibility was demonstrated by the fact that adaptive shape response can be obtained by simple changing the applied field shape.

With all of these advantages, you would think that there would be many applications for magnetostrictors. The lack of applications can be attributed to the need to apply magnetic fields to the structure to induce actuation. Grumman has used it as a means to activate a control surface to optimize the lift performance of an aircraft wing. It has also been considered as an actuator for the cancellation of rotor induced vibrations in helicopters (NASA, 1992). A more recent use of magnetostrictors is for corrosion detection in pipes (Paula, 1996). A picture of this can be seen in Figure

![Figure 2.2 Magnetostrictor for Corrosion Detection (Paula, 1996)](image-url)
2.2. They have also been studied for damping vibrations on space structures (Johnson, 1992).

2.2.3. **Shape Memory Alloys**

Shape memory alloys are materials that exhibit a shape change when heated. This is accomplished through a phase change and can impart large strains, up to 20000 for Nitinol, to the structure. The heat may be applied by any means including electrical resistive heating. However, the bandwidth of the actuator is limited by the speed at which it can be cooled between states.

Nitinol, a nickel titanium alloy, is the most common of the shape memory alloys and can have its transformation temperature set from below -100°C to above +100°C by varying the percentage of nickel in it. The transformation takes place between a weaker, low temperature martensite phase and a stronger, high temperature austenite phase. The stiffness increase between these two phases can be by a factor of three or more, as seen in Table 2.1.

To use this shape memory effect, the material is put in its preferred shape and held above its transformation temperature, in its austenite phase, for a set amount of time. It is then allowed to cool to its usage temperature, where it is in its martensite phase. The material can then be deformed. Upon heating above its transformation temperature, the material will revert to its original shape. This recovery of the preferred shape is what makes shape memory alloys useful as actuators. The process can be seen in Figure 2.3. Extensive research has been done to describe the behavior of

![Figure 2.3 Transformation of Nitinol (Stoeckel, 1989)](image)
this material (Rogers et al., 1991; Thier et al., 1991; Hedayat et al., 1992).

Shape memory alloys are available in a variety of configurations. They come as wire, ribbon, sheets and springs. They can also be conditioned to go between two preset shapes instead of just one. This is called the two-way shape memory effect. They are biocompatible and are used in several biomedical applications. They have been used for everything from eyeglass frames and coffeepot thermostats to satellite release devices and electrical couplings (Shape Memory Applications Inc.). Some of these applications take advantage of the superelastic property of Nitinol, which allows it to recover strains of up to 6 percent without being heated.

Significant problems with using shape memory alloy actuators include their low bandwidth. The reaction time of the actuator may be fast but before it can be actuated a second time, it must be allowed to cool back below its transformation temperature. This cooling time can be decreased by methods such as water cooling, but usually the cost of such techniques outweigh the benefits.

Other problems with SMA actuators include hysteresis in the loading path, required size of the embedded actuator, and transfer of heat to the surrounding structure. The hysteresis problem can be reduced by the addition of copper to the alloy. The size of the embedded actuator must be considered as it can have adverse effects on the mechanical properties of the structure. Care must be taken when selecting the transformation temperature of the SMA so that the heat generated during actuation does not harm the structure.

2.2.4. Electrorheological Fluids

Electrorheological (ER) fluids exhibit a coupling between their fluidic and electrical properties. An ER fluid consists of a suspension of fine semi-conducting particles in a dielectric fluid. Cellulose, cornstarch grain, alumina powder and silica gel have all been used as particles in ER fluids. Without an applied electric field, the ER fluid behaves like a Newtonian fluid, however once a sufficient electric field is applied the fluid undergoes a
dramatic change. When an electric field is applied the particles form into chains that cause resistance to flow or resistance to shear movement. This increase in resistance is closely associated to an increase in yield stress and is proportional to field strength. The main drawback to ER fluids, and the one that keeps them from being used in many applications, is that very high levels of voltage must be applied to actuate them.

ER fluids have been proposed and used in some applications, though. Among them are shock absorbers (Neumann, 1996), engine mounts, and damping of helicopter blades (NASA, 1992). Once the voltage required to actuate an ER fluid drops to an acceptable level, then these fluids will see a more varied use. Until then, they will remain in the domain of the research lab.

2.2.5. Electrostrictive Materials

Like piezoelectrics, electrostrictive materials change shape when an electrical field is applied or generate a voltage when a strain is applied. The difference is that the induced strain is proportional to the square of the electric field, so the displacement is always unidirectional, regardless of the polarity. This process is inherently nonlinear, but is not bothered by hysteresis, as it is in piezoelectrics.

An example of an electrostrictive material is lead magnesium niobate (PMN). The properties of PMN are shown in Table 2.1 and it can be seen that its performance compares favorably to PZT. PMN is available in sheets like PZT and can be embedded or surface mounted. At present most of its applications are in the optics industry, where it is used for micron level adjustment of mirrors. It is also being used in conjunction with PZT on the control of precision structures (Anderson et al.).

2.2.6. Microelectromechanical Systems (MEMS)

MEMS are one of the newest and hottest sensor / actuator technologies available today. MEMS are small (about the width of a human hair) devices made by micro-machining and other processes developed to make integrated circuits. Their small size makes them an
exciting technology for actuators, but also causes them to be too expensive for most applications. At present most of the use for MEMS has been in the automobile and biomedical fields, where the large numbers required have helped to offset manufacturing costs and bring overall cost down. The demand for MEMS actuators for smart structures is still too small, and their price too high, to make them cost effective. As new manufacturing techniques are developed, the cost will come down and MEMS will become economically feasible. At present, generators, pumps, motors, and valves have been made, though the focus is still on sensors. MEMS as sensors will be covered in §2.3.3.

2.3. Sensors

The control system must first gather information on what is going on in its surroundings before it can tell the actuator what to do. This is the role of the sensor. In the analogy of a smart structure to a human body, the sensors play the role of the nervous system. Like actuators, there are many different type of sensors. The sensors of interest in smart structures are those that can sense strain displacement and similar quantities. Table 2.3 shows a comparison of the properties of some common sensor types.

<table>
<thead>
<tr>
<th></th>
<th>Foil\textsuperscript{a}</th>
<th>Semiconductor\textsuperscript{a}</th>
<th>Fiber Optic\textsuperscript{b}</th>
<th>Piezo Film\textsuperscript{c}</th>
<th>Piezoceramic\textsuperscript{c}</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sensitivity</strong></td>
<td>30 V/ε</td>
<td>1000 V/ε</td>
<td>10\textsuperscript{6} deg/ε</td>
<td>10\textsuperscript{4} V/ε</td>
<td>2x10\textsuperscript{4} V/ε</td>
</tr>
<tr>
<td><strong>Localization, in</strong></td>
<td>0.008</td>
<td>0.03</td>
<td>~0.04</td>
<td>&lt;0.04</td>
<td>&lt;0.04</td>
</tr>
<tr>
<td><strong>Bandwidth</strong></td>
<td>0 Hz - acoustic</td>
<td>0 Hz - acoustic</td>
<td>~0 Hz - acoustic</td>
<td>~0.1 Hz - GHz</td>
<td>~0.1 Hz - GHz</td>
</tr>
</tbody>
</table>

\textsuperscript{a} 10V excitation  
\textsuperscript{b} 0.04 in interferometer gauge length  
\textsuperscript{c} 0.001 in sensor thickness

The first two columns are the traditional foil and semiconductor strain gauges used everyday in engineering applications and testing. They both rely on a change in resistivity to detect a change in strain. They will not be covered below, as they are not generally
found in smart structures, due to issues of compatibility and weighting functions. All of the sensors above have a wide enough bandwidth to be useful in structural applications. The sensors that will be covered below include piezoelectric, fiber optic and MEMS.

2.3.1. Piezoelectric Sensors

Piezoelectric sensors are formed and behave the same as piezoelectric actuators. Details on forming piezoelectrics can be found in Chapter 3. Only here it is the direct and not the inverse piezoelectric effect that is sought. The same problems that piezoelectrics had as actuators are valid as sensors. The piezoelectric film in the table above can be made out of either polymer or ceramic. The low modulus of the polymer is not such a problem when sensing, but its lower temperature range is still a problem. It low modulus can actually be an advantage cause it means less loading on the structure. The drawback to a piezo film is that it can not be embedded.

Piezoelectrics are still the most widely used type of sensor used in smart structures though fiber optics is becoming increasingly popular. Their ability to be used as both sensors and actuators is one of the reasons for their wide use. Piezoelectric sensors have been used for vibration control (Miller and Hubbard), disturbance rejection (Collins et al., 1991), and damage detection (Islam and Craig, 1994) among other things. More applications can be found in §2.4.

2.3.2. Fiber Optic Sensors

Fiber optic sensors are on the rise in smart structures. At one point, the necessary hardware to utilize them was considered too bulky and power hungry to make the sensors useful and cost effective. However, as their use in the telecommunications industry has grown, they have become increasingly attractive as sensors for smart structures. The same fibers used to transmit telephone calls and cable TV programs can be used to transmit strain measurements about a structure. This makes the fibers and hardware easy to come by and relatively cheap (less than $0.10 / m for single mode fiber). A good overview of
using fiber optics in smart structures can be found in Udd (1996). Most of the information below comes from there.

Fiber optic sensors are capable of sensing various types of information, including temperature, chemical changes and strain. It is strain though, that is most used in smart structure applications. There are many different ways to interrogate the sensor to get the desired information. The most useful of these are based on the following four arrangements: Bragg gratings on the fiber core, the fiber Fabry-Perot, the two-modal fiber sensor and the polarimetric fiber sensor. All of these, except for the Bragg grating, operate on the idea of optical interference.

The Bragg grating consists of bands of higher reflectivity etched onto the fiber core. These gratings are spaced at a set period. As the fiber undergoes strain or temperature changes, the period of these gratings change and thus the properties, specifically a shift in the peak wavelength, of the light reflected back changes. Bragg gratings are easily manufactured as the wire is made, by imaging two short wavelength laser beams on the core of the fiber. The two beams cause an interference pattern, and thus the grating is imaged on the fiber core. This is shown in Figure 2.4. Bragg gratings can be distributed along the length of the fiber and the individual strain measurements can be recovered by using optical wavelength demultiplexing techniques.

The fiber Fabry-Perot sensor is an interferometric sensor, which uses the phase change between two light beams to make its measurements. The sensor consists of a semireflective splice at one end and a mirrored cleave on the other. In this way, it is only a point sensor, and each sensor must be on its own section of fiber. When light is sent down the fiber some of it is reflected back at the

![Figure 2.4 Formation of a Bragg Grating (Udd, 1996)]
splice, the rest continues the length of the sensor, its gauge length, and is then reflected back. The phase difference between these two beams is caused by any strain or temperature changes in the sensor gauge length. Any strain before the sensor is automatically left out, since both beams had underwent the same phase changes getting to the splice. The splice is a point of structural weakness, but can avoided by using a grating as the semireflective element. Of all the interferometric sensors, this one is the most sensitive to strain.

The two-moded fiber sensor consists of a two-moded fiber bonded to the end of the single mode fiber lead. The two-moded fiber’s splice is offset, so that it acts as a spatial filter, and its end is mirrored. Light passes from the single mode fiber into the two-moded fiber, and it is split into two spatial modes. These modes interfere with each other and reflect off the mirrored end. One lobe of this interference pattern is transferred back into the single mode fiber and travels back to the detector. Changes in the state of the two-moded fiber are made evident as changes in this interference lobe and can thus be sensed as changes in strain or temperature. The main problems with this sensor are the difficulty in making the offset splice and the weakness that this introduces into the fiber.

Polarimetric sensors require high birefringence fibers. These fibers have the ability to propagate two orthogonal linearly polarized modes at two different phase velocities. Strain or temperature acting on the fiber changes the natural phase difference between these two modes. Thus, to measure strain or temperature in a fiber the change in the phase difference of the two modes must be measured.

The Bragg grating is the only one of the above methods that can be used as a distributed sensor, and is at best only a quasi-distributed sensor. All of the others are only point sensors. Fiber optics, though, do have the ability to function as truly distributed sensor. To do this optical time domain reflectometer (OTDR) techniques must be used. These techniques take advantage of the Rayleigh scattering that is inherent in fiber optics. Rayleigh scattering is light scattering in the fiber due to imperfections in the fiber. The time that it takes for this scattered light to return to the instrument gives the location of
the measurement. The attenuation of the pulse energy returned is used to measure strain. Configuring the sensor so that the strain introduces microbending in the fiber, which in turn causes an attenuation of the returned power, does this.

As can be seen above, fiber optics is capable of making strain measurements in many ways. Their advantages include being electrically passive, so they don’t provide a conduction path through the material, they are environmentally rugged and having a high immunity to electromagnetic interference. They also have the ability to carry out measurements over very large spatial distances and the extremely low characteristic loss rate of the medium allows the sensors to be arbitrarily far away from the processing electronics (Spillman, 1996). However, there can be problems though when they are embedded in materials.

The fiber shape and flexibility of a fiber optic strand lend themselves to being embedded in composites. The average fiber optic strand is 125 µm in diameter, it can be over 250 µm with its protective coating, compared to approximately 16 µm for a glass fiber and between 100 and 300 µm for a single layer glass/epoxy laminate. This can lead to resin rich areas around the fiber optic and could possibly degrade the composite’s properties. Tests were done on both tensile (Seo and Lee, 1995) properties and fatigue (Seo et al., 1995) properties of laminates with embedded fiber optics. No significant effect was seen on the tensile properties of the laminate, but the optical fibers did have a significant effect on the fatigue properties.

The acrylate coating usually found on the fibers can also be a source of problems for fiber optics. It can interfere with the transfer of strain from the structure to the fiber because, of its softness and through debonding. It has been suggested that a polyimide coating be used instead (Hadjiprocopiou et al., 1995). Polyimide coatings not only have the advantage of being chemically bonded to the fiber core, thus transferring the loads better, but also are more thermally stable.

A presently fiber optic sensors show the most promise in health monitoring of civil
engineering structures (Culshaw et al., 1996), due to their ability to measure over large distances. They have also been used for damage detection (Asanuma et al., 1996), and vibration control (Measures, 1992).

2.3.3. Microelectromechanical Systems (MEMS)

As stated in §2.2.6, most of the currently available MEMS are sensors. These MEMS have largely been made for the automobile and biomedical industries, though some accelerometers and gyros have been made though that could be used in smart structures. The big drawback to MEMS is the high initial cost to design and build the correct one for the job at hand. If an off the shelf unit can be used, then their small size and low power consumption could make MEMS the sensor of choice.

At present, MEMS are being used as chemical sensors, air bag sensors, anti-lock brake sensors and gas pressure sensors in automobiles, to name a few applications. In the biomedical field, some applications include blood pressure sensors, gas detection sensors, and chemical analysis. New applications of MEMS sensors include high-resolution displays and electrostatically actuated microrelays (Paula, 1996).

2.4. Applications and Research

This section will try to give a brief overview of some of the current applications of smart structures.

2.4.1. Smart Structures Around the House

Smart materials and structures are slowly creeping into the everyday life of the average person. The simplest example of this is the photogrey glasses mentioned at the beginning of this report. Other examples would include windows whose transparency can be adjusted at the touch of a button (Neumann, 1996) and the new anti-scald showerheads made from SMA’s (Ashley, 1996). These new showerheads cut the water supply down to a trickle when the water reaches 116 °F, to prevent burns. The trickle of water that
remains is what is used to cool the SMA down and thus reset the device. Other uses of SMA actuators around the house include thermostats in coffee pots and eyeglass frames that return to their original shape when run under hot water (Shape Memory Applications Inc.). Even skis are becoming smart. K2 has a pair of skis with piezoceramics in them for vibration control (Ashley, 1995). The idea behind them is to improve edge contact with the snow and thus give better control to the skier.

2.4.2. Biomedical Applications

Biomedical applications of smart structures are dominated by the use of SMA’s as actuators. This is probably due to their being biocompatible. They are used as orthodontic wire in braces, bendable surgical tools, and blood clot filters to name a few applications (Shape Memory Applications Inc.). Newer uses of SMA’s include devices to correct scoliosis and as tiny gripping tools to place clotting agents in the brain (Ashley, 1996). MEMS are also being used in biomedical applications as various sensors (Paula, 1996).

2.4.3. Damage Detection

Damage detection and health monitoring is a very important field for the aerospace community. Electrostrictive, piezoelectric and fiber optic materials have all been used to achieve this goal. Electrostrictive materials are being used to detect corrosion in pipes (Thier et al., 1991) by sending elastic waves through the pipes. Piezoceramics are being used to detect damage in composites by detecting changes in the structures natural frequencies (Islam and Craig, 1994). Fiber optics is being used to detect damage in large civil engineering structures (Culshaw et al., 1996), such as bridges and buildings, because of their ability to measure over large distances. They are also being used for damage detection in metal matrix composites (Asanuma et al., 1996).

Two new methods in damage detection involve none of the above materials. These new methods depend on measuring the changes in electrical resistivity as the structure suffers
damage. The first method (Valenti, 1996) involves placing small electrical probes on carbon fiber composites. Carbon conducts electricity, so as fibers are broken the resistivity of the part goes up. This method is also capable of measuring temporary changes due to strain.

The other method involves a new intelligent material called CFGFRP (Carbon Fiber, Glass Fiber Reinforced Plastic) (Takagi, 1996). This material has a built in self-diagnosing function. As the carbon fibers in the material are broken, the electrical resistance of the material increases greatly. The CFGFRP then resists the load due to the presence of the glass fiber.

2.4.4. Vibration Control

Vibration control is another important area for the aerospace community. Especially in the field of space based optics, where vibrations could cause the signal to miss its target. Piezoelectric material are the most often use for vibration suppression due to their high bandwidth. Work has been done on vibration in beams using both piezopolymers (Miller and Hubbard; Bailey and Hubbard, 1985) and piezoceramics (Baz and Poh, 1988; Poh and Baz, 1990; Lazarus and Crawley, 1992; Hanagud et al., 1995). Vibration control has also been applied to airframes (Hanagud and Babu, 1994). Similar vibration control experiments have been tried with fiber optics as sensors and piezoelectrics as actuators (Asanuma et al., 1996; Chien et al., 1996). SMA’s (Rogers et al., 1991) and magnetostrictors (Johnson, 1992) have also been experimented with to try to damp out vibrations.

The structure shown in Figure 2.5 is the ASTREX testbed at USAF Phillips Lab it is supposed to simulate the beam expander of a

![Figure 2.5 ASTREX Testbed](image)
space based weapon. The whole structure is mounted on an air bearing and can be spun with one finger. The long struts are have piezoceramic actuators embedded in them to eliminate vibrations and perform disturbance rejection. New control algorithms and devices are periodically tried out on it. A magnetostrictor actuator was tried out on it by Johnson (1992).

2.4.5. Positioning and Shape Control

Being able to position a structure accurately and be able to reject any disturbances introduced is important for any pointing system. Recently, changing the pitch of helicopter rotor blades actively, using piezoelectrics has been tried (Ashley, 1996). The result was reduced mass and radar cross section over conventionally articulated rotors. Research has also been done on controlling the slewing of an active structure (Garcia and Inman, 1990; Denoyer and Kwak) and on disturbance rejection for precision structures (Anderson et al.; Collins et al., 1991). One of the ways that smart structures have been applied to robotics is by modifying the position of a robotic finger to control the amount of force it imparts (Tanaka et al., 1996).

Being able to keep a preferred shape or being able to change shape can be useful for positioning and other applications. Work has been done on keeping composite columns straight under compressive loads (Thompson and Loughlan, 1995). This was done because imperfections in the composites tend to make them deflect, and thus reduce their buckling loads, when loaded as columns. Shape control in flexible beams has been done using SMA’s (Nagaya and Ryu,
An offshoot of these position and shape control applications are the smart deployment mechanism shown in Figure 2.6. The SMA launch retention mechanism helps to eliminate the shock of separation of a satellite from the launch vehicle. The other devices help to deploy solar array panels and antennae arrays.
3. Historical Review: Piezoelectrics and Modeling of Them

Piezoelectric ceramics (PZT) are among the most widely used and studied of the active materials. From this study have arisen constitutive governing laws (Cady, 1946; Toupin, 1956; Tiersten, 1969; Joshi, 1992; Yu, 1993), analytical and experimental analysis (Lalande, 1995; Zhou et al., 1993; Liang et al., 1993b; Pan et al., 1991; Crawley and Anderson, 1989; Crawley and deLuis, 1987), and methods for the design and implementation of PZT actuators (Bailey et al., 1988, Crawley and Lazarus, 1989; deLuis and Crawley, 1990; Kienholz, 1993; LaPeter et al., 1991; Hagood et al. 1994; Zhou et al., 1995). This chapter will introduce the history, manufacture, use, and modeling of PZTs.

3.1. History

The piezoelectric effect was first discovered in 1880 by the Curie brothers, Pierre and Jacques. In that year they published experiments demonstrating the direct piezoelectric effect in crystals of tourmaline, quartz, topaz, cane sugar and Rochelle salt. The direct piezoelectric effect is the ability of a material to develop an electric charge when subjected to a mechanical strain. The inverse piezoelectric effect, generating a strain when a voltage is applied, was predicted by Lippman in 1881. Later that year the Curie brothers verified this and went on to establish the complete reversibility of these deformations in the crystals.

In 1894, Woldemar Voigt took the relationship a step further, by coming up with a complete thermodynamic model of the phenomenon. Voigt defined the twenty naturally occurring crystal classes having piezoelectric properties and the 18 piezoelectric coefficients need to manipulate the model.

The first practical use of piezoelectricity was made by Paul Langevin in 1917. Langevin used quartz crystals to create a ultrasonic submarine detector. Despite the poor performance and production difficulties associated with piezoelectric crystals of the time, many applications were developed. Among them were microphones, accelerometers,
phonograph needles, signal filters, and transient pressure measurements.

The use of piezoelectric materials was revolutionized with the discovery that poling could induce a piezoelectric effect in materials. This induced piezoelectric effect was found to be many times stronger than that observed in naturally occurring crystals. Barium titanate was developed in 1942 by poling. Lead zirconate titanate (PZT), developed in the early 1950’s, showed a much stronger piezoelectric effect and is widely used today as a piezoelectric actuator. A methodology was developed whereby, these two families of materials could be doped with impurities to obtain the desired material properties. This allows the material to be tailored to the specific application, and has led to current crop of piezoelectric ceramics.

3.2. Material Fabrication

Piezoelectric ceramics are formed in a two step process of sintering and poling (Moulson and Herbert, 1990). The ceramic is formed by placing powdered ceramic into a mold and sintering it at high temperatures, typically 800°C. It is then removed from the oven and electrodes are plated on their poling surfaces. This is done while they are still above their Curie temperature. The Curie is the temperature at which the crystal structure of the material converts from a non-symmetric to symmetric form (Piezo Systems, 1996), and the polarization in the material will be lost.

![Diagram of poled piezoceramic](image)

Figure 3.1 Potentials and their effect on a poled piezoceramic (Morgan Matroc, 1993)
The next step is to pole the ceramic. This is done by placing the ceramic in a bath of transformer oil and applying a large potential across it. The field is typically from one to four megavolts per meter. The oil is slowly cooled until it is 50°C below the Curie temperature. The electric dipoles within the material align themselves so that the negative pole is aligned with the positive poling voltage and the positive pole is aligned with the negative poling voltage. This alignment causes the individual particles to align into a single crystallographic orientation and induces a permanent structural deformation, making the material expand in the direction of the applied potential and contract in directions orthogonal to this poling direction.

The alignment is not complete however, and when a lower voltage is subsequently applied, the dipoles respond by trying to further orient themselves. If the voltage applied is of the same polarity as the poling voltage, the ceramic elongates along the deformation direction. If the applied voltage is of the opposite polarity, the opposite effect is observed. This can be seen in Figure 3.1.

3.3. Constitutive Relations

Figure 3.2 shows the basic deformation configurations for piezoelectric plates. Usually the deformation is a combination of two or more of these actions. These deformations are the result of the relationship between the applied electric field and the resulting response of the piezoceramic. This relationship depends upon the shape and composition of the piezoelectric, its piezoelectric properties, the position of the electrodes, the location of the poling axis, and the polarity of the applied field. This relationship is described by the
constitutive equations for a piezoceramic material. In compressed matrix notation these equations are:

\[ S_{p} = s_{pq}^{E} T_{q} + d_{kp} E_{k} \]  
\[ D_{i} = d_{iq} T_{q} + \varepsilon_{ik}^{T} E_{k} \]  

(3.1)

Where \( p,q,k,i \) take on values 1,2,3, donating the three material axis, and:

\[ T_{q} = \text{the stress tensor} \]
\n\[ s_{pq}^{E} = \text{the elastic compliance matrix} \]
\n\[ S_{p} = \text{the mechanical strain tensor} \]
\n\[ d_{kp} = \text{the piezoelectric strain constant matrix}, \]
\n\[ E_{k} = \text{electric field density} \]
\n\[ D_{i} = \text{the electric displacement} \]
\n\[ \varepsilon_{ik}^{T} = \text{the permittivity matrix} \]

These coefficients have two subscripts. The first subscript refers to the applied voltage and the second subscript refers to the direction of the mechanical strain. The material constants can also have superscripts, which indicate the electrical or mechanical boundary conditions applied when the constant was tested. These superscripts are:

\[ T = \text{constant stress, no strain constraints} \]
\[ E = \text{constant electrical field, short circuit the electrodes} \]
\[ D = \text{constant electrical displacement, open circuit the electrodes} \]
\[ S = \text{constant strain, mechanically clamped condition} \]

The constants relating mechanical strain produced to the applied electric field are the \( d_{kp} \).
or piezoelectric strain coefficients. \( d_{ij} \) can also be viewed as relating the charge on the electrode to the applied mechanical stress. The first view is useful when the indirect piezoelectric effect is used and has typical units of \textit{meters/volt}. The second view is used for the direct piezoelectric effect and has typical units of \textit{coulombs/Newton}.

