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ESTIMATION OF GROUND WATER RECHARGE DUE TO RAINFALL BY MODELLING OF SOIL MOISTURE MOVEMENT

by
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ABSTRACT

The purpose of this study is to estimate the ground water recharge due to rainfall by studying one-dimensional vertical flow of water in the unsaturated zone. A model has been formulated for finite difference solution of the non-linear Richards equation applicable to transient, one-dimensional water flow through the unsaturated porous medium. Implicit scheme with implicit linearization (prediction-correction) has been used for discretization. The ground water has been estimated using appropriate initial and boundary conditions for storm and interstrom periods.

KEY WORDS : Ground water, Recharge, Water content, Soil water pressure, Hydraulic conductivity.

INTRODUCTION

Quantification of ground water recharge is a major problem in many water-resource investigations. It is a complex function of meteorological conditions, soil, vegetation, physiographic characteristics and properties of the geologic material within the paths of flow. The conventional method of estimating recharge as precipitation minus evapotranspiration minus runoff, with allowance for changes in soil moisture storage, is very sensitive to measurement errors and to the time scale of analysis. The customary method of calculating ground water recharge by multiplying a constant specific yield value by the water table rise over a certain time interval may be erroneous, especially in shallow aquifers. The hydraulic approach, based on Darcy's equation, offers the most direct measurement of seepage rates and hence recharge. However, it is highly site specific, laborious and expensive, requiring specialized field equipment and personnel.

The objective of the present study is to estimate the amount and time distribution of ground water recharge due to a series of rainfall events with rain intensities approximately equal to soil infiltrability (i.e., constant pressure head maintained at the soil surface) and these rainfall events separated by interstorm periods. A numerical model (finite difference scheme) is used for solving the nonlinear partial differential equation (Richards equation) describing one-dimensional water flow through the unsaturated porous medium.

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METHODOLOGY

Constitutive Equations

Darcy's equation for vertical flow can be written as

$$q = -K\frac{\partial H}{\partial z} = -K\frac{\partial}{\partial z}(h - z) \qquad (1)$$

where q is the flux, H the total hydraulic head, h the soil water pressure head, z the vertical distance from the soil surface downward (i.e., the depth), and k the hydraulic conductivity. At the soil surface, q = I, the infiltration rate. In an unsaturated soil, h is negative. Combining this formulation of Darcy's Eq. (1) with the continuity equation $\partial\theta / \partial t = -\partial q / \partial z$ gives the general flow equation

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}(K\frac{\partial H}{\partial z}) = \frac{\partial}{\partial z}(K\frac{\partial h}{\partial z}) - \frac{\partial K}{\partial z} \qquad (2)$$

If soil moisture content θ and pressure head h are uniquely related, then the left-hand side of Eq. (2) can be written $\frac{\partial\theta}{\partial t} = \frac{d\theta}{dh} \cdot \frac{\partial h}{\partial t}$ which transforms Eq. (2) into

$$C\frac{\partial h}{\partial t} = \frac{\partial}{\partial z}(K\frac{\partial h}{\partial z}) - \frac{\partial K}{\partial z} \qquad (3)$$

where $C\ (= d\theta / dh)$ is defined as the specific (or differential) water capacity (i.e., the change in water content in a unit volume of soil per unit change in matric potential).

Alternatively, we can transform the right-hand side of Eq. (2) once again using the chain rule to render

$\frac{\partial h}{\partial z} = \frac{dh}{d\theta} \cdot \frac{\partial\theta}{\partial z} = \frac{1}{C} \cdot \frac{\partial\theta}{\partial z}$ we thus obtain $\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}(\frac{K}{C} \cdot \frac{\partial\theta}{\partial z}) - \frac{\partial K}{\partial z}$ or

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}(D\frac{\partial\theta}{\partial z}) - \frac{\partial K}{\partial z} \qquad (4)$$

where D is the soil water diffusivity. Eqs. (2), (3) and (4) can all be considered as forms of the Richards equation.

Initial and Boundary Conditions

For the present study, the initial and the boundary conditions have been defined as follows.

