

**COURSE OUTLINE**  
**BC CALCULUS**  
**2013-2014**

**I. PRE-CALCULUS REVIEW AND LIMITS (~12 days):**

- Solving Polynomial and Fractional Inequalities
- Symmetry
- Horizontal and Vertical Asymptotes
- Limits
  - The Intuitive Definition
  - Computational Techniques
  - Infinite Limits and Limits at Infinity
  - L'Hopital's Rule (limits of the form  $\frac{0}{0}$ ,  $\frac{\infty}{\infty}$ ,  $\infty - \infty$ ,  $0 \cdot \infty$ ,  $0^0$ ,  $1^\infty$ ,  $\infty^0$ )
- Graphing Rational Functions
- Continuity
  - Intermediate Value Theorem
  - Extreme Value Theorem
- Finding the Slope of the Tangent Line to a Curve
- Average vs. Instantaneous Velocity

**II. DIFFERENTIATION TECHNIQUES (~10 days):**

- Definition of the Derivative
- Differentiation Formulas
- The Product Rule
- The Quotient Rule
- Higher Order Derivatives
- The Chain Rule
- What is the derivative of  $\sin x$ ?
- Derivative of Trigonometric Functions
- Implicit Differentiation

**III. APPLICATIONS OF THE DERIVATIVE (~20 days):**

- Continuity and Differentiability
- Rectilinear Motion Problems
- Graphing the Derivative of a Function
- Differentials
- Tangent Line (Linear) Approximations
- Newton's Method
- Geometric Applications
  - Rolle's Theorem
  - Mean Value Theorem
- Curve Sketching
  - Increasing and Decreasing Functions; Critical Points
  - Concavity; Inflection Points
  - Absolute Extrema; Relative Extrema
  - 1<sup>st</sup> and 2<sup>nd</sup> Derivative Tests

- Graphing (include polynomial functions, rational functions, trigonometric functions, functions with rational exponents, and AP type problems)
- Applied Maximum/Minimum Problems
- Related Rates Problems

**IV. INTEGRATION TECHNIQUES (~11 days):**

- Introduction to Antiderivatives
- Integration Formulas
- Integrals of Trigonometric Functions
- Integrals of Absolute Value Functions
- Rectilinear Motion Problems
- U-Substitutions
- Improper Integrals
- Integration By Parts
- Integrating Powers of Trigonometric Functions
- Integrals Involving Trigonometric Substitutions
- Integrating Rational Functions – The Partial Fractions Method (*distinct linear factors*)

**V. APPLICATIONS OF THE INTEGRAL (~18 days):**

- Calculating Areas Using Rectangles (LRAM, MRAM, RRAM)
- The Theory of the Integral
- The First Fundamental Theorem of Calculus
- Evaluating Definite Integrals
- Displacement; Total Distance Traveled
- The Trapezoidal Rule
- The Mean Value Theorem for Integrals (Average Value)
- Area Between Two Curves
- Volumes of Solids of Revolution (Disk, Washer, and Shell Methods)
- Volumes of Solids with Known Cross Sections
- Deriving Volume Formulas Using Calculus
- The Arc Length of a Plane Curve
- Area of a Surface of Revolution
- Work
- The Second Fundamental Theorem of Calculus

**VI. INVERSE FUNCTIONS AND INVERSE TRIG FUNCTIONS (~4 days):**

- Review definition of inverse, graphing inverses, finding inverses
- Derivatives of Inverse Functions
- Review graphs of inverse trigonometric functions
- Derivatives of Inverse Trigonometric Functions
- Integrals Involving Inverse Trigonometric Functions

**VII. LOGARITHMIC AND EXPONENTIAL FUNCTIONS (~4 days):**

- The Natural Logarithm Function (review definition of logarithms, important properties, definition of  $\ln x$ , definition of  $e$ , graph of  $\ln x$  using calculus, graph of  $e^x$  using inverse idea)
- Derivatives and Integrals Involving  $\ln x$

- Logarithmic Differentiation (finding  $\frac{d}{dx}[\text{variable}^{\text{variable}}]$ )
- Exponential Functions
- Derivatives and Integrals Involving Exponential Functions

**VIII. DIFFERENTIAL EQUATIONS (~6 days):**

- Solving First Order Separable Equations
- Exponential Growth and Decay
- Logistic Growth
- Slope Fields
- Euler's Method

**IX. PARAMETRIC EQUATIONS AND VECTOR VALUED FUNCTIONS (~3 days):**

- Review of parametric equations (graphing, eliminating the parameter)
- Finding  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  of Parametric Equations
- Arc Length and Surface Area of Parametric Equations
- Vector Valued Functions

**X. POLAR COORDINATES (~3 days):**

- Review polar graphing (cardioids, limacons, circles, roses)
- Area in Polar Coordinates
- Arc Length of a Polar Curve

**XI. INFINITE SEQUENCES AND INFINITE SERIES (~14 days):**

- What is an Infinite Sequence?
- Monotonic Sequences
- What is an Infinite Series?
- Telescoping Sums
- Harmonic Series
- Geometric Series
- P-Series
- Convergence Tests
  - The Divergence Test
  - The Integral Test
  - The Basic Comparison Test
  - The Limit Comparison Test
  - The Ratio Test
  - The Alternating Series Test
- Absolute and Conditional Convergence
- Power Series
- Interval of Convergence
- Maclaurin Series
- Taylor Series
- Building New Series from Known Power Series
- Differentiation and Integration of Power Series
- Error

After the Advanced Placement Exam, many interesting topics are explored, and may include:

- Introduction to Game Theory (the game of NIM)
- Introduction to Graph Theory (The Konigsburg Bridge Problem)
- Introduction to Number Theory (Congruences and The Chinese Remainder Theorem)
- Integrating Rational Functions – The Partial Fractions Method (for *repeated linear, irreducible quadratic, and repeated irreducible quadratic factors*)
- Solving First Order Linear Differential Equations – The Integrating Factor Method
- Limits – The Formal Definition ( $\varepsilon - \delta$  Proofs)
- Partial Derivatives
- Double Integrals