The constitutive equations can be written in many different forms. The form given in (3.1) is in terms of strain and electric field density. A form in terms of stress and electric displacement is given below:

\[
\begin{align*}
S_p &= s^D_{pq} T_q + g_{kp} D_k \\
E_i &= -g_{iq} T_q + \beta^T_{ik} D_k
\end{align*}
\]

This form introduces new constants, \( g_{kp} \), the voltage constants. These constants relate the electric field to the state of the mechanical stress. Large \( g_{kp} \) constants indicate large voltage output per unit stress, which is a quality indicative of a good sensor. Polymer piezoelectrics, such as PVDF, have high voltage constants, which is why they are often used as sensors.

\subsection*{3.4. Common Configurations}

Piezoelectric ceramics are available in several configurations. Common leveraging configurations, used for positioning, include bimorphs, unimorphs, and stacks. Newer leveraging configurations include Rainbow wafers and Thunder wafers (Ashley, 1995). For vibration and shape control piezoceramics are usually imbedded or surface mounted.

\subsubsection*{3.4.1. Bimorphs}

Bimorphs are among the oldest leveraging configuration. They were first introduced six decades ago by C. B. Sawyer (Morgan Matroc TP-218). They consist of two piezoelectric patches bonded to each other or to each side of a shim. Normally they are configured so that one patch is expanded and another is contracted. This way a large bending displacement can be achieved. A bimorph is shown in Figure 3.3.
Unimorphs are a type of bimorph where the only a single layer of piezoelectric is used. The second layer is replaced by a non-active host. Unimorphs are stiffer than bimorphs, but bimorphs can obtain much larger displacements. Active beams that are completely covered with piezoelectrics, such as those of Bailey (1985), are no more than large unimorphs. Other variations of the bimorph include multimorph in which holes are drilled through the ceramic to shape the polarization of the material. This polarization shaping and the inhomogeneity of the structure lead to a large mechanical output.

Bimorphs can be combined mechanically with the material poles either aligned or opposed and electrically in parallel or series. This way they can be used to form stacks, which are capable of producing large displacements. Bimorphs have found use in the fields of ultrasonics, wave filtering, electromechanical relays, and home speaker systems, and noise cancellation among other ones. A good overview of the use and configurations of bimorphs can found in Morgan Matroc TP-218.

### 3.4.2. Stacks

Stacks consist of multiple layers of piezoceramic bonded together so they are mechanically in series and electrically in parallel. The stack is wired so that when an electrical potential is applied to it is applied across all of the layers. The strain induced in each layer is then summed up to produce the overall induced strain of the stack.
Stacks are capable of microns of expansion, kilonewtons of force, and have a response time in the neighborhood of ten microseconds. The use temperature of piezoelectric stacks is limited to half their Curie temperature. Above this point, the polarization of the material begins to break down. Two common stack configurations can be found in Figure 3.4.

3.4.3. Moonie Actuators

Moonie actuators were first proposed by Q. C. Xu in 1991 (Uchino, 1993) for use as hydrophone sensors. They consist of a multilayer piezoceramic bonded between two metal caps. The moon shaped space between the ceramic and the end cap give it its name. Figure 3.5 shows the geometry of a Moonie actuator.

The actuator is capable of a displacement eight times that of a multilayer stack of the same volume. A moonie actuator five millimeters by five millimeters by two and one half millimeters has been reported to
generate 20\(\mu\)m (0.000787 inches) of displacement, exerting five hundred kilopascals (72.5 psi) of stress under the application of a sixty volt electric field (Uchino, 1993). This large displacement is a combination of an in-of-plane displacement, described by \(d_{33}\), and a flexural motion due to mechanical motion, described by \(d_{31}\).

### 3.4.4. Rainbow and Thunder Wafers

RAINBOW (Reduced And INternally Biased Oxide Wafer) (Heartling, 1994) and Thunder wafers are two new configurations that show promising improvements over the traditional ones. RAINBOW wafers are lead lanthanum zirconium titanate (PLZT) ceramics that have had one side of them rendered inactive. This causes the PLZT to form either a dome or a saddle shape, depending on the original shape. These act similar to unimorphs and are capable of as much as 0.050 inch at applied voltages between 300 and 500V (Ashley, 1995). They can exert stresses of nearly 600 kilopascals and can be stacked to achieve higher displacements. A Thunder wafer is a piezoelectric attached to a metal backing with a polyimide adhesive. The device is then heat-treated and the thermal mismatch between the piezoelectric and the backing causes a dome like structure to form. A thunder wafer with a diameter of seven centimeters has produced a displacement of one centimeter with no load applied. Neither of these configurations is widely available at present.

### 3.4.5. Surface Mounted and Embedded

The actuator configurations presented above have, for the most part, been used for positioning. The fields of vibration and shape control tend to use actuators patches that have been either directly bonded to or embedded in the structure. In these cases, the position of the actuator, the orientation of the patch polarization, and the polarity of the applied excitation determine whether the induced loading is pure bending moments or extensional and compressional loading.

For surface bonded configurations, the deformation of the piezoceramic is constrained to
match the deformation of the structure at the contact surface. The other side of the piezoceramic is free of any constraint. A constraint strain is assumed through the thickness.

Quite often surface mounted actuators are used in pairs, with the patches on opposite sides of the structure. This configuration is seen in Figure 3.6. In these cases, the structure is used as a common ground. If the polarization direction of the patches are aligned (the positive electrode of one patch and the negative electrode of the other is exposed), then the excitation of the patches induces bending moment to the structure. If the polarities are opposed then extension is induced. If the polarities are neither aligned nor opposed, or if different potentials are applied to each patch, then a combination of bending and extension is induced in the structure.

Embedding the actuator allows the designer to place it exactly where he wants it, to get the desired effect. The embedded configuration can be seen in Figure 3.7. The actuator is constrained to match the structures deformation at both the top and bottom surfaces. Again, a constraint strain is assumed through the thickness. A lot of work has been done on embedding piezoceramics in composites and the problems it introduces (Bailey et al., 1988; Hagood et al., 1988; Ikegami et al., 1990; Wang and Rogers, 1991; Ha et al., 1991;
Section 3.5.4 will deal with the problems of embedding piezoceramics in composites.

3.5. **Implementation Issues**

Many factors can affect the use of piezoceramic actuators. Among these are depoling, hysteresis, Curie temperature, aging, non linear response, creep effects, variations with mechanical strain, and effects from embedding them in composites. A good overview of these and other factors was given by Crawley and Anderson (1989). What follows is a brief overview of some of these effects.

### 3.5.1. Depoling

Normally the induced poling of the ceramic is considered permanent, but it can be lost under certain conditions. Both temperature and high field levels can cause the ceramic to be depoled.

A piezoceramic has a maximum use temperature, called its Curie temperature. The Curie temperature is the temperature at which the crystal structure of the material converts from a non-symmetric to symmetric form (Piezo Systems, 1996). If the ceramic exceeds this temperature then the polarization in the material is lost. When making sure that the ceramic stays beneath its Curie temperature, its internal power dissipation must also be taken into account.

![Figure 3.8 Field vs. Strain curve for unconstrained G-1195 (Crawley and Anderson, 1989)](image)
Large potentials can also cause the ceramic to depole. All piezoceramics have a rated coercive field. The material depoles at this field level. If a DC field, greater than the coercive field, is applied opposite to the polarization direction, then the material will depole and repole itself in the direction opposite to its original poling direction. If the field is applied aligned with the polarization direction, the material will not depole, but arcing and brittle fracture may be caused. These fields either can be applied directly or can be induced by large applied stresses. Stresses on the order of 10 MPa to 100 MPa have been found to cause depolarization (Moulson and Herbert, 1990). To avoid depoling it is suggested that the field levels be kept below 80% of the coercive field level.

If the ceramic does depole or have its polarity reversed, it can be fixed. To do this simply a high sustained voltage in the direction of the desired polarity. Generally, 15 to 30 minutes is enough to achieve the desired polarity.

### 3.5.2. Nonlinear Response and Aging

The constitutive equations presented in §3.3 showed a linear relationship between the applied field and the resulting strain. As can be seen in Figure 3.8, this is just an approximation that is only valid for low applied voltages and strain levels. Since most published $d_{ij}$ values are given as the slope of this linearized curve, they are only valid for small fields. If the field level of interest is outside of the linear region, then a new value for $d_{ij}$ must be found by making a linearizing the data in the field level of interest.

The properties of the ceramic decay logarithmically with time. This effect is called aging and will cause the linearization of the data to no longer be valid. After time, a new $d_{ij}$ should be calculated from a new linearized curve. Aging can also be accelerated by exposure to elevated temperatures.
3.5.3. Hysteresis

Figure 3.9 displays the hysteretic field-strain relationship of a piezoceramic for three different levels of applied field. The higher the applied field the more hysteresis is observed. This hysteresis can be a very large problem in positioning applications, as the hysteresis can be as much as 30% of the full stroke of the actuator. For vibration problems, hysteresis can be considered as an unmodeled phase lag. This phase lag will not cause instability so long as the system phase margins are sufficient. For static applications hysteresis must be dealt with explicitly.

3.5.4. Problems with Embedding Piezoceramics

The layered structure of most composites lends themselves to the embedding of piezoceramic patches. Embedding the actuator allows the designer to place it exactly where he wants it to get the desired effect. It allows the designer to fine tune the overall structure more precisely then when the patches are surface mounted.

Some problems with piezoceramics arise, though, when they are embedded in materials. The first is that the cure temperature for composite materials can be above the Curie temperature for the piezoceramic. This requires the use of lower temperature curing composites, but limits the service temperature of the structure.

Another problem is that the inclusion of the piezoelectric patch is usually done by cutting out a hole in a composite layer. This means that some of the fibers are no longer continuous, thus interrupting the load path. This requires that the layer including the piezoelectric be modeled differently or considered a sacrificial layer. This also brings up
the question of what effect the piezoelectric has on the mechanical properties of the composite. Work has been done to study the effect of the piezoceramic on the composite, be it embedded or surface mounted (Wang and Rogers, 1991; Ha et al., 1991; Ha et al., 1992).

Embedding the piezoelectric helps to protect it from the environment outside the structure, but causes a problem with serviceability. Serviceability is a concern since there is no way to access them once the part has been made. The only contact with the patch is through the leads attached to it. If the leads are broken there is no way to reattach them and the patch is useless.

The fragile nature of ceramic piezoelectrics can be a problem. Care must be taken when selecting piezoceramics to make sure that they will survive the manufacturing process and the operating environment of the structure. Their brittleness limits the surface curvatures that can be imposed upon the ceramic. This can be overcome by using piezopolymers, but as mentioned earlier they do not have the stiffness usually required for actuators.

3.6. Modeling of Piezoelectrics

The performance of an application can only be as good as the model it was designed from. For this reason, accurate and easy to use models are desired. However, most of the existing models are either inaccurate or complicated to use. This section will present an overview of four methods for developing models. A very complete and concise view of this area was presented by Lalande (1995). Rather than duplicate his work a brief overview of each method will be. More detail on any of the methods can be obtained by referring to Lalande’s work, which is available on-line at the world wide web site of the Center for Intelligent Materials Systems and Structures at Virginia Polytechnic Institute, or through the same institution’s library.

3.6.1. Static Equivalent Force Method (SEF)

Models derived from the SEF method are the most widely used although they have been
shown to significantly miscalculate the systems resonant frequencies, and the actuator force, stress, and strain (Liang et al., 1993b; Zhou et al., 1996; Fairweather 1998). The reason for their widespread acceptance is their easy formulation and a retention of the physical insight into the system.

In this approach, the actuator is replaced by a force or moment acting on the structure at static equilibrium. This force is calculated for a unit level of actuator excitation. The structure itself is just considered an elastic stiffness against which the actuator acts. The method derives its name from this replacing of the actuator with an equivalent force calculated from a static analysis of the structure. This replacement of the actuator with equivalent forces can be seen in Figure 3.10.

No attempt is made to include the stiffness, mass or frequency response of the actuator in the model. Furthermore, this method has only been applied to simple beam and plate structures. Application to more complex geometries would be very complex.

There are many different SEF models in existence. The differences between them come

Figure 3.10 Actuator replaced by equivalent forces (Fairweather, 1998)

Figure 3.11 Assumed strain fields of the actuator and structure (Fairweather, 1998 after Lalande, 1995)
from the assumed strain field and whether or not the epoxy bonding layer is taken into account. Figure 3.11 shows the assumed strain field for several different models. Lalande gives an overview of these different models and summarizes the assumptions made in each one. He also points out the major features and any errors that were made for each model. Table 3.1 from Lalande gives the extensional equivalent force, and out of phase equivalent moment for each of the models.
<table>
<thead>
<tr>
<th>Analytical Model</th>
<th>Equivalent Force</th>
<th>Equivalent Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Beams</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crawley and de Luis (1987)</td>
<td>( \frac{2 E_t t_s}{2 + \Psi} ) ( \Lambda )</td>
<td>( \frac{E_t t_s^2}{6 + \Psi} ) ( \Lambda )</td>
</tr>
<tr>
<td>Chaudhry and Rogers (1994)</td>
<td>( \frac{2 E_t t_s}{2 + \Psi} ) ( \Lambda )</td>
<td>( \frac{E_t t_s^2}{6 + \Psi + \frac{2}{T^2}} ) ( \Lambda )</td>
</tr>
<tr>
<td>Crawley and Anderson (1989)</td>
<td>( \frac{2 E_t t_s}{2 + \Psi} ) ( \Lambda )</td>
<td>( \frac{E_t t_s^2\left(1 + \frac{1}{T}\right)}{6 + \Psi + \frac{12}{T} + \frac{8}{T^2}} ) ( \Lambda )</td>
</tr>
<tr>
<td>Wang and Rogers (1991)</td>
<td>( \frac{2 E_t t_s}{6 + \Psi} ) ( \Lambda )</td>
<td>( (t_a + t_s) \frac{E_t t_s}{6 + \Psi} ) ( \Lambda )</td>
</tr>
<tr>
<td><strong>Plates</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hagood et al. (1988)</td>
<td>( \frac{E_t t_s}{1 - v} ) ( \frac{2}{2 + \Psi} ) ( \Lambda )</td>
<td>( \frac{E_t t_s^2}{1 - v} ) ( 6 + \Psi )</td>
</tr>
<tr>
<td>Crawley and Lazarus (1989)</td>
<td>( \frac{E_t t_s}{1 - v} ) ( \frac{2}{2 + \Psi} ) ( \Lambda )</td>
<td>( \frac{E_t t_s^2 \Lambda}{1 - v} ) ( \frac{1 + \frac{1}{T}}{6 + \Psi + \frac{12}{T} + \frac{8}{T^2}} )</td>
</tr>
<tr>
<td>Dimitriadis et al. (1991)</td>
<td>N/A</td>
<td>( \frac{E_t t_s^2 \Lambda}{1 - v} ) ( \frac{1 + \frac{1}{T}}{6 + \Psi + \frac{12}{T} + \frac{8}{T^2}} )</td>
</tr>
<tr>
<td>Kim and Jones (1991)</td>
<td>N/A</td>
<td>( \frac{E_t t_s^2 \Lambda}{1 - v} ) ( \frac{1 + \frac{1}{T}}{6 + \Psi + \frac{12}{T} + \frac{8}{T^2}} )</td>
</tr>
<tr>
<td>Wang and Rogers (1991)</td>
<td>( \frac{E_t t_s}{1 - v} ) ( \frac{2}{6 + \Psi} ) ( \Lambda )</td>
<td>( \frac{E_t t_s^2\left(1 + \frac{1}{T}\right)}{1 - v} ) ( 6 + \Psi ) ( \Lambda )</td>
</tr>
<tr>
<td>Wang and Rogers (1991b)</td>
<td>( \frac{2 E_t t_a}{1 - v} ) ( \Lambda )</td>
<td>( \frac{E_t t_s^2\left(1 + T\right)}{1 - v} ) ( \Lambda )</td>
</tr>
<tr>
<td>Lin and Rogers (1992)</td>
<td>( \frac{\Psi E_t t_s^2 h \Lambda}{K_1} \left( K_{10} + K_{11} \frac{\cosh(\eta_1 \xi)}{\cosh(\eta_1 \ell)} \right) )</td>
<td>( \frac{\Psi E_t t_s^2 h \Lambda}{6 K_0} \left( K_8 + K_9 \frac{\cosh(\eta_1 \xi)}{\cosh(\eta_1 \ell)} \right) )</td>
</tr>
</tbody>
</table>

Where \( T = \frac{t_s}{t_a} \) and \( \Psi = \frac{E_t t_s}{E_t t_a} \).
3.6.2. Finite Element Methods (FEM)

One approach to modeling the interaction between the actuator and the structure has been the use of finite element methods (FEM) to include the piezoceramic’s properties into finite element analysis (FEA) programs (Alik and Hughes, 1970; McDearmon, 1984; Crawley and Anderson, 1989; Hagood et al., 1990). Again, Lalande (1995) gives an excellent summary of using FEM to get models. He groups the work into three paths that can be taken to obtain a model. These paths are: directly formulating elements for specific applications, utilizing a thermoelastic analogy, and using commercially available FEA codes having piezoelectric element formulations.

Writing your own elements allows you to exactly tailor your element to the problem at hand. This way you can include both mechanical and electrical effects, as well as including whichever material properties and boundary conditions are deemed necessary. If working with a commercial code that allows custom elements, these elements can be added directly to the program. This can allow for the use of existing pre- and post-processors if they can handle the custom elements. If they cannot, then a custom one would have to be written. These elements are usually free to use, since they can be found in the literature. Table 3.2 shows a list some of the available elements.
Table 3.2 Some Piezoelectric Elements found in the Literature (Fairweather, 1998)

<table>
<thead>
<tr>
<th>Element type</th>
<th>Number of nodes</th>
<th>Degrees of freedom per node</th>
<th>Developer</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laminated brick</td>
<td>8</td>
<td>3 normal displacements</td>
<td>Ha et al. (1990, 1991, 1992)</td>
<td>• Used to model laminated composite.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 electrical potential</td>
<td></td>
<td>• Demonstrated need for three-dimensional incompatible modes.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 electrical potential</td>
<td></td>
<td>• Provided experimentally verified predictions for beams and plates.</td>
</tr>
<tr>
<td>Plate</td>
<td>4</td>
<td>1 normal displacement</td>
<td>Hwang and Park (1993)</td>
<td>• Derivation based on Hamilton’s principle.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 rotational displacements</td>
<td></td>
<td>• Used classical laminate theory.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 electrical potential</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Laminated Shell</td>
<td>12</td>
<td>3 displacements</td>
<td>Tzou and Ye (1994)</td>
<td>• Applied to bi-morph and semi-circular shell.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 electrical potential</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quadrilateral plate</td>
<td>9</td>
<td>3 normal displacements</td>
<td>Chandrashenkhara and Agarwal (1993) Shah et al. (1993, 1993b)</td>
<td>• Includes induced strain load as external moment loading on actuator boundaries. No electrical degree of freedom.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 rotational displacements</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quadrilateral plate</td>
<td>20</td>
<td>3 normal displacements</td>
<td>Detwiler et al. (1994)</td>
<td>• Includes induced strain load as external moment loading on actuator boundaries. No electrical degree of freedom.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 rotational displacements</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Though they are free to use, they can be very expensive in terms of the time required to code and debug them. They require a high degree of familiarity with FEM, which most designers lack, and thus often require someone with specialized knowledge of the FEA code to come in and write them. In addition, using FEA can cause a loss of insight into the problem and finally, every time the patch is moved on the structure a whole new solution must be done. This can become very computationally expensive and time consuming for complex problems.

The thermoelastic analogy treats piezoelectricity as a thermal expansion problem and utilizes existing thermoelastic elements. In this analogy, voltage is represented by temperature and current is represented by heat flux. This approach can be used with any commercial code that supports thermal loading and can incorporate any geometry that the package can handle. Mollenhauer and Griffin (1994) demonstrated this approach and showed that is accurate for static deformation problems. Dynamic problems can only be studied if the package features harmonic thermal analysis. Few packages have this feature.
so this approach is most useful for static problems. In addition, a very fine mesh is needed around the actuator for accurate strain transfer. This leads to large numbers of elements and thus longer solution times. Finally, this approach suffers from the common FEA problems of a loss of insight into the problem and that every time the patch is moved on the structure a whole new solution must be done. This can become very computationally expensive and time consuming for complex problems.

The final path is using commercially available FEA codes having piezoelectric element formulations. The initial cost of these packages is high, but once purchased they have the lowest cost in man-hours, since the elements are already coded and designers already know how to use the packages. In addition, existing structural models can be used, with the addition of the piezoelectric elements representing the actuator. The capabilities of two widely used packages, ABAQUS and Ansys, was done by Lin, Abatan and Rogers (1994). This path, though, also suffers from the common FEA problems of a loss of insight into the problem and that every time the patch is moved on the structure a whole new solution must be done. This can become very computationally expensive and time consuming for complex problems.

3.6.3. Models Derived from First Principles

The use of first principles can allow for both the mechanical and electrical aspects of the actuator to be taken into account. Which aspects and to what degree, depend upon the principle applied. The drawback to this approach is that for anything more complex than a simple beam these models become very high order. This makes them not well suited for typical design studies, such as optimal placement of actuators. Additionally, the dynamics of the structure and actuator are solved simultaneously, so for every change in actuator placement, the whole problem must be solved again.

The Rayleigh-Ritz energy method has been used by Hagood et al. (1990) and Akella (1994). Hagood's model includes the dynamics of an electrical network connected across the patch electrodes and produced a state space formulation that is well suited to control
system development by modern design methods. Akella's model consisted of a beam subjected to arbitrary boundary conditions and was used to formulate an output feedback controller that guaranteed that the closed loop system was passive.

The Bernoulli-Euler beam theory was used by Pan, Hansen, and Snyder (1991) to develop a dynamic beam model that incorporated the stiffness of the patch actuator and the loading on it, but its electrical dynamics were neglected. They then compared their results to experimental measurements and to the SEF model of Crawley and de Luis (1987). The dynamic model's results were in good agreement with the experimental measurements, whereas the SEF model's results were measurably different.

3.6.4. Impedance Method

The impedance method is the latest technique to model the interaction between the actuator and the structure. It was developed at Virginia Polytechnic Institute and most of the work on it has been done there. The technique is best summarized by the following six steps as put forth by Fairweather (1998).

1. A dominant mode of low-frequency motion is assumed in the proposed induced strain actuator. For long, slender actuators with side electrodes, the mode is usually assumed a purely extensional vibration. This is the mode usually used for actuators on beams. For thin rectangular actuators with electrodes on the two largest faces, the motion is assumed a longitudinal extension in two dimensions, and with a contraction through the actuator’s thickness. This is the mode used for actuators on plates.

2. The boundary conditions for the resulting actuator vibration problem are written in terms of the appropriate mechanical impedance of the structure to which the actuator will be attached.

3. The response of the actuator is determined in terms of the impedance of the host structure.
4. The mechanical impedance of the host structure is then determined. Prior to Fairweather (1998), this was done by analytically solving the structural vibration problem of the host structure's geometry.

5. The mechanical impedances are used to determine the response of the actuator, which includes the actuator stress and the actuator displacement.

6. The actuator stress is used to compute the force applied to the structure by the actuator. The response of the structure to the application of this force is then calculated.

The major advantage of this technique is that multiple actuator locations can be tried with just one solution of the host structure's vibration problem. This is possible because the structure's mechanical impedances are formed without the presence of the actuator in the model. This ability to easily try out multiple actuator locations helps the design engineer to maintain a high degree of physical insight into what is happening. In addition, this technique has been shown to have very good agreement with experimental measurements as opposed to the simpler SEF predictions (Fairweather, 1998; Zhou et al., 1996; Sermoneta et al. 1995, Rossi et al. 1993). This good agreement with experimental results allows the design engineer to trust his model. This trust in the model allows it to be viewed as a final or close to final design instead of just a starting point for a design determined by experimentation.

There are problems with this approach though. To begin with, it is a fairly new and novel approach so many people have shied away from it for this reason. The bigger problem though is in the determination of the structure's mechanical impedances. Until recently, these were found be analytically solving the structure's vibration problem. This limits the methods applicability to simple geometries for which a closed form solution of the impedance can be found. Recently, Fairweather (1998) developed a means of generating these impedances from an FEM analysis of the host structure. He uses eigenvalues and mass normalized eigenvectors to compute the frequency response of the structure and to
generate its mechanical impedances.

A good overview of the research on impedance methods, and corrections of errors made in many of the published papers, can be found in Fairweather (1998). Rather than just reiterate his findings, only a brief overview of the published papers will be given. For a more detailed review, consult his work.

The basics of impedance modeling was laid out by Liang et al. (1993), and a comparison between impedance modeling and the SEF method was made for a spring-mass-damper system. Liang et al. (1993b) extended the method to a multiple degree of freedom system by addressing the dynamic modeling of an active beam. Zhou et al. (1993) applied the method to a free cylindrical structure and compared it to a two-dimensional SEF model. Lalande (1995) studied ring and shell structures and included the effects of transverse shear, which Zhou et al. (1993) had left out. Simply supported plates were studied by Zhou et al. (1994) and the results compared to SEF model predictions and experimental results. The mechanical impedances of the host structure in all the above papers were generated analytically.

Fairweather (1998) developed a way to generate the mechanical impedances from the results of an FEA analysis of the host structure. This allows the impedance method to be applied to any structure that can be accurately modeled by FEA. This was done for both beam and simply supported plate problems and the results compared to experimental results and/or the published results from the above papers. Corrections to errors made in many of the above papers were also presented.