I. Initial conditions :

$\theta(z,0) = \theta_i$ for $z \geq 0,\ t = 0$ (5)

(Equilibrium moisture profile with surface moisture content = 0.10)

II. Upper boundary conditions :

(a) during rain infiltration - $\theta(0,t) = (\theta_s - 0.001)$ for $z = 0,\ t \geq 0$ (6)

(b) during interstorm period -

If the relative humidity (f) and the temperature of the air (T) as a function of the time are known, and if it may be assumed that the pressure head at the soil surface is at equilibrium with the atmosphere, then h(0, t) can be derived from the thermodynamic relation

(Edlefson and Anderson, 1943):

$$h(0,t) = \frac{RT(t)}{Mg} \ln [f(t)] \quad (7)$$

where R is the universal gas constant (8.314×10^7 erg/mole/K), T is the absolute temperature (K), g is acceleration due to gravity (980.665 cm/s^2), M is the molecular weight of water (18 gm/mole), f is the relative humidity of the air (fraction) and h is in bars. Knowing h(o, t), $\theta(o,t)$ can be derived from the soil water retention curve.

III. Lower Boundary Conditions :

The phreatic surface acts as lower boundary of the system in case of ground water recharge due to rainfall. The lower boundary condition has therefore been set as

$$\theta(z = L, t) = \theta_s - 0.001 \quad (8)$$

where L is the depth of the ground water table and the subscript s denotes saturated condition.

Soil Moisture Characteristics

For the present study, functional relations, as reported by Haverkamp et al. (1977), for characterizing the hydraulic properties of a soil, were used. The following analytical expressions, obtained by a least square fit through all data points were chosen for characterizing the soil :

$$K = K_s \frac{A}{A + |h|^{\beta_1}}; \quad (9)$$

where, $K_s = 34\,cm/h$, $A = 1.175 \times 10^6$, $\beta_1 = 4.74$ and

$$\theta = \frac{\alpha(\theta_s - \theta_r)}{\alpha + |h|^{\beta_2}} + \theta_r ; \quad (10)$$

where, $\theta_s = 0.287$, $\theta_r = 0.075$, $\alpha = 1.611 \times 10^6$, $\beta_2 = 3.96$; subscript s refers to saturation, i.e. the value of θ for which h = 0, and the subscript r to residual water content.

Figure 1 presents the relationships between the soil water pressure h, the water content θ and the hydraulic conductivity K for the above soil used in this study.

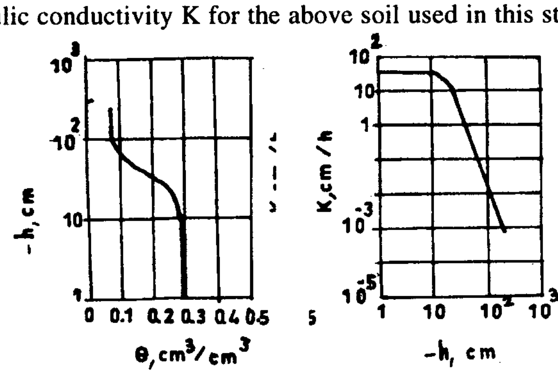

FIG. 1 RELATIONSHIPS BETWEEN THE SOIL WATER PRESSURE h, THE WATER CONTENT θ AND THE HYDRAULIC CONDUCTIVITY K FOR THE SOIL USED IN THE STUDY

Finite Difference Approximation

Equation (3) is a non-linear partial differential equation because the parameters K(h) and C(h) depend on the actual solution of h(z, t). For a given grid point at a given time, the values of the coefficients C(h) and K(h) can be expressed either from their values at the preceding time step (explicit linearization) or from a prediction at time ($t + 0.5\ \Delta t$) using a method (implicit linearization) described by Douglas and Jones (1963).

