

CHAPTER 20

Answer to Checkpoint Questions

1. all but c
2. (a) all tie; (b) 3, 2, 1
3. gas A
4. 5 (greatest change in T), then tie of 1, 2, 3, and 4
5. 1, 2, 3 ($Q_3 = 0$, Q_2 goes into work W_2 , but Q_1 goes into greater work W_1 and increases gas temperature)

Answer to Questions

1. increased but less than doubled
2. the cooler room
3. a, c, b
4. tie of a and c, then b, then d
5. 1180 J
6. 1–4
7. d, tie of a and b, then c
8. 20 J
9. constant-volume process
10. (a) 3; (b) 1; (c) 4; (d) 2; (e) yes
11. (a) same; (b) increases; (c) decreases; (d) increases
12. (a) 1, 2, 3, 4; (b) 1, 2, 3
13. -4 J
14. a, 4; b, 8; c, 1; d, 5; e, 3; f, 7; g, 2; h, 6

15. (a) 1, polyatomic; 2, diatomic; 3, monatomic; (b) more

Solutions to Exercises & Problems

1E

(a) The number of moles is

$$n = \frac{m}{M_{\text{Au}}} = \frac{2.5 \text{ g}}{197 \text{ g/mol}} = 0.0127 \text{ mol}.$$

(b) The number of atoms is

$$N = nN_A = \frac{m}{M}N_A = (0.0127 \text{ mol})(6.02 \times 10^{23} / \text{mol}) = 7.65 \times 10^{21}.$$

2E

Each atom has a mass of $m = M/N_A$, where M is the molar mass and N_A is the Avogadro constant. The molar mass of arsenic is 74.9 g/mol or 74.9×10^{-3} kg/mol. 7.50×10^{24} arsenic atoms have a total mass of $(7.50 \times 10^{24})(74.9 \times 10^{-3} \text{ kg/mol}) / (6.02 \times 10^{23} \text{ mol}^{-1}) = 0.933 \text{ kg}$.

3P

18 g of water has 6.02×10^{23} water molecules. So 1 gram of water has $N = (1/18)(6.02 \times 10^{23})$ water molecules. If these molecules were distributed uniformly over the surface of the Earth then the molecular number density would be

$$D = \frac{N}{4\pi R_e^2} = \frac{6.02 \times 10^{23} / 18}{4\pi(6.4 \times 10^8 \text{ cm})^2} = 6560 / \text{cm}^2,$$

i.e., there would be 6560 water molecules per cm^2 .

4P

Let the volume of water in question be v . The number of molecules it contains is then $\rho v N_A / M$, where ρ is the density of water and M is the molar mass of water. If we denote the total volume of the water in all the oceans as V , then

$$\frac{V}{v} = \frac{\rho v N_A}{M}.$$

Solve for v :

$$v = \sqrt{\frac{VM}{\rho N_A}} \simeq \sqrt{\frac{(0.75)(4\pi)(6.4 \times 10^6 \text{ m})^2(5 \times 10^3 \text{ m})(1.8 \times 10^{-2} \text{ kg/mol})}{(1.0 \times 10^3 \text{ kg/m}^3)(6.0 \times 10^{23} \text{ /mol})}}$$

$$\simeq 8 \times 10^{-6} \text{ m}^3 = (2 \text{ cm})^2.$$

This is approximately the volume of water contained in a tablespoon.

5P

Since the molar mass of the ink is $M = 18 \text{ g/mol}$, the number of molecules in $m = 1 \mu\text{g} = 10^{-6} \text{ g}$ of ink is

$$N = \frac{mN_A}{M} = \frac{(10^{-6} \text{ g})(6.02 \times 10^{23} \text{ mol}^{-1})}{18 \text{ g/mol}} \simeq 3 \times 10^{16}.$$

The number of creatures in our galaxy, with the assumption made in the problem, is about $N' = (5 \times 10^9)(10^{11}) = 5 \times 10^{20}$. So the statement was wrong by a factor of about 20,000.

6E

(a) Use $pV = nRT$ to solve for the volume V :

$$V = \frac{nRT}{p} = \frac{(1.00 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(273 \text{ K})}{1.01 \times 10^5 \text{ Pa}} = 2.24 \times 10^{-3} \text{ m}^3,$$

which is 22.4 L.

(b) The Loschmidt number is

$$n_L = \frac{(1.00 \text{ cm}^3)(6.02 \times 10^{23} \text{ /mol})}{(22.4 \text{ L/mol})(10^3 \text{ cm}^3 \text{ /L})} = 2.69 \times 10^{19}.$$

7E

(a) Solve $pV = nRT$ for n : $n = pV/RT = (100 \text{ Pa})(1.0 \times 10^{-6} \text{ m}^3)/(8.31 \text{ J/mol}\cdot\text{K})(220 \text{ K}) = 5.47 \times 10^{-8} \text{ mol}$.

(b) The number of molecules N is the product of the number of moles n and the number of molecules in a mole N_A (the Avogadro constant). Thus $N = nN_A = (5.47 \times 10^{-6} \text{ mol})(6.02 \times 10^{23} \text{ mol}^{-1}) = 3.29 \times 10^{16}$ molecules.

8E

Use $pV = nRT$. The number of molecules per cubic centimeter is

$$N = \frac{pVN_A}{RT} = \frac{(1.01 \times 10^{-13} \text{ Pa})(1.00 \times 10^{-6} \text{ m}^3)(6.02 \times 10^{23} \text{ /mol})}{(8.31 \text{ J/mol}\cdot\text{K})(293 \text{ K})} = 25.$$

9E

Let $T_1 = 10.0^\circ\text{C} = 283\text{ K}$, $T_2 = 30.0^\circ\text{C} = 303\text{ K}$, $p_1 = 100\text{ kPa}$, $p_2 = 300\text{ kPa}$, and $V_1 = 2.50\text{ m}^3$.

(a) Use $p_1V_1/T_1 = nR$ to solve for n :

$$n = \frac{p_1V_1}{T_1R} = \frac{(100 \times 10^3\text{ Pa})(2.50\text{ m}^3)}{(283\text{ K})(8.31\text{ J/mol}\cdot\text{K})} = 106\text{ mol}.$$

(b) For the same amount of ideal gas, when the pressure changes to p_2 , its temperature changes to T_2 . So $p_2V_2 = nRT_2$, which we solve for V_2 :

$$\begin{aligned} V_2 &= \frac{nRT_2}{p_2} = \left(\frac{T_2}{T_1}\right)\left(\frac{p_1}{p_2}\right)V_1 \\ &= \left(\frac{303\text{ K}}{283\text{ K}}\right)\left(\frac{100 \times 10^3\text{ Pa}}{300 \times 10^3\text{ Pa}}\right)(2.50\text{ m}^3) = 0.892\text{ m}^3. \end{aligned}$$

10E

(a) Solve $pV = nRT$ for n . First convert the temperature to the Kelvin scale: $T = 40.0 + 273.15 = 313.15\text{ K}$. Also convert the volume to m^3 : $1000\text{ cm}^3 = 1000 \times 10^{-6}\text{ m}^3$. Then $n = pV/RT = (1.01 \times 10^5\text{ Pa})(1000 \times 10^{-6}\text{ m}^3)/(8.31\text{ J/mol}\cdot\text{K})(313.15\text{ K}) = 3.88 \times 10^{-2}\text{ mol}$.

(b) Solve $pV = nRT$ for T : $T = pV/nR = (1.06 \times 10^5\text{ Pa})(1500 \times 10^{-6}\text{ m}^3)/(3.88 \times 10^{-2}\text{ mol})(8.31\text{ J/mol}\cdot\text{K}) = 493\text{ K} = 220^\circ\text{C}$.

11E

Let $V_1 = 1000\text{ in.}^3$, $V_2 = 1020\text{ in.}^3$, $p_1 = 24.0\text{ lb/in.}^2$, $T_1 = 0.00^\circ\text{C} = 273\text{ K}$, and $T_2 = 27.0^\circ\text{C} = 300\text{ K}$. Since the number of moles of the gas remains the same, from $pV = nRT$ we have $n = p_1V_1/RT_1 = p_2V_2/RT_2$. Thus

$$p_2 = \left(\frac{T_2}{T_1}\right)\left(\frac{V_1}{V_2}\right)p_1 = \left(\frac{300.15\text{ K}}{273.15\text{ K}}\right)\left(\frac{1000\text{ in.}^3}{1020\text{ in.}^3}\right)(24.0\text{ lb/in.}^2) = 27.0\text{ lb/in.}^2.$$

12E

The work done is

$$\begin{aligned} W &= - \int_{v_i}^{v_f} p dV = p(V_i - V_f) \\ &= (1.01 \times 10^5\text{ Pa})(22.4\text{ L} - 16.8\text{ L})(10^{-3}\text{ m}^3/\text{L}) = 653\text{ J}. \end{aligned}$$

13P

(a) The number of moles of the air molecules is $n = pV/RT = N/N_A$. So

$$\begin{aligned} N &= \left(\frac{pV}{RT}\right)N_A = \frac{(1.01 \times 10^5 \text{ Pa})(1 \text{ m}^3)(6.02 \times 10^{23} \text{ mol}^{-1})}{(8.31 \text{ J/mol} \cdot \text{K})(293.15 \text{ K})} \\ &= 2.5 \times 10^{25}, \end{aligned}$$

where $T = 20^\circ\text{C} = 293 \text{ K}$ and $P = 1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$ were substituted.

(b) Let M_{air} be the molar mass of the air. Then $M_{\text{air}} = \frac{3}{4}M_{\text{N}_2} + \frac{1}{4}M_{\text{O}_2}$. So the mass of the air is

$$\begin{aligned} m &= M_{\text{air}}n = \frac{pV}{RT} \left(\frac{3}{4}M_{\text{N}_2} + \frac{1}{4}M_{\text{O}_2} \right) \\ &= \frac{(1.01 \times 10^5 \text{ Pa})(1 \text{ m}^3)}{(8.31 \text{ J/mol} \cdot \text{K})(293.15 \text{ K})} \left[\frac{3}{4}(14 \text{ g/mol}) + \frac{1}{4}(32 \text{ g/mol}) \right] \\ &= 1.2 \text{ kg}. \end{aligned}$$

14P

Since the pressure is constant the work is given by $W = p(V_2 - V_1)$. The initial volume is $V_1 = (AT_1 - BT_1^2)/p$, where T_1 is the initial temperature. The final volume is $V_2 = (AT_2 - BT_2^2)/p$. Thus $W = A(T_2 - T_1) - B(T_2^2 - T_1^2)$.