The ability of the impedance approach to accurately predict the response of a beam with multiple actuators was shown by Sermoneta et al. (1995). A method to determine the impedance of a piezoceramic stack actuator was presented by Lomenzo et al. (1993). This is important since stack actuators are widely used in active structures to generate high displacements and high forces.
The impedance method was put to a practical use by Sumali and Cudney (1994) in the design of an active engine mount. It was shown how the impedance method in parametric investigations of dynamic system behavior. This paper also showed it is easy to maintain physical insight into what is happening in the system while using the impedance method.
4. Finite Element Derived Impedance Method Applied to Plates

This chapter will develop the necessary equations to generate the response of an active plate to an actuator. First, the vibration of the piezoceramic is modeled analytically. Then a method of determining the mechanical moment impedances from FEA will be given. The FEA program that will be used is MSC/Nastran. These impedances can then be used to calculate the moment exerted by the piezoceramic patch and the displacement of the patch.

This development comes from the work of Fairweather (1998). Who in turn based his work off Zhou et al. (1996) and Norton (1989). At the end of the chapter how to adjust the equations to allow for off axis actuators and the use of shape functions to describe the eigenvectors.
The following table shows the variables used in this chapter:

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Symbol</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complex modulus of elasticity of the piezoceramic</td>
<td>( \tilde{\gamma}_{11}^E )</td>
<td>Pa</td>
</tr>
<tr>
<td>Poisson ratio of the piezoceramic</td>
<td>( \nu_p )</td>
<td>-</td>
</tr>
<tr>
<td>Density of the piezoceramic material</td>
<td>( \rho_p )</td>
<td>kg/m(^3)</td>
</tr>
<tr>
<td>Piezoelectric strain coefficient</td>
<td>( d_{31} )</td>
<td>m/N</td>
</tr>
<tr>
<td>Piezoelectric strain coefficient</td>
<td>( d_{32} )</td>
<td>m/N</td>
</tr>
<tr>
<td>Thickness of the piezoceramic</td>
<td>( h_p )</td>
<td>m</td>
</tr>
<tr>
<td>Thickness of the plate</td>
<td>( t_p )</td>
<td>m</td>
</tr>
<tr>
<td>Length of the piezoceramic</td>
<td>( a_p )</td>
<td>m</td>
</tr>
<tr>
<td>Width of the piezoceramic</td>
<td>( b_p )</td>
<td>m</td>
</tr>
<tr>
<td>Electric field applied to the piezoceramic</td>
<td>( E_3 )</td>
<td>V/m</td>
</tr>
<tr>
<td>Cross sectional area of piezoceramic in YZ plane</td>
<td>( A_x )</td>
<td>m(^2)</td>
</tr>
<tr>
<td>Cross sectional area of piezoceramic in XZ plane</td>
<td>( A_y )</td>
<td>m(^2)</td>
</tr>
<tr>
<td>Mechanical impedance</td>
<td>( Z_{ij} )</td>
<td>N*sec/m</td>
</tr>
<tr>
<td>Mechanical admittance</td>
<td>( H_{ij} )</td>
<td>M/N*sec</td>
</tr>
<tr>
<td>In-plane force admittance</td>
<td>( Q_{kl} )</td>
<td></td>
</tr>
<tr>
<td>Elastic Compliance Matrix</td>
<td>( s_{ij} )</td>
<td>1/MPa</td>
</tr>
<tr>
<td>Spatial frequency of the oscillations</td>
<td>( \Omega )</td>
<td></td>
</tr>
<tr>
<td>Mass matrix</td>
<td>[M]</td>
<td></td>
</tr>
<tr>
<td>Stiffness matrix</td>
<td>[K]</td>
<td></td>
</tr>
<tr>
<td>Identity matrix</td>
<td>[I]</td>
<td></td>
</tr>
<tr>
<td>Matrix of eigenvectors</td>
<td>[( \Phi )]</td>
<td></td>
</tr>
<tr>
<td>Matrix of eigenvalues</td>
<td>[( \Lambda )]</td>
<td></td>
</tr>
<tr>
<td>Displacement of piezoceramic in X-direction</td>
<td>( u(x_p, y_p, t) )</td>
<td>m</td>
</tr>
<tr>
<td>Displacement of piezoceramic in Y-direction</td>
<td>( v(x_p, y_p, t) )</td>
<td>m</td>
</tr>
</tbody>
</table>

### 4.1. Vibration of the Piezoceramic Actuator

The piezoceramic actuator is assumed to undergo an extensional vibration in two dimensions. The boundary conditions are written in terms of the mechanical stresses applied to the actuator. These stresses are written in terms of the mechanical impedance of the host structure. The coefficients of the actuator displacement solution are determined using the constitutive equations of the ceramic.
Consider Figure 4.1, which depicts schematically the piezoceramic patch anchored to a host structure via mechanical impedance elements.

The force exerted by the actuator on the structure is written in terms of the structure's mechanical impedance as:

\[
\begin{bmatrix}
F_x \\
F_y
\end{bmatrix} =
\begin{bmatrix}
Z_{xx} & Z_{xy} \\
Z_{yx} & Z_{yy}
\end{bmatrix}
\begin{bmatrix}
\dot{u} \\
\dot{v}
\end{bmatrix}
\]

(4.1)

The stress caused by this force is written as:

\[
\begin{bmatrix}
\sigma_{xp} \\
\sigma_{yp}
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{\lambda_x} & 0 \\
0 & \frac{1}{\lambda_y}
\end{bmatrix}
\begin{bmatrix}
Z_{xx} & Z_{xy} \\
Z_{yx} & Z_{yy}
\end{bmatrix}
\begin{bmatrix}
\dot{u} \\
\dot{v}
\end{bmatrix}
\]

(4.2)

The strain developed in the actuator results from the mechanical force and the electrically induced strain of the piezoelectric effect. This strain is described by the constitutive equations as:

\[
\begin{bmatrix}
\varepsilon_{xp} \\
\varepsilon_{yp}
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial u}{\partial x_p} \\
\frac{\partial v}{\partial y_p}
\end{bmatrix} =
\begin{bmatrix}
s_{11} & s_{12} \\
s_{21} & s_{22}
\end{bmatrix}
\begin{bmatrix}
\sigma_{xp} \\
\sigma_{yp}
\end{bmatrix} +
\begin{bmatrix}
d_{31} \\
d_{32}
\end{bmatrix}
E_3
\]

(4.3)

Figure 4.1 Piezoceramic connected by impedance elements to structure (Zhou, Liang and Rogers, 1996)
If the piezoceramic material of the actuator is assumed both homogeneous and isotropic, then the elastic compliance is then written as:

\[ s_{ij} = \frac{1}{\tilde{Y}_{11}^E} \]

And the constitutive relations are rewritten as:

\[
\begin{bmatrix}
\frac{\partial u}{\partial x_p} \\
\frac{\partial v}{\partial y_p}
\end{bmatrix} = \frac{1}{\tilde{Y}_{11}^E} \begin{bmatrix} 1 & -\nu_p & 1 \\ -\nu_p & 1 & 0 \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} + \begin{bmatrix} d_{31} \\ d_{32} \end{bmatrix} E_3
\]

If (4.2) is substituted for the mechanical stress in (4.5), then the constitutive relations can be expressed in terms of the impedance of the host structure as:

\[
\begin{bmatrix}
\frac{\partial u}{\partial x_p} \\
\frac{\partial v}{\partial y_p}
\end{bmatrix} = \frac{1}{\tilde{Y}_{11}^E} \begin{bmatrix} 1 & -\nu_p & 1 \\ -\nu_p & 1 & 0 \end{bmatrix} \begin{bmatrix} Z_{xx} & Z_{xy} & \frac{1}{\lambda} \\ Z_{yx} & Z_{yy} & \frac{1}{\mu} \end{bmatrix} \begin{bmatrix} u \\ v \\ \dot{\gamma} \end{bmatrix} + \begin{bmatrix} d_{31} \\ d_{32} \end{bmatrix} E_3
\]

In (4.6), the strain in the ceramic is written in terms of the velocity of the ceramic and the applied electric field. For (4.6) to be useful, expressions for the time varying displacement of the ceramic, \( u(x_p,y_p,t) \) and \( v(x_p,y_p,t) \), must be derived. To obtain these expressions, a differential area element (Figure 4.2) of the ceramic material is examined.
Summing of the forces in the $x_p$ and $y_p$ directions results in the following dynamic equations of motion:

$$
\sum F_x = -\sigma_{x_p} h_x dy_p + \left( \sigma_{x_p} + \frac{\partial \sigma_{x_p}}{\partial y_p} \right) h_x dy_p - \tau_{x_p y_p} h_y dx_p + \left( \tau_{x_p y_p} + \frac{\partial \tau_{x_p y_p}}{\partial x_p} \right) h_y dx_p = \rho_s h_x dx_p dy_p \frac{\partial^2 u}{\partial t^2} \tag{4.7}
$$

$$
\sum F_y = -\sigma_{y_p} h_y dx_p + \left( \sigma_{y_p} + \frac{\partial \sigma_{y_p}}{\partial x_p} \right) h_y dx_p - \tau_{y_p x_p} h_x dy_p + \left( \tau_{y_p x_p} + \frac{\partial \tau_{y_p x_p}}{\partial y_p} \right) h_x dy_p = \rho_s h_y dx_p dy_p \frac{\partial^2 v}{\partial t^2} \tag{4.8}
$$

These differential equations are simplified into the following form (Zhou, Liang and Rogers, 1996):

$$
\frac{\partial \sigma_{x_p}}{\partial x_p} + \frac{\partial \tau_{x_p y_p}}{\partial y_p} = \rho_s \frac{\partial^2 u}{\partial t^2} \tag{4.9}
$$

$$
\frac{\partial \sigma_{y_p}}{\partial y_p} + \frac{\partial \tau_{y_p x_p}}{\partial x_p} = \rho_s \frac{\partial^2 v}{\partial t^2} \tag{4.10}
$$

Expressions for the normal stresses $\sigma_{x_p}$ and $\sigma_{y_p}$ are determined from the constitutive equations given in (4.5) as:
An expression for the shear stress is obtained from its definition as:

\[
\tau_{\gamma p} = \tilde{G}_{\gamma p} \left( \frac{\tilde{Y}_{11}^E}{2(1+\upsilon_p)} \left( \frac{\partial u}{\partial y_p} + \frac{\partial v}{\partial x_p} \right) \right)
\]  

(4.12)

Assume that the electrical field applied to the ceramic material is spatially uniform over the electrodes of the patch. Differentiating (4.11) and (4.12) and substituting the results into (4.9) and (4.10) leads to the following, coupled, differential equations of motion for the piezoceramic patch.

\[
\frac{Y^E}{1-\upsilon_p^2} \frac{\partial^2 u}{\partial x_p^2} + \frac{Y^E}{2(1-\upsilon_p)} \frac{\partial^2 v}{\partial x_p \partial y_p} + \frac{Y^E}{2(1+\upsilon_p)} \frac{\partial^2 u}{\partial y_p^2} = \rho_p \frac{\partial^2 u}{\partial t^2}
\]  

(4.13)

\[
\frac{Y^E}{1-\upsilon_p^2} \frac{\partial^2 v}{\partial y_p^2} + \frac{Y^E}{2(1-\upsilon_p)} \frac{\partial^2 u}{\partial x_p \partial y_p} + \frac{Y^E}{2(1+\upsilon_p)} \frac{\partial^2 v}{\partial x_p^2} = \rho_p \frac{\partial^2 v}{\partial t^2}
\]  

(4.14)

These coupled equations describe a classical problem in elasticity, one that was shown by Love (1944) to have a complete analytical solution if the material domain is bounded by a circle.

However, rectangular or square patches are being analyzed here so the solution of (4.13) and (4.14) is more complex. There have been several solutions given for this problem if there are free - free boundary conditions on the plate (Petržílka, 1935; Bechmann, 1941; and Eckstein, 1944). The present analysis has different boundary conditions, though, so these solutions are not applicable.

The boundary conditions on the patch are described in terms of the impedance of the plate structure to which the patch is attached. This makes solution of the problem harder as the boundary conditions are variable and can't be used to decouple the equations. What is
needed is a way to decouple the equations.

Zhou, Liang and Rogers (1996) proposed a solution to this problem. They neglect the change of rate of shear strain terms in (4.13) and (4.14) to obtain

\[
\frac{Y^E}{1-\nu_p^2} \frac{\partial^2 u}{\partial t^2} = \rho_p \frac{\partial^2 u}{\partial t^2}
\]

(4.15)

\[
\frac{Y^E}{1-\nu_p^2} \frac{\partial^2 v}{\partial t^2} = \rho_p \frac{\partial^2 v}{\partial t^2}
\]

(4.16)

and then using a separation of variables approach to achieve a solution at individual frequencies.

There has been some question as to whether or not this assumption if enough to produce (4.15) and (4.16) from (4.13) and (4.14). Fairweather (1998) claims that to do this one must also assume the Poisson effect normal stresses to be negligible. However a close inspection of those equations that make up (4.13) and (4.14) show that the assumption is complete.

Since the same reasoning can be used for both equations, only one needs to be examined. Let us look at how (4.13) becomes (4.15) with this assumption. To do this we actually must start with (4.9). If we assume a constant electric field and substitute (4.11) and (4.12) into (4.13) and do not combine terms, we obtain:

\[
\frac{Y^E}{1-\nu_p^2} \frac{\partial^2 u}{\partial x^2} + \frac{Y^E}{1-\nu_p^2} \frac{\partial^2 u}{\partial y^2} + \frac{Y^E}{2(1+\nu_p)} \frac{\partial^2 v}{\partial x \partial y} + \frac{Y^E}{2(1+\nu_p)} \frac{\partial^2 v}{\partial y^2} = \rho_p \frac{\partial^2 u}{\partial t^2}
\]

(4.17)

The first two terms comes from the substitution of (4.11) and the second two from (4.12). The first term is the term that is retained in (4.15). The fourth term is apparent as one of the terms the assumption drops going from (4.13) to (4.15). What happens to the second and third terms is from where the confusion about the assumption arises.
Combined they make the second term in (4.13). The assumption removes the third term. So why doesn’t the second term remain by itself in (4.15)? Magnitude is the reason. If you look at the terms it is obvious that they contain the same differentials of the same variable. The difference is their coefficients. Examination of these coefficients shows that they are of the same magnitude over the range of acceptable values of $v_p$. At $v_p = \gamma$ they are exactly equal in magnitude. Since the contribution of third term is considered negligible, we must also consider the second term’s contribution to be negligible. Thus in this way, the assumption, that the change of rate of shear strain is negligible, allows us to go from (4.13) to (4.15).

The solutions to (4.15) and (4.16) are found through a separation of variables approach. The process is identical for each so only the solution to (4.15) is performed in detail.

Assume that the problem is variable separable, in which case there exists a solution of the form $u(x_p, t) = \phi(x_p)q(t)$. Substituting this solution into the governing differential equation and defining a separation constant, $k$, produces the following equation.

$$
\frac{\phi''(x_p)}{\phi(x_p)} = \frac{\rho_p (1 - v_p^2)}{Y^E} \ddot{q}(t) = k \tag{4.18}
$$

The separation constant is solved for by taking the temporal portion to the frequency domain:

$$
k = -\omega^2 \frac{\rho_p (1 - v_p^2)}{Y^E} \tag{4.19}
$$

This is used to separate the governing differential equation into the following ordinary, homogeneous differential equations.

$$
\phi''(x_p) + \frac{\omega^2 \rho_p (1 - v_p^2)}{Y^E} \phi(x_p) = 0 \tag{4.20}
$$

$$
\ddot{q}(t) + \omega^2 q(t) = 0 \tag{4.21}
$$
The equation for \( \nu(x_p, t) \) is separated using a similar procedure. Assume solutions of the form
\[
\begin{align*}
\phi(x_p)q(t) &= \left( A \sin(\Omega y_p) + B \cos(\Omega y_p) \right) e^{j\omega t} \quad (4.22) \\
\phi(y_p)q(t) &= \left( C \sin(\Omega y_p) + D \cos(\Omega y_p) \right) e^{j\omega t} \quad (4.23)
\end{align*}
\]

The constant \( \Omega \) is the spatial frequency of the oscillations and is given as
\[
\Omega = \omega \sqrt{\frac{\rho_p (1 - \nu^2)}{Y^E}} \quad (4.24)
\]

The boundary conditions of the piezoceramic patch are
\[
u(0, t) = 0 \quad \nu(0, t) = 0 \quad (4.25)
\]

which lead to \( B = D = 0 \).

The coefficients \( A \) and \( C \) are determined by substituting the assumed solutions into the constitutive equations given by (4.6) and evaluating them at \( x_p = a_p \) and \( y_p = b_p \).

\[
\begin{pmatrix}
A \\
C
\end{pmatrix} = \begin{pmatrix}
\Omega \cos(\Omega b_p) + \frac{j \alpha Z_m \sin(\Omega a_p)}{Y^E A_x} - \frac{j \nu \nu Z_m \sin(\Omega a_p)}{Y^E A_x} & \frac{j \alpha Z_m \sin(\Omega b_p)}{Y^E A_x} - \frac{j \nu \nu Z_m \sin(\Omega b_p)}{Y^E A_x} \\
\frac{j \alpha Z_m \sin(\Omega a_p)}{Y^E A_y} - \frac{j \nu \nu Z_m \sin(\Omega a_p)}{Y^E A_y} & \Omega \cos(\Omega b_p) + \frac{j \alpha Z_m \sin(\Omega b_p)}{Y^E A_y} - \frac{j \nu \nu Z_m \sin(\Omega b_p)}{Y^E A_y}
\end{pmatrix}^{-1} \begin{pmatrix}
d \alpha \\
\nu \nu
\end{pmatrix} \quad (4.26)
\]

Substituting the displacement solutions (4.22) and (4.23) into (4.1) gives the force output in terms of the impedance of the mechanical structure as
\[
\begin{pmatrix}
F_{a_p} \\
F_{b_p}
\end{pmatrix} = -j \omega \begin{pmatrix}
Z_{xx} & Z_{xy} \\
Z_{yx} & Z_{yy}
\end{pmatrix} \begin{pmatrix}
A \sin(\Omega a_p) \\
C \sin(\Omega b_p)
\end{pmatrix} e^{j\omega t} \quad (4.27)
\]

where coefficients \( A \) and \( C \) are given by (4.26). The moment exerted by the patch on the structure is found by using the geometric relationship between force and moment.
\[
\begin{bmatrix}
M_{a_p} \\
M_{b_p}
\end{bmatrix} = -j\omega (t_p + \hat{t}_p) \begin{bmatrix}
Z_{xx} & Z_{xy} \\
Z_{yx} & Z_{yy}
\end{bmatrix} \begin{bmatrix}
A\sin(\Omega a_p) & 0 \\
0 & C\sin(\Omega b_p)
\end{bmatrix} e^{j\omega t}
\] (4.28)

Now that the force and moment exerted by the piezoceramic patch has been determined in terms of the host structure's mechanical moment impedance, the mechanical moment impedances themselves must be determined.

4.2. **Determination of the Moment Impedances from FEA**

Since the impedances are to be determined using MSC/Nastran, the first step is to develop a means to extract the frequency response functions from MSC/Nastran. To do this it is necessary to develop a relationship between the eigenvectors of a node and the forces, moments, velocities and displacements associated with that node.

Consider the following sets of undamped spring mass systems

\[
\begin{bmatrix}
M \\
K
\end{bmatrix} \ddot{x} + \begin{bmatrix}
K

\end{bmatrix} x = f(t)
\] (4.29)

where \(x\) and \(f(t)\) are \((n \times 1)\) vectors describing the physical positions and time-varying forcing functions applied to each degree of freedom of the system and assume that the mass matrix is non-singular and that the mass matrix and stiffness matrices are symmetric.

Taking the equation to the Laplace domain yields

\[
\begin{bmatrix}
M \\
K
\end{bmatrix} s^2 X(s) + \begin{bmatrix}
K

\end{bmatrix} X(s) = F(s)
\] (4.30)

Since

\[
\begin{bmatrix}
M \\
K
\end{bmatrix} = \begin{bmatrix}
M

\end{bmatrix}^T \\
\begin{bmatrix}
K

\end{bmatrix} = \begin{bmatrix}
K

\end{bmatrix}^T
\] (4.31)

the coordinate transformation \(x = \begin{bmatrix}
M

\end{bmatrix}^{-\frac{1}{2}} z\) can be made yielding
\[ [M]^\frac{1}{2}s^2Z(s) + [K][M]^\frac{1}{2}Z(s) = F(s) \]  

(4.32)

Pre-multiplying (4.32) by \([M]^{-\frac{1}{2}}\) and collecting terms produces

\[
\left( [I]s^2 + [M]^{-\frac{1}{2}}[K][M]^{-\frac{1}{2}} \right)Z(s) = [M]^{-\frac{1}{2}}F(s)
\]

(4.33)

where \([I]\) is the identity matrix. The eigenvalues of this system can be determined by finding the roots of the equation

\[
\left( [I] + [M]^{-\frac{1}{2}}[K][M]^{-\frac{1}{2}} \right) = 0
\]

(4.34)

Through inspection it is noted that \(\omega = \sqrt{-\lambda}\). The eigenvectors of the system are given in the conventional manner as those, which satisfy the equation

\[
\left( [I] + [M]^{-\frac{1}{2}}[K][M]^{-\frac{1}{2}} \right)\phi_i = 0
\]

(4.35)

If a matrix of these eigenvectors is defined as

\[
[\Phi] = \begin{bmatrix} \phi_1 & \phi_2 & \ldots & \phi_n \end{bmatrix}
\]

(4.36)

and a matrix of the eigenvalues is defined as

\[
[\Lambda] = \begin{bmatrix}
-\omega_1^2 & 0 & \ldots & 0 \\
0 & -\omega_2^2 & \ldots & 0 \\
\vdots & \ddots & \ddots & 0 \\
0 & \ldots & 0 & -\omega_n^2
\end{bmatrix}
\]

(4.37)

then (4.35) can be rewritten in as

\[
[M]^{-\frac{1}{2}}[K][M]^{-\frac{1}{2}}[\Phi] = -[\Phi][\Lambda]
\]

(4.38)

MSC/Nastran provides \(\Phi\) as a matrix of orthonormal vectors so \(\Phi^{-1} = \Phi^T\). Pre-multiplying (4.38) by \(\Phi^T\) and utilizing the above property produces the following result.
\[
\begin{align*}
\Phi^T [M]^{-\frac{1}{2}} [K] [M]^{-\frac{1}{2}} \Phi &= -[\Lambda] \\
\text{(4.39)}
\end{align*}
\]

If the coordinate transformation \( z = [\Phi]q \) is made, then (4.33) can be rewritten in these new modal coordinates as

\[
Q(s)s^2 + [\Phi]^T [M]^{-\frac{1}{2}} [K] [M]^{-\frac{1}{2}} [\Phi] Q(s) = [\Phi]^T [M]^{-\frac{1}{2}} F(s)
\]
\[
\text{(4.40)}
\]

which can be rewritten based on the results of (4.39) as

\[
Q(s)s^2 - [\Lambda]Q(s) = [\Phi]^T [M]^{-\frac{1}{2}} F(s)
\]
\[
\text{(4.41)}
\]

Define a mass normalized eigenvector as

\[
[\Phi]_m = [M]^{-\frac{1}{2}} [\Phi]
\]
\[
\text{(4.42)}
\]

Since \((AB)^T = B^T A^T\) and \(M = M^T\) then

\[
[\Phi]_m^T = [\Phi]^T [M]^{-\frac{1}{2}}
\]
\[
\text{(4.43)}
\]

Substituting this into (4.41) yields:

\[
Q(s)s^2 - [\Lambda]Q(s) = [\Phi]_m^T F(s)
\]
\[
\text{(4.44)}
\]

The following equation can be obtained from (4.44)

\[
Q(s) = ([I)s^2 - [\Lambda])^{-1} [\Phi]_m^T F(s)
\]
\[
\text{(4.45)}
\]

Using \( X(s) = [M]^{-\frac{1}{2}} [\Phi] Q(s) = [\Phi]_m Q(s) \) to transform (4.45) back to the original coordinate system yields:

\[
X(s) = [\Phi]_m ([I)s^2 - [\Lambda])^{-1} [\Phi]_m^T F(s)
\]
\[
\text{(4.46)}
\]

This is the equation relating the displacement vector in physical coordinates to force.
applied in physical coordinates. For a specific physical coordinate $X_j(s)$ and force $F_k(s)$

\[
\frac{X_j(s)}{F_k(s)} = \sum_{i=1}^{\infty} \frac{[\Phi(j,i)]_{n_k} [\Phi(k,i)]_{n_f}}{s^2 + \omega_i^2} \tag{4.47}
\]

Differentiating (4.47) yields the mechanical admittance, the inverse of the mechanical impedance, of the structure:

\[
\frac{V_j(s)}{F_k(s)} = \sum_{i=1}^{\infty} \frac{[\Phi(j,i)]_{n_k} [\Phi(k,i)]_{n_f} s}{s^2 + \omega_i^2} \tag{4.48}
\]

This equation for the mechanical admittance makes no requirements on the type of structure, so it can be used for any structure. Since we are interested in plates, we will now show how to get the moment impedances for a plate from (4.48).

