FIXED INCOME – EQUITY - ECONOMICS

1. ***Spot Yields and Spot Yield Curves***

a) Yield to Maturity is a popular way of characterizing the return that should be expected from an investment in a bond. But, unless the YIELD CURVE is FLAT and COUPON REINVESTMENT rates can be expected to equal a bond's YTM, HORIZON RETURN is a Better measure of the probable investment portfolio from a bond

b) YTM is ONE SINGLE DISCOUNT Rate that if used to discount all of a bond's future cash flows to their present value, will produce a present value that equals the price of a bond. But, this is appropriate ONLY when Yield Curves are FLAT

c) When Yield Curves are NOT FLAT, then the 1st year's Cash Flow should be discounted by the 1-year rate, the 2nd year's Cash Flow by the 2-year rate, etc.

d) These SERIES of Discount Rates, each of which should be Applied ONLY to the Cash Flows generated in a Particular year are called **SPOT RATES**

For Example: Note the difference between YTM (r) and Spot Yields (r_n)

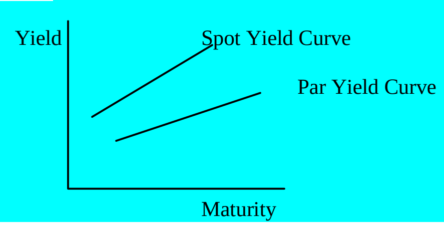
YTM → $P_B = [CF_1/(1+r)^1] + [CF_2/(1+r)^2] + [CF_3/(1+r)^3] + \ldots + [CF_n/(1+r)^n]$

SPOT→ $P_B = [CF_1/(1+r_1)^1] + [CF_2/(1+r_2)^2] + [CF_3/(1+r_3)^3] + \ldots + [CF_n/(1+r_n)^n]$

- Since the price of the bond is the price quoted in the market, both of these valuation models should produce the same result. Thus, a YTM is actually some sort of AVERAGE of the Series of SPOT RATES that can also be used to value the bond.
- Since YTM is some sort of weighted average of the spot rates, YTM tends to be MORE STABLE than the spot yields
- The **PAR YIELD CURVE** is the pictorial representation of the yields that theoretical newly issued bonds of different maturities should command in the market As newly issued bonds sell at par, this is the PAR YIELD CURVE (coupons correspond to YTM)
- The **SPOT YIELD CURVE** is the pictorial representation of the spot rates that apply to theoretical, newly issued ZERO COUPON bonds of various maturities.
- If YTM are an average of spot rates, the Par Yield Curve should be BELOW the SPOT Yield Curve when the term structure of interest rates is POSITIVELY SLOPED (Long-term rates are higher than short-term interest rates)

For Example:

Maturity	**Spot Rate**	**Average Yield**
1	4.0%	
2	5.0%	
3	6.0%	
4	7.0%	
5	8.0%	6%

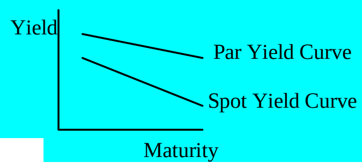


- NOTE: This is a positively sloped Interest Rate Structure
- WHEN the TERM STRUCTURE of Interest Rates is POSITIVELY SLOPED, the YTM on an n-year bond will be LESS than the Nth Year Spot Rate. This will be true across the entire yield curve

- When the term structure of interest Rates is INVERTED (long-term interest rates are LOWER than short-term rates), then →

For Example:

Maturity	Spot Rate	Average Yield
1	8.0%	
2	7.0%	
3	6.0%	
4	5.0%	
5	4.0%	6.0%



- When the term structure of interest rates is inverted, the n-year YTM will be above the n-year spot rate: an INVERTED Spot rate yield curve will lie below the corresponding inverted par yield curve

2. ***Calculating Spot Rates from Par Yields***

a) The IDEAL way to price bonds is to use the spot yields as the discounting factors applicable to the coupon and principal payment of cash flows. But, except for zero coupon bonds, spot yields are NOT directly observable. But, it is possible to calculate the Spot rate indirectly from a series of current coupon bonds of different maturities through the process of **BOOTSTRAPPING**