15P

Suppose the gas expands from volume V_1 to volume V_2 during the isothermal portion of the process. The work it does is

$$W = \int_{V_1}^{V_2} p dV = nRT \int_{V_1}^{V_2} \frac{dV}{V} = nRT \ln\left(\frac{V_2}{V_1}\right),$$

where $pV = nRT$ was used to replace p with nRT/V . Now $V_1 = nRT/p_1$ and $V_2 = nRT/p_2$, so $V_2/V_1 = p_2/p_1$. Also replace nRT with p_1V_1 to obtain

$$W = p_1V_1 \ln\left(\frac{p_1}{p_2}\right).$$

Since the initial gauge pressure is $1.03 \times 10^5 \text{ Pa}$, $p_1 = 1.03 \times 10^5 \text{ Pa} + 1.01 \times 10^5 \text{ Pa} = 2.04 \times 10^5 \text{ Pa}$. The final pressure is atmospheric pressure: $p_2 = 1.01 \times 10^5 \text{ Pa}$. Thus

$$W = (2.04 \times 10^5 \text{ Pa})(0.14 \text{ m}^3) \ln\left(\frac{2.04 \text{ Pa}}{1.01 \text{ Pa}}\right) = 2.00 \times 10^4 \text{ J}.$$

During the constant pressure portion of the process the work done by the gas is $W = p_2(V_1 - V_2)$. Notice that the gas starts in a state with pressure p_2 , so this is the pressure

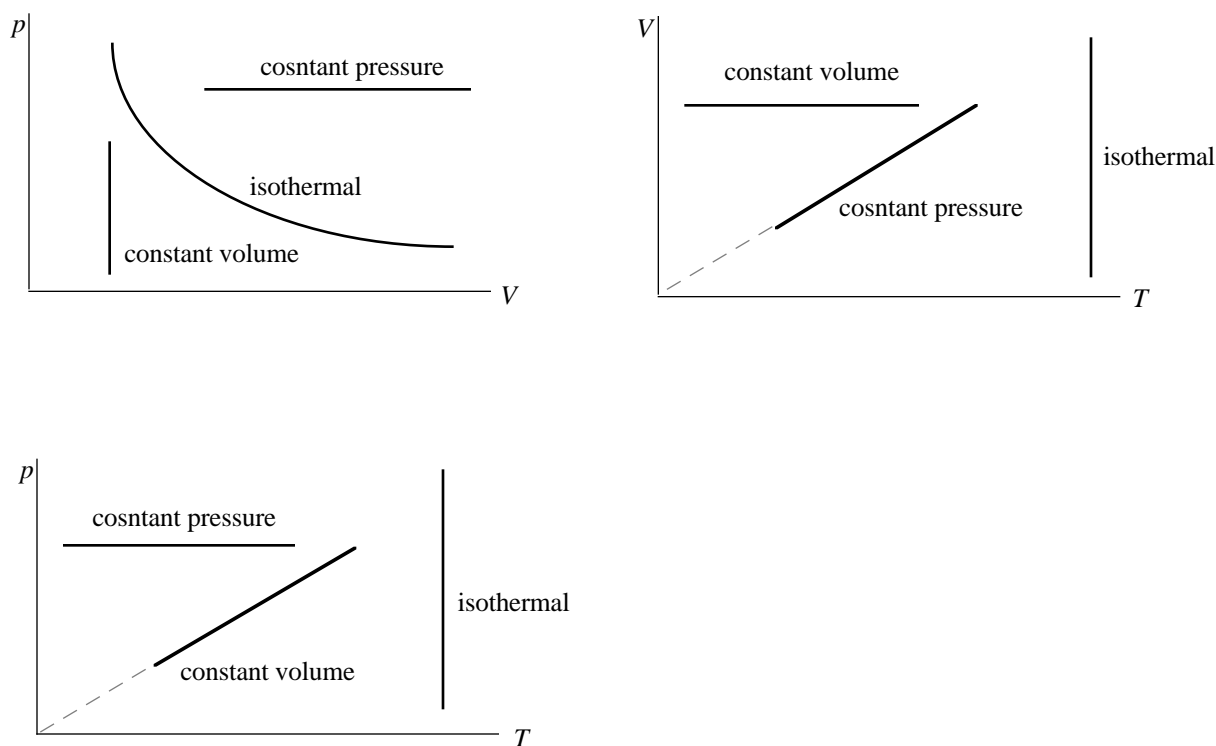
throughout this portion of the process. Also note that the volume decreases from V_2 to V_1 . Now $V_2 = p_1 V_1 / p_2$, so

$$\begin{aligned} W &= p_2 \left(V_1 - \frac{p_1 V_1}{p_2} \right) = (p_2 - p_1) V_1 \\ &= (1.01 \times 10^5 \text{ Pa} - 2.04 \times 10^5 \text{ Pa})(0.14 \text{ m}^3) = -1.44 \times 10^4 \text{ J}. \end{aligned}$$

The total work done by the gas over the entire process is $W = 2.00 \times 10^4 \text{ J} - 1.44 \times 10^4 \text{ J} = 5.6 \times 10^3 \text{ J}$.

16P

The diagrams are shown below.



(d) You can use $pV = nRT$ to analyze each case. Note that n is proportional to the mass M of the ideal gas. For example, the slope of the constant-volume line in the p - T diagram is $p/T = nR/V \propto$ mass of the ideal gas.

17P

The pressure p_1 due to the first gas is $p_1 = n_1 RT/V$, and the pressure p_2 due to the second gas is $p_2 = n_2 RT/V$. So the total pressure on the container wall is

$$p = p_1 + p_2 = \frac{n_1 RT}{V} + \frac{n_2 RT}{V} = (n_1 + n_2) \frac{RT}{V}.$$

The fraction of P due to the second gas is then

$$\frac{p_2}{p} = \frac{n_2 RT/V}{(n_1 + n_2)(RT/V)} = \frac{n_2}{n_1 + n_2} = \frac{0.5}{2 + 0.5} = \frac{1}{5}.$$

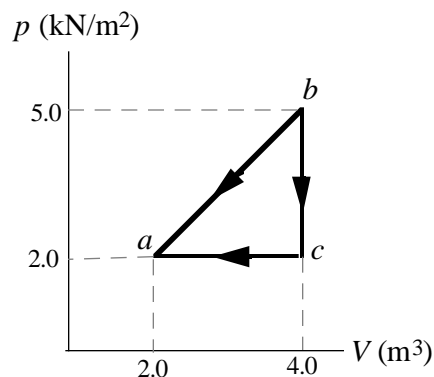
18P

(a) Use $pV = nRT$ to solve for n :

$$\begin{aligned} n &= \frac{p_1 V_1}{RT_1} = \frac{(2.5 \times 10^3 \text{ Pa})(1.0 \text{ m}^3)}{(8.31 \text{ J/mol}\cdot\text{K})(200 \text{ K})} \\ &= 1.5 \text{ mol}. \end{aligned}$$

(b)

$$\begin{aligned} T_B &= \frac{p_B V_B}{nR} \\ &= \frac{(7.5 \times 10^3 \text{ Pa})(3.0 \text{ m}^3)}{(1.5 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})} \\ &= 1.8 \times 10^3 \text{ K}. \end{aligned}$$



(c)

$$T_C = \frac{p_C V_C}{nR} = \frac{(2.5 \times 10^3 \text{ Pa})(3.0 \text{ m}^3)}{(1.5 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})} = 6.0 \times 10^2 \text{ K}.$$

(d) Use $Q = \Delta E_{\text{int}} + W$. For the cyclic process which starts at a and ends at a we have $\Delta T = 0$, so $\Delta E_{\text{int}} = 0$. Thus

$$\begin{aligned} Q = W &= \frac{1}{2}(V_c - V_a)(p_b - p_c) \\ &= \frac{1}{2}(3 \text{ m}^3 - 1 \text{ m}^3)(7.5 \times 10^3 \text{ Pa} - 2.5 \times 10^3 \text{ Pa}) = 5.0 \times 10^3 \text{ J}. \end{aligned}$$

19P

(a) The work done by the gas in the process is $W = p\Delta V = (25 \text{ N/m}^2)(1.8 \text{ m}^3 - 3.0 \text{ m}^3) = -30 \text{ J}$. Thus the change in internal energy of the gas is

$$\Delta E_{\text{int}} = Q - W = -75 \text{ J} - (-30 \text{ J}) = -45 \text{ J}.$$

(b) The final temperature is

$$T_f = \left(\frac{p_f V_f}{p_i V_i} \right) T_i = \frac{(1.8 \text{ m}^3)(300 \text{ K})}{3.0 \text{ m}^3} = 180 \text{ K}.$$

20P

Denote the lower and higher elevations with subscripts 1 and 2, respectively. Then

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}.$$

The gas volume V_2 at the higher elevation is then

$$V_2 = \left(\frac{p_1 T_2}{p_2 T_1} \right) V_1 = \left[\frac{(76 \text{ cmHg})(273 - 48) \text{ K}}{(38 \text{ cmHg})(273 + 20) \text{ K}} \right] (2.2 \text{ m}^3) = 3.4 \text{ m}^3.$$

21P

Assume that the pressure of the air in the bubble is essentially the same as the pressure in the surrounding water. If d is the depth of the lake and ρ is the density of water, then the pressure at the bottom of the lake is $p_1 = p_0 + \rho g d$, where p_0 is atmospheric pressure. Since $p_1 V_1 = n R T_1$, the number of moles of gas in the bubble is $n = p_1 V_1 / R T_1 = (p_0 + \rho g d) V_1 / R T_1$, where V_1 is the volume of the bubble at the bottom of the lake and T_1 is the temperature there. At the surface of the lake the pressure is p_0 and the volume of the bubble is $V_2 = n R T_2 / p_0$. Substitute for n to obtain

$$\begin{aligned} V_2 &= \frac{T_2}{T_1} \left(\frac{p_0 + \rho g d}{p_0} \right) V_1 \\ &= \left(\frac{293 \text{ K}}{277 \text{ K}} \right) \left[\frac{1.013 \times 10^5 \text{ Pa} + (0.998 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(40 \text{ m})}{1.013 \times 10^5 \text{ Pa}} \right] (20 \text{ cm}^3) \\ &= 100 \text{ cm}^3. \end{aligned}$$

22P

Consider the open end of the pipe. The balance of the pressures inside and outside the pipe requires that $p + \rho_w (L/2)g = p_0 + \rho_w h g$, where p_0 is the atmospheric pressure, and p is the pressure of the air inside the pipe, which satisfies $p(L/2) = p_0 L$, or $p = 2p_0$. Solve for h :

$$h = \frac{p - p_0}{\rho_w g} + \frac{L}{2} = \frac{1.01 \times 10^5 \text{ Pa}}{(1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} + \frac{25.0 \text{ m}}{2} = 22.8 \text{ m}.$$

23P

Let the mass of the balloon be m_b and that of the lift be m_l . According to Archimedes's Principle the buoyant force is $F = \rho_{\text{air}} g V_b$. The equilibrium of forces for the balloon requires $m_l g = F - (m_b + m_{\text{air}})g$. Solve for the weight of the air in the balloon:

$$\begin{aligned} m_{\text{air}} g &= \rho_{\text{air}} g V_b - m_l g - m_b g \\ &= (7.56 \times 10^{-2} \text{ lb/ft}^3)(77,000 \text{ ft}^3) - 600 \text{ lb} - 500 \text{ lb} \\ &= 4.7 \times 10^3 \text{ lb} = 2.1 \times 10^4 \text{ N}. \end{aligned}$$

So $m_{\text{air}} = 2.1 \times 10^4 \text{ N}/9.80 \text{ m/s}^2 = 2.1 \times 10^3 \text{ kg}$.