Consider Figure 4.3 which shows piezoceramic patch attached to a plate structure. Inserting the eigenvectors associated with rotation for both the force input and velocity output into (4.48), one obtains the transfer function between rotational velocity and applied moment. For example, if the eigenvectors associated with rotation about the $y$-axis at $(x_1, y_2)$ are substituted for both the input and output, one obtains a moment
admittance about the y-axis of the structure at the center of the left edge of the patch.

\[
\frac{s\Theta_{(x_1, y_2)}(s)}{m_{(x_1, y_2)}(s)} = \sum_{i=1}^{\infty} \frac{[\Phi(R_2(x_1, y_2), i)]_{xy}[\Phi(R_2(x_1, y_2), i)]_{yp}}{s^2 + \omega_i^2} s
\]

(4.49)

Using the principal of superposition, one can obtain the admittance transfer function between the differential rotational velocity about the y-axis at points \((x_1, y_2)\) and \((x_3, y_2)\) and counter-posed moments applied at the same points as

\[
\frac{s[\Theta_{(x_3, y_2)}(s) - \Theta_{(x_1, y_2)}(s)]}{m_{(x_1, y_2)}(s) - m_{(x_3, y_2)}(s)} = \sum_{i=1}^{\infty} \frac{[\Phi(R_2(x_3, y_2), i) - \Phi(R_2(x_1, y_2), i)]_{xy}[\Phi(R_2(x_3, y_2), i) - \Phi(R_2(x_1, y_2), i)]_{yp}}{s^2 + \omega_i^2} s = H_{xy}
\]

(4.50)

and the admittance transfer function between the differential rotational velocity at points \((x_2, y_1)\) and \((x_5, y_2)\) and counter-posed moments applied at \((x_2, y_1)\) and \((x_3, y_2)\) as

\[
\frac{s[\Theta_{(x_5, y_2)}(s) - \Theta_{(x_2, y_1)}(s)]}{m_{(x_2, y_1)}(s) - m_{(x_5, y_2)}(s)} = \sum_{i=1}^{\infty} \frac{[\Phi(R_2(x_5, y_2), i) - \Phi(R_2(x_2, y_1), i)]_{xy}[\Phi(R_2(x_5, y_2), i) - \Phi(R_2(x_2, y_1), i)]_{yp}}{s^2 + \omega_i^2} s = H_{sy}
\]

(4.51)

The differential angular velocity between points \((x_1, y_2)\) and \((x_3, y_2)\) can then be written in terms of the mechanical admittances as

\[
\frac{s[\Theta_{(x_3, y_2)}(s) - \Theta_{(x_1, y_2)}(s)]}{m_{(x_2, y_2)}(s) - m_{(x_2, y_2)}(s)} = H_{xy} [m_{(x_3, y_2)}(s) - m_{(x_2, y_2)}(s)] + H_{xy} [m_{(x_2, y_2)}(s) - m_{(x_2, y_2)}(s)]
\]

(4.52)

If the following definitions are made

\[
\Delta\theta_x(s) = \Theta_{(x_1, y_2)}(s) - \Theta_{(x_2, y_2)}(s)
\]

\[
\Delta\theta_y(s) = \Theta_{(x_2, y_1)}(s) - \Theta_{(x_2, y_2)}(s)
\]

\[
\Delta m_x(s) = m_{(x_2, y_2)}(s) - m_{(x_3, y_2)}(s)
\]

\[
\Delta m_y(s) = m_{(x_2, y_2)}(s) - m_{(x_2, y_2)}(s)
\]

then the matrix equation relating differential rotational velocities to the mechanical moment admittances and applied counter-posed moments is written as

\[
\begin{bmatrix}
s\Delta\theta_x(s) \\
s\Delta\theta_y(s)
\end{bmatrix} =
\begin{bmatrix}
H_{xy} & H_{xy} \\
H_{xy} & H_{xy}
\end{bmatrix}
\begin{bmatrix}
\Delta m_x(s) \\
\Delta m_y(s)
\end{bmatrix}
\]

(4.54)
Pre-multiplying the previous expression by the inverse of the mechanical admittance matrix yields the following relationship between angular velocity and applied moment

\[
\begin{bmatrix}
\Delta m_x(s) \\
\Delta m_y(s)
\end{bmatrix} =
\begin{bmatrix}
H_{xx} & H_{xy} \\
H_{yx} & H_{yy}
\end{bmatrix}^{-1}
\begin{bmatrix}
s\Delta \theta_x(s) \\
s\Delta \theta_y(s)
\end{bmatrix}
\tag{4.55}
\]

The force applied to the structure is written in terms of the moment exerted by the piezoceramic as \( \Delta m_x(s) = -\Delta F_x(s)(t_p + h_a) \) and the rotational velocity of the plate is written in terms of the linear velocity of the piezoceramic patch as \( s\Delta \theta_x(s) = s\frac{2}{(t_p + h_a)} \Delta u(s) \). Equation (4.55) is then rewritten as

\[
\begin{bmatrix}
\Delta F_x(s) \\
\Delta F_y(s)
\end{bmatrix} =
\frac{2}{(t_p + h_a)^2}
\begin{bmatrix}
H_{xx} & H_{xy} \\
H_{yx} & H_{yy}
\end{bmatrix}^{-1}
\begin{bmatrix}
s\Delta u(s) \\
s\Delta v(s)
\end{bmatrix}
\tag{4.56}
\]

The patch model assumed zero linear displacement, and therefore velocity, in the x-direction along the line \( x_p = 0 \) and the y-direction along the line \( y_p = 0 \). In addition, (4.27) shows that the force, \( F_x \), is zero when evaluated along the line \( x_p = 0 \) and the force, \( F_y \), is zero when evaluated along the line \( y_p = 0 \). Making these observations, the above is simplified as

\[
\begin{bmatrix}
F_{x_{xy}}(s) \\
F_{y_{xy}}(s)
\end{bmatrix} =
\frac{2}{(t_p + h_a)^2}
\begin{bmatrix}
H_{xx} & H_{xy} \\
H_{yx} & H_{yy}
\end{bmatrix}^{-1}
\begin{bmatrix}
su_{x_{xy}}(s) \\
sv_{y_{xy}}(s)
\end{bmatrix}
\tag{4.57}
\]

which is recognized as the frequency domain equivalent of (4.1) where

\[
\begin{bmatrix}
Z_{xx} & Z_{xy} \\
Z_{yx} & Z_{yy}
\end{bmatrix} =
\frac{2}{(t_p + h_a)^2}
\begin{bmatrix}
H_{xx} & H_{xy} \\
H_{yx} & H_{yy}
\end{bmatrix}^{-1}
\begin{bmatrix}
Q_{xx} & Q_{xy} \\
Q_{yx} & Q_{yy}
\end{bmatrix}^{-1}
\tag{4.58}
\]

In (4.58) \( Q_{kl} \) is defined to be the in plane force admittance, and is related to \( H_{kl} \) as

\[
Q_{kl} = \frac{(t_p + h_a)^2}{2} H_{kl}
\]

Utilizing the preceding, results the moments exerted by the piezoceramic can be computed from (4.28) based on the impedances determined from the
eigenvalues and eigenvectors of the FEA.

4.3. **Determination of the Structure's Response**

Now that we have the structure's impedances in terms of its eigenvalues and eigenvectors, we need to develop an equation that will give us the displacement in the z-direction. The governing differential equation for this type of motion is given by Soedel (1981) as:

$$D \left( \frac{\partial^4 w(x, y, t)}{\partial x^4} + 2 \frac{\partial^4 w(x, y, t)}{\partial x^2 \partial y^2} + \frac{\partial^4 w(x, y, t)}{\partial y^4} \right) + \rho h \ddot{w}(x, y, t) = -Q(x, y, t)$$

(4.59)

Where \( D \) represents the stiffness of the plate, \( h \) is the thickness of the plate, \( \rho \) is the density of the plate material, \( w(x, y, t) \) is the transverse displacement of the plate, and \( Q(x, y, t) \) is the distributed loading function. To solve (4.59), first look at the homogeneous form of the differential equation and use a separation of variables approach so that the response \( w(x, y, t) \) is written as:

$$w(x, y, t) = q(t) \psi(x, y)$$

(4.60)

Using this is in the homogeneous form of (4.59) gives:

$$D \left( \frac{\partial^4 \psi(x, y)}{\partial x^4} + 2 \frac{\partial^4 \psi(x, y)}{\partial x^2 \partial y^2} + \frac{\partial^4 \psi(x, y)}{\partial y^4} \right) \psi(x, y) = -\frac{\rho h q(t)}{q(t)} = k$$

(4.61)

Where the separation constant \( k \) is found to be:

$$k = \rho h \omega_n^2$$

(4.62)

This gives us the following separation into temporal and spatial domains:

$$D \left( \frac{\partial^4 \psi(x, y)}{\partial x^4} + 2 \frac{\partial^4 \psi(x, y)}{\partial x^2 \partial y^2} + \frac{\partial^4 \psi(x, y)}{\partial y^4} \right) - \rho h \omega_n^2 \psi(x, y) = 0$$

(4.63)

$$\rho h \ddot{q}(t) + \rho h \omega_n^2 q(t) = 0$$

(4.64)
The boundary conditions are to be satisfied by the eigenfunctions \( \phi_{mn}(x, y) \) that are determined from the FEA normal mode analysis. This is the main benefit of the approach. Any boundary condition that can be modeled by FEA can be used.

This solution can be represented as a summation of many admissible plate configurations, each with independent spatial frequencies. If the forcing function is assumed to be of the variable separable form:

\[
Q(x, y, t) = Q_s(x, y)Q_t e^{j\omega t}
\]  

(4.65)

Then for each of the spatial modes there exists a corresponding temporal solution of the form:

\[
q_{mn}(t) = A_{mn} e^{j\omega t + \phi_{mn}}
\]  

(4.66)

The complete vibration response of the plate (4.60) is rewritten as

\[
w(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{mn}(t) \phi_{mn}(x, y)
\]  

(4.67)

Substituting this into (4.59) yields:

\[
D \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{mn}(t) \left( \frac{\partial^2 \phi_{mn}(x, y)}{\partial x^2} + 2 \frac{\partial^2 \phi_{mn}(x, y)}{\partial x \partial y} + \frac{\partial^2 \phi_{mn}(x, y)}{\partial y^2} \right) + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \rho \beta \phi_{mn}(x, y) \phi'_{mn}(t) = -Q_s(x, y)Q_t e^{j\omega t}
\]  

(4.68)

If we now follow Norton (1989) and:

1. Multiply through the equation by an orthogonal basis function
2. Substitute appropriate results from the previous separation of variables
3. Integrate over the length and width of the plate.
4. Use the properties of orthogonality to eliminate terms
The following equation is produced:

\[
\ddot{q}_{mn}(t) + \omega_{mn}^2 q_{mn}(t) = -\frac{Q_{e} e^{i\omega t}}{\rho h \int_0^L \int_0^W \phi_{mn}^2(x, y) \, dx \, dy} \int_0^L \int_0^W \phi_{mn}(x, y) Q_{e}(x, y) \, dx \, dy
\]  

(4.69)

Defining the modal force integral:

\[
F_{mn} = -\frac{1}{\rho h \int_0^L \int_0^W \phi_{mn}^2(x, y) \, dx \, dy} \int_0^L \int_0^W \phi_{mn}(x, y) Q_{e}(x, y) \, dx \, dy
\]  

(4.70)

Then (4.69) can be rewritten as:

\[
\ddot{q}_{mn}(t) + \omega_{mn}^2 q_{mn}(t) = F_{mn} Q_{e} e^{i\omega t}
\]  

(4.71)

Each modal equation is then written as:

\[
q_{mn}(t)(\omega_{mn}^2 - \omega^2) = F_{mn} Q_{e} e^{i\omega t}
\]  

(4.72)

Solving for the unknown \( A_{mn} \) in (4.66) as:

\[
A_{mn} = \frac{F_{mn} Q_{e}}{(\omega_{mn}^2 - \omega^2) e^{\phi_{mn}}}
\]  

(4.73)

And substituting this result into (4.67) gives the vibration response as:

\[
w(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{F_{mn} \phi_{mn}(x, y)}{(\omega_{mn}^2 - \omega^2)} Q_{e} e^{i\omega t}
\]  

(4.74)

If the temporal amplitude, \( Q_{e} \), is assumed to be unity, the only undetermined quantity is \( F_{mn} \), since the eigenvectors come from FEA. The factor pre-multiplying the integral in (4.70) is from mass normalization. If \( \phi_{mn}(x, y) \) is chosen to be the mass normalized eigenvector \( \phi_{mn}(x, y) \), then this factor can be dropped.

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The load distribution is given by \( Q(x, y) = \left( \frac{\partial M_x}{\partial x} + \frac{\partial M_y}{\partial y} \right) \), where the moment expressions are

\[
M_x = \overline{M}_x \left( \delta (x-x_i) - \delta (x-x_j) \right) \left( h(y-y_i) - h(y-y_j) \right) e^{i\omega t}
\]  \( (4.75) \)

\[
M_y = \overline{M}_y \left( \delta (y-y_i) - \delta (y-y_j) \right) \left( h(x-x_i) - h(x-x_j) \right) e^{i\omega t}
\]  \( (4.76) \)

The integral of \( (4.70) \) is rewritten with these substitutions as

\[
F_{mn} = -\int_0^b \int_0^a \phi_{mn} (x,y) \left[ \overline{M}_x \left( \delta '(x-x_i) - \delta '(x-x_j) \right) \left( h(y-y_i) - h(y-y_j) \right) \right] dy \ dx
\]

\[
-\int_0^b \int_0^a \phi_{mn} (x,y) \left[ \overline{M}_y \left( \delta '(y-y_i) - \delta '(y-y_j) \right) \left( h(x-x_i) - h(x-x_j) \right) \right] dx \ dy
\]

Using the property of the Dirac function, assuming that the actuator is within the bounds of the structure, \( 0 \leq x_i \leq x_j \leq a \) and \( 0 \leq y_i \leq y_j \leq b \), and that moment is constant along the edge of the actuator, \( (4.77) \), can be rewritten as:

\[
F_{mn} = -\overline{M}_x \int_0^b \left[ -\frac{\partial \phi_{mn} (x,y)}{\partial x} \bigg|_{y=y_i} + \frac{\partial \phi_{mn} (x,y)}{\partial x} \bigg|_{y=y_j} \right] dy
\]

\[
-\overline{M}_y \int_0^a \left[ -\frac{\partial \phi_{mn} (x,y)}{\partial y} \bigg|_{y=y_i} + \frac{\partial \phi_{mn} (x,y)}{\partial y} \bigg|_{y=y_j} \right] dx
\]

\[
(4.78)
\]

To perform this integration it is necessary to have an expression for the spatial derivatives of the eigenvectors in the domains of interest.

Under the Kirchoff hypothesis, the rotational eigenvectors \( R1 \) and \( R2 \) (MacNeal, 1994) are:

\[
R1 = -\frac{\partial \phi (x,y)_n}{\partial y}
\]  \( (4.79) \)

\[
R2 = \frac{\partial \phi (x,y)_n}{\partial x}
\]  \( (4.80) \)

MSC/Nastran reports these at discrete points (nodes) along the edge of the actuator, so they were fit to a sixth order polynomial so that the integration could be carried out.
explicitly. Alternatively the integration could have been carried out numerically on the values at the nodes.

The polynomial was then integrated to obtain the individual modal force projection in terms of $\vec{M}_x$ and $\vec{M}_y$.

$$w(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{(E_{x_m} \vec{M}_x + E_{y_n} \vec{M}_y) \phi_{mn}(x, y) e^{j \omega t}}{(\omega_{mn}^2 - \omega^2)}$$  \hspace{1cm} (4.81)

$E_{x_n}$ and $E_{y_n}$ are the results of the spatial integration of the rotations about the x-axis and y-axis respectively.

4.4. Center Points vs. Shape Functions

MSC/Nastran reports the eigenvectors at the nodes. These are discrete points along the sides of the actuator. The eigenvectors used in section 4.2 are reported at the center points of the patch sides. The question arises though, is the center point a good representation of what is happening on the side of the patch.

To investigate this question shape functions can be used instead of just the eigenvector at the center point. In this case a sixth order polynomial is fit to the eigenvectors at the nodes along the patch sides. The same polynomial is used for the shape functions as was used to get the modal force projection. This way the extra work required to use shape functions instead of center points is minimized.

In Chapter 6 comparisons will be made between using the eigenvectors at only the center points and using shape functions to represent the eigenvectors along the patch side. It will be shown that the shape functions do provide a better result than the center points.

4.5. Aligned vs. Angled

To this point the actuator has always been assumed to have its axis aligned with the axis of
the plate. What happens when this is no longer true? It is entirely possible that it could be advantageous to situate an actuator such that it isn't aligned with the structures axis. So we need to know the answer to this question.

A careful examination of the equations presented in this chapter shows that when the eigenvectors are used they are always used in the local coordinate system of the patch. So we must ensure that the eigenvectors are in this coordinate system. MSC/Nastran reports the eigenvectors in the global coordinate system, though. So the eigenvectors have to be rotated from the global system of the plate to the patch's local coordinate system. Since the Z-axis is the same for both the patch and the plate there is no need to rotate any of the Z quantities back to the global system to find the transverse displacement.

Now that we have an equation for the transverse displacement at a point it is time to see if this equation matches up with experimental results.


5. FEA Models

The first step in testing the method is to generate accurate FEA models. The results of the method can only be as accurate as the FEA model from which the eigenvectors are extracted. The models were generated in FEMAP and then run in MSC/NASTRAN. This chapter will give the details of the FEA models and how they were validated.

5.1. FEA Model for the Aluminum Plate

The first model made was for a 6061-T6-aluminum plate bolted at all four corners. CQUAD4 elements were used to model the plate and a normal mode analysis was run to extract the first 35 modes. The properties used in the FEA model are shown in Table 5.1.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of elasticity</td>
<td>68.9</td>
<td>GPa</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>2700</td>
<td>kg/m³</td>
</tr>
<tr>
<td>Thickness of the Al plate</td>
<td>1.5637</td>
<td>mm</td>
</tr>
<tr>
<td>Width of the Al plate</td>
<td>0.203</td>
<td>m</td>
</tr>
<tr>
<td>Length of the Al plate</td>
<td>0.304</td>
<td>m</td>
</tr>
</tbody>
</table>

Remember, that the piezoelectric actuator does not enter into the FEA model. A picture of the finite element mesh used can be seen in Figure 5.1. The nodes around the bolt holes were fixed so as to allow no rotations or displacements.

To make sure that this model accurately represents the plate, an aluminum plate was made and a modal analysis was conducted with an impact hammer. The results were compared to the FEA results. This was done before the piezoelectric actuators...
were bonded to the plate, since the FEA model does not contain the actuator. The experimental setup is shown in Figure 5.2.

The displacement sensor was placed at a location and then the hammer was used to excite the plate. After taking a set of data, the displacement sensor was then moved to another location and the process repeated.

The location of the displacement sensor was a site of maximum displacement for one of the modes as predicted by FEA.

The steps used to perform this experiment are summarized below:

1. Using a PCB Piezotronics Impulse Hammer (Model: 086B01, Serial #: 3195) with a medium (plastic) impact tip, the plate was struck. If the ringing died out before the sample was done another impact was done.

2. The signal generated by the impact hammer was fed into a PCB Piezotronics Power Unit / Signal Conditioner (Model: 480D06, Serial #: 9256) with a gain of 1.

3. This signal was then fed into input channel one of a DSPT SigLab (Model: 20-22A, Serial #: 1501).

4. The displacement of the plate was measured by a Bently Nevada Proximitor. The Proximitor consists of a probe (Model: 30000-00-32-36-02, Serial #: H432587) and a sending unit (Model: CP-0171857-01, Serial #: KC50510) with a gain of 200 mV/mil.

5. The output from the sending unit has a range of from zero to eighteen volts. This range is not compatible with the DSPT SigLab so an offset circuit was built. The signal from the Sending Unit was fed into this circuit so that its full range was
adjusted to be between positive and negative nine volts. The circuit can be seen in Figure 5.3.

6. This signal was then fed into input channel two of the DSPT SigLab.

7. The DSPT SigLab was controlled by a virtual instrument network analyzer (VNA) module inside of Matlab. This compares the two input functions and generates the transfer function between them. VNA was set up to take 120 averages of a 4096 point spectrum at a measurement bandwidth of one kilohertz with the hammer input as the applied input and the displacement as the response.

After all of the data had been collected the modal frequencies were extracted and compared to those predicted by the FEA. Since the data was collected at locations representing the maximum predicted displacement, the FEA value used for comparison was taken from the maximum displacement for that mode. There were three FEA models generated and their predicted responses were averaged to get the predicted FEA response. The difference between the experimental and predicted FEA modes is shown in Table 5.2.

![Figure 5.3: Offset Circuit](image-url)
Table 5.2: Difference between Predicted and measured Modes

<table>
<thead>
<tr>
<th>Mode</th>
<th>Measured Frequency (rad/sec)</th>
<th>Predicted Frequency (rad/sec)</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>81.8750</td>
<td>81.40003</td>
<td>-0.58%</td>
</tr>
<tr>
<td>2</td>
<td>137.5000</td>
<td>137.2957</td>
<td>-0.15%</td>
</tr>
<tr>
<td>3</td>
<td>210.6250</td>
<td>205.8459</td>
<td>-2.27%</td>
</tr>
<tr>
<td>4</td>
<td>240.6250</td>
<td>234.2385</td>
<td>-2.65%</td>
</tr>
<tr>
<td>5</td>
<td>334.3750</td>
<td>329.3206</td>
<td>-1.51%</td>
</tr>
<tr>
<td>6</td>
<td>338.7500</td>
<td>332.9155</td>
<td>-1.72%</td>
</tr>
<tr>
<td>7</td>
<td>356.2500</td>
<td>348.4398</td>
<td>-2.19%</td>
</tr>
</tbody>
</table>

This was deemed an acceptable fit, so additional models were made. There were three different sets of boundary conditions: one with bolts on four corners, a second with the four bolts plus two additional bolts in the interior, and a third with the corner bolts plus the holes for the interior bolts but no constraints on the holes.

To make extraction of the correct eigenvectors easier, nodes were placed along the edges of where the patch would be. This is not necessary for the method but made implementing it easier. So for each boundary condition there was a 90-degree and a 45-degree patch model made. Finally for each of these configurations both a coarse and a fine mesh were made. This way it could be determined if any of the modes were only numerical and if mesh density had an effect on results. In all twelve models were made for the aluminum plate.

Figure 5.4: FEA models with corner holes (left) and interior holes (right)
Figure 5.4 shows two of the final coarse FEA models. The one on the left only has corner holes and has nodes positioned along the boundaries of where 90-degree patch will be. The one on the right has the two interior holes added and has nodes positioned for a 45-degree patch. The final models also contain CTRIA3 elements in addition to the CQUAD4 elements in the models used for the modal analysis. The CTRIA3 elements were added automatically by FEMAP whenever the aspect ratio of the CQUAD4 elements was unacceptable.

For the coarse mesh there were 48 elements along the top and bottom sides, 32 elements along the left and right sides, 10 elements along each side of the patch and eight elements around each bolt hole. For the models with interior bolts or holes there was a square section of elements placed around the sensor area. This was to make sure that there was a node at the sensor location. FEMAP's auto mesh generation capability was used to generate the actual mesh. The mesh sizes varied from 2031 elements to 4158 elements.

For the fine mesh there were 72 elements along the top and bottom sides, 48 elements along the left and right sides, 16 elements along each side of the patch and 12 elements around each bolt hole. For the models with interior bolts or holes there was a square section of elements placed around the sensor area. This was to make sure that there was a node at the sensor location. For the models with 90-degree patches and interior bolts or holes there were only 12 elements along each side of the patch. This change was necessary because otherwise the models became too big for the computer to handle. FEMAP's auto mesh generation capability was used to generate the actual mesh. The mesh sizes varied from 4831 elements to 7624 elements.

5.2. **FEA Model for the Composite Plate**

The same basic procedure was followed for the composite plates. The plates were

![Figure 5.5: FEA mesh used for modal analysis of the composite plate](image)
made out of a graphite cross-ply cloth with a lay up of [0/90, +45/-45]. This was done so that any errors in manufacturing the plates would have a minimal impact on their performance. The initial results of the modal testing were very unsatisfactory. Differences were found on the order of 50%. It was discovered that the quoted properties for the graphite were for a unidirectional prepreg, whereas the material used was a bi-directional cloth used in a wet lay-up. Unfortunately no testing equipment was available within a reasonable time frame to obtain accurate material properties. The best test that could be performed was a beam-bending test on an extra piece of the laminate. This test provided a bending modulus for the laminate. This bending modulus the PC Laminate computer program was then used to find the property values needed by FEMAP. The laminate was entered as 2-D orthotropic material instead of as individual plies. Ply values were available from PC Laminate but it was decided that, since the test had been performed on the laminate and not on an individual ply, laminate data should be used.

Additionally an aluminum button was added into the model. The aluminum button was needed because the displacement sensor only works on aluminum. The button was approximately a one-inch square and was centered under the sensor. As in the aluminum plates the sensor was placed at several different locations during the testing. The button was attached with two-way tape so as to make moving it possible. In the FEA model the area of the button was modeled as two ply laminate, one ply being the graphite laminate and the other being the aluminum button. The bottom surface of these elements was shifted so that it lined up with the rest of the model and the aluminum layer was above the composite. It was found though that the presence of the button in the FEA model had very little effect on the predicted mode frequencies.
Table 5.3: Physical Properties used in FEA analysis of composite plate

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of elasticity for Al</td>
<td>68.9</td>
<td>GPa</td>
</tr>
<tr>
<td>Poisson ratio for Al</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Density of Al</td>
<td>2700</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>Thickness of the Al button</td>
<td>1.5637</td>
<td>mm</td>
</tr>
<tr>
<td>Width of the Al button</td>
<td>0.0254</td>
<td>m</td>
</tr>
<tr>
<td>Length of the Al button</td>
<td>0.0254</td>
<td>m</td>
</tr>
<tr>
<td>Modulus of elasticity for the composite</td>
<td>27.8</td>
<td>GPa</td>
</tr>
<tr>
<td>Shear modulus for the composite</td>
<td>10.5</td>
<td>GPa</td>
</tr>
<tr>
<td>Poisson ratio for the composite</td>
<td>0.32</td>
<td></td>
</tr>
<tr>
<td>Density of the composite</td>
<td>1374.63</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>Thickness of the composite plate</td>
<td>1.397</td>
<td>mm</td>
</tr>
<tr>
<td>Width of the composite plate</td>
<td>0.203</td>
<td>m</td>
</tr>
<tr>
<td>Length of the composite plate</td>
<td>0.304</td>
<td>m</td>
</tr>
</tbody>
</table>

One of the FEA models used for the modal testing of the composite plates can be seen in Figure 5.5. The button can be seen as a square at the center of the plate. Unlike the aluminum plates these models started with a mixture of CQUAD4 and CTRIA3 elements. Again, the nodes around the bolt holes were fixed so as to allow no rotations or displacements and a normal mode analysis was run to extract the first 35 modes. The properties used in the FEA analysis can be seen in Table 5.3. The composite was entered into FEMAP as two-dimensional orthotropic material. However, due to the quasi-isotropic nature of the lay-up the laminate has the same value for both the transverse and axial modulus of elasticity.