Equation (3) is solved by a finite difference technique with appropriate initial and boundary conditions. We have

$$C\frac{\partial h}{\partial t} = \frac{\partial}{\partial z}[K(\frac{\partial h}{\partial z} - 1)] \text{ or } C\frac{\partial h}{\partial t} = \frac{\partial K}{\partial z}(\frac{\partial h}{\partial z} - 1) + K\frac{\partial^2 h}{\partial z^2}$$

$$\text{or } \frac{C}{K}\frac{\partial h}{\partial t} = \frac{\partial^2 h}{\partial z^2} + \frac{1}{K}\frac{\partial K}{\partial z}(\frac{\partial h}{\partial z} - 1) \quad (11)$$

Using implicit evaluation of the coefficients C(h) and K(h) at time ($t + 0.5\ \Delta t$), pressure distribution is evaluated at time ($t + \Delta t$). The above partial differential equation is approximated to the following finite difference equations replacing ∂t and ∂z by Δt and Δz, respectively.

Prediction (Estimation of C_i^j and K_i^j)

From Eq. (11), by taking time step as $\Delta t / 2$, we have

$$\frac{2C_i^j}{K_i^j} \cdot \frac{h_i^{j+1/2} - h_i^j}{\Delta t} = \frac{h_{i+1}^{j+1/2} - 2h_i^{j+1/2} + h_{i-1}^{j+1/2}}{(\Delta z)^2} + \frac{1}{K_i^j} \cdot \frac{K_{i+1}^j - K_{i-1}^j}{2\Delta z}[\frac{h_{i+1}^j - h_{i-1}^j}{2\Delta z} - 1]$$

where i refers to depth and j refers to time. Rearranging the terms, we get

$$-\frac{\Delta t}{(\Delta z)^2} h_{i-1}^{j+1/2} + [\frac{2C_i^j}{K_i^j} + \frac{2\Delta t}{(\Delta z)^2}] h_i^{j+1/2} - \frac{\Delta t}{(\Delta z)^2} h_{i+1}^{j+1/2}$$

$$= \frac{2C_i^j}{K_i^j} h_i^j + \frac{1}{2}\frac{K_{i+1}^j - K_{i-1}^j}{K_i^j}\frac{\Delta t}{\Delta z}[\frac{h_{i+1}^j - h_{i-1}^j}{2\Delta z} - 1] \quad (12)$$

Correction (estimation of h_i^j)

From equation (11), by taking time step as Δt, we have

$$\frac{C_i^{j+1/2}}{K_i^{j+1/2}} \cdot \frac{h_i^{j+1} - h_i^j}{\Delta t} = \frac{1}{2}[\frac{h_{i+1}^{j+1} - 2h_i^{j+1} + h_{i-1}^{j+1}}{(\Delta z)^2} + \frac{h_{i+1}^j - 2h_i^j + h_{i-1}^j}{(\Delta z)^2}]$$

$$+ \frac{1}{K_i^{j+1/2}} \cdot \frac{K_{i+1}^{j+1/2} - K_{i-1}^{j+1/2}}{2\Delta z}[\frac{h_{i+1}^{j+1/2} - h_{i-1}^{j+1/2}}{2\Delta z} - 1]$$

Rearranging the terms, we get

$$-\frac{1}{2}\frac{\Delta t}{(\Delta z)^2} h_{i-1}^{j+1} + [\frac{C_i^{j+1/2}}{K_i^{j+1/2}} + \frac{\Delta t}{(\Delta z)^2}] h_i^{j+1} - \frac{1}{2}\frac{\Delta t}{(\Delta z)^2} h_{i+1}^{j+1}$$

$$= \frac{C_i^{j+1/2}}{K_i^{j+1/2}} h_i^j + \frac{1}{2} \frac{\Delta t}{(\Delta z)^2} [h_{i+1}^j - 2 h_i^j + h_{i-1}^j] + \frac{1}{2} \frac{K_{i+1}^{j+1/2} - K_{i-1}^{j+1/2}}{K_i^{j+1/2}} \frac{\Delta t}{\Delta z} [\frac{h_{i+1}^{j+1/2} - h_{i-1}^{j+1/2}}{2\Delta z} - 1] \quad (13)$$

When Eqs. (12) or (13) is applied at all nodes, the result is a system of simultaneous linear algebraic equations with a tridiagonal coefficient matrix with zero elements outside the diagonals and unknown values of h. In solving this system of equations, the so-called direct method was used by applying a tridiagonal algorithm of the kind discussed by Remson et al. (1971).