Now for the air inside the balloon $p_{\text{in}}V_b = nRT_{\text{in}} = (m_{\text{air}}/M_{\text{air}})RT_{\text{in}}$, where M_{air} is the molar mass of the air and $p_{\text{in}} = p_{\text{out}} = 1 \text{ atm}$. So the temperature inside the balloon is

$$\begin{aligned} T_{\text{in}} &= \frac{p_{\text{in}}V_bM_{\text{air}}}{Rm_{\text{air}}} \\ &= \frac{(1.01 \times 10^5 \text{ Pa})(77,000 \text{ ft}^3)(2.832 \times 10^{-2} \text{ m}^3/\text{ft}^3)(0.028 \text{ kg/mol})}{(8.31 \text{ J/mol} \cdot \text{K})(2142.8 \text{ kg})} \\ &= 365 \text{ K} = 92 \text{ }^\circ\text{C} = 198 \text{ }^\circ\text{F}. \end{aligned}$$

24P

(a) Use $pV = nRT$. The volume of the tank is

$$\begin{aligned} V &= \frac{nRT}{p} = \frac{(300 \text{ g}/17 \text{ g/mol})(8.31 \text{ J/mol} \cdot \text{K})(273 \text{ }^\circ\text{C} + 77 \text{ }^\circ\text{C})}{1.35 \times 10^6 \text{ Pa}} \\ &= 3.8 \times 10^{-2} \text{ m}^3 = 38 \text{ L}. \end{aligned}$$

(b) The number of moles of the remaining gas is

$$n' = \frac{p'V}{RT'} = \frac{(8.7 \times 10^5 \text{ Pa})(3.8 \times 10^{-2} \text{ m}^3)}{(8.31 \text{ J/mol} \cdot \text{K})(273 \text{ }^\circ\text{C} + 22 \text{ }^\circ\text{C})} = 13.5 \text{ mol}.$$

The mass of the gas that leaked out is then $\Delta m = 300 \text{ g} - (13.5 \text{ mol})(17 \text{ g/mol}) = 71 \text{ g}$.

25P

When the valve is closed the number of moles of the gas in container A is $n_A = p_A V_A / RT_A$ and that in container B is $n_B = 4p_B V_A / RT_B$. The total number of moles in both containers is then

$$n = n_A + n_B = \frac{V_A}{R} \left(\frac{p_A}{T_A} + \frac{4p_B}{T_B} \right) = \text{const.}$$

After the valve is opened the pressure in container A is $p'_A = Rn'_A T_A / V_A$ and that in container B is $p'_B = Rn'_B T_B / 4V_A$. Equate p'_A and p'_B to obtain $Rn'_A T_A / V_A = Rn'_B T_B / 4V_A$, or $n'_B = (4T_A / T_B)n'_A$. Thus

$$n = n'_A + n'_B = n'_A \left(1 + \frac{4T_A}{T_B} \right) = n_A + n_B = \frac{V_A}{R} \left(\frac{p_A}{T_A} + \frac{4p_B}{T_B} \right).$$

Solve the above equation for n'_A :

$$n'_A = \frac{V}{R} \frac{(p_A/T_A + 4p_B/T_B)}{(1 + 4T_A/T_B)}.$$

Substitute this expression for n'_A into $p'V_A = n'_AR T_A$ to obtain the final pressure p' :

$$p' = \frac{n'_A R T_A}{V_A} = \frac{p_A + 4p_B T_A/T_B}{1 + 4T_A/T_B},$$

which gives $p' = 2.0 \times 10^5$ Pa.

26E

The rms speed of helium atoms at 1000 K is

$$v_{\text{rms}} = \sqrt{\frac{3RT}{mN_A}} = \sqrt{\frac{3(8.31 \text{ J/mol}\cdot\text{K})(1000 \text{ K})}{4.00 \times 10^{-3} \text{ kg/mol}}} = 2.50 \times 10^3 \text{ m/s}.$$

27E

According to the kinetic theory the rms speed is

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}},$$

where T is the temperature and M is the molar mass. According to Table 20-1 the molar mass of molecular hydrogen is $2.02 \text{ g/mol} = 2.02 \times 10^{-3} \text{ kg/mol}$, so

$$v_{\text{rms}} = \sqrt{\frac{3(8.31 \text{ J/mol}\cdot\text{K})(2.7 \text{ K})}{2.02 \times 10^{-3} \text{ kg}}} = 180 \text{ m/s}.$$

28E

The rms speed of argon atoms at 313 K is

$$v_{\text{rms}} = \sqrt{\frac{3RT}{mN_A}} = \sqrt{\frac{3(8.31 \text{ J/mol}\cdot\text{K})(313 \text{ K})}{39.9 \times 10^{-3} \text{ kg/mol}}} = 442 \text{ m/s}.$$

29E

Use $v_{\text{rms}} = (3kT/m_e)^{1/2}$, where m_e is the mass of the electron and k is the Boltzmann constant. So for electrons inside the Sun

$$v_{\text{rms}} = \left[\frac{3(1.38 \times 10^{-23} \text{ J/K})(2.00 \times 10^6 \text{ K})}{9.11 \times 10^{-31} \text{ kg}} \right]^{1/2} = 9.53 \times 10^6 \text{ m/s}.$$

30E

(a)

$$v_{\text{rms}} = \sqrt{\frac{3RT}{mN_A}} = \sqrt{\frac{3(8.31 \text{ J/mol}\cdot\text{K})(273^\circ\text{C} + 20.0^\circ\text{C})}{28.0 \times 10^{-3} \text{ kg/mol}}} = 511 \text{ m/s}.$$

(b) Since $v_{\text{rms}} \propto \sqrt{T}$, the temperature at which v_{rms} will be half that value is $T_1 = (1/2)^2 T = 293 \text{ K}/4 = 73.3 \text{ K} = -200^\circ\text{C}$, and the temperature at which v_{rms} will be twice that value is $T_2 = 2^2 T = 4(293 \text{ K}) = 1.17 \times 10^3 \text{ K} = 899^\circ\text{C}$.

31E

Use $v_{\text{rms}} = \sqrt{3RT/M}$, where M is the molar mass of the ideal gas. So

$$\left(\frac{3RT_{\text{H}_2}}{M_{\text{H}_2}}\right)^{1/2} = \left(\frac{3RT_{\text{He}}}{M_{\text{He}}}\right)^{1/2},$$

which yields

$$T_{\text{He}} = \left(\frac{M_{\text{He}}}{M_{\text{H}_2}}\right) T_{\text{H}_2} = \left(\frac{4.00 \text{ g/mol}}{2.00 \text{ g/mol}}\right) (20.0 + 273) \text{ K} = 586 \text{ K} = 313^\circ\text{C}.$$

32P

Use $pV = nRT = (m/M)RT$, where m is the mass of the gas and M is its molar mass. The density of the gas is then $\rho = m/V = pM/RT$.

(a) Now

$$\begin{aligned} v_{\text{rms}} &= \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3p}{\rho}} \\ &= \sqrt{\frac{3(1.00 \times 10^{-2} \text{ atm})(1.01 \times 10^5 \text{ Pa/atm})}{1.24 \times 10^{-2} \text{ kg/m}^3}} = 494 \text{ m/s}. \end{aligned}$$

(b) The molar mass of the ideal gas is

$$M = \frac{\rho RT}{p} = \frac{(1.24 \times 10^{-2} \text{ kg/m}^3)(8.31 \text{ J/mol}\cdot\text{K})(273 \text{ K})}{(1.0 \times 10^{-2} \text{ atm})(1.01 \times 10^5 \text{ Pa/atm})} = 0.028 \text{ g/mol}.$$

The gas is therefore nitrogen (N_2).

33P

On reflection only the normal component of the momentum changes, so for one molecule the change in momentum is $2mv \cos \theta$, where m is the mass of the molecule, v is its speed, and θ is the angle between its velocity and the normal to the wall. If N molecules collide

with the wall the change in their total momentum is $\Delta P = 2Nmv \cos \theta$ and if the total time taken for the collisions is Δt then the average rate of change of the total momentum is $\Delta P/\Delta t = 2(N/\Delta t)mv \cos \theta$. This the average force exerted by the N molecules on the wall and the pressure is the force per unit area:

$$\begin{aligned} p &= \frac{2}{A} \frac{N}{\Delta t} mv \cos \theta \\ &= \frac{2}{2.0 \times 10^{-4} \text{ m}^2} (10^{23} \text{ s}^{-1})(3.3 \times 10^{-27} \text{ kg})(1.0 \times 10^3 \text{ m/s}) \cos 55^\circ \\ &= 1.9 \times 10^3 \text{ Pa.} \end{aligned}$$

Notice that the value given for the mass was converted to kg and the value given for the area was converted to m^2 .

34E

(a) The average translational energy is given by $\bar{K} = \frac{3}{2}kT$, where k is the Boltzmann constant ($1.38 \times 10^{-23} \text{ J/K}$) and T is the temperature on the Kelvin scale. Thus $\bar{K} = \frac{3}{2}(1.38 \times 10^{-23} \text{ J/K})(1600 \text{ K}) = 3.31 \times 10^{-20} \text{ J}$. Round to $3.3 \times 10^{-20} \text{ J}$.

(b) Since $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$, $\bar{K} = (3.31 \times 10^{-20} \text{ J})/(1.60 \times 10^{-19} \text{ J/eV}) = 0.21 \text{ eV}$.

35E

The translational kinetic energy of a particle is $\bar{K} = \frac{3}{2}kT$ and one electron-volt is equal to $1.6 \times 10^{-19} \text{ J}$.