There was one model made for each sensor location that was going to be tested. The locations were selected to correspond with the point of maximum displacement for a mode. Multiple models were required because the location of the aluminum button had to be moved to wherever the sensor was located. After these models were made the modal testing was performed following the same procedure outlined in §5.1.

After all of the data had been collected the modal frequencies were extracted and
compared to those predicted by the FEA. The experimental value at each location was compared to the value for the FEA model with the button at that location. Furthermore, each of these locations was picked since it is the location of the maximum displacement for a certain mode, so it was this experimentally found mode that was compared to the FEA value at this location. The difference between the experimental and predicted FEA modes is shown in Table 5.4.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Measured Frequency (rad/sec)</th>
<th>Predicted Frequency (rad/sec)</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>65.5357</td>
<td>65.7899</td>
<td>0.39%</td>
</tr>
<tr>
<td>2</td>
<td>111.7708</td>
<td>110.595</td>
<td>-1.06%</td>
</tr>
<tr>
<td>3</td>
<td>130.4167</td>
<td>166.8</td>
<td>21.81%</td>
</tr>
<tr>
<td>4</td>
<td>175.0000</td>
<td>188.9572</td>
<td>7.39%</td>
</tr>
<tr>
<td>5</td>
<td>224.6429</td>
<td>266.1453</td>
<td>15.59%</td>
</tr>
<tr>
<td>6</td>
<td>276.9906</td>
<td>267.9463</td>
<td>-3.38%</td>
</tr>
<tr>
<td>7</td>
<td>294.1667</td>
<td>282.0647</td>
<td>-4.29%</td>
</tr>
</tbody>
</table>

These results were not as good as those for the aluminum plate but were the best that could be obtained without having better material property data. As in the case of the aluminum plates, models were then made with three different boundary conditions: bolts on four corners, corner bolts plus two additional bolts in the interior, and corner bolts plus the holes for the interior bolts but no constraints on the holes. Again coarse and fine mesh versions were made and nodes were placed along the boundaries of the patch. This gives a total of 12 models for the composite plates. The composite models though had to have the aluminum button added to them like the models used for modal testing. The models shown in Figure 5.4 are actually models for the composite plates. The button is represented by the square in the upper right hand quadrant.

For the coarse mesh there were 48 elements along the top and bottom sides, 32 elements along the left and right sides, 10 elements along each side of the patch, four elements along each side of the button and eight elements around each bolt hole. FEMAP's auto mesh generation capability was used to generate the actual mesh. The mesh sizes varied.
from 2534 elements to 3599 elements.

For the fine mesh there were 72 elements along the top and bottom sides, 48 elements along the left and right sides, 16 elements along each side of the patch and 12 elements around each bolt hole. For the models with only corner bolts there were eight elements along each side of the button. For models with interior bolts or holes there were six elements along each side of the button. This was done to make sure that the models did not get too big for the computer to handle. FEMAP's auto mesh generation capability was used to generate the actual mesh. The mesh sizes varied from 7085 elements to 7785 elements.
6. Experiments and Results

In Chapter 4 the necessary equations were developed to predict the response of an active plate to an actuator, using Eigen vectors and values. These were to be found from an FEA modal analysis of the plate. In Chapter 5 the FEA models were developed and validated. Now that these two steps have been done it is time to put the method to the test. In this chapter the experiments will be described and the results will be reported.

6.1. Creation of Plates

Instead of just one aluminum and one composite plate, as in Chapter 5, four aluminum and two composite plates were made. Two of the aluminum plates had PZT patches aligned with the plate's axis and two had patches at 45 degrees to the plate's axis. For the composite plates there was one aligned and one at 45 degrees. Initially all plates had only the four holes drilled at the corners. A schematic of the plates can be seen in Figure 6.1.
with the dimensions listed in Table 6.1 and Table 6.2.

<table>
<thead>
<tr>
<th>Dim</th>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>Left Edge of 45º Patch</td>
<td>37/64</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0403</td>
</tr>
<tr>
<td>X2</td>
<td>Left Edge of 90º Patch</td>
<td>0.0508</td>
</tr>
<tr>
<td>X3</td>
<td>Middle of 90º and 45º Patches</td>
<td>0.0762</td>
</tr>
<tr>
<td>X4</td>
<td>Right Edge of 90º Patch</td>
<td>0.1016</td>
</tr>
<tr>
<td>X5</td>
<td>Right Edge of 45º Patch</td>
<td>13/32</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.1121</td>
</tr>
<tr>
<td>X6</td>
<td>Left Edge of Al Button</td>
<td>7 1/2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.1905</td>
</tr>
<tr>
<td>X7</td>
<td>Right Edge of Al Button</td>
<td>8 1/2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.2159</td>
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<tr>
<td>X8</td>
<td>Right Edge of Plate</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>0.3048</td>
</tr>
<tr>
<td>Y1</td>
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<tr>
<td></td>
<td></td>
<td>0.0254</td>
</tr>
<tr>
<td>Y2</td>
<td>Center of 90º Patch</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0508</td>
</tr>
<tr>
<td>Y3</td>
<td>Center of 45º Patch</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0613</td>
</tr>
<tr>
<td>Y4</td>
<td>Top Edge of 90º Patch</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0762</td>
</tr>
<tr>
<td>Y5</td>
<td>Top Edge of 45º Patch</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0972</td>
</tr>
<tr>
<td>Y6</td>
<td>Bottom Edge of Al Button</td>
<td>5 1/2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.1397</td>
</tr>
<tr>
<td>Y7</td>
<td>Top Edge of Al Button</td>
<td>6 1/2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.1651</td>
</tr>
<tr>
<td>Y8</td>
<td>Top Edge of Plate</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>0.2032</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hole</th>
<th>X Location</th>
<th>Y Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1/2</td>
<td>0.0127</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1/2</td>
</tr>
<tr>
<td>B</td>
<td>1/2</td>
<td>0.0127</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7 1/2</td>
</tr>
<tr>
<td>C</td>
<td>11 1/2</td>
<td>0.2921</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7 1/2</td>
</tr>
<tr>
<td>D</td>
<td>11 1/2</td>
<td>0.2921</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1/2</td>
</tr>
<tr>
<td>E</td>
<td>10 1/2</td>
<td>0.2667</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 1/2</td>
</tr>
<tr>
<td>F</td>
<td>2 1/2</td>
<td>0.0635</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4 1/2</td>
</tr>
</tbody>
</table>

All holes have a diameter of 1/4 in (6.35 mm)

The aluminum plates were made of 60 mil (1.524 mm) thick 6061-T6 aluminum. The composite plates were made out of a graphite cross-ply cloth with a lay up of [0/90, +45/-45]. The composite plates were manufactured using a wet lay-up technique with room
temperature curing epoxy and a vacuum bag. The resulting plate had a nominal thickness of 55 mils (1.397 mm). The aluminum buttons were only needed on the composite plates and were affixed with Super Glue.

After the plates had been cut to size and the corner holes drilled the piezoceramic patches were affixed. The patches were Piezo System's Part #: T107-A3-502. This is a single sheet of PSI-5A-S3 material 7.5 mils (0.1905 mm) thick and 2.03 in (51.562 mm) square. Separating the two sides of the patch is a border that extends 0.12 in (3.048 mm) beyond the edge of the patch. The process for bonding the patches can be found in Appendix A. This process has been used in the lab for several years now with excellent results.

When the patches are bonded to the plates their polarities are aligned as shown in Figure 6.2. With this configuration, when a common voltage is applied to the outer surfaces of the patches, one patch undergoes compression and the other extension. This causes a bending moment to be induced in the plate. The plate itself is used as a common ground.

This works well for the aluminum plates but is more complicated for the composite ones. The graphite fibers are electrically conductive, however the epoxy matrix is not. Since the matrix surrounds the fibers this poses a problem. To solve this the area of the plate where the patch is to be bonded was sanded to expose the graphite fibers. Then they patches

![Diagram of Piezoceramic Patches](image)
were bonded onto this sanded area.

After the initial experiments were performed on the corner bolted configuration, the internal bolts were drilled. When doing this care had to be taken to ensure the integrity of the patches. Being ceramic and thin they are extremely delicate. Clamping the plates for drilling was done such that the patches were outside of the clamped area and were not resting directly on anything.

6.2. Experimental Setup and Procedure

The same basic experimental procedure was used for all of the plates. Basically this consisted of sending a sine wave to the actuator and measuring the response at a specific point. The initial experimental setup can be seen in Figure 6.3 and detailed description of the procedure follows.

1. The bolts were tightened to torque of 10 ft lbs (13.56 N m) using a Snap-on Torque Wrench (Model #: QJFR275D, Serial #: 10860034810). The corner bolts were tightened by starting in one corner and then going diagonally to the next one. This ensured that the plate was tightened evenly. For the plates with interior bolts, the two interior bolts were tightened last.

2. The displacement sensor was then adjusted so that it was in the middle of its output range. This was done raising or lowering the sensor until it output 9 V. After the sensor was in the middle of its range, the potentiometer in the offset circuit was
adjusted till the circuit's output was zero.

3. A DSPT SigLab (Model #: 20-22A, Serial #: 1501) Virtual Sine Sweep Analyzer (VSS), which runs inside of Matlab, was used to generate a sine sweep over multiple frequency ranges. These frequency bands were set up by doing a quick sweep over the entire frequency range to identify the areas of interest and then set up more detailed sweeps in these areas. The output voltage was varied over these bands to account for the reduced amplitude of the response at the higher frequencies without saturating the sensor at the lower ones. Appendix B gives the final settings used for each experimental run.

4. For the plates with just corner bolts, the signal was then passed through a Trek high-voltage amplifier (Model #: 50/750-2, Serial #: 249) with a fixed gain of 20. However, when this was done for the plates with interior bolts it was noticed that the stiffer configuration was causing loading effects on the amplifier. To fix this a Piezo Systems high-voltage amplifier (Model #: 60.102, Serial #: 035375) was used instead of the Trek one for the plates with interior bolts and for those with interior holes. This final setup was connected the same as the initial one and can be seen in Figure 6.4.

5. The signal from the DSPT SigLab was also fed into input channel one of the DSPT SigLab. This way the DSPT SigLab could verify that it was sending out what it was supposed to be.

![Figure 6.4: Final experimental setup](image-url)
6. The output from the amplifier was then connected to the electrodes of the piezoceramic actuator as shown in Figure 6.3 and Figure 6.4. The positive wire from the amplifier was connected to the patch, while the negative one was connected to the bolt at the edge of the plate. This bolt was connected to the tabletop, which acted as a common ground.

7. The displacement of the plate was measured by a Bently Nevada Proximitor. The Proximitor consists of a probe (Model #: 30000-00-32-36-02, Serial #: H432587) and a sending unit (Model #: CP-0171857-01, Serial #: KC50510) with a gain of 200 mV/mil. The sensor was located at \( x = 8 \) in (0.2032 m) and \( y = 6 \) in (0.1524 m) as measured from the origin shown in Figure 6.1.

8. The output from the sending unit has a range of from zero to eighteen volts. This range is not compatible with the DSPT SigLab so an offset circuit was built. The signal from the Sending Unit was fed into this circuit so that its full range was adjusted to be between positive and negative nine volts. The circuit can be seen in Figure 6.5.

![Figure 6.5: Offset Circuit](image)

9. This signal was then fed into input channel two of the DSPT SigLab.

10. The VSS module then computes the transfer function between the applied sine sweep voltage and the output from the offset circuit.
This procedure was done twice for each plate in each of the three configurations: corner bolts, interior bolts, and interior holes. Between each run the bolts were loosened and then tightened again in case the vibrations had accidentally loosened or tightened the bolts. Also the displacement sensor was zeroed before each run. This was necessary because there was a drift in the output from the offset circuit. The data was then saved to a file so that it could later be compared to the predicted response.

6.3. Comparison of Experimental and Predicted Responses

After collecting all of the data, the experimental transfer functions were then compared to the ones predicted by the method. Several MATLAB m-files were created for this purpose, which can be found in Appendix C. Comparisons were made between fine and coarse FEA meshes and between evaluating the eigenvectors at the center points versus using shape functions. This comparison followed the following steps:

1. The needed eigenvectors are extracted from the FEA model. They are exported into a standard format and a program is then run on them. This program removes all of the text and leaves only the numbers in a table. Each column is a specific eigenvector component with each row corresponding to one mode.

2. The cleaned eigenvector files are then run through rotate.m or rename.m. Rotate.m takes the eigenvectors along each side of the patch and rotates them from the global to the local coordinate system. This is used for the 45-degree patches. For the 90-degree patches, rename.m is used to simply rename the eigenvectors to the names expected by the other m-files.

3. The results of rename.m or rotate.m are then used by shaper.m to generate the shape functions needed. Shaper.m also figures out the moments applied to the plates by the actuators.

4. Rename.m or Rotate.m is then run again. This time it is only run on the eigenvectors at the center of each side of the patch and at the sensor location. The
eigenvector at the sensor location is not rotated but simply renamed so that it can be used in the other m-files.

5. RespCenter.m then takes the results of shaper.m plus the results of the last running of rename.m or rotate.m and uses it to predict the response at the sensor. It only uses the moments figured out by shaper.m and not the shape functions of the eigenvectors.

6. RespShape.m then takes the results of shaper.m plus the results of the last running of rename.m or rotate.m and uses it to predict the response at the sensor. It uses both the moments and the shape functions generated by shaper.m.

7. GenPlots.m then takes the results of RespCenter.m, RespShape.m and the experimental results and plots them for comparison. They are all plotted on the same set of axis to make the comparison easy.

The experimental results were compared to center point and shape function projections for both coarse and fine FEA meshes. The graphs were done as bode plots with magnitude and phase angle shown on separate sub plots. The phase plots at first look like there is a large difference between the measured and predicted phases. The difference is due to damping. In §4.2 we started with an undamped system so there is no damping in the predictions. This means that the predicted poles and zeroes lie on the imaginary axis. The real system has damping however, so its poles and zeroes will lie on the left hand side of the plane. For the real system there will be a plus 180 degrees change for the zeroes and a minus 180-degree change in phase for the poles. Lying on the axis means that for the predicted poles and zeroes both plus and minus 180 degree changes are valid. This is why the phase for the predicted poles and zeroes does not always change in the same direction as the phase does for the real system.

Each graph shows one experimental run, one center point projection and one shape function projection. This resulted in a total of 86 different plots. In the following sections
one plot for each configuration versus both coarse and fine FEA meshes will be shown. It must be remembered that correctly predicting the poles is more important than correctly predicting the zeroes. This is because the poles are a product of the system, whereas the zeroes depend heavily on the sensor and its dynamics.

6.4. **Aluminum Plate Results**

First the results for the aluminum plates will be examined. The case of a plate bolted at four corners will be presented first, then a plate with interior bolts, and finally a plate with interior holes. It is useful to keep in mind the accuracy of the initial FEA model without an actuator as shown in Table 5.2. Obviously the accuracy of the method is limited by the accuracy of the underlying FEA.

![Figure 6.6: Al plate with 90-degree patch vs. Coarse (left) and Fine (right) FEA](image)

Figure 6.6 shows the experimental results versus both coarse and fine FEA for a patch aligned with the plate's axis. As can be seen from the plots there is very little difference between using the eigenvectors at the center points of the patch sides and using shape functions to describe the eigenvectors along the patch sides. For the first pole both methods and both mesh densities are within 1% of the experimentally found ones. For the second pole they are within 2%. In both cases the difference between measured and predicted is of a similar magnitude as the differences between experimental runs. The differences between the two methods are the magnitudes of the poles. The locations of the poles remain the same in both approaches. Both the coarse and fine meshes produce
results that closely track the experimental response. The fine mesh though does do a better job at matching the magnitude of the experimental results and more closely predicts the first zero. The discrepancies in magnitude between the poles may be due to the way in which the data was taken. As can be seen by looking at the settings in Appendix B the frequency range between the poles or zeroes was transversed very quickly. This may account for the error in magnitude between them.

Figure 6.7 shows the experimental results versus both coarse and fine FEA for a patch at 45 degrees to the plate's axis. Unlike in the 90-degree case the differences between center point vs. shape functions and coarse mesh versus fine mesh are more apparent. However like the 90-degree case there is practically no difference in predicting the poles between the methods and mesh sizes. For the first pole the difference was between 1.5 and 2%. For the second pole it was between 2 and 3%. Though this seems like a larger range than the 90-degree case, the range is due to differences between experimental runs and not predictions. Also, though it is not readily apparent on the center point approach predicts extra poles and zeroes that the shape function approach does not. These poles and zeroes are not found experimentally.

For the coarse mesh there is a noticeable difference between the predicted magnitudes for using center points vs. shape functions. Both approaches follow the experimental response well up to about 350 Hz. The shape function approach though does a much better job at predicting the correct magnitude of the response. Above 350 Hz though both
approaches underestimate the magnitude of the response. In the experimental setup though there was a change in step size around 350 Hz so the problem could arise from this. Using a finer FEA mesh reduces the differences in magnitude between the two approaches.

Figure 6.8 shows the experimental results versus both coarse and fine FEA for a patch aligned with the plate's axis and having interior bolts. There is little difference between the center point and shape function approaches. For the first two poles both methods and mesh types are less than 1% off from the experimental results. The difference being that the shape function approach predicts slightly larger magnitudes and earlier poles and zeroes, for the later modes. Both under predict the pole at 300 Hz by about 5 Hz. However, the shape function approach correctly predicts the next pole, whereas the center point approach misses it by a few Hz. Above 450 Hz both approaches under predict the occurrence of the poles and zeroes. This may be due to the way the data was collected. It appears that whenever there is a large frequency range that is transversed quickly before a pole, that pole occurs at a higher frequency than predicted. In this case the setting show a large frequency range was transversed just before the pole was found.

As for the coarse vs. fine FEA mesh issue, in this case there isn't much gained by using a finer mesh. The fine mesh does not do a better job at predicting the occurrence of poles and zeroes but it does do a better job at predicting the magnitudes of them. Other than the magnitude differences at the poles and zeroes the two approaches pretty much mirror
each other.

Figure 6.9 shows the experimental results versus both coarse and fine FEA for a patch at 45 degrees to the plate's axis and interior bolts. For this case the results are not as good as the previous cases. This is to be expected though since this is the hardest case. It not only has interior bolts but also the patch at an angle to the plate's axis. Looking at the comparison to the coarse FEA mesh you can see that both methods do a good job of predicting the poles. The locations of the poles are close however their magnitude gets worse as the frequency increases. For the coarse mesh, both methods were 3% off for the first pole and less than 1% off for the second pole.

The zeroes are the problem with this case though. The prediction is for two poles followed by a zero. However experimentally we find a pole, a zero and then another pole. This could be due to the way the data was taken though, as the zeroes are a function of the sensor. So zeroes being missed or predicted incorrectly is not unusual or unexpected.

Something very weird happened with the fine FEA for this case. For some unknown reason the center point prediction is giving magnitudes several orders of magnitude too high. It is like a decimal point is missing from someplace. For this case the center point and shape function predictions give very different results. The shape function approach gets the magnitude range correct but over predicts the poles by several Hz at each mode. It is off by 4% at the first pole and 9% off at the second pole. The center point approach
predicts the poles better, 2% for the first pole and 1% for the second one, but as stated
above is way off in the magnitude area. Since the other interior bolts and off axis patch
cases have showed good results I believe the underlying FEA model to be at fault here.
Unfortunately, the license on NASTRAN has expired so a new model cannot be
constructed and run.

Figure 6.10 shows the experimental results versus both coarse and fine FEA for a patch
aligned with the plate's axis and interior holes. As would be expected these results are
close to those for the corner plate with a 90-degree patch. Both approaches track the
experimental results quite well. They are off by 3% at the first pole and less than 1% at
the second pole. The shape function approach tends to better predict the magnitude
though not by much. Using a finer mesh further improves the performance of the shape
function approach though whether or not it is enough to warrant the extra computational
time (3 min for the coarse versus 30 min for the fine) is debatable.
Figure 6.11 shows the experimental results versus both coarse and fine FEA for a patch at 45 degrees to the plate's axis with interior holes. Like the previous case this one is very similar to its earlier counterpart without interior holes. Both approaches and both mesh densities do a good job of following the trend of the experimental response. Both methods and meshes were of by between 2.5 and 3% for the first pole and less than 1% for the second pole. The use of shape functions does give a better prediction of the magnitude though in most cases the difference between the two approaches is negligible. The use of the finer FEA mesh does once again further improve the performance of the shape function approach.

As has been stated before the zeroes depend largely on the sensor dynamics. So mispredicting them is not unusual. However extra poles are not expected. The center point method does just this. It predicts extra pairs of poles and zeroes not seen in the shape function approach. These may arise from the rotation of the eigenvectors. Since the shape function approach fits the eigenvectors to a polynomial after the rotation these effects are most likely averaged out.

Taking all of the aluminum plate results into consideration, it is readily apparent that the method does a very good job at predicting the response of the plate. In general both the center point and shape function approaches perform well, though when one is better it is always the shape function approach. The performance increase is more in correctly
predicting the magnitude than in predicting the locations of the poles and zeroes.

The use of a finer FEA mesh does further help the shape function approach to correctly predict the magnitude. It does not however have much effect on the center point approach. The increase in the shape function approach's accuracy most likely comes from having more data points to generate the function from. Since the center point approach only uses one point per side, no matter the FEA model, it does not receive a similar benefit. Any benefit it receives comes from the overall improvement of the model itself.

A few problems are apparent from the data though. First the zeroes appear to be harder to predict than the poles. If one or the other is missed it is usually the zeroes. This is most likely due to sensor dynamics and was to be expected.

The second problem is the prediction of poles when there is a large frequency range without anything happening preceding them. When this happens the model often predicts a pole before it actually occurs. This could be attributable to the experimental set up. In order to collect the data in a timely fashion, the sine sweep moved rapidly through areas where there was thought to be little happening. This rapid change in frequency may have caused the poles to be detected at higher frequencies than they should have. Even with this problem the predicted poles are generally with in 5 to 10% of the experimentally detected ones.

The last problem is with just the center point approach. For aligned actuators its performance is close to that of the shape function approach though the other approach does do slightly better with off resonance magnitude. However, when off axis actuators are used, the center point method introduces extra pole / zero pairs. These pairs are not found in either the shape function approach or in the experimental results. They most likely are caused by the rotation of the eigenvectors. Fitting the rotated eigenvectors to shape functions averages out these effects.
6.5. Composite Plate Results

Now the results for the composite plates will be examined. The case of a plate bolted at four corners will be presented first, then a plate with interior bolts, and finally a plate with interior holes. It is useful to keep in mind the accuracy of the initial FEA model without an actuator as shown in Table 5.4. Obviously the accuracy of the method is limited by the accuracy of the underlying FEA. In this case the first two modes were good, but the third fourth and fifth modes were off by anywhere from 7 to 22%. Also the use of the aluminum button can be a source of experimental error. Though it was included in the model, it was viewed as being perfectly attached to the surface. In reality it was held in place by super glue, so this joint is a possible source of error. The vibration of the plate affected this joint so much that in at least one instance the button came loose and vibrated right off the plate.

![Figure 6.12: Composite plate with 90-degree patch vs. Coarse (left) and Fine (right) FEA](image)

Figure 6.12 shows the experimental results versus both coarse and fine FEA for a patch aligned with the plate's axis. Overall the predicted response shows the same trends as the experimental results. The agreement is quite good below 175 Hz. To this point the curves lie almost on top of each other. For the coarse mesh the first pole is off by 6% and the second by 2%, for both approaches. For the fine mesh the first pole is off by 4.5% and the second by 2%, again for both approaches. This difference is within the variation seen between experimental runs.
The next pole zero pair though is under predicted by about 20%. This could be due to the errors in the higher modes in the underlying FEA model. The next pole though is predicted correctly, so it could also be from the experimental setup, as the pole zero pair occurs just after a span change where the sweep went from fast to slow. Looking at the span settings shown in Appendix B, it is apparent that all useful data was taken below about 300 Hz, so the later data is not very reliable. The graph showing the results using the fine FEA has been limited to 300 Hz to get a better view of the usable data.

As was seen with the aluminum plates when there is a difference between the center point and shape function approaches, it is the shape function approach that shows the better results. Both follow a similar trend but the shape function predictions are closer to the experimental magnitude. The use of a finer FEA mesh further helps the shape function approach to match the experimental magnitude.

![Graphs showing experimental measurements vs. dynamic impedance model for coarse and fine FEA](image)

**Figure 6.13: Comp plate with 45-degree patch vs. Coarse (left) and Fine (right) FEA**

Figure 6.13 shows the experimental results versus both coarse and fine FEA for a patch at 45 degrees to the plate's axis. These results show a similar degree of agreement as was seen in the 90-degree results. The first few poles and zeroes are matched fairly well but then the predictions constantly under predict the remaining poles and zeroes. Both meshes and approaches are 6% off for the first pole. For the second pole predictions are by 6% when using the coarse mesh and 2% when using the fine mesh. This difference is due to the 5 Hz difference between the two experimental runs shown. There is less than 1 Hz difference between the actual predictions.
As stated the experimental runs being looked at are different from one graph to the next. Looking at the two graphs it can be seen how the experimental setup and span settings can affect the results. From the settings in Appendix B it can be seen that the two runs had very different span settings and this could account for the different locations for the poles and zeroes. As stated above the second pole varies by as much as 5 Hz from one run to the next.

