CARRIER ESTIMATION FOR QAM RECEIVERS USING HIGHER-ORDER CUMULANTS

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ABSTRACT
This paper explores the use of Higher Order Statistics for the recovery of the carrier phase and frequency offset in a QAM receiver operating in an additive Gaussian noise channel. The technique is based on newly established properties of the 3rd and 4th-order cumulants of the demodulated QAM signal. The resulting method may be thought of as higher-order extension of the second order automatic frequency estimators (AFE) commonly reported in the literature [2][3]. The performance is analyzed and simulated for a QAM system in the presence of channel noise. The results are compared to the autocorrelation-based estimators for various frequency offset values and SNR. The issues related to the bias and variance of the higher order moment estimators are addressed and their effect on the estimation accuracy is analyzed in light of the results.

Keywords
Higher order statistics; Frequency estimation. QAM receivers.

1. INTRODUCTION
In QAM receivers such as those used in wireless communications, satellite and cable modems [6] each data burst includes a sequence of training bits (preamble) that are exploited by the receiver for burst identification as well as carrier and symbol timing recovery.

Carrier recovery schemes based on feedback and feed forward structures and operating on a single sample per symbol are widely used in burst receivers. Feed forward schemes are well suited for burst-mode systems due to their short acquisition time. They also avoid some of the problems associated with feedback systems, such as hang up issue during acquisition [5]. The implementation complexity of such schemes as the ML estimators for frequency offset can be quite high when high performance is desired.

To remedy the complexity issue, simplified schemes, based on the ML estimators have been derived; such as the frequency offset method in [3]. These estimators encompass computing the autocorrelation of the baseband signal using the match filter output data. The actual offset may then be derived directly from the angle of the complex autocorrelation, averaged over a number of symbols.

A main caveat in estimators based on second order statistics is their sensitivity to additive noise, which causes performance to degrade in low SNR conditions [2].

Higher-order statistics (HOS) have shown promising potential in a number of signal processing applications, and are of particular value when dealing with a mixture of Gaussian and non-Gaussian processes [8]. Their inherent blindness to Gaussian noise (both white and color) makes them particularly attractive in estimation and detection problems in low SNR conditions.

In this paper we investigate the effectiveness of a new approach for estimating the carrier frequency offset of a QAM signal using the fourth-order cumulant (FOC) of the demodulated baseband signal. The estimation scheme is based on multiple samples per symbol and thus does not rely on a proper sampling of the output of the matched filter. It does however assume that the symbol frequency is correctly available. Since only the higher order cumulant is used, the effect of additive Gaussian noise is theoretically less pronounced than in second-order methods. The estimator does not require an apriori pre-amble sequence, but rather any sequence with a known symbol statistics – for instance rotating symbols, or alternating diagonal symbols. The results for various frequency offsets and various noise levels are generated and compared (section 3.2). The issues related to the bias and variance of the higher order moment estimators are addressed and their effect on the estimation accuracy is analyzed in light of the results in section 3.4.

2. CARRIER ESTIMATION

2.1 Signal Model
The baseband samples of a demodulated QAM signal can be represented as:

$$Z_n = S_n \cdot e^{j\left(2\pi\cdot\Delta f\cdot T_{sa}+\theta\right)} + \upsilon_n \;;\; n = 0\ldots N-1 \quad \textbf{Equ 1}$$

Where S_n is the complex symbol, Δf is the frequency offset that needs to be estimated; T_{sa} is the sampling frequency of the matched filter, assumed to be an integer multiple *K* of the symbol rate; *n* is the sample index and *N* is the total number of samples in the observation window. The carrier phase θ is an unknown phase error and υ_n is an additive Gaussian noise with independent in-phase and quadrature components, each with zero mean and a given variance. For ease of notation, the frequency offset is written as an incremental phase error normalized by the sampling rate $\Omega = 2\pi \cdot \Delta f \cdot T_{sa}$. Thus Equ 1 is written as:

$$Z_n = S_n \cdot e^{j[\Omega n+\theta]} + \upsilon_n \qquad \textbf{Equ 2}$$

Both phase and frequency offsets are assumed unknown but non-random parameters, and are constants for the duration of the burst. The symbol frequency is assumed to be known at the receiver, though the actual timing phase is not required.