(a) At 0°C (273.15 K),

$$\begin{aligned} \bar{K} &= \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23} \text{ J/K})(273.15 \text{ K}) \\ &= 5.65 \times 10^{-21} \text{ J} = \frac{5.65 \times 10^{-21} \text{ J}}{1.6 \times 10^{-19} \text{ J/eV}} = 0.0353 \text{ eV}; \end{aligned}$$

and at $T = 100^\circ \text{C}$ (373 K),

$$\begin{aligned} \bar{K} &= \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23} \text{ J/K})(373.15 \text{ K}) \\ &= 7.72 \times 10^{-21} \text{ J} = \frac{7.72 \times 10^{-21} \text{ J}}{1.6 \times 10^{-19} \text{ J/eV}} = 0.0483 \text{ eV.} \end{aligned}$$

(b) For one mole of ideal gas at $T = 273 \text{ K}$, $\bar{K} = \frac{3}{2}RT = \frac{3}{2}(8.31 \text{ J/K} \cdot \text{mol})(273.15 \text{ K}) = 3.40 \times 10^3 \text{ J}$. At $T = 373 \text{ K}$, $\bar{K} = \frac{3}{2}RT = \frac{3}{2}(8.31 \text{ J/K} \cdot \text{mol})(373.15 \text{ K}) = 4.65 \times 10^3 \text{ J}$.

36E

Let the temperature be T , then $\bar{K} = 3kT/2 = 1.00 \text{ eV}$. Thus

$$T = \frac{2\bar{K}}{3k} = \frac{2(1.00 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})}{3(1.38 \times 10^{-23} \text{ J/K})} = 7.73 \times 10^3 \text{ K.}$$

37E

The gravitational kinetic energy of one oxygen molecule is $U = mgh$, and the translational kinetic energy of the same molecule is $\bar{K} = \frac{3}{2}kT$. So

$$\frac{U}{\bar{K}} = \frac{3mgh}{3kT/2} = \frac{(0.032 \text{ kg}/N_A)gh}{3kT/2} = \frac{2(0.032 \text{ kg})(9.80 \text{ m/s}^2)(0.1 \text{ m})}{3(8.31 \text{ J/mol} \cdot \text{K})(273 \text{ K})} = 9.1 \times 10^{-6}.$$

Here the relation $N_A k = R$ was used.

38P

(a) See 32P.

(b) $pV = nRT = n(kN_A)T = (nN_A)kT = NkT$.

39P

(a) Use $\epsilon = L/N$, where L is the heat of vaporization and N is the number of molecules per gram. The molar mass of atomic hydrogen is 1 g/mol and the molar mass of atomic oxygen is 16 g/mol so the molar mass of H_2O is $1 + 1 + 16 = 18$ g/mol. There are $N_A = 6.02 \times 10^{23}$ molecules in a mole so the number of molecules in a gram of water is $(6.02 \times 10^{23} \text{ mol}^{-1})/(18 \text{ g/mol}) = 3.34 \times 10^{22}$ molecules/g. Thus $\epsilon = (539 \text{ cal/g})/(3.34 \times 10^{22} \text{ g}^{-1}) = 1.61 \times 10^{-20}$ cal. This is $(1.61 \times 10^{-20} \text{ cal})(4.186 \text{ J/cal}) = 6.76 \times 10^{-20}$ J.

(b) The average translational kinetic energy is

$$\bar{K} = \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23} \text{ J/K})[(32.0 + 273.13) \text{ K}] = 6.32 \times 10^{-21} \text{ J}.$$

The ratio of $\bar{K}/\epsilon = (6.76 \times 10^{-20} \text{ J})/(6.32 \times 10^{-21} \text{ J}) = 10.7$.

40P

They are not equivalent. Avogadro's law does not tell how the pressure, volume, and temperature are related, so you cannot use it, for example, to calculate the change in volume when the pressure increases at constant temperature. The ideal gas law, however, implies Avogadro's law. It yields $N = nN_A = (pV/RT)N_A = pV/kT$, where $k = R/N_A$ was used. If the two gases have the same volume, the same pressure, and the same temperature, then pV/kT is the same for them. This implies that N is also the same.

41E

The mean free path of a particle is given by $\lambda = (\sqrt{2}\pi d^2 n)^{-1}$. Solve for d :

$$\begin{aligned} d &= \sqrt{\frac{1}{\sqrt{2}\pi\lambda n}} = \sqrt{\frac{1}{\sqrt{2}\pi(0.80 \times 10^{-7} \text{ m})(2.7 \times 10^{25} / \text{m}^3)}} \\ &= 3.2 \times 10^{-10} \text{ m} = 0.32 \text{ nm}. \end{aligned}$$

42E

(a) According to Eq. 20-22 the mean free path for molecules in a gas is given by

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V},$$

where d is the diameter of a molecule and N is the number of molecules in volume V . Substitute $d = 2.0 \times 10^{-10}$ m and $N/V = 1 \times 10^6$ molecules/m³ to obtain

$$\lambda = \frac{1}{\sqrt{2}\pi(2.0 \times 10^{-10} \text{ m})^2(1 \times 10^6 \text{ m}^{-3})} = 6 \times 10^{12} \text{ m}.$$

(b) At this altitude most of the gas particles are in orbit around the Earth and do not suffer randomizing collisions. The mean free path has little physical significance.

43E

Substitute $d = 1.0 \times 10^{-2}$ m and $N/V = 15/(1.0 \times 10^{-3} \text{ m}^3) = 15 \times 10^3$ beans/m³ into

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V}$$

to obtain

$$\lambda = \frac{1}{\sqrt{2}\pi(1.0 \times 10^{-2} \text{ m})^2(15 \times 10^3 \text{ m}^{-3})} = 0.15 \text{ m}.$$

The conversion $1.00 \text{ L} = 1.00 \times 10^{-3} \text{ m}^3$ was used.

44E

The average frequency is

$$f = \frac{\bar{v}}{\lambda} = \frac{\sqrt{2}\pi d^2 \bar{v} N}{V}.$$

45P

(a) Use $pV = nRT = NkT$, where p is the pressure, V is the volume, T is the temperature, n is the number of moles, and N is the number of molecules. The substitutions $N = nN_A$ and $k = R/N_A$ were made. Since 1 cm of mercury = 1333 Pa, the pressure is $p = (10^{-7})(1333) = 1.333 \times 10^{-4}$ Pa. Thus

$$\begin{aligned} \frac{N}{V} &= \frac{p}{kT} = \frac{1.333 \times 10^{-4} \text{ Pa}}{(1.38 \times 10^{-23} \text{ J/K})(295 \text{ K})} \\ &= 3.27 \times 10^{16} \text{ molecules/m}^3 = 3.27 \times 10^{10} \text{ molecules/cm}^3. \end{aligned}$$

(b) The molecular diameter is $d = 2.00 \times 10^{-10}$ m, so the mean free path is

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V} = \frac{1}{\sqrt{2}\pi(2.00 \times 10^{-10} \text{ m})^2(3.27 \times 10^{16} \text{ m}^{-3})} = 172 \text{ m}.$$

46P

Let the speed of sound be v_s and its frequency be f . Then

$$\frac{v_s}{f} = \lambda = \frac{1}{\sqrt{2}\pi d^2 N/V},$$

which gives

$$\begin{aligned} f &= \sqrt{2}\pi d^2 v_s \left(\frac{N}{V}\right) = \sqrt{2}\pi d^2 v_s N_A \rho = \sqrt{2}\pi d^2 v_s N_A \left(\frac{pM}{RT}\right) \\ &= \sqrt{2}\pi(3.0 \times 10^{-10} \text{ m})^2(331 \text{ m/s})(6.02 \times 10^{23} / \text{mol}) \frac{(1.01 \times 10^5 \text{ Pa})(0.032 \text{ kg/mol})}{(8.31 \text{ J/mol}\cdot\text{K})(273 \text{ K})} \\ &= 3.7 \times 10^9 \text{ Hz} = 3.7 \text{ GHz}. \end{aligned}$$

47P

For an ideal gas $pV = nRT$.

(a) When $n = 1$, $V = V_m = RT/p$, where V_m is the molar volume of the gas. So

$$V_m = \frac{RT}{p} = \frac{(8.31 \text{ J/mol}\cdot\text{K})(273.15 \text{ K})}{1.01 \times 10^5 \text{ Pa}} = 22.5 \text{ L}.$$

(b) Use $v_{\text{rms}} = \sqrt{3RT/M}$. The ratio is given by

$$\frac{v_{\text{rms,He}}}{v_{\text{rms,Ne}}} = \sqrt{\frac{M_{\text{Ne}}}{M_{\text{He}}}} = \sqrt{\frac{20 \text{ g}}{4.0 \text{ g}}} = 2.25.$$

(c) Use $\lambda_{\text{He}} = (\sqrt{2}\pi d^2 n^*)^{-1}$, where n^* is the number of particles per unit volume given by $n^* = N/V = N_A n/V = N_A p/RT = p/kT$. So

$$\begin{aligned} \lambda_{\text{He}} &= \frac{1}{\sqrt{2}\pi d^2 (p/kT)} = \frac{kT}{\sqrt{2}\pi d^2 p} \\ &= \frac{(1.38 \times 10^{-23} \text{ J/K})(273.15 \text{ K})}{1.414\pi(1 \times 10^{-10} \text{ m})^2(1.01 \times 10^5 \text{ Pa})} = 0.84 \mu\text{m}. \end{aligned}$$

(d) $\lambda_{\text{Ne}} = \lambda_{\text{He}} = 0.84 \mu\text{m}$.

48P(a) Since $\lambda \propto d^{-2}$,

$$\frac{d_{\text{Ar}}}{d_{\text{N}_2}} = \sqrt{\frac{\lambda_{\text{N}_2}}{\lambda_{\text{Ar}}}} = \sqrt{\frac{27.5 \times 10^{-6} \text{ cm}}{9.9 \times 10^{-6} \text{ cm}}} = 1.7.$$

(b) Since $\lambda \propto (N/V)^{-1} \propto (p/T)^{-1}$,

$$\lambda_{\text{Ar}}(20^\circ\text{C}, 15 \text{ cmHg}) = \left(\frac{15 \text{ cmHg}}{75 \text{ cmHg}}\right)^{-1} (9.9 \times 10^{-6} \text{ cm}) = 5.0 \times 10^{-5} \text{ cm},$$

$$\lambda_{\text{Ar}}(-40^\circ\text{C}, 75 \text{ cmHg}) = \left(\frac{273^\circ\text{C} - 40^\circ\text{C}}{273^\circ\text{C} + 20^\circ\text{C}}\right) (9.9 \times 10^{-6} \text{ cm}) = 7.9 \times 10^{-6} \text{ cm}.$$

49P

(a) Using a ruler, we find the diameter of the period D to be roughly 0.5 mm. So its area is $A = \pi D^2/4 \approx 2 \times 10^{-7} \text{ m}^2$. Meanwhile, we estimate the diameter d of an air molecule to be roughly $2 \times 10^{-10} \text{ m}$. So the area an air molecule covers is $a = \pi d^2/4 \approx 3 \times 10^{-20} \text{ m}^2$. Thus

$$\frac{A}{a} \approx \frac{2 \times 10^{-7}}{3 \times 10^{-20}} \approx 10^{13}.$$

This tells us that 10^{13} air molecules are needed to cover the period.

