Prediction of Evaporation Losses from Shallow Water Table using a Numerical Model

C P Kumar, *Member*

The purpose of the study, reported in this paper, is to estimate the steady state evaporation rates from bare soils under conditions of high water table. A finite difference numerical scheme based upon the one-dimensional Richards equation has been employed to attain the steady state moisture profiles and estimate the evaporation rates under conditions of high water table. The procedure takes into account the relevant atmospheric factors and the soil's capability to conduct water. Field data required include soil water retention curves, water table depth, and a record of air temperature and air humidity. Results obtained with the method demonstrate how the soil water evaporation rates depend on water table depth.

Keywords : Evaporation, Water table, Soil moisture, Suction head, Hydraulic conductivity

INTRODUCTION

In the absence of vegetation, and when the soil surface is subject to radiation and wind effects, evaporation occurs directly and entirely from the soil. Under annual field crops, the soil surface may remain largely bare throughout the periods of tillage, planting, germination, and early seedling growth, periods in which evaporation can deplete the moisture of the surface soil and hamper the growth of young plants during their most vulnerable stage. Rapid drying of a seedbed can thwart germination and doom an entire crop from the start. The problem can also be acute in young orchards, where the soil surface is often kept bare continuously for several years; and in dryland farming in arid zones, where the land is regularly fallowed for several months to collect and conserve rainwater from one season to the next.

Evaporation of water from soil can also lead to soil salinization. This danger is felt most in regions where irrigation water is scarce (and possibly brackish) and where annual rainfall is low, as well as in regions with a high groundwater table. Where a groundwater table occurs close to the surface, continual flow may take place from the saturated zone beneath, through the unsaturated soil, to the surface. If this flow is more or less steady, continued evaporation can occur without materially changing the soil moisture content (though cumulative salinization may take place at the surface). In the absence of shallow groundwater, on the other hand, the loss of water at the surface and the resulting upward flow of water in the profile will necessarily be a transient state process causing the soil to dry. A proper formulation of an evaporation process should account for spatial and temporal variability, as well as for interactions with the environment above and below ground.

It is desirable to estimate the evaporation rates from bare land surfaces and to predict the variation of these rates with meteorological conditions, or with man-imposed changes in the water table level. This estimate might be important in certain regions during the appraisal of groundwater availability. For such purposes, it is often both permissible and useful to assume steady state of the hydraulic gradient driven upward flux of water and to neglect certain effects of soil temperature and of solute accumulations.

C P Kumar is with National Institute of Hydrology, Roorkee 247 667.

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PROBLEM DEFINITION

The objective of the present study is to determine the evaporation from shallow water tables through a homogeneous soil profile under isothermal conditions on the basis of solutions of the water flow equation. The steady state upward water flow from a shallow water table through the soil toward its surface is described by the nonlinear Richards equation. A numerical model (finite difference scheme) is used for solving the partial differential equation describing one-dimensional water flow through the unsaturated porous medium. Steady state moisture profile is obtained for the given initial and boundary conditions and the steady state evaporation rate is estimated by using Darcy's law. The evaporation rate can be limited either by the external evaporative conditions or by the maximal rate at which the soil can transmit water to its surface. If the water table is near the soil surface, the external conditions will govern the evaporation rate. However, if the water table becomes deeper, the evaporation rate approaches a limiting value which is determined by the soil profile capabilities of water transmission regardless of the external conditions. The effect of water table depth on the actual steady evaporation rate is examined by varying depth of the water table for the given values of temperature and humidity of the air.

METHODOLOGY

GENERAL

Most of the process involving soil water flow in the field, and in the rooting zone of most plant habitats, occur while the soil is in an unsaturated condition. Unsaturated flow processes are in general complicated and difficult to describe quantitatively, since they often entail changes in the state and content of soil water during flow. Such changes involve complex relations among the variable water content, suction, and conductivity, which may be affected by hysteresis. The formulation and solution of unsaturated flow problems very often require the use of indirect methods of analysis, based on approximations or numerical techniques.