2.2 Frequency Offset Estimation with FOC

For a given complex sequence Z_n, the horizontal slice of the 4th-order cumulant is a one-dimensional function of lag and may be expressed in terms of the second order moments as [4] [8]:

$$C_4(L) = E\left[z_n^2 z_{n+L}^{2*}\right] - \left\{E\left[z_n^2\right]\right\}^2 - 2\left\{E\left[z_n z_{n+L}^*\right]\right\}^2 \quad \textbf{Equ 3}$$

Where the signal value at lag *L*, the autocorrelation and the 4th-order moment functions are given by :

$$Z_{n+L}^2 = S_{n+L}^2 \cdot e^{j(2\Omega n + 2\Omega L + 2\theta)}$$

$$E[Z_n Z_{n+L}^*] = E[S_n S_{n+L}^*] \cdot e^{-j\Omega L}$$

$$E[Z_n^2 Z_n^{2*}] = E[S_n^2 S_n^{2*}] \cdot e^{-2j\Omega L}$$

respectively. It is assumed that *L* represents a number of samples spanning an integer number of symbols. As a result, the 4th order moment function may be evaluated for various data patterns. For instance, if *L* represents 1 symbol and given the 4 corner symbols of a QAM constellation:

$$S_1 = a + ja \; ; \qquad S_2 = a - ja \; ;$$

$$S_3 = -a - ja \; ; \qquad S_4 = -a + ja$$

and assuming the preamble consists of 2 alternating diagonal corner symbols, and a raised cosine Nyquist filter is used, then it may be shown that higher order correlations may be expressed in terms of the symbol energy:

$$E[S_n S_n^*] = E_s \quad ; \quad E[S_n S_{n+L}^*] = -E_s$$

$$E[S_n^2 S_n^{2*}] = 1.5 E_s^2 \quad ; \quad E[S_n^2 S_{n+L}^{2*}] = 1.5 E_s^2$$

Substituting in Equ 3 above, we get:

$$C_4(L) = 1.5 E_s^2 \cdot e^{-j2\Omega L} - E_s^2 - 2\left(-E_s e^{-j\Omega L}\right)^2$$

after grouping terms, the normalized kurtosis is:

$$K = \frac{C_4(L)}{C_4(0)} = \frac{1}{1.5}\left\{1 + 0.5 e^{-j2\Omega L}\right\}$$

, and after expansion,

$$K = \frac{1}{1.5}\left\{\left[0.5 + \cos^2(\Omega L)\right] - j\sin(\Omega L)\cos(\Omega L)\right\}.$$

The frequency offset estimate is then:

$$\hat{\Omega} = -\frac{1}{L}\arctan\left[\frac{imag(K)}{real(K) - 0.5/1.5}\right] \quad \textbf{Equ 4}$$

This estimator may be thought of as a higher-order extension to the one based on the 2nd order (autocorrelation) function given in [2] as:

$$\Omega = \frac{1}{L}\arctan\left[\frac{imag\ (R)}{real\ (R)}\right] \quad \textbf{Equ 5}$$

where *R* is the autocorrelation function at lag *L*:

$$R(L) = E[Z_n Z_{n+L}^*]$$

2.3 Phase Offset Estimation

For a given complex sequence Z_n, the horizontal slice of the 3rd-order cumulant is a one-dimensional function of lag given by [4][8]:

$$C_3(L) = E[Z_n^* Z_{n+L}^2] \quad \textbf{Equ 6}$$

Given the 4 corner symbols of a QAM constellation (described above) and assuming the effect of the Nyquist filter were ignored for now, then it may be shown that:

- If any 2 diagonal symbols or if all 4 symbols are sent, and regardless of the value of the phase and frequency offset, then the skewness (i.e. the value of $C_3(L)$ at lag *L*=0) is always null: $C_3(0) = 0$.
- If 2 diagonal symbols are sent (for instance, S_1, S_3), and *L* represent the number of samples 1 symbol away, then the 3rd-order cumulant becomes:

$$C_3(L) = E[S_n^* S_{n+L}^2] \cdot e^{j(2\Omega L + \Omega n + \theta)}$$

$$C_3(L) = 2a^3\left\{\left[\cos(x) - \sin(x)\right] + j\left[\sin(x) + \cos(x)\right]\right\}$$

where : $x = 2\Omega L + \Omega n + \theta$

If the real and imaginary parts of are added, we get:

$$T \equiv real\left[C_3(L)\right] + imag\left[C_3(L)\right]$$

$$T = 4a^3 \cdot \cos(2\Omega L + \Omega n + \theta)$$

Since the average symbol energy is: $E_s = 2a^2$ then we can write the normalized versions of the above entities as:

The normalized 3rd-order cumulant:

$$\frac{C_3(L)}{E_s^{3/2}} = \frac{\sqrt{2}}{4}\{[\cos(x) - \sin(x)] + j[\sin(x) + \cos(x)]\}$$

The normalized sum of the real and imaginary:

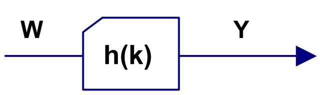
$$\frac{T}{E_s^{3/2}} = \left(\sqrt{2}/2\right)\cdot \cos(2\Omega L + \Omega n + \theta)$$

If the frequency offset is zero (or known), then the phase may be estimated as:

$$\hat{\theta} = arcsos\left[\frac{2}{\sqrt{2}}\frac{T}{E_s^{3/2}}\right] \qquad \textbf{Equ 7}$$

2.4 Effect of the Nyquist Filter

The results of the previous section did not account for the presence of a raised cosine filter. In general [9], the cumulant of order P at the output Y of a filter $h\ (n)$ (denoted by $C_P[y]$) may be expressed in terms of the cumulant or order P at the input W of the filter (denoted by $C_W[y]$), and the sum of the P^{th} power of the coefficients:



$$C_P[y] = C_P[W]\cdot \sum_k (h_k)^P$$

Similarly, the normalized cumulant, for example the 4th order normalized by the 2nd order, and defined as: $K_{(4,2)}[y] \equiv C_4[y]/C_2[y]$ can be expressed as:

$$K_{(4,2)}[y] = \left[\sum_k (h_k)^4 \Big/ \left|\sum_k (h_k)^2\right|^{4/2}\right] K_{(4,2)}[y]$$

3. Performance Analysis: Frequency Estimate

3.1 Simulation Data

A passband QPSK transmitter/receiver system (Figure 1) is modeled. The carrier frequency was set to 5 MHz and the symbol rate to 160 Ksymb/sec. The transmitter and receiver Nyquist filters are root-raised cosine with a rolloff of 0.25. The frequency estimation is done using blocks of 100 symbols at the output of the receiver's Nyquist. An oversampling of 8 samples / symbol is used. The frequency offset was varied from 200 to 10000 Hz. Noise is added to cover a spectrum SNR range from 8 to 40 dB.

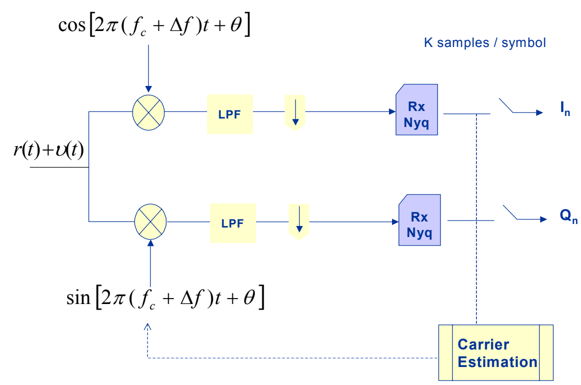


Figure 1: Carrier recovery at the receiver

3.2 Simulation Results

The table below is a summary of the frequency estimation using both 2nd and 4th order methods.