(b) Assume that every second there are N air molecules which collide with the period. If each one of them bounces back elastically after the collision then the change in linear momentum per molecule per collision is $2mv_x$, where m is the molecular mass and v_x is the component of the molecular velocity in the direction perpendicular to the surface of the paper containing the period. Thus the pressure exerted by the air molecules on the period is

$$p = \frac{2mN\bar{v}_x}{A\Delta t},$$

where $\Delta t = 1 \text{ s}$ and $\bar{v}_x \approx (\bar{v}_x^2)^{1/2} = v_{\text{rms}}/\sqrt{3}$. Also $m = M/N_A$, where M is the average molar mass of the air molecules. Solve for N :

$$\begin{aligned} N &= \frac{\sqrt{3}pAN_A\Delta t}{2Mv_{\text{rms}}} = \frac{pAN_A\Delta t}{2\sqrt{MRT}} \\ &= \frac{(1.01 \times 10^5 \text{ Pa})(2 \times 10^{-7} \text{ m}^2)(6.02 \times 10^{23} / \text{mol})(1 \text{ s})}{2\sqrt{(0.028 \text{ kg/mol})(8.31 \text{ J/mol}\cdot\text{K})(300 \text{ K})}} \approx 7 \times 10^{20}. \end{aligned}$$

50P

(a) The average speed is

$$\bar{v} = \frac{\sum v}{N},$$

where the sum is over the speeds of the particles and N is the number of particles. Thus

$$\bar{v} = \frac{(2.0 + 3.0 + 4.0 + 5.0 + 6.0 + 7.0 + 8.0 + 9.0 + 10.0 + 11.0) \text{ km/s}}{10} = 6.5 \text{ km/s}.$$

(b) The rms speed is given by

$$v_{\text{rms}} = \sqrt{\frac{\sum v^2}{N}}.$$

Now

$$\begin{aligned} \sum v^2 &= \left[(2.0)^2 + (3.0)^2 + (4.0)^2 + (5.0)^2 + (6.0)^2 \right. \\ &\quad \left. + (7.0)^2 + (8.0)^2 + (9.0)^2 + (10.0)^2 + (11.0)^2 \right] \text{ km}^2/\text{s}^2 \\ &= 505 \text{ km}^2/\text{s}^2, \end{aligned}$$

so

$$v_{\text{rms}} = \sqrt{\frac{505 \text{ km}^2/\text{s}^2}{10}} = 7.1 \text{ km/s}.$$

51E

(a)

$$\bar{v} = \frac{\sum n_i v_i}{\sum n_i} = \frac{2(1.0) + 4(2.0) + 6(3.0) + 8(4.0) + 2(5.0)}{2 + 4 + 6 + 8 + 2} = 3.2 \text{ cm/s}.$$

(b) From $v_{\text{rms}} = \sqrt{\sum n_i v_i^2 / \sum n_i}$ we get

$$v_{\text{rms}} = \sqrt{\frac{2(1.0)^2 + 4(2.0)^2 + 6(3.0)^2 + 8(4.0)^2 + 2(5.0)^2}{2 + 4 + 6 + 8 + 2}} = 3.4 \text{ cm/s}.$$

(c) There are eight particles at $v = 4.0$ cm/s, more than the number of particles at any other single speed. So 4.0 cm/s is the most probable speed.

52E

(a)

$$\bar{v} = \frac{1}{N} \sum_{i=1}^N v_i = \frac{1}{10} [4(200 \text{ m/s}) + 2(500 \text{ m/s}) + 4(600 \text{ m/s})] = 420 \text{ m/s},$$

$$v_{\text{rms}} = \sqrt{\frac{1}{N} \sum_{i=1}^N v_i^2} = \sqrt{\frac{1}{10} [4(200 \text{ m/s})^2 + 2(500 \text{ m/s})^2 + 4(600 \text{ m/s})^2]} = 458 \text{ m/s}.$$

Indeed we see that $v_{\text{rms}} > \bar{v}$.

(b) You may check the validity of the inequality $v_{\text{rms}} \geq \bar{v}$ for any distribution you can think of. For example, imagine that the ten particles are divided into two groups of five particles each, with the first group of particles moving at speed v_1 and the second group at v_2 . Obviously now $\bar{v} = (v_1 + v_2)/2$ and

$$v_{\text{rms}} = \sqrt{\frac{v_1^2 + v_2^2}{2}} \geq \frac{v_1 + v_2}{2} = \bar{v}.$$

(c) If and only if all the individual molecular speeds are the same does v_{rms} equal \bar{v} .

53E

(a) $\bar{v} < v_{\text{rms}} < v_P$.

(b) For Maxwell distribution $v_P < \bar{v} < v_{\text{rms}}$.

54E

Use $v_p = \sqrt{2RT_2/M}$ and $v_{\text{rms}} = \sqrt{3RT_1/M}$. The ratio is

$$\frac{T_2}{T_1} = \frac{3}{2} \left(\frac{v_p}{v_{\text{rms}}} \right)^2 = \frac{3}{2}.$$

55P

(a) The rms speed of molecules in a gas is given by $v_{\text{rms}} = \sqrt{3RT/M}$, where T is the temperature and M is the molar mass of the molecules. The speed required for escape from the Earth's gravitational pull is $v = \sqrt{2gr_e}$, where g is the acceleration due to gravity at the Earth's surface and r_e ($= 6.37 \times 10^6$ m) is the radius of the Earth. This expression can be derived easily. Take the zero of gravitational potential energy to be at infinity. Then energy of a particle with speed v and mass m at the Earth's surface is $E = -mgr_e + \frac{1}{2}mv^2$. If the particle is just able to travel far away its kinetic energy must tend toward 0 as its distance from the Earth tends to ∞ . This means $E = 0$ and $v = \sqrt{2gr_e}$.

Equate the expressions for the speeds to obtain $\sqrt{3RT/M} = \sqrt{2gr_e}$. The solution for T is $T = 2gr_e M/3R$. The molar mass of hydrogen is 2.02×10^{-3} kg/mol, so for that gas

$$T = \frac{2(9.8 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})(2.02 \times 10^{-3} \text{ kg/mol})}{3 \times (8.31 \text{ J/mol} \cdot \text{K})} = 1.0 \times 10^4 \text{ K}.$$

The molar mass of oxygen is 32.0×10^{-3} kg/mol, so for that gas

$$T = \frac{2(9.8 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})(32.0 \times 10^{-3} \text{ kg/mol})}{3 \times (8.31 \text{ J/mol} \cdot \text{K})} = 1.6 \times 10^5 \text{ K}.$$

(b) Now $T = 2g_m r_m M / 3R$, where r_m ($= 1.74 \times 10^6$ m) is the radius of the moon and g_m ($= 0.16g$) is the acceleration due to gravity at the moon's surface. For hydrogen

$$T = \frac{2(0.16)(9.8 \text{ m/s}^2)(1.74 \times 10^6 \text{ m})(2.02 \times 10^{-3} \text{ kg/mol})}{3(8.31 \text{ J/mol}\cdot\text{K})} = 4.4 \times 10^2 \text{ K}.$$

For oxygen

$$T = \frac{2(0.16)(9.8 \text{ m/s}^2)(1.74 \times 10^6 \text{ m})(32.0 \times 10^{-3} \text{ kg/mol})}{3(8.31 \text{ J/mol}\cdot\text{K})} = 7.0 \times 10^3 \text{ K}.$$

(c) The temperature high in the Earth's atmosphere is great enough for a significant number of hydrogen atoms in the tail of the Maxwellian distribution to escape. As a result the atmosphere is depleted of hydrogen. Very few oxygen atoms, on the other hand, escape.

56P

(a) The root-mean-square speed is given by $v_{\text{rms}} = \sqrt{3RT/M}$. The molar mass of hydrogen is 2.02×10^{-3} kg/mol, so

$$v_{\text{rms}} = \sqrt{\frac{3(8.31 \text{ J/mol}\cdot\text{K})(4000 \text{ K})}{(2.02 \times 10^{-3} \text{ kg/mol})}} = 7.0 \times 10^3 \text{ m/s}.$$

(b) When the surfaces of the spheres that represent an H_2 molecule and an A atom are touching the distance between their centers is the sum of their radii: $d = r_1 + r_2 = 0.5 \times 10^{-8} \text{ cm} + 1.5 \times 10^{-8} \text{ cm} = 2.0 \times 10^{-8} \text{ cm}$.

(c) The argon atoms are essentially at rest so in time t the hydrogen atom collides with all the argon atoms in a cylinder of radius d and length vt , where v is its speed. That is, the number of collisions is $\pi d^2 vt N/V$, where N/V is the concentration of argon atoms. molr of collisions per unit time is $\pi d^2 v N/V = \pi(2.0 \times 10^{-10} \text{ m})^2(7.0 \times 10^3 \text{ m/s})(4.0 \times 10^{25} \text{ m}^{-3}) = 3.5 \times 10^{10}$ collisions/s.

57E

The formulas for the average speed and v_{rms} are $\bar{v} = \sqrt{8kT/\pi m}$ and $v_{\text{rms}} = \sqrt{3kT/m}$. In this case

$$\bar{v}_2 = \sqrt{\frac{8kT}{\pi m_2}} = 2v_{\text{rms}_1} = 2\sqrt{\frac{3kT}{m_1}},$$

so $m_1/m_2 = 3\pi/2 = 4.7$.

58P

(a) From the normalization condition

$$\int_0^\infty P(v) dv = \int_0^{v_0} cv^2 dv = \frac{1}{3}cv_0^3 = N$$

we obtain $c = 3N/v_0^3$.