For Example: Suppose there are a series of current coupon bonds with maturities shown below. One can calculate the spot yields from the reiterative procedure

Maturity (yrs)	Coupon	Price	Par Yield
0.5	6.0%	100	6.00%
1.0	6.5%	100	6.50%
1.5	7.0%	100	7.00%
2.0	7.5%	99	8.05%

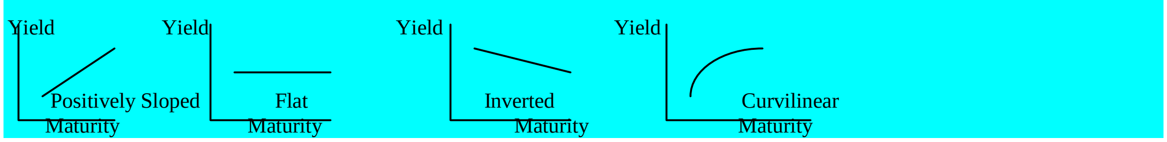
1. The ANNUALIZED SPOT YIELD on the 6 month bond is 6.00% because it is Effectively a ZERO COUPON Bond
2. The ANNUALIZED SPOT YIELD on the 1 year bond is calculated as follows (letting X stand for the PERIODIC SPOT Rate)
   $\boxed{P_B = [C/(1+r_{6\text{ month}})^1] + [C/(1+r_{1\text{ year}})^2]}$ → 100 = $[3.25/(1+.03)^1] + [(3.25+100)/(1+X)^2]$ → 1+X =1.032545
   The Annualized 1 year Spot Rate is 6.51%
3. The ANNUALIZED SPOT YIELD on the 1 ½ year bond is
   $\boxed{P_B = [C/(1+r_{6\text{ month}})^1] + [C/(1+r_{1\text{ year}})^2] + [C/(1+r_{1\frac{1}{2}\text{ year}})^3]}$ →100 = $[3.5/(1.03)] + [3.5/(1.03254)^2] + [(3.5+100)/(1+X)^3]$
   1+X =1.035119
   The Annualized 1 ½ Year Spot Rate is 7.02%
4. The ANNUALIZED SPOT YIELD on the 2 year bond is
   $\boxed{P_B = [C/(1+r_{6\text{ month}})^1] + [C/(1+r_{1\text{ year}})^2] + [C/(1+r_{1\frac{1}{2}\text{ year}})^3] + [C/(1+r_{2\text{ year}})^4}$
   $99=[3.75/1.03]+[3.75/(1.03254)^2]+[3.75/(1.035119)^3]+[103.75/(1+X)^4]$
   The Annualized Spot Yield is 8.13 %

- With the YIELD CURVE POSITIVELY SLOPED (YTM rise as Maturity increases), the Spot Yield Curves are ABOVE the YTM. This is the same conclusion determined from the logic previously. Conversely, when the Yield Curve is INVERTED (YTM declines as Maturity increases), the Spot Yields will be BELOWY the YTM

3. *Forward Rates, Spot Rates and the Pure Expectations Theory of the Term Structure of Interest Rates*

- The Term Structure of Interest Rates describes the Shape of the Yield Curve

Yield Curve Shapes

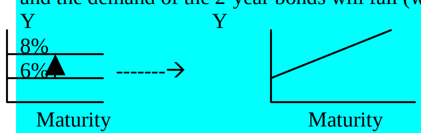


- The Most popular theory to explain why the Yield curve takes on various shapes at different times is the **Pure Expectations Theory**
- The Pure Expectations Theory asserts that the Shape of the Yield Curve results from Bond Prices Adjusting to INVESTOR Expectations about what the Future Interest Rates will be.