SOIL WATER FLOW

A proper physical description of water flow in the soil requires that three parameters be specified : flux, hydraulic gradient, and conductivity. Knowledge of any two of these allows the calculation of the third, according to Darcy's law which states that the flux equals the product of conductivity and the hydraulic gradient. Darcy's law has been found to apply for unsaturated as well as for saturated soils, but the pressure gadient at unsaturation becomes a suction gradient, and the hydraulic conductivity is no longer constant, but a function of water content or suction. Since the conductivity depends on the number, sizes, and shapes of the conducting pores, its value is greatest when the soil is saturated, and decreases steeply when the soil water suction increases and the soil looses moisture. Darcy's law suffices to describe water flow under steady state conditions, but must be combined with the continuity equation to describe unsteady (transient state) flow. According to Darcy's law, for one-dimensional vertical flow, the volumetric flux q(cm^3/cm^2/h) can be written as

$$q = -K\frac{\partial}{\partial Z}(h - Z) \quad \text{(cm/h)}$$

$$\text{or, } q = -K\left(\frac{\partial h}{\partial Z} - 1\right) \quad \text{(cm/h)} \qquad (1)$$

where K is the hydraulic conductivity, cm/h; h, the soil water pressure head (relative to the atmosphere), cm of water; and Z, the gravitational head, cm, (considered positive in downward direction).

In order to get a complete mathematical description for unsaturated flow, the continuity principle (Law of Conservation of Matter) is applied

$$\frac{\partial \theta}{\partial t} = -\frac{\partial q}{\partial Z} \quad (/h) \qquad (2)$$

where θ is soil moisture content, cm^3/cm^3; and t, time, h.

Substitution of equation (1) in equation (2) yields the partial differential equation

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial Z}\left[K\left(\frac{\partial h}{\partial Z} - 1\right)\right] \qquad (3)$$

Equation (3) is a second order, parabolic type of partial differential equation (Richards equation) which is non-linear because of the dependency of K and h on θ (linearity means that the coefficients in a differential equation are only functions of the independent variables Z and t). To avoid the problem of the two dependent variables θ and h, the derivative of θ with respect to h can be introduced, which is known as the specific water capacity C

$$C = \frac{d\theta}{dh} \quad (/\text{cm}) \qquad (4)$$

In equation (4) a normal, instead of a partial derivative notation is used, because h is considered here as a single value function of θ (no hysteresis).

Writing

$$\frac{\partial \theta}{\partial t} = \frac{d\theta}{dh} \cdot \frac{\partial h}{\partial t} \qquad (5)$$

and substituting equation (4) in equation (3) yields

$$C(h)\frac{\partial h}{\partial t} = \frac{\partial}{\partial Z}\left[K(h)\left(\frac{\partial h}{\partial Z} - 1\right)\right] \qquad (6)$$

In equation (6) the coefficients C and K are functions of the dependent variable h, but not functions of the derivatives $\partial h/\partial t$ and $\partial h/\partial Z$. Written in this form, equation (6) provides the basis for predicting soil water movement in layered soils of which each layer may have different physical properties.

Inital and Boundary Conditions

To obtain a solution for the one-dimensional vertical flow equation, equation (6) must be supplemented by appropriate initial and boundary conditions.

As initial condition (at t =0) the pressure head is specified as a function of the depth Z

$$h(Z, t = 0) = h_0 \qquad (7)$$

As hysteresis is not considered in this study, this condition is equivalent to

$$\theta(Z, t = 0) = \theta_0 \qquad (8)$$

One can then easily obtain the value of h (and vice versa) from the expression : $h = f(\theta)$.

To describe the boundary conditions one can distinguish between three types:

(a) Dirichlet condition: specification of the dependent variable, the pressure head

$$\left\{\begin{matrix} h(Z = 0, t) = h_u \\ h(Z = L, t) = h_1 \end{matrix}\right\} \qquad (9)$$

These conditions are equivalent to

$$\left\{\begin{matrix} \theta(Z = 0, t) = \theta_u \\ \theta(Z = L, t) = \theta_1 \end{matrix}\right\} \qquad (10)$$

(b) Neumann condition: specification of the derivative of the pressure head. For the soil water problem this condition means a specification of the flow through the boundaries

$$q(t) = -K(h)\left(-\frac{\partial h}{\partial Z} - 1\right) \qquad (11)$$

(c) 'Mixed' condition, a combination of the first two types. In particular this can specify h at the lower boundary and q at the upper boundary.