Again the shape function approach is shown to be better when a difference between the two can be detected. Also the center point approach predicts more poles and zeroes than the shape function approach. The fact that these extra poles and zeroes disappear when using the whole side of the patch instead of just one point leads to the idea that they may be numerical in nature. It also shows an advantage to the shape function approach, since it looks at more than just one point, the effect of discrepancies at any one point are minimized. Thus the use of a finer mesh and the shape function approach together should further improve the prediction by giving it more points to fit the function to. This has been seen in previous results, as the shape approach always gets better with a finer mesh. The trade off is still though is the improvement worth the increase in computing time.

![Figure 6.14: Composite plate with 90-degree patch and interior bolts vs. Coarse (left) and Fine (right) FEA](image)

Figure 6.14 shows the experimental results versus both coarse and fine FEA for a patch aligned with the plate's axis and having interior bolts. As would be expected when the system gets stiffer and more complicated the errors in the underlying FEA model have
more of an effect on the results. The predicted closely spaced poles were seen experimentally but the size of the graph makes them too small to be seen. Experimentally they were found to be only 3 Hz apart, though they were predicted to be 10 Hz apart. For the first pole the prediction was off by 18%. For the second it was off by 12%. There was no appreciable difference between the different approaches or meshes in the accuracy of the predictions.

As far as which approach and mesh size works best, the same conclusions can be drawn as from the earlier results. When one is better than the other it is always the shape function approach that is better at predicting the off resonance magnitude. When the mesh is made finer the shape function approach benefits from it. The center point approach doesn't and in some cases becomes worse.

![Figure 6.15](image)

Figure 6.15: Composite plate with 45-degree patch and interior bolts vs. Coarse (left) and Fine (right) FEA

Figure 6.15 shows the experimental results versus both coarse and fine FEA for a patch at 45 degrees to the plate's axis and interior bolts. Again these results were not expected to be as good as those for the aluminum plate. Besides the problems faced by the previous case this one has the added complexity of the patch being at 45-degrees. At first glance, it appears that there is no correlation between the predicted and experimental results. However a closer examination shows that the predicted response again follows the same trends as the experimental one. The experimental response is generally smaller in magnitude and higher in frequency than the predicted response. The first two predicted
poles are present in the experimental result (look at the left plot) but are smaller and more closely spaced. Some of the difference in magnitude could be due to the button and its imperfect interface with the plate. As with the 90-degree case, there was no appreciable difference between the different approaches or meshes in the accuracy of the predictions. For the first pole the prediction was off by 17%. For the second it was off by 12%.

Comparing the shape function and center point approaches just reinforces what has been stated before. Both approaches predict the same poles, though the shape function does better at matching off resonance magnitudes. The use of a finer mesh helps to make this match better. Also as has been seen in other off axis cases the center point approach predicts extra pole / zero pairs not seen in the other approach.

Figure 6.16 shows the experimental results versus both coarse and fine FEA for a patch aligned with the plate's axis and interior holes. As would be expected these results are close to those for the corner plate with a 90-degree patch. Again both approaches do a good job of tracking the experimental response at low frequencies, with both the first and second poles being predicted within one 1%. Though they again mistakenly predict a pole - zero pair at about 180 Hz. Experimentally this pair is found at about 230 Hz. Though the pole after this pair is predicted correctly. The useful data for this run ends at about 340 Hz so the fine graph only goes to 300 Hz to give a better view of the lower frequency range. As before the shape function is a little better than center points and benefits from
the finer mesh.

![Figure 6.17: Comp plate with 45-degree patch and interior holes vs. Coarse (left) and Fine (right) FEA](image)

Figure 6.17 shows the experimental results versus both coarse and fine FEA for a patch at 45 degrees to the plate's axis with interior holes. Like the previous case this one is very similar to its earlier counterpart without interior holes, though not quite as good. For the first pole the predictions are off by 6%. For the second pole they are off by 3%.

What is interesting about this case though are the differences between the center point and shape function approach. For the third pole the center point approach predicts two closely spaced poles as opposed to the one pole predicted by the shape function approach and found by experiment. Also a couple of extra pole zero pairs are predicted by the center point approach but not by the shape function approach. These extra poles and zeros have been mentioned before but they are more apparent with this case.

The overall results for the composite plates do not show the same predictive capability for the method that the results for the aluminum plates did. They were not expected to though as the underlying FEA model was shown to be flawed after the first couple of modes back in §5.2. Since the whole method is based off of the FEA model, the results can only be as good as the model.

Also the matter of the aluminum button not being perfectly bonded to the plate could have influenced the results. The button did break free of the plate at least once, so there is
reason to believe that during other runs it may not have been rigidly attached and may have been dampening the plate's response. This could account for the experimental response being at time much lower in magnitude than predicted.

The bolts used to anchor the plates could also be a source of problems. The surface of the composite was softer than the aluminum plates used previously. When the bolts were tightened it was found that they actually cut into the plate's surface. Washers were used to try and fix this but there was a question as to how well the matched up on the top and bottom of the plate. Also they cut into the plate as well, though to a lesser degree. The bolts cutting into the plate and/or the misalignment of the washers would have changed the boundary conditions from what they were in the FEA model. Also this situation may have introduced moments into the plate, the very types that were supposed to be eliminated by evenly tightening the bolts. It was noticed during the experiments that if this happened the results could be very different from what was expected. The bolts would all have to be loosened and then tightened down again. This situation could possibly be avoided in the future by using nylon or rubber washers.

Even with all these sources of error the results were not completely without merit. They should still be enough to convince anyone that given a good FEA model the method can produce acceptable results. Also some insights into the differences between the center point and shape function approaches and the effects of mesh density were gained. These might not have been as readily apparent if the results had been as good as the ones for the aluminum plates.

With the aluminum plates there wasn't much difference between the two approaches. With the composite plates though the shape function approach was shown to clearly be better. Often the center point approach would predict extra poles and zeroes that were not seen experimentally. The shape function approach though did not produce anywhere near as many false poles and zeroes. Also it improves as the mesh gets finer, which is not always true for the center point approach. Often the extra zeroes and poles appear when the mesh gets finer. So the shape function approach should be used whenever possible as
there is little extra cost to using it.

As for mesh density the composite results did point out a couple of interesting things. First off even the coarse mesh produced decent results. So even a fairly simple model can produce good results quickly. The coarse mesh models only took about 3 minutes to run as compared to 30 min on average for the fine mesh ones. The increase in mesh density though did improve the results. A trade off though must be made on just how accurate the results need to be versus the time required to run the model. No really large models could be run, as the hard disk of the computer was not very large.

The more important point though was the emergence of numerical poles and zeroes. These were seen mostly with the center point approach but are possible with shape function approach as well. The best way to check for them is to run the same problem but with a different mesh and see if they are still there. The shape function approach helps to minimize their influence by using the results from several nodes.

6.6. Overall Results

The results presented show that the method does a good job of predicting the response of the plate. The results were shown under different boundary conditions, different material types, different approaches, different mesh densities, and with the patch in different orientations. The composite plate results were not as good as the aluminum ones but that was too be expected for the reasons mentioned above. How well the method predicts the response of the plate depends greatly upon how good the underlying FEA model is. The better the model the better the results.

The off axis patch results were almost as good as those of the on axis patches. This shows that by simply rotating the eigenvectors from the global coordinate system, which is how they are reported by NASTRAN, to the local coordinate system, the method can be applied to patches at different angles to the plate. This approach is possible because the equations presented in Chapter 4 are all in the local coordinate system. The final result
does not have to be rotated back to the global coordinate system because it is in the out of plane direction and this direction is common to both coordinate systems.

The effect of mesh density was explored. It was shown that, while the method worked fine with a coarse mesh, a finer mesh generated better results for matching the magnitude of the response. This was especially true for the shape function approach. The center point approach though is not always improved as it sometimes predicts poles and zeroes that are numerical in nature. How fine to make the mesh becomes a trade off between accuracy and computational cost.

The use of shape functions versus center points to describe the eigenvectors along the patch sides was investigated. There was little difference in predicting the locations of the poles between the two approaches. However, the shape function approach does a better job of predicting the of resonance magnitudes, if there is any difference between the two. The center point approach was seen to predict extra pole zero pairs when off axis actuators were used. This was not seen with the shape function approach. This is most likely because the shape function approach uses the entire side of the patch instead of just one node. This minimizes the effect of any one node and any purely numerical results it might have. In the case of off axis actuators these purely numerical results are most likely due to the rotation of the eigenvectors. It was stated in §4.4, the same polynomial is used for the shape functions as was used to get the modal force projection. So the work of creating the shape functions has already been done when $\vec{M}_x$ and $\vec{M}_y$ are computed, thus using the shape functions requires no extra computational effort. So to make the method as robust as possible the shape function method should always be used.
7. Discussion and Future Work

7.1. Discussion

For a modeling method to be useful it must not only give an accurate prediction, but also be easy to use and retain a physical insight into the system. Until recently, all the methods of modeling piezoceramic actuators are lacking in one or more of these areas.

SEF models are the easiest to use and they still retain a physical insight into the system but are not very accurate. FEA models are accurate, but require training in specific programs or the ability to write your own custom elements. Also, physical insight can become obscured. First principles give accurate models and retain physical insight. However, for anything more complex than a simple beam these models become very high order, which makes them ill suited for typical design studies.

Impedance methods are accurate and retain insight, but until recently, they have relied on finding the impedances analytically. This has limited their use to structures for which this can be done, such as beams, curved shells, and plates. Even then, with these simple structures, only certain boundary conditions could be solved for. Fairweather (1998) developed a method whereby the impedances could be found from FEA. He calculated the impedances for a beam and a simply supported plate using FEA and compared his results to the SEF model, analytically computed impedances and, for the beam, experimental measurements.

There were several issues left open, though, that had to be addressed before the method was ready for general use. These were the use different boundary conditions, material symmetries, methods of computing the applied moment, and off axis actuator placement. Also there was no experimental data to show that the method actually works.

This work sought to address these issues. Furthermore, the issue of the effect of mesh density on the results was dealt with and the underlying assumptions of Fairweather's
work were clarified. The final result is a version of the method that can deal with various boundary conditions, off axis actuator placement, and materials, and is backed up by experimental results. To accomplish this a straight forward and logical approach was taken.

First the field of smart structures was looked at to get an idea of how pervasive the use of active materials has become in our society. This wide spread use of active materials makes it important to be able to accurately model the effect of an actuator on a structure. This was done in Chapter 2. However this was a broad look at the entire field of smart structures and active materials. What was needed was a more detailed look at one particular actuator type.

Since the method is aimed at piezoceramic actuators they were looked at in detail in Chapter 3. Their history, how they are made, common configurations and implementation issues were all presented. Then the different methods for modeling the effect of the actuator on the structure were presented. The pluses and minuses for each of the methods were presented. Comparison of the methods shows that the impedance method comes closest to possessing qualities desired in a method. However it was found to not be widely used due to its reliance on analytically deriving impedances and its somewhat novel approach. This ended the review of what had been done before and why the present work was needed.

The next step was to present the impedance method as outlined by Fairweather (1998). Chapter 4 does this and adds onto his method. First the solution to the vibration problem of the actuator is presented and solved in terms of the structure's impedance. Some of the assumptions made by Fairweather are revisited and clarified. Then the equations for extracting the structural impedances from the FEA analysis are presented. These two solutions are then used together to generate the structure's response to the actuator.

These equations were presented for patches aligned with the plate's axis and using eigenvectors from the center of each patch side. However it may be desirable to not have
the patch aligned with the plate's axis. To adjust the method for this case it was determined that the eigenvectors had to be rotated from the global coordinate system, where they are reported by NASTRAN, to the patch's coordinate system. This is simply a two dimensional rotation. This simple solution is possible because all of the equations were done in the patch's coordinate system and the final result is in the out of plane direction, which is the same for both coordinate systems. Also there was a question as to whether or not the center of the patch side is an accurate representation of the entire patch side. To address this question shape functions were developed to represent the eigenvectors along the entire patch side. This adjustment requires no extra work as the shape functions are already calculated when the modal force projection is calculated.

After the equations were developed the next step was to generate the FEA models that were to be used. To ensure these models were accurate they were compared to actual plates. Chapter 5 shows how the models were made and how this testing was done. For the aluminum plates a good agreement was found between the modes predicted by FEA and those recovered by modal testing. The same level of agreement was not possible for the composite plates though due a lack of accurate material property data.

Once the modal testing was complete the final FEA models were made. For each material type there was three sets of boundary conditions: bolted at the four corners, corner bolts plus two interior bolts, and corner bolts plus two interior holes. To make extraction of the eigenvectors easier nodes were places along the outline of the patch. So for each of the above cases both a 90-degree and a 45-degree patch model was made. Finally each model had both a coarse and fine mesh version made. This brought the total number of models up to 24. For the composite plates the aluminum button needed for the sensor to work was added to the models.

Now all of the items were in place to perform the experiments and see how the method performed against real data. Chapter 6 is where this happens and where the results are reported. First the method for creating the plates was presented. Then the experimental setup and the method of collecting data were gone through. Next the method for taking
the FEA results, using them to predict the plate's response and comparing them to the experimental data is presented. Then the results themselves are given.

The results for the aluminum plate show good agreement between the predicted and measured response of the plate. The results for the composite plates were not as good but did help to make clear the differences between shape function and center point calculations, and the effects of mesh density. Shape function calculations were found to give better results of off resonance magnitudes than center point calculations and since there is no extra work required to use them it was decided that they should be used whenever possible. A finer mesh was found to produce better results in off resonance magnitude, though at increased computational costs. Also what appeared to be purely numerical poles and zeroes were found with the center point approach, when off axis actuators are used.

There were several possible reasons for the poor performance of the composite plate predictions. First the underlying FEA model itself was not very good. This was pointed out originally in Chapter 5. The material values used were not correct but were the best that could be obtained at the time. Also the displacement sensor used works on eddy currents so an aluminum button had to be added to the plate so that it could be used. This button was included in the FEA model. In the model the interface between the button and the plate was assumed to be perfect. However the button was held on with super glue so it was far from perfect. At least once the button came off during a test, so at other times it might have been loose enough to have an effect on the results. Another possible source of error is the bolts that held the plate in place. These cut into the plate when they were tightened and may have induced unwanted forces into the plate. Washers were used to try and fix this but they may also have done the same thing. Even with all these sources of error the composite results were still good enough to show promise.

Taken together the aluminum and composite results show that the method is accurate. The method can be used with different boundary conditions and with off axis actuators with out substantial loss of accuracy. The results show that the shape function method of
calculating the applied moment should be used and that increasing mesh density increases
the accuracy of the result. How fine the mesh should be results in a trade off between
increased accuracy and increased computational costs.

7.2. Future Work

This work extended the capabilities of the impedance method and experimentally verified
its accuracy. However it is not quite ready to be handed over to the design engineers as a
design tool. There are several issues that still need work. Among these are the equations
of motion for the actuator, the streamlining of the analysis process, and application of the
method to a case study.

7.2.1. Equations of Motion for the Actuator

In §4.1 the vibration of the actuator was studied. The equations of motion for the patch
were found to be:

\[ \frac{Y^E}{1-\nu_p^2} \frac{\partial^2 u}{\partial x_p^2} + \frac{Y^E}{2(1-\nu_p)} \frac{\partial^2 v}{\partial x_p \partial y_p} + \frac{Y^E}{2(1+\nu_p)} \frac{\partial^2 v}{\partial y_p^2} = \rho_p \frac{\partial^2 u}{\partial t^2} \]  

(7.1)

\[ \frac{Y^E}{1-\nu_p^2} \frac{\partial^2 v}{\partial y_p^2} + \frac{Y^E}{2(1-\nu_p)} \frac{\partial^2 u}{\partial x_p \partial y_p} + \frac{Y^E}{2(1+\nu_p)} \frac{\partial^2 u}{\partial x_p^2} = \rho_p \frac{\partial^2 v}{\partial t^2} \]  

(7.2)

To solve these equations assumptions were made that the rate of shear strain was
negligible and that \( \nu \) was approximately 0.3. This allowed for the equations to be solved
by separation of variables. However certain characteristics of the actuator's vibration may
be missed by these assumptions. To investigate this, other solution methods should be
tried to solve (7.1) and (7.2). A solution, which does not require these assumptions,
would of course be preferred over the present approach.

One possible approach is to decouple (7.1) and (7.2) to form a fourth order linear partial
differential equation in either \( u \) or \( v \). The equation for \( u \) is found as:
\[
\frac{\partial^4 u}{\partial t^4} = \frac{Y^E}{\rho_p (1+\gamma_p)} \left( \frac{\partial^4 u}{\partial^4 x} + \frac{\partial^4 u}{\partial^4 y} \right) + \frac{Y^E}{2\rho_p^2 (1+\gamma_p) (1-\gamma_p^2)} \left( \frac{\partial^4 u}{\partial^4 x^2} + \frac{\partial^4 u}{\partial^4 y^2} \right) = 0
\]  
(7.3)

This can be factored so as to produce two second order linear differential operators.

\[
\left( \frac{\partial^2}{\partial t^2} - \frac{Y^E}{2\rho_p (1+\gamma_p)} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \right) \left( \frac{\partial^2}{\partial t^2} - \frac{Y^E}{\rho_p (1-\gamma_p^2)} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \right) u = 0
\]  
(7.4)

The solution for the fourth order linear partial differential equation for \( u \) can then be found as the solution of the following two equations:

\[
\frac{\partial^2 u}{\partial t^2} - \frac{Y^E}{2\rho_p (1+\gamma_p)} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) = 0
\]  
(7.5)

\[
\frac{\partial^2 u}{\partial t^2} - \frac{Y^E}{\rho_p (1-\gamma_p^2)} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) = u
\]  
(7.6)

The solution of course has to satisfy the initial and boundary conditions for \( u \). The solution for \( u \) can then be substituted into (7.1) and this in turn solved to get \( v \). Another, less general, solution for (7.3) can be found by separately solving the left hand sides of both (7.5) and (7.6) set equal to zero, then adding these two solutions together.

Another solution to (7.3) can be written as:

\[
u = f(at - x_p - y_p) + g(at + x_p + y_p) + h(bt - x_p - y_p) + j(bt + x_p + y_p)
\]  
(7.7)

where:

\[
a = \sqrt{\frac{Y^E}{\rho_p (1+\gamma_p)}} \quad b = \sqrt{\frac{2Y^E}{\rho_p (1-\gamma_p^2)}}
\]  
(7.8)

and \( f, g, h \) and \( j \) are arbitrary functions. These functions are selected so that they satisfy the initial and boundary conditions. Once again once \( u \) is found it is substituted into (7.1)
and this in turn solved to get \( v \). This solution though is only useful for special geometries. The actuator geometry best suited for it appears to be an infinite 2-D strip.

Another approach to solving (7.1) and (7.2) is perturbation methods (Holmes, 1995). The solution found in §4.1 satisfies equations (7.1) and (7.2). However it leaves out any coupling between the \( x \) and \( y \) displacements. The coupling can be added through small correction terms in either the terms dropped when (4.13) and (4.14) were reduced to (4.15) and (4.16), or the initial and / or boundary conditions. In perturbation analysis one wants to use the smallness of a coefficient or the slowness of the derivatives to one's advantage.

First let us consider using smallness to or advantage. In (7.1) and (7.2), the coefficients of all the terms are of the same order, so there is no smallness that can be used. One way to introduce some smallness into the problem is through a small perturbation of the boundary and / or initial conditions. An example of a perturbed initial condition for \( u \) is shown in (7.9).

\[
\text{u}_{ic}(x, y) = u_0(x) + \varepsilon u_1(x, y)
\]  

(7.9)

The first term is the current initial condition. The second term is a small correction term, where the size of this term would have to be specified. For a boundary condition the terms would also depend on \( t \) and would be evaluated at specific value of \( x \) or \( y \). The solution would then have the form:

\[
u = u_0(x, t) + \varepsilon u_1(x, y, t) + ...
\]

(7.10)

The first term is the solution found in §4.1. The second and later terms being the correction terms. These terms are where coupling between the \( x \) and \( y \) displacements are brought into the solution.

Now let us consider using the slowness of the derivatives in (7.1) and (7.2) to our advantage. The slowness of these derivatives can be quantified by \( \varepsilon \). To use the
slowness the assumed solution for $u$ and $v$ are changed to:

\[
\begin{align*}
    u &= u_0(x, \varepsilon x, y, t) + \varepsilon u_1(x, \varepsilon x, y, t) + \ldots \tag{7.11} \\
    v &= v_0(\varepsilon x, y, t) + \varepsilon v_1(\varepsilon x, y, t) + \ldots \tag{7.12}
\end{align*}
\]

The first term is a modified version of the current assumed solution. The appearance of $\varepsilon x$ and $\varepsilon y$ in the assumed solutions takes care of the coupling by adding a small dependence on the variable perpendicular to the displacement. The second and subsequent terms are additional correction terms.

The assumed solutions of (7.11) and (7.12) are then substituted into (7.1) and (7.2). Terms are then grouped by powers of $\varepsilon$. The $O(1)$ problem (those terms not multiplied by $\varepsilon$ or one of its powers) is solved first. This solution will be the original solution modified by a function of the slow variable. After solving the $O(1)$ problem, the $O(\varepsilon)$ problem is solved, followed by the $O(\varepsilon^2)$, and so on until the desired order of solution is reached.

### 7.2.2. Streamlining of the Analysis Process

At present, it takes several Matlab m-files and manually exporting the eigenvectors from FEMAP to predict the response of the plate. The m-files used have been standardized to some degree but Rotate.m or rename.m must still be edited manually for every model. Shaper.m must be modified every time the number of nodes along each patch side changes. RespCenter.m and RespShape.m are standardized and should not need to be edited unless the output from the other files is renamed. In addition to this, after each eigenvector is manually extracted from FEMAP, it must be put into a form that is usable by the m-files. Also, if the physical dimensions of the plate or the location of the patch change, then the files have to be edited to reflect this. So, as you can see, a large amount of manual file editing and manipulation must be done for each predicted response.
The same steps are performed for each model, so it should be possible to combine all of
the existing m-files into one m-file. The input for this m-file would contain:

- Physical properties of the plate
- Nodes along the patch sides
- Node at the sensor location
- Patch angle
- Location of the patch
- List of modal frequencies from the FEA
- Whether to use center point or shape functions

The eigenvectors themselves could either be part of the file or the file could include the
names of other files containing them. With this information, the m-file should be able to
give the response at the sensor location. This process still requires the user to manually
extract the eigenvectors from the FEA results, to put them into a form usable by the m-
files, and to construct the input files.

The next step beyond this would be to take all of the necessary data directly from the FEA
analysis with limited user interaction. Looking at what was suggested for an input file
above, it becomes apparent that most of the info is available in the FEA model. So it
would be possible to skip the intermediate step of removing the eigenvectors from the
FEA model and formatting them and do this automatically. Ideally, the program would
ask the user for the following information, either interactively or by the use of an input
file:

- Nodes along the patch sides
- Location of the corners of the patch
- Sensor location
The program would then use this information to extract what it needs from the FEA model and generate the response of the structure. The properties of the piezoceramic patch would either have to be hard-coded or have to be entered as part of the input process. At this point, the method would be useful as the results could be viewed mere minutes after the FEA model has been run instead of the hours it can now take to manually export and manipulate all the data.

One step beyond this would be to incorporate this process directly into the post processor. This way, the designer could simply select the nodes that define the patch and the response node. Since the piezoceramic is not in the FEA model, it would either have to be hard-coded or be selected from a list of available piezoelectric materials. Most pre / post processors have a user-definable material library so the latter option would not be hard to implement and would offer more flexibility. The rest of the required information is already in the model so there is no need or further interaction. It would even be possible to recover the response of the entire structure instead of just one node. However, this may prove too computationally expensive. The greatest advantage of this type of implementation though is that the designer would not have to leave his familiar program to get the response. Also the responses from multiple actuator locations could be compared quickly and easily. This ability to easily examine multiple actuator locations would allow for design optimization studies. At this point the program could be sold, either as an add-on pack or to the pre / post processor company for inclusion in later releases.

7.2.3. Case Study

The biggest drawback to this method's acceptance though is its lack of application to real-world structures. So far the method has only been tested under laboratory conditions and on structures created for just this purpose. This is fine for initially proving that the method works, but not enough for it to gain wide acceptance. A case study is needed to overcome this.

Since the method gives the dynamic response of the structure / actuator system it is best
suited for problems where this is needed. Such problems include noise cancellation and vibration suppression, so one of these areas should be chosen. Problems such as fan and motor noise cancellation in enclosures are possible applications. Examples of such enclosures include:

- Copiers
- Overhead projectors
- Computer cases
- Air conditioners

For vibration suppression, possible candidates include:

- Helicopter rotors
- Pointing devices (such as optical booms on satellites, gun turrets, etc.)
- Car body panels (they can be made thinner and lighter if wind vibrations can be dampened).

Any problem for which the dynamic response of the structure to the actuator is desired can be treated by this method. The only real requirement, for the equations to work as presented, is that the structure can be analyzed using plate elements. This does not limit the method's applicability much though since most structures of interest can be analyzed using plate elements. For the equations to work, as is, though the out-of-plane coordinate axis at the sensor location must be the same as the one used by the patch's coordinate system. If this were not the case (due to surface curvature or other reasons), then the appropriate coordinate transformations would need to be performed to express the sensor location's eigenvectors in the patch's coordinate system. Additionally, it has always been assumed that the patch and sensor are located on the same surface of the structure. While this should be the case, if it is not, then extra care should be taken with the coordinate transformations. Also the method has not been tested under this condition, so it may not work as is.
No matter what problem is selected for a case study, a two-step approach is suggested. First, a case where a piezoceramic actuator is already being used should be examined. It should be shown that using the method provides the same or better results for this case. A comparison between existing design methods and the FEA-based impedance method for this case could be conducted to show any advantages in ease of use, insight into the problem, performance, and time-savings. After this case the method should be applied to a case for which an existing solution is not available. This would be a truer test of the method's usability, as a possible solution is not known from the beginning. A structure similar to that used in the first part would allow for some comparison's to be made, but it is not necessary. The results from these two cases, taken together, should help the method to gain acceptance in the design community.
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Appendix A. Process for Bonding Actuator to Structure

The process used in the Active Materials and Smart Structures Laboratory at Rensselaer for surface bonding piezoceramic actuators to structures was given by Fairweather (1998). His process will be reiterated here for reference.