Method	SNR	Δf=200 Hz	2 kHz	10 kHz
2nd order	40	199 Hz	2000 Hz	10000 Hz
4th order	dB	200	2007	10026
2nd order	20	190	2002	9999
4th order		198	2009	10024
2nd order	15	205	1982	9992
4th order		202	1990	10016
2nd order	8	221	1954	9992
4th order		198	1972	10016

3.3 Analysis

It is observed that in clean condition, the 2nd-order estimator is more accurate for large values of ΔF, and the 4th order estimator is more accurate for smaller values. As the channel gets noisy, both estimators degrade. However the level of degradation of the 4th order one is not as pronounced at lower values of ΔF.

The above suggests that even though the 4th order estimator is theoretically more robust to Gaussian noise, the high variance of that estimator when computed from a block of 100 symbols is too high and outweighs the benefit of noise robustness. It was also observed as different runs of the same setup leads to a relatively wide range in the frequency estimation. A possible explanation for the lower accuracy of the 4th order case is the bias and variance of the estimators of the 4th-order statistics when computed from a finite set of data points, as detailed in the next section.

3.4 Bias and Variance of the HOS estimators

When estimating statistical entities using time averages (in Equ 3 and 6), a variance and sometimes a bias is introduced and is more pronounced in the higher order moments than it is in the 2nd order ones.

3.4.1 Case of the baseband QAM signal

The demodulated baseband signal given in Equ 2 may be thought of as a complex sinusoid whose frequency is the result of inter-modulation between the symbol rate and the frequency offset (Ω). We've shown in [4] that the estimators for the 3rd and 4th-order statistics of a deterministic sine wave are biased whenever the segment size is not an integer number of periods. This bias term consists of higher frequency cosine and sine terms and may be significantly reduced if the estimators are computed for a few overlapping segments and then averaged. Therefore computing the higher moments from a single block of data is not effective in reducing the bias, even if a large segment size is used. We also note that there is only 1 bias term in the second moment of a sine wave, 2 terms in the 3rd moment and 3 terms in the 4th moment, thus the estimation error based on the 4th order is at least 3 times as high as the one based on the 2nd order.

3.4.2 Case of Gaussian noise

The time-average estimator of the 4th-order statistics of a Gaussian process is only asymptotically unbiased, thus it contains a bias term for a finite segment size. As shown in [4], an unbiased estimator for the kurtosis (i.e. $C_4(0)$) may be devised, but it is not easy to device one for the general expression of $C_4(L)$. As a result, computing Equ 3 from a finite data segment will always contain a bias term that is function of the noise energy, and therefore increases with decreasing SNR. In addition to the bias, the variance of the time-average estimators of the 2nd, 3rd, and 4th -order statistics are function of the process variance and increase exponentially with the order. We've shown in [4] these variances to be:

$$Var[M_2] = \frac{2\upsilon^2}{N}; \quad Var[M_3] = \frac{15\upsilon^3}{N}; \quad \text{and}$$

$$Var[M_4] = \frac{96\upsilon^4}{N};$$

where υ is the variance of the underlying process (i.e the Gaussian noise energy). As a result, the segment size N has to be significantly increased (by $48\upsilon^2$) in order to bring the variance of the 4th order moment at par with the 2nd order.

4. CONCLUSION

We've explored the use of higher order statistics for the general problem of carrier recovery in a QAM receiver. New estimators for the carrier phase and frequency offset were developed based on newly established expressions of the 3rd and 4th-order cumulants of the demodulated QAM signal. The performance in noise, compared to 2nd-order estimator shows mixed results for small and large frequency offset values; however we did verify that the higher order estimator is more robust to noise for small values of frequency offsets, though it is not the case for larger ones. Clearly the improvement depends on the ability to find better ways to compute the higher order statistics in a way to reduce the large bias and variance, when computing them from a finite data set. If this can be done, then more noise-robust methods for carrier estimation may be developed based on higher order statistics.

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