If the relative humidity (f) and the temperature of the air (T) as a function of time are known, and if it may be assumed that the pressure head at the soil surface is at equilibrium with the atmosphere, then $h(0, t)$ can be derived from the thermodynamic relation[1] :

$$h(0, t) = \frac{RT(t)}{Mg} \ln [f(t)] \qquad (12)$$

where R is the universal gas constant (8.314×10^7 erg/mole/K); T, the absolute temperature, K; g, acceleration due to gravity (980.665 cm/s^2); M, the molecular weight of water (18 g/mole); f, the relative humidity of the air (fraction); and h, bars. Knowing $h(0, t)$, $\theta(0, t)$ can be derived from the soil water retention curve.

For the present study, initial condition has been defined by equation (8) as

$$\theta(Z, t = 0) = 0.10 \tag{13}$$

and the upper boundary condition has been obtained by equation (12). The phreatic surface acts as lower boundary of the system in case of a shallow ground water table. The lower boundary condition has therefore been set as (Dirichlet type, equation (10)) :

$$\theta(Z = L, t) = \theta_s \tag{14}$$

where L is the depth of the ground water table; and the subscript s denotes saturated condition.

Soil Moisture Characteristics

For the present study, functional relations, as reported by Haverkamp, *et al*[2], for characterizing the hydraulic properties of a soil, were used. They compared six models, employing different ways of discretization of the non-linear infiltration equation in terms of execution time, accuracy, and programming considerations. The models were tested by comparing water content profiles calculated at given times by each of the model with results obtained from an infiltration experiment carried out in the laboratory. All models yielded excellent agreement with water content profiles measured at various times.

The infiltration experiments were done in the laboratory using a plexiglass column, 93.5 cm long and with 6 cm inside diameter uniformly packed with sand to a bulk density of 1.66 g/cm^3. The column was equipped with tensiometers at depths of 7, 22, 37, 52, 67 and 82 cm below the soil surface. Each tensiometer had its own pressure transducer. The changes of water content at different depths were obtained by gamma ray attenuation using a source of americium-241. A constant water pressure (θ = 0.10) was maintained at the lower end of the column, a constant flux (13.69 cm/h) was imposed at the soil surface (Z = 0) and initial condition as θ = 0.10 throughout the depth. The hydraulic conductivity and water content relationship of the soil was obtained by analysis of the water content and water pressure profiles during transient flow. The soil water pressure and water content relationship was obtained at each tensiometer depth by correlating tensiometer readings and water content measurements during the experiments. The following analytical expressions, obtained by a least square fit through all data points were chosen for characterizing the soil:

$$K = K_s \frac{A}{A + |h|^{\beta_1}}; \tag{15}$$

K_s = 34 cm/h ; $A = 1.175 \times 10^6$; and $\beta_1 = 4.74$

$$\text{and } \theta = \frac{\alpha(\theta_s - \theta_r)}{\alpha + |h|^{\beta_2}} + \theta_r \tag{16}$$

$\theta_s = 0.287$; $\theta_r = 0.075$; $\alpha = 1.611 \times 10^6$; and $\beta_2 = 3.96$

where subscript s refers to saturation, *ie*, the value of θ for which h = 0, and the subscript r to residual water content.

FIGURE 1
RELATIONSHIP BETWEEN SOIL WATER PRESSURE, WATER CONTENT AND HYDRAULIC CONDUCTIVITY FOR THE SOIL USED IN THE STUDY

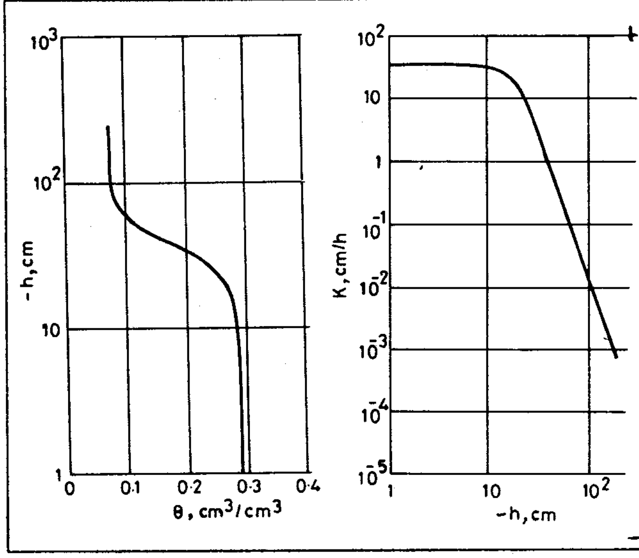


Fig 1 present the relationships between the soil water pressure h, the water content θ and the hydraulic conductivity K for the above soil used in this study.