When bonding the actuator there are three things that should be strived for:

1. The thinnest possible bond layer (minimizing any lag in shear transfer)
2. A low electrical impedance at the interface between the actuator electrode and the structure
3. Assurance that the electrical connectivity does not result in a short of the opposing electrodes of a single ceramic actuator.

To achieve these goals the following steps below were followed. The use of a conductive and a non conductive epoxy addresses the problems of spill over and bond stiffness.

1. The polarization of the ceramic material must be identified. Often, one pole on each patch is marked with a small dot, or a colored stripe, to indicate common polarizations between actuators.

2. The structure is taken to be a common ground for the electrical network. One actuator’s positive electrode, and the other actuator’s negative electrode must be bonded to the structure, as per Figure 6.2. This allows pure-moments to be exerted on the structure under the application of a common polarity to the exposed electrodes of the actuators.

3. The exposed electrode of each actuator was covered by 3M’s Scotch™ Long-Mask™ masking tape. This tape has adhesive properties that allow it to be removed from a surface after seven days of application without leaving adhesive on the target. The masking tape was applied to the actuator such that 1” of tape extended past the edges of the actuator on all four of its edges.

4. Two epoxies were used in the bonding process. The first, BA-2129, is a non-conductive, high-stiffness clear bond material. The other, BA-2902, is an electrically conductive silver epoxy adhesive. The epoxies used were produced by Tra-Con, Inc.

   Tra-Con, Inc.
   45 Wiggins Avenue
   Bedford, MA 01730
   1-800-872-2661
5. The surface of the structure was prepared by lightly sanding the area to which the actuator was to be applied, and then thoroughly washing this area with isopropyl alcohol. The specific area to which the actuator was to be bonded was marked off on each side of the structure using a permanent marking pen.

6. A vacuum bag was prepared, using a clear vinyl with “dum-dum” to create a vacuum pocket.

7. The epoxies were prepared, and were applied to the ceramic in the following manner

![Figure A.1 Placement of the epoxies on the ceramic actuator](Fairweather, 1998)

8. The method of application shown above ensured that epoxy leaking from edges of the actuator would be of the non-conductive type, and would not be able to short the upper and lower electrodes. The masking tape applied to the surface of the patch provided an extra level of precaution with regard to this leakage effect. In addition, the use of the high modulus non-conductive epoxy over the majority of the bond interface provides a stiffer bond layer than is possible with conductive epoxy.

9. The actuators were then placed in position on the structure, and the tape extending over the edges of the actuator was used to hold the devices in place until the epoxy began to set.

10. The entire structure was then wrapped in peel-ply to prevent epoxy from bonding it to the vinyl bag. It was placed in the vacuum bag, and attached to the vacuum pumps for 24 hours.

11. The structure was then removed from the bag and the peel-ply. The masking tape was removed from the actuator electrodes, and lead solder with high silver content was used to attach conductors to the actuator surface. The use of solder with silver prevents the soldering process from pulling silver from the electrodes.

12. A multimeter was used to check that the bonding process did not short the electrodes.
Appendix B. DSP Sig Lab Virtual Sine Sweep Settings

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**Figure B.1: Aluminum 90-Degree Corner Plate 1 Run 1**

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**Figure B.2: Aluminum 90-Degree Corner Plate 1 Run 2**
Figure B.3: Aluminum 90-Degree Corner Plate 1 Run 3

Figure B.4: Aluminum 90-Degree Corner Plate 2 Run 1

Figure B.5: Aluminum 90-Degree Corner Plate 2 Run 2
Figure B.6: Aluminum 90-Degree Interior Bolts Plate 1 Run 1

Figure B.7: Aluminum 90-Degree Interior Bolts Plate 1 Run 2

Figure B.8: Aluminum 90-Degree Interior Bolts Plate 2 Run 1

125
| ADD SPAN | Frequency start: 2 127 154 303 313 346 373 383 393 463 473 |
| DEL SPAN | Frequency end: 127 154 303 313 346 373 383 393 463 473 1000 |
| Sweep type | Lin Lin Lin Lin Lin Lin Lin Lin Lin Lin |
| Tracking bandwidth (Hz) | 2 0.5 2 0.5 2 0.5 1 0.5 2 0.5 5 |
| Number of averages | 4 10 4 10 4 10 4 10 4 10 4 |
| Number of steps | 10 25 10 10 10 27 10 10 10 10 100 |
| Inter step delay (mS) | 0 0 0 0 0 0 0 0 0 0 0 |
| Acquisition time (sec) | 40 1000 40 400 40 1080 80 400 40 400 160 |
| Level control channel | Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 |
| Control level (volts) | 1 2.5 1 2.5 1 2 2 3 3 3 2 |
| Ch1 AC - | 1.2v 2.5v 1.2v 2.5v 1.2v 2.5v 2.5v 5v 5v 5v 2.5v |
| Ch2 AC - | 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 1.2v 1.2v 1.2v 1.2v |

Figure B.9: Aluminum 90-Degree Interior Bolts Plate 2 Run 2

| ADD SPAN | Frequency start: 3 75 85 133 143 200 210 235 245 327 366 |
| DEL SPAN | Frequency end: 75 85 133 143 200 210 235 245 327 366 1000 |
| Sweep type | Lin Lin Lin Lin Lin Lin Lin Lin Lin Lin Lin |
| Tracking bandwidth (Hz) | 2 0.5 2 0.5 2 0.5 2 0.5 2 0.5 5 |
| Number of averages | 4 10 4 10 4 10 4 10 4 10 4 |
| Number of steps | 10 10 10 10 10 10 10 10 10 10 39 |
| Inter step delay (mS) | 0 0 0 0 0 0 0 0 0 0 0 |
| Acquisition time (sec) | 40 400 40 400 40 400 40 400 40 1560 160 |
| Level control channel | Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 |
| Control level (volts) | 1 1 1 1 1 1 1 2.5 1 1 2 |
| Ch1 AC - | 1.2v 1.2v 1.2v 1.2v 1.2v 1.2v 1.2v 2.5v 1.2v 1.2v 1.2v |
| Ch2 AC - | 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v |

Figure B.10: Aluminum 90-Degree Interior Holes Plate 1 Run 1

| ADD SPAN | Frequency start: 3 75 85 133 143 200 210 235 255 327 366 |
| DEL SPAN | Frequency end: 75 85 133 143 200 210 235 255 327 366 1000 |
| Sweep type | Lin Lin Lin Lin Lin Lin Lin Lin Lin Lin Lin |
| Tracking bandwidth (Hz) | 2 0.5 2 0.5 2 0.5 2 0.5 2 0.5 5 |
| Number of averages | 4 10 4 10 4 10 4 10 4 10 4 |
| Number of steps | 10 10 10 10 10 10 10 10 20 10 39 |
| Inter step delay (mS) | 0 0 0 0 0 0 0 0 0 0 0 |
| Acquisition time (sec) | 40 400 40 400 40 400 40 800 40 1560 160 |
| Level control channel | Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 |
| Control level (volts) | 1 1 1 1 1 1 1 2.5 1 1 2 |
| Ch1 AC - | 1.2v 1.2v 1.2v 1.2v 1.2v 1.2v 1.2v 2.5v 1.2v 1.2v 1.2v |
| Ch2 AC - | 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v |

Figure B.11: Aluminum 90-Degree Interior Holes Plate 1 Run 2
| ADD SPAN | Frequency start: | 3 74 84 133 143 202 212 235 265 327 366 |
| DEL SPAN | Frequency end:  | 74 84 133 143 202 212 235 255 327 366 1000 |
| Sweep type | | Lin Lin Lin Lin Lin Lin Lin Lin Lin Lin |
| Tracking bandwidth (Hz) | 2 0.5 2 0.5 2 0.5 2 0.5 2 0.5 5 |
| Number of averages | 4 10 4 10 4 10 4 10 4 10 4 |
| Number of steps | 10 10 10 10 10 10 10 10 20 10 39 100 |
| Inter step delay (mS) | 0 0 0 0 0 0 0 0 0 0 0 0 |
| Acquisition time (sec) | 40 400 40 400 40 400 800 800 800 800 1560 160 |
| Level control channel | Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 |
| Control level (volts) | 1 1 1 1 1 1 1 2.5 1 1 2 |
| Ch1 AC | - | 1.2v 1.2v 1.2v 1.2v 1.2v 1.2v 2.5v 1.2v 1.2v 2.5v |
| Ch2 AC | - | 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v |

**Figure B.12: Aluminum 90-Degree Interior Holes Plate 2 Run 1**

| ADD SPAN | Frequency start: | 2 78 88 136 146 206 216 239 249 329 375 |
| DEL SPAN | Frequency end:  | 78 88 136 146 206 216 239 249 329 375 1000 |
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| Tracking bandwidth (Hz) | 2 0.5 2 0.5 2 0.5 2 0.5 2 0.5 5 |
| Number of averages | 4 10 4 10 4 10 4 10 4 10 4 |
| Number of steps | 10 10 10 10 10 10 10 10 20 10 39 100 |
| Inter step delay (mS) | 0 0 0 0 0 0 0 0 0 0 0 0 |
| Acquisition time (sec) | 40 400 40 400 40 400 800 800 800 800 1560 160 |
| Level control channel | Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 |
| Control level (volts) | 1 1 1 1 1 1 1 2.5 1 1 2 |
| Ch1 AC | - | 1.2v 1.2v 1.2v 1.2v 1.2v 1.2v 2.5v 1.2v 1.2v 2.5v |
| Ch2 AC | - | 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v |

**Figure B.13: Aluminum 90-Degree Interior Holes Plate 2 Run 2**

| ADD SPAN | Frequency start: | 2 78 88 136 146 206 216 239 249 329 375 |
| DEL SPAN | Frequency end:  | 78 88 136 146 206 216 239 249 329 375 1000 |
| Sweep type | | Lin Lin Lin Lin Lin Lin Lin Lin Lin Lin |
| Tracking bandwidth (Hz) | 2 0.5 2 0.5 2 0.5 2 0.5 2 0.5 5 |
| Number of averages | 4 10 4 10 4 10 4 10 4 10 4 |
| Number of steps | 10 10 10 10 10 10 10 10 20 10 39 100 |
| Inter step delay (mS) | 0 0 0 0 0 0 0 0 0 0 0 0 |
| Acquisition time (sec) | 40 400 40 400 40 400 800 800 800 800 1560 160 |
| Level control channel | Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 |
| Control level (volts) | 1 1 1 1 1 1 1 2.5 1 1 2 |
| Ch1 AC | - | 1.2v 1.2v 1.2v 1.2v 1.2v 1.2v 2.5v 1.2v 1.2v 2.5v |
| Ch2 AC | - | 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v |

**Figure B.14: Aluminum 45-Degree Corner Plate 1 Run 1 & 2**
Figure B.15: Aluminum 45-Degree Corner Plate 2 Run 1

Figure B.16: Aluminum 45-Degree Corner Plate 2 Run 2

Figure B.17: Aluminum 45-Degree Interior Bolts Plate 1 Run 1
Figure B.18: Aluminum 45-Degree Interior Bolts Plate 1 Run 2

Figure B.19: Aluminum 45-Degree Interior Bolts Plate 2 Run 1

Figure B.20: Aluminum 45-Degree Interior Bolts Plate 2 Run 2
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Figure B.24: Composite 90-Degree Corner Plate Run 1

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Figure B.25: Composite 90-Degree Corner Plate Run 2

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Figure B.26: Composite 90-Degree Interior Holes Plate Run 1
### Figure B.27: Composite 90-Degree Interior Holes Plate Run 2

| ADD SPAN Frequency start: | 2 132 138 288 305 320 360 438 471 548 558 |
| DEL SPAN Frequency end:  | 132 138 288 305 320 360 438 471 548 558 1000 |
| Sweep type                | Lin Lin Lin Lin Lin Lin Lin Lin Lin Lin |
| Tracking bandwidth (Hz)   | 2 0.5 2 0.5 1 0.5 2 0.5 2 0.5 5 |
| Number of averages        | 4 10 4 10 4 10 4 10 4 10 4 |
| Number of steps           | 10 30 10 17 15 40 10 33 10 10 100 |
| Inter step delay (mS)     | 0 0 0 0 0 0 0 0 0 0 0 |
| Acquisition time (sec)    | 40 1200 40 680 120 1600 40 1320 40 400 160 |
| Level control channel     | Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 |
| Control level (volts)     | 2 3 2 3 2 2 3 2 2 2 |
| Ch1 AC                    | 2.5v 5v 2.5v 5v 2.5v 2.5v 5v 2.5v 2.5v 2.5v 2.5v |
| Ch2 AC                    | 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v |

### Figure B.28: Composite 90-Degree Interior Holes No Washers Plate Run 1

| ADD SPAN Frequency start: | 2 124 130 272 282 316 326 338 348 412 445 |
| DEL SPAN Frequency end:  | 124 130 272 282 316 326 338 348 412 445 1000 |
| Sweep type                | Lin Lin Lin Lin Lin Lin Lin Lin Lin Lin Lin |
| Tracking bandwidth (Hz)   | 2 0.5 2 0.5 1 0.5 2 0.5 2 0.5 5 |
| Number of averages        | 4 10 4 10 4 10 4 10 4 10 4 |
| Number of steps           | 10 30 10 10 10 10 10 10 10 10 100 |
| Inter step delay (mS)     | 0 0 0 0 0 0 0 0 0 0 0 |
| Acquisition time (sec)    | 40 1200 40 400 40 80 400 40 400 40 400 160 |
| Level control channel     | Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 |
| Control level (volts)     | 2 3 2 3 2 2 3 2 2 3 2 |
| Ch1 AC                    | 2.5v 5v 2.5v 5v 2.5v 2.5v 5v 2.5v 5v 2.5v 2.5v |
| Ch2 AC                    | 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v |

### Figure B.29: Composite 90-Degree Interior Holes Plate Run 1

<p>| ADD SPAN Frequency start: | 2 71 81 120 130 181 191 238 248 285 346 |
| DEL SPAN Frequency end:  | 71 81 120 130 181 191 238 248 285 346 1000 |
| Sweep type                | Lin Lin Lin Lin Lin Lin Lin Lin Lin Lin Lin |
| Tracking bandwidth (Hz)   | 2 0.5 2 0.5 2 0.5 2 0.5 2 0.5 5 |
| Number of averages        | 4 10 4 10 4 10 4 10 4 10 4 |
| Number of steps           | 10 10 10 10 10 10 10 10 10 10 100 |
| Inter step delay (mS)     | 0 0 0 0 0 0 0 0 0 0 0 |
| Acquisition time (sec)    | 40 400 40 400 40 400 40 400 40 2440 160 |
| Level control channel     | Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 |
| Control level (volts)     | 1 0.7 1 0.7 1 0.7 1 0.7 1 0.7 2 |
| Ch1 AC                    | 1.2v 1.2v 1.2v 1.2v 1.2v 1.2v 1.2v 1.2v 1.2v 1.2v 2.5v |
| Ch2 AC                    | 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v 2.5v |</p>
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<th>Number of Steps</th>
<th>Inter Step Delay (mS)</th>
<th>Acquisition Time (sec)</th>
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<th>Control Level (volts)</th>
<th>Ch1 AC</th>
<th>Ch2 AC</th>
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<td>0 0 0 0 0 0 0</td>
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<td>2 0.5 2 0.5 2 0.5 2 0.5 2 0.5 2</td>
<td>10 10 10</td>
<td>0 0 0 0</td>
<td>40 400 800 40 400 40 400 40</td>
<td>Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1</td>
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<tr>
<td>Acquisition Time (sec)</td>
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<td>2 0.5 2 0.5 2 0.5 2 0.5 2 0.5 2</td>
<td>10 10 10</td>
<td>0 0 0 0</td>
<td>40 400 800 40 400 40 400 40</td>
<td>Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1 Out1</td>
<td>1 0.7 1 1 0.7 1 1 1 1 1</td>
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<td>10 10 10</td>
<td>0 0 0 0</td>
<td>40 400 800 40 400 40 400 40</td>
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<td>10 10 10</td>
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<td>40 400 800 40 400 40 400 40</td>
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**Figure B.30: Composite 90-Degree Interior Holes Run 2**

**Figure B.31: Composite 90-Degree Plate Interior Holes No Washers Run 1**

**Figure B.32: Composite 45-Degree Corner Plate Run 1**
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<td>Inter step delay (mS)</td>
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**Figure B.33: Composite 45-Degree Corner Plate Run 2**

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**Figure B.34: Composite 45-Degree Interior Bolts Plate Run 1**

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**Figure B.35: Composite 45-Degree Interior Bolts No Washers Plate Run 1**

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Figure B.36: Composite 45-Degree Interior Bolts No Washers Plate Run 2

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Figure B.37: Composite 45-Degree Interior Holes Plate Run 1

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Figure B.38: Composite 45-Degree Interior Holes No Washer Plate Run 1
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<td>4</td>
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<td>10</td>
<td>10</td>
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<tr>
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<td>2480</td>
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<td>Out1</td>
<td>Out1</td>
<td>Out1</td>
<td>Out1</td>
<td>Out1</td>
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<td>Out1</td>
<td>Out1</td>
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<td></td>
</tr>
<tr>
<td>Control level (volts)</td>
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<td>1</td>
<td>2</td>
<td>1</td>
<td>0.7</td>
<td>1</td>
<td>1.5</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Ch1 AC</td>
<td>1.2v</td>
<td>1.2v</td>
<td>1.2v</td>
<td>2.5v</td>
<td>1.2v</td>
<td>1.2v</td>
<td>2.5v</td>
<td>1.2v</td>
<td>1.2v</td>
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<td>2.5v</td>
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<tr>
<td>Ch2 AC</td>
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<td>2.5v</td>
<td>2.5v</td>
<td>2.5v</td>
<td>2.5v</td>
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<td>2.5v</td>
<td>2.5v</td>
<td>2.5v</td>
<td>2.5v</td>
<td>Auto</td>
</tr>
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</table>

Figure B.39: Composite 45-Degree Interior Holes No Washers Plate Run 2
Appendix C. Matlab M-Files

C.1 Rotate.m

This m-file rotates the eigenvectors at the centers of the patch sides from the global to the local coordinate system. A similar file is used to rotate all the eigenvectors along each side of the patch. That file simply goes through the same steps but for more eigenvectors. If no rotation is needed, as in the case of the 90-degree patch, then the m-file is modified so as to simply rename the eigenvectors and save them to rotated.mat.

% m-file to rotate the x and y rotational components of the eigenvectors
% by Andrew Littlefield
% 14 Jul 99

clear all;
close all;

% Angle to rotate by
angle=-pi/4;

%transformation matrix for a rotation about the z-axis
transform=[cos(angle) -sin(angle) 0; sin(angle) cos(angle) 0; 0 0 1];

%load eigenvector along top of patch and assign to meaningful name
load n165.eig
topc=n165;

%load eigenvectors along right side of patch and assign to meaningful name
load n2054.eig
rightc=n2054;

%load eigenvectors along bottom of patch and assign to meaningful name
load n174.eig
bottomc=n174;

%load eigenvectors along left side of patch and assign to meaningful name
load n170.eig
leftc=n170;

%load eigenvector at sensor and assign to meaningful name
load n674.eig
sensor=n674;

%rotate the components of top
temp=topc;
for mode=1:35
    topc(mode,4)=transform(1,1)*temp(mode,4)+transform(2,1)*temp(mode,5);
    topc(mode,5)=transform(1,2)*temp(mode,4)+transform(2,2)*temp(mode,5);
end
% rotate the components of right
temp = rightc;
for mode = 1:35
    rightc(mode,4) = transform(1,1)*temp(mode,4) + transform(2,1)*temp(mode,5);
    rightc(mode,5) = transform(1,2)*temp(mode,4) + transform(2,2)*temp(mode,5);
end

% rotate the components of bottom
temp = bottomc;
for mode = 1:35
    bottomc(mode,4) = transform(1,1)*temp(mode,4) + transform(2,1)*temp(mode,5);
    bottomc(mode,5) = transform(1,2)*temp(mode,4) + transform(2,2)*temp(mode,5);
end

% rotate the components of left
temp = leftc;
for mode = 1:35
    leftc(mode,4) = transform(1,1)*temp(mode,4) + transform(2,1)*temp(mode,5);
    leftc(mode,5) = transform(1,2)*temp(mode,4) + transform(2,2)*temp(mode,5);
end

% rotate the components of sensor
temp = sensor;
for mode = 1:35
    sensor(mode,4) = transform(1,1)*temp(mode,4) + transform(2,1)*temp(mode,5);
    sensor(mode,5) = transform(1,2)*temp(mode,4) + transform(2,2)*temp(mode,5);
end

save rotated.mat topc rightc bottomc leftc sensor
This m-file curve fits the eigenvectors along the sides to get the shape functions needed to describe the moments induced by the actuator. Also it generates the moments applied to the plate by the actuators. The eigenvectors have already been rotated if necessary by Rotate.m.

C.2. Shaper.m

% m-file to do interpolation to get shape functions to describe moment eigenvectors
% Original by Jim Fairweather
% 15 Jul 99 - Modification by Andrew Littlefield

clear all;
close all;

% Setup required constants in local coordinate system
x1=-0.015;
x2=0.036;
y1=0.072;
y2=0.123;

ypos=linspace(y1,y2,11);
xpos=linspace(x1,x2,11);

% define eigenvectors rotated to local coordinates
load rotated;

%define components relating to the moments
my2=[top1(:,4) top2(:,4) top3(:,4) top4(:,4) top5(:,4) top6(:,4) top7(:,4) top8(:,4) top9(:,4) top10(:,4) top11(:,4)];
my1=[bottom1(:,4) bottom2(:,4) bottom3(:,4) bottom4(:,4) bottom5(:,4) bottom6(:,4) bottom7(:,4) bottom8(:,4) bottom9(:,4) bottom10(:,4) bottom11(:,4)];
mx1=[left1(:,5) left2(:,5) left3(:,5) left4(:,5) left5(:,5) left6(:,5) left7(:,5) left8(:,5) left9(:,5) left10(:,5) left11(:,5)];
mx2=[right1(:,5) right2(:,5) right3(:,5) right4(:,5) right5(:,5) right6(:,5) right7(:,5) right8(:,5) right9(:,5) right10(:,5) right11(:,5)];

%curve fit the Y eigenvectors
for n=1:35

    %Find the coefficients
    coeff=polyfit(xpos,my1(n,:),6);
    A6=coeff(1);
    A5=coeff(2);
    A4=coeff(3);
    A3=coeff(4);
    A2=coeff(5);
    A1=coeff(6);
    A0=coeff(7);

    %Build the equation using the coefficients
    intpor1=-(.2497388615e-4*A3+.2097539989e-5*A4+.1583987769e-
7*A6+.508000000e-1*A0+.3870960000e-2*A1+.3058918603e-3*A2+.18045610e-6*A5);

%Find the coefficients
coeff=polyfit(xpos,my2(n,:),6);
A6=coeff(1);
A5=coeff(2);
A4=coeff(3);
A3=coeff(4);
A2=coeff(5);
A1=coeff(6);
A0=coeff(7);

%Build the equation using the coefficients
intpor2=(.2497388615e-4*A3+.2097539989e-5*A4+.1583987769e-7*A6+.508000000e-1*A0+.3870960000e-2*A1+.3058918603e-3*A2+.18045610e-6*A5);

%Calculate the desired components
totalintY(n)=(intpor1+intpor2);
momy1(n)=intpor1;
momy2(n)=-intpor2;
end

%Save the desired components
save dismy.mat momy1 momy2 totalintY

%curve fit the X eigenvectors
for n=1:35
    %Find the coefficients
    coeff=polyfit(ypos,mx1(n,:),6);
    A6=coeff(1);
    A5=coeff(2);
    A4=coeff(3);
    A3=coeff(4);
    A2=coeff(5);
    A1=coeff(6);
    A0=coeff(7);

    %Build the equation using the coefficients
    intpor1=(.83246286e-5*A3+.5116982622e-6*A4+.2130042e-8*A6+.5080000000e-1*A0+.2580640000e-2*A1+.1420212217e-3*A2+.3258235190e-7*A5);

    %Find the coefficients
    coeff=polyfit(ypos,mx2(n,:),6);
    A6=coeff(1);
    A5=coeff(2);
    A4=coeff(3);
    A3=coeff(4);
    A2=coeff(5);
    A1=coeff(6);
    A0=coeff(7);

    %Build the equation using the coefficients
    intpor2=(-.83246286e-5*A3+.5116982622e-6*A4+.2130042e-8*A6+.5080000000e-1*A0+.2580640000e-2*A1+.1420212217e-3*A2+.3258235190e-7*A5);
%Calculate the desired components
momx1(n)=intpor1;
momx2(n)=-intpor2;
totalintX(n)=(intpor1+intpor2);

end

%Save the desired components
save dismx.mat momx1 momx2 totalintX
This m-file calculates the frequency response of the plate based on the NASTRAN eigenvectors and values from the middle of the patch sides. These eigenvectors have been rotated by Rotate.m if necessary.

```matlab
% m-file to compute frequency response based on Nastran eigenvectors and values
% james fairweather
% 5.27.97
% 6.24.97 major modification
% 15 Jul 99 - modification by Andrew Littlefield
% uses eigenvectors from middle of patch sides

clear all;
close all;

hb=0.0014224; % thickness of Al plate (meters)
%hb=0.001397 %thickness of Comp plate (meters)
ha=0.0075*0.0254; % thickness of actuator (meters)
w=logspace(log10(1*2*pi),log10(500*2*pi),2000)';

%Load eigenvectors of nodes at center of patch sides
%They have previously been rotated to local coord system
%load rotated

%assign these to the variables originally used by Jim.
top=topc;
right=rightc;
left=leftc;
bottom=bottomc;
center=sensor;

%Load in list of frequencies as reported by FEA analysis
load freq.lst

% Required constants, using PSI-5A-S3
Vm=1; % voltage applied to PZT (volts)
hp=ha; % thickness of the actuator (meters)
E=Vm/hp; % electric field (V/m)
d31=-1.80e-10; % strain coefficient between 3 and 1 direction (m/volt)
d32=-1.80e-10; % strain coefficient between 3 and 2 direction (m/volt)
rho_piezo=7700; % density of the patch (kg/m^3)
nu_piezo=0.3; % poisson's ratio the patch (unitless)
neta=0.001; % structural loss factor
Y_E=5.7e10*(1+neta*i); % complex young's modulus of the patch (N/m^2)
Kp=sqrt(rho_piezo*(1-nu_piezo^2)/Y_E); % wavenumber
ap=2.03*0.0254; % length of patch in x-direction (meters)
bp=2.03*0.0254; % length of patch in y-direction (meters)

onew=ones(size(w));
k=w*Kp;
```
%compute FEA impedance

num_modes=35; %Number of modes generated by FEA Analysis

omega=freq*2*pi;  % convert frequencies from Hz to rad/sec

for n=1:num_modes
    den(n,:)=[1 2*0.00*omega(n) omega(n)^2]; %(s^2+omega^2)
end