Estimation of Ground Water Recharge

After obtaining the pressure (and soil moisture) distribution at each time step, the ground water recharge due to rainfall was estimated by the following two methods:

(i) Darcian Flux Method

The ground water recharge due to rainfall (RR) was estimated by applying the finite difference form of Eq. (1) for two vertically adjacent nodal points (at and above the water table) for each time step.

$$RR = - K_{i+1/2}^j \left(\frac{h_{i+1}^j - h_i^j}{\Delta z} - 1\right) \quad (14)$$

where, $K_{i+1/2}^j = \sqrt{(K_i^j K_{i+1}^j)}$

Geometric mean of K was taken following suggestions of Haverkamp and Vauclin (1979).

(ii) Water Balance of the Unsaturated Zone

The soil water balance of the unsaturated zone can be represented as follows :

RECH = RAIN - EVAP - DELSM (15)

where, RECH = ground water recharge ;

RAIN = rain infiltration ;

EVAP = evaporation from the soil ; and

DELSM = change in soil moisture storage of the unsaturated zone.

Eqs. (14) and (15) provide a means of estimating ground water recharge due to rainfall during each time step. Rain infiltration and evaporation from the soil (assumed as zero during the storm period) were also computed from Eq. (1) for two vertically adjacent nodal points (at and below the ground surface).

RESULTS

The numerical model described above was tested by comparing water content profiles calculated at given times with results obtained from quasi-analytical solution of Philip subject to condition of a constant pressure at the soil surface ($\theta = 0.267 cm^3 / cm^3$). Haverkamp et al. (1977) have reported the infiltration profiles at various times for infiltration in the sand (under consideration) obtained by quasi-analytical solution of Philip. The model yielded good agreement with water content profiles at various times (Kumar and Mishra, 1991).

The present study was carried out for bare-surface (i.e no vegetation) and therefore transpiration by plants was not taken into account. The sub-surface profile was divided into 75 layers of thickness 4 cm each (depth interval, Δz) down to the water table position assumed at a depth of 3 metres. Keeping in view the stability of the numerical scheme, the time step (Δt) was taken as 3 seconds during the entire study period. Three rainfall events of 3 hours duration each separated by interstorm periods of 3 hours duration were considered for the study (Fig. 2). Uniform evaporative conditions (temperature = 25°C, relative humidity = 0.75) were assumed during the interstorm periods. The upper boundary condition during the rain infiltration was defined as

$$\theta(0,t) = 0.286 \qquad \text{for } z = 0,\ t \geq 0$$

implying that a constant pressure head corresponding to $\theta = 0.286$ ($h = -9.56$ cm) was maintained at the soil surface during the rain infiltration. The lower boundary condition was defined as

$$\theta(z=L,t) = 0.286$$

The ground water recharge due to rainfall was estimated for a total duration of 30 hours by Darcian flux method and through water balance of the unsaturated zone.

TABLE-1

GROUND WATER RECHARGE DUE TO RAINFALL

Hour	Rain Infiltration (cm)	Evaporation from the Soil (cm)	Change in Soil Moisture Storage (cm)	Ground Water Recharge (Water Balance) (cm)	Ground Water Recharge (Darcy's Law) (cm)
1	35.26	0	36.49	- 1.23	0.
2	32.78	0	14.47	18.31	17.93
3	32.76	0	0	32.76	32.76
4	0	0.17	-18.56	18.39	16.93
5	0	0.05	- 6.54	6.49	6.15
6	0	0.03	- 4.30	4.27	3.99
7	35.15	0	29.36	5.79	4.73
8	32.76	0	0.04	32.72	32.72
9	32.76	0	0	32.76	32.76
10	0	0.17	- 18.56	18.39	16.93
11	0	0.05	- 6.54	6.49	6.15
12	0	0.03	- 4.30	4.27	3.99
13	35.15	0	29.36	5.79	4.73
14	32.77	0	0.04	32.73	32.72
15	32.78	0	0	32.78	32.76
16	0	0.17	- 18.56	18.39	16.93
17	0	0.05	- 6.54	6.49	6.15
18	0	0.03	- 4.30	4.27	3.99
19	0	0.03	- 3.24	3.21	2.96

Cont.