Finite Difference Approximation

Equation (6) is a non-linear partial differential equation (PDE) because the parameters $K(h)$ and $C(h)$ depend on the actual solution of $h(Z, t)$. The non-linearity of the equation causes problems in its solution. Analytical solutions are known for special cases only. The majority of practical field problems can only be solved by numerical methods. In this respect one can use either explicit or implicit methods. Although an implicit approach is more complicated, it is preferable because of its better stability and convergence. Moreover, it permits relatively large time steps thus keeping computer costs low. For a given grid point at a given time, the values of the coefficients $C(h)$ and $K(h)$ can be expressed either from their values at the preceding time step (explicit linearization), or from a prediction at time (t+1/2 Δt) using a method described by Douglas and Jones[3] (implicit linearization).

Solving equaion (6) by a finite difference technique and appropriate initial and boundary conditions.

$$C \frac{\partial h}{\partial t} = \frac{\partial}{\partial Z}\left[K\left(\frac{\partial h}{\partial Z} - 1\right)\right]$$

$$\text{or } C \frac{\partial h}{\partial t} = \frac{\partial K}{\partial Z}\left(\frac{\partial h}{\partial Z} - 1\right) + K \frac{\partial^2 h}{\partial Z^2} \tag{17}$$

$$\text{or } \frac{C}{K} \frac{\partial h}{\partial t} = \frac{\partial^2 h}{\partial Z^2} + \frac{1}{K} \frac{\partial K}{\partial Z}\left(\frac{\partial h}{\partial Z} - 1\right)$$

Using implicit evaluation of the coefficients at time (t+1/2 Δt), that is, values for K and C are obtained at time (t+1/2 Δt), then pressure distribution is evaluated at time (t+Δt). The partial differential equation is approximated by a finite difference equation replacing ∂t and ∂Z by Δt and ΔZ, respectively.

Prediction (estimation of C_i^j and K_i^j)

From equation (17), by taking time step as $\Delta t/2$,

$$\frac{2C_i^j}{K_i^j}\cdot\frac{h_i^{j+1/2}-h_i^j}{\Delta t}=\frac{h_{i+1}^{j+1/2}-2h_i^{j+1/2}+h_{i-1}^{j+1/2}}{(\Delta Z)^2}$$

$$+\frac{1}{K_i^j}\cdot\frac{K_{i+1}^j-K_{i-1}^j}{2\Delta Z}\left[\frac{h_{i+1}^j-h_{i-1}^j}{2\Delta Z}-1\right]$$

where i refers to depth and j refers to time. Rearranging the terms,

$$-\frac{\Delta t}{(\Delta Z)^2}h_{i-1}^{j+1/2}+\left[\frac{2C_i^j}{K_i^j}+\frac{2\Delta t}{(\Delta Z)^2}\right]h_i^{j+1/2}-\frac{\Delta t}{(\Delta Z)^2}h_{i+1}^{j+1/2}$$

$$=\frac{2C_i^j}{K_i^j}h_i^j+\frac{1}{2}\frac{K_{i+1}^j-K_{i-1}^j}{K_i^j}\frac{\Delta t}{\Delta Z}\left[\frac{h_{i+1}^j-h_{i-1}^j}{2\Delta Z}-1\right] \quad (18)$$

Correction (estimation of h_i^j)

From equation (17), by taking time step as Δt,

$$\frac{C_i^{j+1/2}}{K_i^{j+1/2}}\cdot\frac{h_i^{j+1}-h_i^j}{\Delta t}=\frac{1}{2}\left[\frac{h_{i+1}^{j+1}-2h_i^{j+1}+h_{i-1}^{j+1}}{(\Delta Z)^2}+\frac{h_{i+1}^j-2h_i^j+h_{i-1}^j}{(\Delta Z)^2}\right]$$

$$+\frac{1}{K_i^{j+1/2}}\cdot\frac{K_{i+1}^{j+1/2}-K_{i-1}^{j+1/2}}{2\Delta Z}\left[\frac{h_{i+1}^{j+1/2}-h_{i-1}^{j+1/2}}{2\Delta Z}-1\right]$$