% The impedances to be computed (actually admittances are computed and then inverted for impedance)
% imp=1 compute Zxx
% imp=2 compute Zyy
% imp=3 compute Zxy
% imp=4 compute Zyx

for imp=1:4
    if imp==1
        drive_point_one=-left(:,5); % isolate R2 or rotations about the
        Y axis at the first patch location
        drive_point_two=-right(:,5); % isolate R2 or rotations about the
        Y axis at the second patch location
        recover_point_one=-left(:,5); % isolate R2 or rotations about the
        Y axis at the first patch location
        recover_point_two=-right(:,5); % isolate R2 or rotations about the
        Y axis at the second patch location
    elseif imp==2
        drive_point_two=top(:,4); % isolate R2 or rotations about the X
        axis at the first patch location
        drive_point_one=bottom(:,4); % isolate R2 or rotations about the
        X axis at the second patch location
        recover_point_two=top(:,4); % isolate R1 or rotations about the
        X axis at the first patch location
        recover_point_one=bottom(:,4); % isolate R1 or rotations about the
        X axis at the second patch location
    elseif imp==3
        drive_point_one=bottom(:,4); % isolate R2 or rotations about the
        X axis at the first patch location
        drive_point_two=top(:,4); % isolate R2 or rotations about the X
        axis at the second patch location
        recover_point_one=-left(:,5); % isolate R2 or rotations about the
        Y axis at the first patch location
        recover_point_two=-right(:,5); % isolate R2 or rotations about the
        Y axis at the second patch location
    elseif imp==4
        drive_point_one=-left(:,5); % isolate R2 or rotations about the
        Y axis at the first patch location
        drive_point_two=-right(:,5); % isolate R2 or rotations about the
        Y axis at the second patch location
        recover_point_one=bottom(:,4); % isolate R1 or rotations about the
        X axis at the second patch location
        recover_point_two=top(:,4); % isolate R1 or rotations about the
X axis at the first patch location

end

basedr=drive_point_one-drive_point_two;  % compute the basedrive differential.  (R_1-R_2)
phio=recover_point_one-recover_point_two;  % compute response point differential (R_1-R_2)

adm_num=[phio.*basedr zeros(max(size(phio.*basedr)),1)];  % (R1-R2)*(R1-R2)s

%Put first two modes in parallel

[adm_numt,adm_dent]=parallel(adm_num(1,:),den(1,:),adm_num(2,:),den(2,:));

%Put the rest of the modes in parallel
for n=3:num_modes
    [adm_numt,adm_dent]=parallel(adm_numt,adm_dent,adm_num(n,:),den(n,:));
end

%recover magnitude and phase and use them to form admittance
[adm_mag,adm_phase,w]=bode(adm_numt,adm_dent,w);
TF_adm(:,imp)=(adm_mag.*exp(j*adm_phase*pi/180));
end

%get impedance from admittance
for n=1:max(size(w));
    imped=inv([TF_adm(n,1) TF_adm(n,4);TF_adm(n,3) TF_adm(n,2)]);
    %TF_imp(n,1)=imag(imped(1,1))*j;
    %TF_imp(n,2)=imag(imped(2,2))*j;
    %TF_imp(n,3)=imag(imped(1,2))*j;
    %TF_imp(n,4)=imag(imped(2,1))*j;
    TF_imp(n,1)=imped(1,1);
    TF_imp(n,2)=imped(2,2);
    TF_imp(n,3)=imped(1,2);
    TF_imp(n,4)=imped(2,1);
end;

%Eqn 8.39 form Jim
TF_imp=TF_imp*2/((hb+ha)^2);

% portion of code to compute coefficients A and C of the assumed displacement solution Eqn 8.25 from Jim's thesis

N_11=(k.*cos(k*ap)+j*TF_imp(:,1).*w.*sin(k*bp)/(bp*hp*Y_E)-j*nu_piezo*TF_imp(:,1).*w.*sin(k*ap)/(bp*hp*Y_E));
N_12=1/Y_E*(j*TF_imp(:,3).*w.*sin(k*bp)/(bp*hp)-j*nu_piezo*TF_imp(:,3).*w.*sin(k*bp)/(bp*hp));
N_21=1/Y_E*(j*TF_imp(:,4).*w.*sin(k*bp)/(ap*hp)-j*nu_piezo*TF_imp(:,4).*w.*sin(k*bp)/(ap*hp));
N_22=(k.*cos(k*bp)+j*TF_imp(:,2).*w.*sin(k*bp)/(ap*hp*Y_E)-j*nu_piezo*TF_imp(:,2).*w.*sin(k*bp)/(ap*hp*Y_E));
for n=1:max(size(w))
    coeff=[([N_11(n) N_12(n);N_21(n) N_22(n)],[d31,d32])*E];
    A(n)=coeff(1);
    C(n)=coeff(2);
end;

%Eqn 8.26 from Jim's thesis
for n=1:max(size(w))
    Fx(n)=-j*w(n)*(TF_imp(n,1)*A(n)*sin(Kp*w(n)*ap)+TF_imp(n,3)*C(n)*sin(Kp*w(n)*bp));
    Fy(n)=-j*w(n)*(TF_imp(n,4)*A(n)*sin(Kp*w(n)*ap)+TF_imp(n,2)*C(n)*sin(Kp*w(n)*bp));
end;

Fx=Fx.';
Fy=Fy.';

%Eqn 8.27 from Jim's thesis
Mx=(hb+hp)/ap*Fx;
My=(hb+hp)/bp*Fy;

phio=center(:,3); % isolate T3 or translations along the Z axis at the center of the plate

%Compute Part of displacement due to X
load dismx;
basedr=totalintX';

num=[phio.*basedr];
[numt,dent]=parallel(num(1,:),den(1,:),num(2,:),den(2,:));
for n=3:num_modes
    [numt,dent]=parallel(numt,dent,num(n,:),den(n,:));
end

[mag,phase,w]=bode(numt,dent,w);
TF_disx=mag.*exp(j*phase*pi/180);

disx=Mx.*TF_disx;

%Compute part of displacement due to Y
load dismy;
basedr=totalintY';

num=[phio.*basedr];
[numt,dent]=parallel(num(1,:),den(1,:),num(2,:),den(2,:));
for n=3:num_modes
    [numt,dent]=parallel(numt,dent,num(n,:),den(n,:));
end

[mag,phase,w]=bode(numt,dent,w);
TF_disy=mag.*exp(j*phase*pi/180);

disy=My.*TF_disy;

%Calculate final values and save them eqn 8.91 from Jim
Total_Dis = disx + disy;
Total_Static_Mod_dis = TF_disx * Mx(2) + TF_disy * My(2);

save ResCenter.mat Total_Dis Total_Static_Mod_dis w
This m-file calculates the frequency response of the plate based on the NASTRAN eigenvectors and values. These eigenvectors and values have been fit to shape functions by Shaper.m. Also, these eigenvectors have been rotated by Rotate.m if necessary.

% m-file to compute frequency response based on Nastran eigenvectors and values
% james fairweather
% 5.27.97
% 6.24.97 major modification
% 15 Jul 99 - modification by Andrew Littlefield
% uses curve fit eigenvectors

clear all;
close all;

hb=0.0014224;  % thickness of Al plate (meters)
%hb=0.001397  % thickness of Comp plate (meters)
ha=0.0075*0.0254;  % thickness of actuator (meters)

% load values from shaper.m including curve fit eigenvectors
load dismx;
load dismy;
momx1=momx1';
momx2=momx2';
momy1=momy1';
momy2=momy2';

w=logspace(log10(1*2*pi),log10(500*2*pi),2000)';

% Load eigenvectors of nodes at center of patch sides
% They have previously been rotated to local coord system
load rotated

% assign these to the variables originally used by Jim.
top=topc;
right=rightc;
left=leftc;
bottom=bottomc;
center=sensor;

load freq.lst % load list of eigenfrequencies

% Compute the impedance of the patch
% Required constants, using PSI-5A-S3

Vm=1;  % voltage applied to PZT (volts)
hp=ha;  % thickness of the actuator (meters)
E=Vm/hp;  % electric field (V/m)
d31=-1.80e-10;  % strain coefficient between 3 and 1 direction (m/volt)
d32=-1.80e-10;  % strain coefficient between 3 and 2 direction (m/volt)
rho_piezo=7700;  % density of the patch (kg/m^3)
nu_piezo=0.3;  % poisson's ratio the patch (unitless)
neta=0.001;  % structural loss factor
Y_E=5.7e10*(1+neta*i);  % complex young's modulus of the patch (N/m^2)
Kp=sqrt(rho_piezo*(1-nu_piezo^2)/Y_E);  % wavenumber
ap=2.03*0.0254;  % length of patch in x-direction (meters)
bp=2.03*0.0254;  % length of patch in y-direction (meters)

onew=ones(size(w));
k=w*Kp;

%compute FEA impedance

num_modes=35;  %Number of modes from FEA analysis

omega=freq*2*pi;  % convert frequencies from Hz to rad/sec
damp=zeros(size(omega));
for n=1:num_modes
    den(n,:)=[1 2*damp(n)*omega(n) omega(n)^2];  % (s^2+omega^2)
end

% The impedances to be computed (actually admittances are computed and then inverted for impedance)
% imp=1 compute Zxx
% imp=2 compute Zyy
% imp=3 compute Zxy
% imp=4 compute Zyx

for imp=1:4
    if imp==1
        drive_point_one=momx1;  % isolate R2 or rotations about the Y axis at the first patch location
        drive_point_two=momx2;  % isolate R2 or rotations about the Y axis at the second patch location
        recover_point_one=momx1;  % isolate R2 or rotations about the Y axis at the first patch location
        recover_point_two=momx2;  % isolate R2 or rotations about the Y axis at the second patch location
    elseif imp==2
        drive_point_one=momy1;  % isolate R2 or rotations about the X axis at the first patch location
        drive_point_two=momy2;  % isolate R2 or rotations about the X axis at the second patch location
        recover_point_one=momy1;  % isolate R2 or rotations about the X axis at the first patch location
        recover_point_two=momy2;  % isolate R2 or rotations about the X axis at the second patch location
    elseif imp==3
        drive_point_one=momy1;  % isolate R2 or rotations about the X axis at the first patch location
        drive_point_two=momy2;  % isolate R2 or rotations about the X axis at the second patch location
        recover_point_one=momx1;  % isolate R2 or rotations about the Y axis at the first patch location
        recover_point_two=momx2;  % isolate R2 or rotations about the Y axis at the second patch location
    elseif imp==4
        drive_point_one=momy1;  % isolate R2 or rotations about the X axis at the first patch location
        drive_point_two=momy2;  % isolate R2 or rotations about the X axis at the second patch location
        recover_point_one=momx1;  % isolate R2 or rotations about the Y axis at the first patch location
        recover_point_two=momx2;  % isolate R2 or rotations about the Y axis at the second patch location
    end
axis at the second patch location
    recover_point_two=momx2;  % isolate R2 or rotations about the Y axis at the first patch location

else if imp==4
    drive_point_one=momx1;  % isolate R2 or rotations about the Y axis at the first patch location
    drive_point_two=momx2;  % isolate R2 or rotations about the Y axis at the second patch location
    recover_point_one=momy1; % isolate R1 or rotations about the X axis at the second patch location
    recover_point_two=momy2; % isolate R1 or rotations about the X axis at the first patch location
end

basedr=drive_point_one-drive_point_two;  % compute the basedrive differential.  \(R_1-R_2\)
phio=recover_point_one-recover_point_two; % compute response point differential \(R_1-R_2\)

adm_num=[phio.*basedr zeros(max(size(phio.*basedr)),1)];%(R1-R2)*(R1-R2)

%Put first two modes in parallel
[adm_numt,adm_dent]=parallel(adm_num(1,:),den(1,:),adm_num(2,:),den(2,:));

%Put the rest of the modes in parallel
for n=3:num_modes
    [adm_numt,adm_dent]=parallel(adm_numt,adm_dent,adm_num(n,:),den(n,:));
end

%recover magnitude and phase and use them to form admittance
[adm_mag,adm_phase,w]=bode(adm_numt,adm_dent,w);
TF_adm(:,imp)=(adm_mag.*exp(j*adm_phase*pi/180));
end

%get impedance from admittance
for n=1:max(size(w));
    imped=inv([TF_adm(n,1) TF_adm(n,4);TF_adm(n,3) TF_adm(n,2)]);
    TF_imp(n,1)=imag(imped(1,1))*j;
    TF_imp(n,2)=imag(imped(2,2))*j;
    TF_imp(n,3)=imag(imped(1,2))*j;
    TF_imp(n,4)=imag(imped(2,1))*j;
    TF_imp(n,1)=imped(1,1);
    TF_imp(n,2)=imped(2,2);
    TF_imp(n,3)=imped(1,2);
    TF_imp(n,4)=imped(2,1);
end;

%Eqn 8.39 form Jim
TF_imp=TF_imp*2/((hb+ha)^2)*ap;

% portion of code to compute coefficients A and C of the assumed
displacement solution

\[ N_{11} = \left( k \cdot \cos(k \cdot a_p) + j \cdot \text{TF}_\text{imp}(;1) \cdot w \cdot \sin(k \cdot a_p) / (b_p \cdot h_p \cdot Y_E) \right) - j \cdot \nu_{\text{piezo}} \cdot \text{TF}_\text{imp}(;4) \cdot w \cdot \sin(k \cdot a_p) / (a_p \cdot h_p \cdot Y_E) \] 
\[ N_{12} = 1 / Y_E \cdot \left( j \cdot \text{TF}_\text{imp}(;3) \cdot w \cdot \sin(k \cdot b_p) / (b_p \cdot h_p) \right) - j \cdot \nu_{\text{piezo}} \cdot \text{TF}_\text{imp}(;2) \cdot w \cdot \sin(k \cdot b_p) / (a_p \cdot h_p) \] 
\[ N_{21} = 1 / Y_E \cdot \left( j \cdot \text{TF}_\text{imp}(;4) \cdot w \cdot \sin(k \cdot a_p) / (a_p \cdot h_p) \right) - j \cdot \nu_{\text{piezo}} \cdot \text{TF}_\text{imp}(;1) \cdot w \cdot \sin(k \cdot a_p) / (b_p \cdot h_p) \] 
\[ N_{22} = \left( k \cdot \cos(k \cdot b_p) + j \cdot \text{TF}_\text{imp}(;2) \cdot w \cdot \sin(k \cdot b_p) / (a_p \cdot h_p \cdot Y_E) \right) - j \cdot \nu_{\text{piezo}} \cdot \text{TF}_\text{imp}(;3) \cdot w \cdot \sin(k \cdot b_p) / (b_p \cdot h_p \cdot Y_E) \]

for \( n = 1: \max(\text{size}(w)) \)
\[
\text{coeff} = \begin{bmatrix} N_{11}(n) & N_{12}(n) \\ N_{21}(n) & N_{22}(n) \end{bmatrix} \begin{bmatrix} d_{31} \\ d_{32} \end{bmatrix} \]
\[ A(n) = \text{coeff}(1); \]
\[ C(n) = \text{coeff}(2); \]
end;

% Eqn 8.26 from Jim's thesis
for \( n = 1: \max(\text{size}(w)) \)
\[
\text{Fx}(n) = -j \cdot w(n) \cdot (\text{TF}_\text{imp}(n,1) \cdot A(n) \cdot \sin(Kp \cdot w(n) \cdot a_p) + \text{TF}_\text{imp}(n,3) \cdot C(n) \cdot \sin(Kp \cdot w(n) \cdot b_p));
\]
\[ \text{Fy}(n) = -j \cdot w(n) \cdot (\text{TF}_\text{imp}(n,4) \cdot A(n) \cdot \sin(Kp \cdot w(n) \cdot a_p) + \text{TF}_\text{imp}(n,2) \cdot C(n) \cdot \sin(Kp \cdot w(n) \cdot b_p)); \]
end;

\[ \text{Fx} = \text{Fx}.'; \]
\[ \text{Fy} = \text{Fy}.'; \]

% Eqn 8.27 from Jim's thesis
\[
\text{Mx} = (h_b + h_p) / a_p \cdot \text{Fx};
\]
\[ \text{My} = (h_b + h_p) / b_p \cdot \text{Fy}; \]

\[ \text{phio} = \text{center}(:,3); \] % isolate T3 or translations along the Z axis at the center of the plate

% Compute Part of displacement due to X
load dismx;
\[ \text{basedr} = \text{totalintX'}; \]
\[ \text{num} = [\text{phio} \cdot \text{basedr}]; \]
\[ [\text{numt}, \text{dent}] = \text{parallel}(\text{num}(1,:), \text{den}(1,:), \text{num}(2,:), \text{den}(2,:)); \]
for \( n = 3: \text{num\_modes} \)
\[ [\text{numt}, \text{dent}] = \text{parallel}(\text{numt}, \text{dent}, \text{num}(n,:), \text{den}(n,:)); \]
end
\[
[\text{mag}, \text{phase}, \text{w}] = \text{bode}(\text{numt}, \text{dent}, \text{w});
\]
\[ \text{TF\_disx} = \text{mag} \cdot \exp(j \cdot \text{phase} \cdot \pi/180); \]
\[ \text{disx} = \text{Mx} \cdot \text{TF\_disx}; \]

% Compute Part of displacement due to Y
load dismy;
\[ \text{basedr} = \text{totalintY'}; \]
\[ \text{num} = [\text{phio} \cdot \text{basedr}]; \]
[numt,dent]=parallel(num(1,:),den(1,:),num(2,:),den(2,:));
for n=3:num_modes
    [numt,dent]=parallel(numt,dent,num(n,:),den(n,:));
end

[mag,phase,w]=bode(numt,dent,w);
TF_disy=mag.*exp(j*phase*pi/180);

disy=My.*TF_disy;

%Calculate final values and save them eqn 8.91 from Jim
Total_Dis=disx+disy;
Total_Static_Mod_dis=TF_disx*Mx(2)+TF_disy*My(2);

save ResShape.mat Total_Dis Total_Static_Mod_dis w
This m-file takes the results from RespCenter.m and RespShape.m and plots them versus the experimental data. The m-file is modified for each plate configuration.

```matlab
C.5. GenPlots.m

% m-file to generate plots from analytical and experimental results
% 16 Jul 99 - Andrew Littlefield

% Load in analytical results for calculation at center points
load ResCenter
CentDis=Total_Dis;
CenterSEF=Total_Static_Mod_dis;

% Load in analytical results for calculation using shape functions
load ResShape
ShapeDis=Total_Dis;
ShapeSEF=Total_Static_Mod_dis;

% Plot results for Sensor Displacement based on this FEA model and Center Point
figure
subplot(211)
semilogy(w/2/pi,abs(CentDis),'k-',w/2/pi,abs(CenterSEF),'k:')
axis([10 500 10e-11 10e-3])
ylabel('Magnitude (m/V)')
title('Displacement of Sensor using Center Points; Dynamic Impedance vs SEF; Al Coarse FEA 45')
subplot(212)
plot(w/2/pi,unwrap(angle(CentDis))*180/pi,'k-',w/2/pi,unwrap(angle(CenterSEF))*180/pi,'k:')
axis([10 500 -1300 0])
ylabel('Phase (degrees)')
xlabel('frequency (Hz)')

% Plot results for Sensor Displacement based on this FEA model and Shape Functions
figure
subplot(211)
semilogy(w/2/pi,abs(ShapeDis),'k-',w/2/pi,abs(ShapeSEF),'k:')
axis([10 500 10e-11 10e-3])
ylabel('Magnitude (m/V)')
title('Displacement of Sensor using Shape Functions; Dynamic Impedance vs SEF; Al Coarse FEA 45')
subplot(212)
plot(w/2/pi,unwrap(angle(ShapeDis))*180/pi,'k-',w/2/pi,unwrap(angle(ShapeSEF))*180/pi,'k:')
axis([10 500 -1300 0])
ylabel('Phase (degrees)')
xlabel('frequency (Hz)')

% Plot results for Sensor Displacement based on this FEA model, Center Point vs Shape Functions
figure
subplot(211)
semilogy(w/2/pi,abs(CentDis),'k-',w/2/pi,abs(ShapeDis),'k:')
axis([10 500 10e-11 10e-3])
```

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ylabel('Magnitude (m/V)')
title('Displacement of Sensor using Center Points; Center Points vs Shape Functions; Al Coarse FEA 45')
subplot(212)
plot(w/2/pi,unwrap(angle(CentDis))*180/pi,'k-'
',w/2/pi,unwrap(angle(ShapeDis))*180/pi,'k:')
axis([10 500 -1300 0])
ylabel('Phase (degrees)')
xlabel('frequency (Hz)')

%load first experimental results and plot it
load Al_Plate45_1.vss -mat
p1r1=2*XferDat;
Angp1r1=unwrap(angle(p1r1))*180/pi;
figure
subplot(211)
semilogy(w/2/pi,abs(CentDis),'k-'
',Fvec,abs(p1r1)*0.00012401*0.049056,'k:','w/2/pi,abs(ShapeDis),'k--')
axis([10 500 1e-11 1e-3])
ylabel('Magnitude (m/V)')
title('Experimental Measurements vs. Dynamic Impedance Model; Al 45 Plate 1 Run 1 vs. Coarse FEA')
subplot(212)
plot(w/2/pi,unwrap(angle(CentDis))*180/pi,'k-',Fvec,Angp1r1-180,'k:',w/2/pi,unwrap(angle(ShapeDis))*180/pi,'k--')
axis([10 500 -1300 0])
ylabel('Phase (degrees)')
xlabel('frequency (Hz)')

%load next experimental results and plot it
load Al_Plate45_1b.vss -mat
p1r2=2*XferDat;
Angp1r2=unwrap(angle(p1r2))*180/pi;
figure
subplot(211)
semilogy(w/2/pi,abs(CentDis),'k-
',Fvec,abs(p1r2)*0.00012401*0.049056,'k:','w/2/pi,abs(ShapeDis),'k--')
axis([10 500 1e-11 1e-3])
ylabel('Magnitude (m/V)')
title('Experimental Measurements vs. Dynamic Impedance Model; Al 45 Plate 1 Run 2 vs. Coarse FEA')
subplot(212)
plot(w/2/pi,unwrap(angle(CentDis))*180/pi,'k-',Fvec,Angp1r2-180,'k:',w/2/pi,unwrap(angle(ShapeDis))*180/pi,'k--')
axis([10 500 -1300 0])
ylabel('Phase (degrees)')
xlabel('frequency (Hz)')

%load next experimental results and plot it
load Al_Plate45_2.vss -mat
p2r1=2*XferDat;
Angp2r1=unwrap(angle(p2r1))*180/pi;
figure
subplot(211)
semilogy(w/2/pi,abs(CentDis),'k-
',Fvec,abs(p2r2)*0.00012401*0.049056,'k:','w/2/pi,abs(ShapeDis),'k--')
axis([10 500 1e-11 1e-3])
ylabel('Magnitude (m/V)')
title('Experimental Measurements vs. Dynamic Impedance Model; Al 45 Plate 1 Run 3 vs. Coarse FEA')
subplot(212)
plot(w/2/pi,unwrap(angle(CentDis))*180/pi,'k-',Fvec,Angp2r1-180,'k:',w/2/pi,unwrap(angle(ShapeDis))*180/pi,'k--')
axis([10 500 -1300 0])
ylabel('Phase (degrees)')
xlabel('frequency (Hz)')
Plate 2 Run 1 vs. Coarse FEA')
subplot(221)
plot(w/2/pi,unwrap(angle(CentDis))*180/pi,'k-',Fvec,Angp2r1-180,'k:',w/2/pi,unwrap(angle(ShapeDis))*180/pi,'k--')
axis([10 500 -1300 0])
ylabel('Phase (degrees)')
xlabel('frequency (Hz)')

%load next experimental results and plot it
load Al_Plate45_2b.vss -mat
p2r2=2*XferDat;
Angp2r2=unwrap(angle(p2r2))*180/pi;
figure
subplot(211)
semilogy(w/2/pi,abs(CentDis),'k-'
',Fvec,abs(p2r2)*0.00012401*0.049056,'k:',w/2/pi,abs(ShapeDis),'k--')
axis([10 500 10e-11 10e-3])
ylabel('Magnitude (m/V)')
title('Experimental Measurements vs. Dynamic Impedance Model; Al 45 Plate 2 Run 2 vs. Coarse FEA')

subplot(222)
plot(w/2/pi,unwrap(angle(CentDis))*180/pi,'k-',Fvec,Angp2r2-180,'k:',w/2/pi,unwrap(angle(ShapeDis))*180/pi,'k--')
axis([10 500 -1300 0])
ylabel('Phase (degrees)')
xlabel('frequency (Hz)')