20	0	0.02	- 2.58	2.56	2.34
21	0	0.02	- 2.14	2.12	1.91
22	0	0.01	- 1.82	1.81	1.61
23	0	0.01	- 1.57	1.56	1.37
24	0	0.01	- 1.37	1.36	1.19
25	0	0.01	- 1.21	1.20	1.04
26	0	0.01	- 1.08	1.07	0.92
27	0	0.01	- 0.96	0.95	0.82
28	0	0.01	- 0.87	0.86	0.73
29	0	0.01	- 0.78	0.77	0.66
30	0	0.01	- 0.71	0.70	0.59
Total	302.17	0.91	3.23	298.03	288.46

Table 1 presents the hourly values of rain infiltration, evaporation from the soil, change in soil moisture storage of the unsaturated zone and ground water recharge by the two methods for the study period. It can be observed that the ground water recharge due to rainfall estimated by Darcian flux method and water balance are in reasonable agreement with each other. The variation of cumulative ground water recharge with time is presented in Fig. 2.

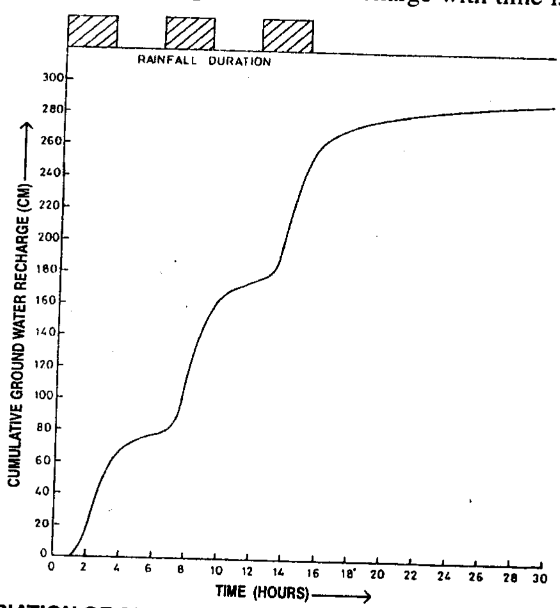

FIG. 2. VARIATION OF CUMULATIVE GROUND WATER RECHARGE WITH TIME

The above method for estimation of ground water recharge due to rainfall is applicable under many conditions by incorporating the appropriate modifications in the initial and boundary conditions. The application of several independent or different ground water recharge estimation methods can complement one another and is likely to improve our knowledge of aquifer recharge, provided that an adequate hydrogeologic database and soil characteristics exist.

CONCLUSIONS

A numerical solution using an implicit finite-difference technique is presented for a mathematical model of one-dimensional, vertical, unsteady, unsaturated flow above a water table. The solution is applicable only for homogeneous and isotropic soils. The model has been applied for upper boundary condition of rain (infiltration equal to soil infiltrability) separated by interstorm periods and ground water recharge due to rainfall has been estimated. The model can furnish information useful in quantification of the rate of ground water recharge for soils with known moisture parameters and rains of a given intensity pattern by suitably modifying the initial and boundary conditions. However, the method is not utilizable when the soil exhibits significant air compression, parameter hysteresis, fabric transformations, or areal heterogeneity.

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APPENDIX : DEFINITION OF SYMBOLS

C = Specific (or differential) water capacity
f = Relative humidity of the air (fraction)
h = Soil water pressure head
i = refers to depth
j = Refers to time
K_s = Hydraulic conductivity at saturation
M = Molecular weight of water
R = Universal gas constant
T = Absolute temperature (^{0}K)
θ = Soil moisture content
θ_r = Residual moisture content
D = Soil water diffusivity
g = Acceleration due to gravity
H = Total hydraulic head
I = Infiltration rate
K = Hydraulic conductivity
L = Depth of the ground water table
q = Flux
t = Time
z = Vertical distance from the soil surface downward (i.e, the depth)
θ_i = Initial moisture content
θ_s = Saturated moisture content