Rearranging the terms,

$$-\frac{1}{2}\frac{\Delta t}{(\Delta Z)^2}h_{i-1}^{j+1}+\left[\frac{C_i^{j+1/2}}{K_i^{j+1/2}}+\frac{\Delta t}{(\Delta Z)^2}\right]h_i^{j+1}-\frac{1}{2}\frac{\Delta t}{(\Delta Z)^2}h_{i+1}^{j+1}$$

$$=\frac{C_i^{j+1/2}}{K_i^{j+1/2}}\frac{h_i^{j+1}}{2}\frac{\Delta t}{(\Delta Z)^2}\left[h_{i+1}^j-2h_i^j+h_{i-1}^j\right]$$

$$+\frac{1}{2}\frac{K_{i+1}^{j+1/2}-K_{i-1}^{j+1/2}}{K_i^{j+1/2}}\frac{\Delta t}{\Delta Z}\left[\frac{h_{i+1}^{j+1/2}-h_{i-1}^{j+1/2}}{2\Delta Z}-1\right] \quad (19)$$

When equation (18) or (19) is applied at all nodes, the result is a system of simultaneous linear algebraic equations with a tridiagonal coefficient matrix with zero elements outside the diagonals and unknown values of h. In solving this system of equations, a so-called direct method was used by applying a tridiagonal algorithm of the kind discussed by Remson, *et al*[4].

The above numerical scheme was tested by comparing the simulated water content profiles at various times with the quasi-analytical solution of Philip, for the condition of a constant pressure at the soil surface (as reported by Haverkamp, *et al*[2]). The model yielded close agreement with water content profiles obtained through quasi-analytical solution of Philip.

Steady state evaporation rates were estimated by applying equation (1) for top two vertically adjacent nodal points after obtaining the equilibrium moisture profile.

$$q=K_{1+1/2}^j\left(\frac{h_2^j-h_1^j}{\Delta Z}-1\right) \quad (20)$$

where, $K_{1+1/2}^j=\sqrt{K_1^j\,K_2^j}$

Geometric mean of K was taken following suggestions of Haverkamp and Vauclin[5].

RESULTS AND DISCUSSIONS

The actual evaporation rate is governed by the atmospheric conditions, transmitting properties of the soil and the water table depth. While the maximum possible (potential) rate of evaporation from a given soil depends only on atmospheric conditions, the actual flux across the soil surface is limited by the ability of the porous medium to transmit water from below.

For the given external evaporative conditions and the water table depth, the equilibrium moisture profile was obtained by using the numerical scheme presented in the paper and assuming the pressure head at the soil surface to be in equilibrium with the surrounding atmosphere. The initial and boundary conditions were defined by the equations (13), (12), and (14), respectively. The rate of loss of water (Darcian flux q) served as a measure of the evaporation rate once the steady state was attained. The evaporation rates for the given external conditions (temperature and humidity of air) and various water table depths were evaluated.

Since the steady state evaporation rates are estimated by considering the two vertically adjacent nodal points (equation (20)), the size of the depth interval plays an important role. It was found that the numerical scheme is stable only when

$$\frac{\Delta t}{(\Delta Z)^2}\le 2.5 \quad (21)$$

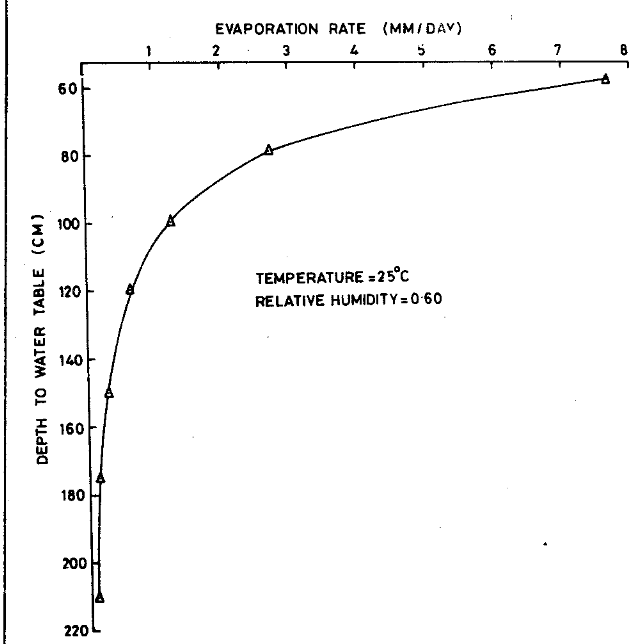
where Δt is the time step, s; and ΔZ, the depth interval, cm. At greater water table depths, matrix dimensions (number of nodes, number of time steps) may become quite large (insufficient virtual address space to complete the link). Also it takes more time to attain the equilibrium moisture profile in case of a deeper water table. Therefore, different sets of ΔZ and Δt had to be used for different water table depths, for condition of stability (equation (21)).