(b) The average speed is

$$\bar{v} = \frac{1}{N} \int_0^\infty P(v)v \, dv = \frac{1}{N} \int_0^{v_0} cv^3 \, dv = \frac{cv_0^4}{4N} = \frac{3}{4}v_0.$$

(c) The rms speed is

$$v_{\text{rms}} = \sqrt{\frac{1}{N} \int_0^\infty P(v)v^2 \, dv} = \sqrt{\frac{1}{N} \int_0^{v_0} cv^4 \, dv} = \sqrt{\frac{cv_0^5}{5N}} = \sqrt{\frac{3}{5}} \simeq 0.775v_0.$$

59P

(a) The integral of the distribution function is the number of particles: $\int P(v) \, dv = N$. The area of the triangular portion is half the product of the base and altitude, or $\frac{1}{2}av_0$. The area of the rectangular portion is the product of the sides, or av_0 . Thus $\int P(v) \, dv = \frac{1}{2}av_0 + av_0 = \frac{3}{2}av_0$ and $\frac{3}{2}av_0 = N$, so $a = 2N/3v_0$.

(b) The number of particles with speeds between $1.5v_0$ and $2v_0$ is given by $\int_{1.5v_0}^{2v_0} P(v) \, dv$. The integral is easy to evaluate since $P(v) = a$ throughout the range of integration. Thus the number of particles is $a(2.0v_0 - 1.5v_0) = 0.5av_0 = N/3$, where $2N/3v_0$ was substituted for a .

(c) The average speed is given by

$$\bar{v} = \frac{1}{N} \int vP(v) \, dv.$$

For the triangular portion of the distribution $P(v) = av/v_0$ and the contribution of this portion is

$$\frac{a}{Nv_0} \int_0^{v_0} v^2 \, dv = \frac{a}{3Nv_0} v_0^3 = \frac{av_0^2}{3N} = \frac{2}{9}v_0,$$

where $2N/3v_0$ was substituted for a . $P(v) = a$ in the rectangular portion and the contribution of this portion is

$$\frac{a}{N} \int_{v_0}^{2v_0} v \, dv = \frac{a}{2N} (4v_0^2 - v_0^2) = \frac{3a}{2N}v_0^2 = v_0.$$

Thus $\bar{v} = 2v_0/9 + v_0 = 1.22v_0$.

(d) The mean-square speed is given by

$$v_{\text{rms}}^2 = \frac{1}{N} \int v^2 P(v) \, dv.$$

The contribution of the triangular section is

$$\frac{a}{Nv_0} \int_0^{v_0} v^3 dv = \frac{a}{4Nv_0} v_0^4 = \frac{1}{6} v_0^2.$$

The contribution of the rectangular portion is

$$\frac{a}{N} \int_{v_0}^{2v_0} v^2 dv = \frac{a}{3N} (8v_0^3 - v_0^3) = \frac{7a}{3N} v_0^3 = \frac{14}{9} v_0^2.$$

Thus

$$v_{\text{rms}} = \sqrt{\frac{1}{6}v_0^2 + \frac{14}{9}v_0^2} = 1.3v_0.$$

60E

(a) The internal energy is

$$E_{\text{int}} = \frac{3}{2}NkT = \frac{3}{2}nRT = \frac{3}{2}(1.0 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(273 \text{ K}) = 3.4 \times 10^3 \text{ J}.$$

(b) For an ideal gas, as long as T is fixed, so is E_{int} , regardless the values of p and V .

61E

According to the first law of thermodynamics $\Delta E_{\text{int}} = Q - W$. Since the process is isothermal $\Delta E_{\text{int}} = 0$ (the internal energy of an ideal gas depends only on the temperature) and $Q = W$. The work done by the gas as its volume expands from V_i to V_f at temperature T is

$$W = \int_{V_i}^{V_f} p dV = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT \ln\left(\frac{V_f}{V_i}\right).$$

For $n = 1 \text{ mol}$, $Q = W = RT \ln(V_f/V_i)$.

62E

The specific heat is

$$C_V = \frac{3R/2}{mN_A} = \frac{3(8.31 \text{ J/mol}\cdot\text{K})/2}{(6.66 \times 10^{-27} \text{ kg})(6.02 \times 10^{23} \text{ /mol})} = 3.11 \times 10^3 \text{ J/kg}\cdot\text{K}.$$

63P

(a) According to the first law of thermodynamics $\Delta Q = \Delta E_{\text{int}} + \Delta W$. When the pressure is a constant $\Delta W = p\Delta V$. So

$$\begin{aligned} \Delta E_{\text{int}} &= \Delta Q - p\Delta V \\ &= 20.9 \text{ J} - (1.0 \times 10^5 \text{ Pa})(100 \text{ cm}^3 - 50 \text{ cm}^3)(10^{-6} \text{ m}^3/\text{cm}^3) \\ &= 15.9 \text{ J}. \end{aligned}$$

(b)

$$C_p = \frac{\Delta Q}{n\Delta T} = \frac{\Delta Q}{n(p\Delta V/nR)} = \frac{R}{p} \frac{\Delta Q}{\Delta V}$$

$$= \frac{(8.31 \text{ J/mol}\cdot\text{K})(20.9 \text{ J})}{(1.01 \times 10^5 \text{ Pa})(50 \times 10^{-6} \text{ m}^3)} = 34.4 \text{ J/mol}\cdot\text{K}.$$

(c) $C_V = C_p - R = 34.4 \text{ J/mol}\cdot\text{K} - 8.31 \text{ J/mol}\cdot\text{K} = 26.1 \text{ J}.$ **64P**

The straight line on the p - V diagram can be expressed as $(p - p_1)/(V - V_1) = p_1/V_1$, where $p_1V_1 = nRT_1$. Solve for p : $p = (p_1/V_1)V = (p_1^2/nRT_1)V$. Thus

(a)

$$W = \int_{V_1}^{2V_1} p dV = \int_{V_1}^{2V_1} \frac{p_1^2}{nRT_1} V dV = \frac{p_1^2}{2nRT_1 - 1} V^2 \Big|_{V_1}^{2V_1} = \frac{3}{2} nRT_1.$$

(b)

$$\Delta E_{\text{int}} = \frac{3}{2} nRT_2 - \frac{3}{2} nRT_1 = \frac{3}{2} (2p_1)(2V_1) - \frac{3}{2} nRT_1$$

$$= (4 - 1) \frac{3}{2} nRT_1 = \frac{9}{2} nRT_1.$$

(c)

$$Q = W + \Delta E_{\text{int}} = \frac{3}{2} nRT_1 + \frac{9}{2} nRT_1 = 6nRT_1.$$

(d) The molar specific heat would be

$$C = \frac{Q}{n\Delta T} = \frac{6RT_1}{T_2 - T_1} = \frac{6RT_1}{(2p_2)(2V_2)/(nR) - T_1} = \frac{6RT_1}{4T_1 - T_1} = 2R.$$

65P

When the temperature changes by ΔT the internal energy of the first gas changes by $n_1 C_1 \Delta T$, the internal energy of the second gas changes by $n_2 C_2 \Delta T$, and the internal energy of the third gas changes by $n_3 C_3 \Delta T$. The change in the internal energy of the composite gas is $\Delta E_{\text{int}} = (n_1 C_1 + n_2 C_2 + n_3 C_3) \Delta T$. This must be $(n_1 + n_2 + n_3) C \Delta T$, where C is the molar specific heat of the composite. Thus

$$C = \frac{n_1 C_1 + n_2 C_2 + n_3 C_3}{n_1 + n_2 + n_3}.$$

66P

(a) Use $C_V = 3R/(2mN_A)$ (see 62E). The mass of the argon atom follows as

$$\begin{aligned} m &= \frac{3R}{2C_V N_A} = \frac{3(8.31 \text{ J/mol}\cdot\text{K})}{2(75 \text{ cal/kg}\cdot^\circ\text{C})(4.18 \text{ J/cal})(6.0 \times 10^{23} / \text{mol})} \\ &= 6.6 \times 10^{-26} \text{ kg}. \end{aligned}$$

(b) The molar mass of argon is

$$M = mN_A = (6.6 \times 10^{-26} \text{ kg})(6.0 \times 10^{23} / \text{mol}) = 40 \text{ g/mol}.$$

67P

For diatomic ideal gas $C_V = \frac{5}{2}R = 20.8 \text{ J}$.

(a)

$$\begin{aligned} \Delta E_{\text{int},ac} &= nC_V(T_c - T_a) = nC_V \left(\frac{p_c V_c}{R} - \frac{p_a V_a}{R} \right) \\ &= \left(\frac{20.8 \text{ J}}{8.31 \text{ J/mol}\cdot\text{K}} \right) [(2.0 \times 10^3 \text{ Pa})(4.0 \text{ m}^3) - (5.0 \times 10^3 \text{ Pa})(2.0 \text{ m}^3)] \\ &= -5.0 \times 10^3 \text{ J}. \end{aligned}$$

(b) The work done from a to c is

$$\begin{aligned} W_{ac} &= \frac{1}{2}(V_c - V_a)(p_b + p_c) \\ &= \frac{1}{2}(4.0 \text{ m}^3 - 2.0 \text{ m}^3)(5.0 \times 10^3 \text{ Pa} + 2.0 \times 10^3 \text{ Pa}) \\ &= 7.0 \times 10^3 \text{ J}, \end{aligned}$$

and the heat added to the gas from a to c is

$$Q_{ac} = \Delta E_{\text{int},ac} + W_{ac} = -5.0 \text{ kJ} + 7.0 \text{ kJ} = 2.0 \times 10^3 \text{ J}.$$

(c) Now $Q_{abc} = \Delta E_{\text{int},abc} + W_{abc}$, where

$$W_{abc} = p_a(V_c - V_a) = (5.0 \times 10^2 \text{ Pa})(4.0 \text{ m}^3 - 2.0 \text{ m}^3) = 1.0 \times 10^4 \text{ J}$$

and $\Delta E_{\text{int},abc} = \Delta E_{\text{int},ac} = -5.0 \times 10^3 \text{ J}$, since ΔE is path-independent. So $Q_{abc} = 1.0 \times 10^4 \text{ J} - 5.0 \times 10^3 \text{ J} = 5.0 \times 10^3 \text{ J}$.

68E

As the volume doubles at constant pressure, so does the temperature in Kelvin. Thus $\Delta T = (2 - 1)(273 \text{ K}) = 273 \text{ K}$. The heat required is then

$$\begin{aligned} Q &= nC_p\Delta T = n(C_V + R)\Delta T = n\left(\frac{5}{2}R + R\right)\Delta T \\ &= (1 \text{ mol})\left(\frac{7}{2}\right)(8.31 \text{ J/mol}\cdot\text{K})(273 \text{ K}) \\ &= 8 \times 10^3 \text{ J}. \end{aligned}$$

69E

(a) The molar mass of O_2 is 32 g so $n = 12 \text{ g}/32 \text{ g} = 0.375 \text{ mol}$.