In order to study the relation between evaporation rate and depth to the water table, solutions were obtained by varying the water table depth and keeping the evaporative conditions same (temperature = 25°C, relative humidity = 0.60). Ample time was allowed for steady state to be attained at each depth. The values of soil water pressures at different nodes during consecutive time steps gave assurance that steady state had been attained. At steady state the rate of loss of water (the flux q), which is approximately the same at every depth, equals the evaporation rate. Table 1 presents the data on depth to water table, depth interval, time step and the estimated evaporation rates. The results are also shown graphi-

TABLE 1
STEADY STATE EVAPORATION RATES AS A FUNCTION OF WATER TABLE DEPTH
(T = 25° C, f = 0.60, $h(0, t)$ = – 703.41 cm)

Depth to Water Table, L, cm	Depth Interval, ΔZ, cm	Time Step, Δt, s	$\frac{\Delta t}{(\Delta Z)^2}$	Number of Vertical Nodes	Evaporation Rate, mm/day
60	2	10	2.5	31	7.659
80	4	40	2.5	21	2.690
100	4	40	2.5	26	1.257
120	4	40	2.5	31	0.672
150	5	60	2.4	31	0.298
175	5	60	2.4	36	0.172
210	6	90	2.5	36	0.086

FIGURE 2
RELATION BETWEEN EVAPORATION RATE AND DEPTH TO WATER TABLE



cally in Fig 2, where the steady state evaporation rates are plotted against the depth to the water table.

It can be expected that for a particular soil and meteorological condition, the evaporation rate remains essentially constant and fixed by weather, if the water table depth does not exceed a certain value. With the water table at greater depths, the evaporative flux decreases markedly because the soil becomes the limiting factor. Fig 2 shows that under the conditions studied, evaporation was soil limited. This figure demonstrates that as the water table is lowered from 60 cm to 100 cm, the evaporation rate decreases markedly with depth. However, further lowering reduces the evaporation rate only slightly.

Upward movement and evaporation of water is possible with the water table as deep as 150 cm and although the rate will be slow, accumulation of harmful amounts of soluble salts is possible if the groundwater is sufficiently saline and if sufficient time is allowed. In such a case it may be feasible to either lower the water table or periodically leach out the salts by the application of excess water at the surface.

When the evaporation rate is low and is limited by external conditions, a large increase in the evaporation rate causes only a small increase in the suction at the soil surface. Evaporation under such conditions is virtually independent of the depth to water table and the capillary conductivity of the soil. The range of external conditions for which this is the case depends upon the depth-to the water table. The shallower the water table the greater the range over which evaporation is controlled by external conditions.

CONCLUSIONS

A numerical model study has been carried out to examine the steady state evaporation from shallow water table through a soil. The evaporation rate is shown to be related to the depth to water table for a particular soil. The dependence of the actual steady state evaporation rate on water table depth (as demonstrated in Fig 2), which can be computed with the aid of the approach presented in the preceding pages, might be most useful in hydrologic practice. The extent to which the above results can be applied quantitatively to the field depends upon the correspondence between capillary conductivity values and those existing in the field. The soil data employed might be less precise than desirable. In addition, it might be impossible to take into account adequately the variability of field soils.

Steady state conditions were assumed throughout this study. In nature, however, the systems considered are seldom in such a state, principally because of the variations in meteorological conditions, in soil salt content, and in water table depth. The changes in soil salt content and water table are relatively slow, and therefore, their short-period effects might be negligible. Their long-range influences, however, could be of very considerable importance and should be taken into account, with different experimentally determined soil parameters and measured or predicted water table depths. Also under various conditions, the thermal transfer of water might significantly change the evaporation rate. In this study, the thermal transfer of liquid water was entirely neglected.

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