(b) For diatomic ideal gas without oscillation $C_p = C_V + R = \frac{5}{2}R + R = \frac{7}{2}R$. Thus

$$\Delta Q = nC_p\Delta T = \frac{7}{2}(0.375 \text{ mole})(8.31 \text{ J/mol}\cdot\text{K})(100 \text{ K}) = 1090 \text{ J}.$$

(c) Use $\Delta E_{\text{int}} = nC_V\Delta T = \frac{5}{2}nR\Delta T$. The ratio is

$$\frac{\Delta E_{\text{int}}}{Q} = \frac{5nR\Delta T/2}{7nR\Delta T/2} = \frac{5}{7} = 0.714.$$

70P

(a) Since the process is at constant pressure the heat added to the gas is given by $Q = nC_p\Delta T$, where n is the number of moles in the gas, C_p is the molar specific heat at constant pressure, and ΔT is the increase in temperature. For a diatomic ideal gas $C_p = \frac{7}{2}R$. Thus $Q = \frac{7}{2}nR\Delta T = \frac{7}{2}(4.00 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(60.0 \text{ K}) = 6.98 \times 10^3 \text{ J}$.

(b) The change in the internal energy is given by $\Delta E_{\text{int}} = nC_V\Delta T$. For a diatomic ideal gas $C_V = \frac{5}{2}R$, so $\Delta E_{\text{int}} = \frac{5}{2}nR\Delta T = \frac{5}{2}(4.00 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(60.0 \text{ K}) = 4.99 \times 10^3 \text{ J}$.

(c) According to the first law of thermodynamics $\Delta E_{\text{int}} = Q - W$, so $W = Q - \Delta E_{\text{int}} = 6.98 \times 10^3 \text{ J} - 4.99 \times 10^3 \text{ J} = 1.99 \times 10^3 \text{ J}$.

(d) The change in the total translational kinetic energy is

$$\Delta K = \frac{3}{2}nR\Delta T = \frac{3}{2}(4.00 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(60.0 \text{ K}) = 2.99 \times 10^3 \text{ J}.$$

71E

(a) Let p_i , V_i , and T_i represent the pressure, volume, and temperature of the initial state of the gas. Let p_f , V_f , and T_f represent the pressure, volume, and temperature of the final state. Since the process is adiabatic $p_iV_i^\gamma = p_fV_f^\gamma$, so $p_f = (V_i/V_f)^\gamma p_i = [(4.3 \text{ L})/(0.76 \text{ L})]^{1.4} (1.2 \text{ atm}) = 14 \text{ atm}$. Round to 14 atm. Notice that since V_i and V_f have the same units, their units cancel and p_f has the same units as p_i .

(b) The gas obeys the ideal gas law $pV = nRT$, so $p_i V_i / p_f V_f = T_i / T_f$ and

$$T_f = \left(\frac{p_f V_f}{p_i V_i} \right) T_i = \frac{(13.6 \text{ atm})(0.76 \text{ L})}{(1.2 \text{ atm})(4.3 \text{ L})} (310 \text{ K}) = 620 \text{ K}.$$

Note that the units of $p_i V_i$ and $p_f V_f$ cancel since they are the same.

72E

(a) For the adiabatic process from state 1 to state 2 $p_1 V_1^\gamma = p_2 V_2^\gamma$, so

$$p_2 = p_1 \left(\frac{V_1}{V_2} \right)^\gamma = (1.0 \text{ atm}) \left(\frac{V_1}{V_1/2} \right)^{1.3} = 2.5 \text{ atm}.$$

Now use the ideal gas equation $pV = nRT$ to eliminate p in the equation for adiabatic processes above to get the temperature of the gas after the expansion:

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1} = (273 \text{ K}) \left(\frac{V_1}{V_1/2} \right)^{1.3-1} = 340 \text{ K}.$$

(b) At constant pressure $V_2/T_2 = V_3/T_3$. So the final volume is

$$V_3 = \left(\frac{T_3}{T_2} \right) V_2 = \left(\frac{273 \text{ K}}{340 \text{ K}} \right) V_2 = \frac{1}{2} (0.8) V_1 = 0.40 \text{ L}.$$

73E

Use the first law of thermodynamics: $\Delta E_{\text{int}} = Q - W$. The change in internal energy is $\Delta E_{\text{int}} = nC_V(T_2 - T_1)$, where C_V is the molar heat capacity for a constant volume process. Since the process is reversible $Q = 0$. Thus $W = -\Delta E_{\text{int}} = nC_V(T_1 - T_2)$.

74E

For adiabatic processes $pV^\gamma = \text{const.} = C$, and for any ideal gas $pV = nRT$. So the value of the constant is

$$\begin{aligned} C &= pV^\gamma = p \left(\frac{nRT}{p} \right)^\gamma \\ &= (1.0 \times 10^5 \text{ Pa})^{1-\gamma} [2(8.31 \text{ J/mol}\cdot\text{K})(300 \text{ K})]^\gamma \\ &= (1.0 \times 10^5 \text{ Pa})^{-0.4} [2(8.31 \text{ J/mol}\cdot\text{K})(300 \text{ K})]^{1.4} \\ &= 1500 \text{ N}\cdot\text{m}^{2.2}. \end{aligned}$$

Here $\gamma = C_p/C_V = 1.4$ for diatomic ideal gases without oscillation was substituted.

75E

(a) For adiabatic processes $pV^\gamma = \text{const.} = C$. Thus

$$B = -V \frac{dp}{dV} = -V \frac{d}{dV} \left(\frac{C}{V^\gamma} \right) = \gamma C V^{-\gamma} = \gamma p.$$

(b) The speed of sound is

$$v_s = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{\gamma(nRT/V)}{nM/V}} = \sqrt{\frac{\gamma RT}{M}}.$$

76E

The speed of sound is $v_s = \sqrt{\gamma p/\rho}$. So

$$\gamma = \frac{v_s^2 \rho}{p} = \frac{(331 \text{ m/s})^2 (1.29 \times 10^{-3} \text{ g/cm}^3)(10^3 \text{ kg/m}^3)(1 \text{ cm}^3/1 \text{ g})}{1.0 \times 10^5 \text{ Pa}} = 1.40.$$

77E

Use $v_s = \sqrt{\gamma RT/M}$ to calculate the ratio:

$$\frac{v_1}{v_2} = \sqrt{\frac{\gamma RT_1/M_1}{\gamma RT_2/M_2}} = \sqrt{\frac{M_2}{M_1}}.$$

78P

(a) The speed of sound in an ideal gas is $v = \sqrt{\gamma p/\rho}$ which, when combined with the ideal gas law equation $pV = nRT = (m/M)RT$, can be re-written as $v = \sqrt{\gamma RT/M}$. Thus

$$\gamma = \frac{Mv^2}{RT}.$$

Since the nodes of the standing wave are 6.77 cm apart the wavelength of the sound wave is $\lambda = 2(6.77 \text{ cm}) = 13.5 \text{ cm}$. So the speed of sound can be obtained from $v = \lambda f = (0.135 \text{ m})(1400 \text{ Hz}) = 189 \text{ m/s}$. Thus

$$\gamma = \frac{C_p}{C_V} = \frac{(0.127 \text{ kg/mol})(189 \text{ m/s})^2}{(8.31 \text{ J/mol}\cdot\text{K})(400 \text{ K})} = 1.4.$$

(b) Since $\gamma = 1.4$ for iodine, it is a diatomic gas.

79E

Use $v_s = \sqrt{\gamma RT/M}$ and $v_{\text{rms}} = \sqrt{3RT/M}$. The ratio is

$$\frac{v_s}{v_{\text{rms}}} = \sqrt{\frac{\gamma RT/M}{3RT/M}} = \sqrt{\frac{\gamma}{3}} = \sqrt{\frac{C_p}{3C_V}} = \sqrt{\frac{C_V + R}{3C_V}} = \sqrt{\frac{5.0R + R}{3(5.0R)}} = 0.63.$$

80P

In the free expansion from state 0 to state 1 we have $Q = W = 0$, so $\Delta E_{\text{int}} = 0$, which means that the temperature of the ideal gas has to remain unchanged. Thus the final pressure is

$$p_1 = \frac{p_0 V_0}{V_1} = \frac{p_0 V_0}{3V_0} = \frac{1}{3}p_0.$$

(b) For the adiabatic process from state 1 to 2 we have $p_1 V_1^\gamma = p_2 V_2^\gamma$, i.e.,

$$\frac{1}{3}p_0(3V_0)^\gamma = (3.00)^\frac{1}{3}p_0 V_0^\gamma,$$

which gives $\gamma = 4/3$. The gas is therefore polyatomic.

(c) From $T = pV/nR$ we get

$$\frac{\bar{K}_2}{\bar{K}_1} = \frac{T_2}{T_1} = \frac{p_2}{p_1} = (3.00)^\frac{1}{3} = 1.44.$$

81P

(a) Use $p_i V_i^\gamma = p_f V_f^\gamma$ to compute γ :

$$\gamma = \frac{\log(p_i/p_f)}{\log(V_f/V_i)} = \frac{\log(1.0 \text{ atm}/1.0 \times 10^5 \text{ atm})}{\log(1.0 \times 10^3 \text{ L}/1.0 \times 10^6 \text{ L})} = \frac{5}{3}.$$

Therefore the gas is monatomic.

(b) The final temperature is

$$T_f = \frac{p_f V_f}{nR} = \frac{p_f V_f}{p_i V_i/T_i} = \frac{(1.0 \times 10^5 \text{ atm})(1.0 \times 10^3 \text{ L})(273 \text{ K})}{(1.0 \text{ atm})(1.0 \times 10^6 \text{ L})} = 2.7 \times 10^4 \text{ K}.$$

(c) The number of moles of gas present is

$$n = \frac{p_i V_i}{RT_i} = \frac{(1.0 \text{ atm})(1.0 \times 10^6 \text{ L})}{(8.31 \text{ J/mol}\cdot\text{K})(273 \text{ K})} = 4.5 \times 10^4 \text{ mol}.$$

(d) The total translational energy per mole before the compression is

$$K_i = \frac{3}{2}RT_i = \frac{3}{2}(8.31 \text{ J/mol}\cdot\text{K})(273 \text{ K}) = 3.4 \times 10^3 \text{ J}.$$

After the compression,

$$K_f = \frac{3}{2}RT_f = \frac{3}{2}(8.31 \text{ J/mol}\cdot\text{K})(2.7 \times 10^4 \text{ K}) = 3.4 \times 10^5 \text{ J}.$$

(e) Since $v_{\text{rms}}^2 \propto T$, we have

$$\frac{v_{\text{rms},i}^2}{v_{\text{rms},f}^2} = \frac{T_i}{T_f} = \frac{273 \text{ K}}{2.7 \times 10^4 \text{ K}} = 0.01.$$

82P

(a) For the isothermal process the final temperature of the gas is $T_f = T_i = 300 \text{ K}$. The final pressure is

$$p_f = \frac{p_i V_i}{V_f} = \frac{(32 \text{ atm})(1.0 \text{ L})}{4.0 \text{ L}} = 8.0 \text{ atm},$$

and the work done is

$$\begin{aligned} W &= nRT_i \ln\left(\frac{V_f}{V_i}\right) = p_i V_i \ln\left(\frac{V_f}{V_i}\right) \\ &= (32 \text{ atm})(1.01 \times 10^5 \text{ Pa/atm})(1.0 \times 10^{-3} \text{ m}^3) \ln\left(\frac{4.0 \text{ L}}{1.0 \text{ L}}\right) = 4.4 \times 10^3 \text{ J}. \end{aligned}$$

(b) For the adiabatic process $p_i V_i^\gamma = p_f V_f^\gamma$. Thus

$$\begin{aligned} p_f &= p_i \left(\frac{V_i}{V_f}\right)^\gamma = (32 \text{ atm}) \left(\frac{1.0 \text{ L}}{4.0 \text{ L}}\right)^{5/3} = 3.2 \text{ atm}, \\ T_f &= \frac{p_f V_f T_i}{p_i V_i} = \frac{(3.2 \text{ atm})(4.0 \text{ L})(300 \text{ K})}{(32 \text{ atm})(1.0 \text{ L})} = 120 \text{ K}, \end{aligned}$$

and

$$\begin{aligned} W &= Q - \Delta E_{\text{int}} = -\Delta E_{\text{int}} = -\frac{3}{2}nR\Delta T = -\frac{3}{2}(p_f V_f - p_i V_i) \\ &= -\frac{3}{2}[(3.2 \text{ atm})(4.0 \text{ L}) - (32 \text{ atm})(1.0 \text{ L})](1.01 \times 10^5 \text{ Pa/atm})(10^{-3} \text{ m}^3/\text{L}) \\ &= 2.9 \times 10^3 \text{ J}. \end{aligned}$$

(c) Now $\gamma = 1.4$ so

$$p_f = p_i \left(\frac{V_i}{V_f} \right)^\gamma = (32 \text{ atm}) \left(\frac{1.0 \text{ L}}{4.0 \text{ L}} \right)^{1.4} = 4.6 \text{ atm},$$

$$T_f = \frac{p_f V_f T_i}{p_i V_i} = \frac{(4.6 \text{ atm})(4.0 \text{ L})(300 \text{ K})}{(32 \text{ atm})(1.0 \text{ L})} = 170 \text{ K},$$

and

$$W = Q - \Delta E_{\text{int}} = -\frac{5}{2} n R \Delta T = -\frac{5}{2} (p_f V_f - p_i V_i)$$

$$= -\frac{5}{2} [(4.6 \text{ atm})(4.0 \text{ L}) - (32 \text{ atm})(1.0 \text{ L})] (1.01 \times 10^5 \text{ Pa/atm}) (10^{-3} \text{ m}^3/\text{L})$$

$$= 3.4 \times 10^3 \text{ J}.$$

83P

Label the various states of the ideal gas as follows: it starts expanding adiabatically from state 1 until it reaches state 2, with $V_2 = 4 \text{ m}^3$; then continues onto state 3 isothermally, with $V_3 = 10 \text{ m}^3$; and eventually getting compressed adiabatically to reach state 4, the final state. For the adiabatic process $1 \rightarrow 2$ $p_1 V_1^\gamma = p_2 V_2^\gamma$, for the isothermal process $2 \rightarrow 3$ $p_2 V_2 = p_3 V_3$, and finally for the adiabatic process $3 \rightarrow 4$ $p_3 V_3^\gamma = p_4 V_4^\gamma$. These equations yield

$$p_4 = p_3 \left(\frac{V_3}{V_4} \right)^\gamma = p_2 \left(\frac{V_2}{V_3} \right) \left(\frac{V_3}{V_4} \right)^\gamma = p_1 \left(\frac{V_1}{V_2} \right)^\gamma \left(\frac{V_2}{V_3} \right) \left(\frac{V_3}{V_4} \right)^\gamma.$$

Substitute this expression for p_4 into the equation $p_1 V_1 = p_4 V_4$ (since $T_1 = T_4$) to obtain $V_1 V_3 = V_2 V_4$. Now solve for V_4 :

$$V_4 = \frac{V_1 V_3}{V_2} = \frac{(2 \text{ m}^3)(10 \text{ m}^3)}{4 \text{ m}^3} = 5 \text{ m}^3.$$

84P

For the constant volume process $W = 0$. The change in internal energy is $\Delta E_{\text{int}} = n C_V \Delta T = (3.0 \text{ mol})(6.00 \text{ cal/mol}\cdot\text{K})(50 \text{ K}) = 900 \text{ cal}$, the heat added is $Q = W + \Delta E_{\text{int}} = 900 \text{ cal}$, and the change in total translational kinetic energy is

$$\Delta K = \frac{3}{2} n R \Delta T = \frac{3}{2} (3.0 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(50 \text{ K}) = 1.87 \times 10^3 \text{ J} = 450 \text{ cal}.$$

For constant pressure we have $\Delta E_{\text{int}} = 900 \text{ cal}$, $\Delta K = 450 \text{ cal}$,

$$Q = C_p \Delta T = n(C_V + R)\Delta T$$

$$= (3.0 \text{ mol})(6.00 \text{ cal/mol}\cdot\text{K} + 8.31 \text{ J/mol}\cdot\text{K})(50 \text{ K}) = 1.2 \times 10^3 \text{ cal},$$

and $W = Q - \Delta E_{\text{int}} = 1.2 \times 10^3 \text{ cal} - 900 \text{ cal} = 300 \text{ cal}$.

For adiabatic process we have $Q = 0$, $\Delta E_{\text{int}} = 900 \text{ cal}$, $\Delta K = 450 \text{ cal}$, and $W = Q - \Delta E_{\text{int}} = -900 \text{ cal}$ (where the minus sign indicate that the gas does negative work on the environment).

85P

In the following $C_V (= \frac{3}{2}R)$ is the molar specific heat at constant volume, $C_p (= \frac{5}{2}R)$ is the molar specific heat at constant pressure, ΔT is the temperature increase, and n is the number of moles.

(a) The process $1 \rightarrow 2$ takes place at constant volume. The heat added is

$$\begin{aligned} Q &= nC_V \Delta T = \frac{3}{2}nR \Delta T \\ &= \frac{3}{2}(1.00 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(600 \text{ K} - 300 \text{ K}) = 3.74 \times 10^3 \text{ J}. \end{aligned}$$

The change in the internal energy is $\Delta E_{\text{int}} = nC_V \Delta T = 3.74 \times 10^3 \text{ J}$. The work done by the gas is $W = Q - \Delta E_{\text{int}} = 0$.

The process $2 \rightarrow 3$ is adiabatic. The heat added is 0. The change in the internal energy is

$$\begin{aligned} \Delta E_{\text{int}} &= nC_V \Delta T = \frac{3}{2}nR \Delta T \\ &= \frac{3}{2}(1.00 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(455 \text{ K} - 600 \text{ K}) = -1.81 \times 10^3 \text{ J}. \end{aligned}$$

The work done by the gas is $W = Q - \Delta T = +1.81 \times 10^3 \text{ J}$.

The process $3 \rightarrow 1$ takes place at constant pressure. The heat added is

$$\begin{aligned} Q &= nC_p \Delta T = \frac{5}{2}nR \Delta T \\ &= \frac{5}{2}(1.00 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(300 \text{ K} - 455 \text{ K}) = -3.22 \times 10^3 \text{ J}. \end{aligned}$$

The change in the internal energy is

$$\begin{aligned} \Delta E_{\text{int}} &= nC_V \Delta T = \frac{3}{2}nR \Delta T \\ &= \frac{3}{2}(1.00 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(300 \text{ K} - 455 \text{ K}) = -1.93 \times 10^3 \text{ J}. \end{aligned}$$

The work done by the gas is $W = Q - \Delta E_{\text{int}} = -3.22 \times 10^3 \text{ J} + 1.93 \times 10^3 \text{ J} = -1.29 \times 10^3 \text{ J}$.

For the entire process the heat added is $Q = 3.74 \times 10^3 \text{ J} + 0 - 3.22 \times 10^3 \text{ J} = 520 \text{ J}$, the change in the internal energy is $\Delta E_{\text{int}} = 3.74 \times 10^3 \text{ J} - 1.81 \times 10^3 \text{ J} - 1.93 \times 10^3 \text{ J} = 0$, and the work done by the gas is $W = 0 + 1.81 \times 10^3 \text{ J} - 1.93 \times 10^3 \text{ J} = 520 \text{ J}$.

(b) First find the initial volume. Since $p_1V_1 = nRT_1$,

$$V_1 = \frac{nRT_1}{p_1} = \frac{(1.00 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(300 \text{ K})}{(1.013 \times 10^5 \text{ Pa})} = 2.46 \times 10^{-2} \text{ m}^3.$$

Since $1 \rightarrow 2$ is a constant volume process $V_2 = V_1 = 2.46 \times 10^{-2} \text{ m}^3$.

The pressure for state 2 is

$$p_2 = \frac{nRT_2}{V_2} = \frac{(1.00 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(600 \text{ K})}{2.46 \times 10^{-2} \text{ m}^3} = 2.02 \times 10^5 \text{ Pa}.$$

This is equivalent to 2.00 atm.

Since $3 \rightarrow 1$ is a constant pressure process, the pressure for state 3 is the same as the pressure for state 1: $p_3 = p_1 = 1.013 \times 10^5 \text{ Pa}$ (1.00 atm). The volume for state 3 is

$$V_3 = \frac{nRT_3}{p_3} = \frac{(1.00 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(455 \text{ K})}{1.013 \times 10^5 \text{ Pa}} = 3.73 \times 10^{-2} \text{ m}^3.$$

86P

(b) work done by environment: $7.72 \times 10^4 \text{ J}$; heat absorbed: $5.47 \times 10^4 \text{ J}$;

(c) $5.17 \text{ J/mol}\cdot\text{K}$;

(d) work done by environment: $4.32 \times 10^4 \text{ J}$; heat absorbed: $8.87 \times 10^4 \text{ J}$; molar specific heat: $8.38 \text{ J/mol}\cdot\text{K}$