To Illustrate: Suppose the yield curve is FLAT with bonds of ALL maturities selling to yield 6%. Now, suppose investors come to expect that interest rates are going to increase over the next year to 8%. With the yield curve flat at 6%, but expected to be shifting to 8% in a year, investors with a 2-year INVESTMENT HORIZON would reason as follows:

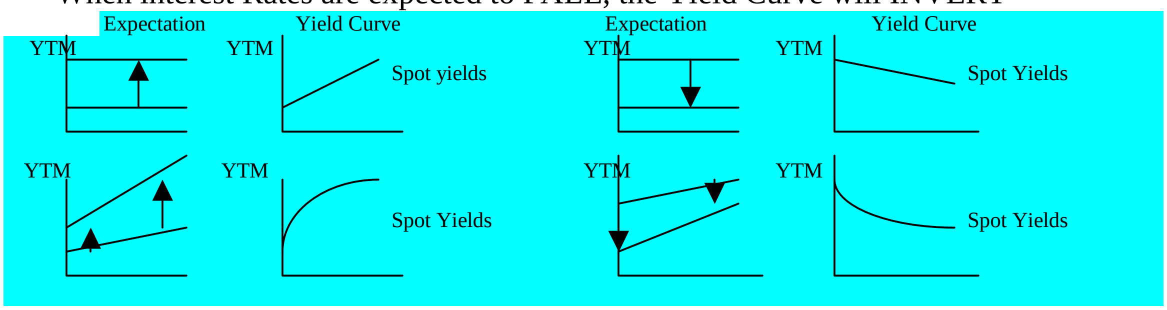
1. Buy a 2-year bond and earn a YTM of 6% over the next 2 years, OR
2. Buy a 1-year bond at the current rate of 6% and then roll it over, 1 year hence, into another 1-year bond whose yield is expected to be 8% at that time, to earn approximately 7% over the 2 years.

After reasoning as such, rational investors with 2 year investment horizons would choose option 2. As all rational investors with 2-year time horizons do this at the same time, the demand (and price) of 1-year bonds will rise (with falling yields) and the demand of the 2-year bonds will fall (with rising yields). The shape of the yield curve will then tilt upward.



When Expectations shift; Yield Curve will Reshape as above

- The Changing expectations about future interest rates will set dynamic forces into motion within the bond market that will result in a change in the shape of the yield curve.
- When interest Rates are expected to RISE, the Yield Curve will become POSITIVELY SLOPED
- When interest Rates are expected to FALL, the Yield Curve will INVERT



a) The Relationship Between Forward Rates and Spot Rates

- If the Bond Market behaves in accordance with the PURE EXPECTATION Theory of Term Structure of Interest Rates, then the Shape of the Yield Curve can tell us what market participants expect interest rates will be in the future

For Example: Assume the Currently Observable Spot Yield Curve is INVERTED with the following term Structure

Maturity	Spot Yield
1 Yr.	8.05%
2 Yrs.	7.90%
5 Yrs.	7.70%
10 Yrs.	745%

Given this Yield Curve, Investors with 2-year time horizons could make either of 2 alternative investments:

1. Buy 2-year bonds and hold them until maturity, thereby ending up with the following amount per dollar invested
   Ending Value = $(1+r_2)^2 = (1.079)^2$
2. Buy 1-year bonds, hold until maturity, and then roll the proceeds over into a new 1-year bond and hold until maturity. The ending value realized will be:
   Ending Value = $(1+r_1)^1 + (1+r_2)^2 = (1.0805)({}_1f_1)$ where $({}_1f_1)$ is the 1 year forward rate on 1 year bonds, it is the rate expected on 1-year bonds1 year into the future

Unfortunately, it is unknown what the 1-year interest rate will be one year in the future. But if the bond market is efficient and the pure expectations theory is correct, then investing in a 1-year bond and rolling the proceeds over for an additional year should produce the same result as if the investor invests in a 2-year bond and holds it until maturity. Therefore, according to the PURE EXPECTATIONS THEORY, both alternatives should produce the same result.

$(1+r_{m+n})^{m+n} = (1+r_m)^m (1+{}_mf_n)^n$ where n is the number of years in the future (m is the # years rate)

Here

$(1+r_2)^2 = (1+r_1)^1(1+{}_1f_1)^1$

$(1.079)^2 = (1.0805)(1+{}_1f_1)^1$

${}_1f_1 = 7.75\%$

- In effect, the pure expectations theory asserts that the current slope of the spot yield curve between 1 and 2 year maturities implicitly contains a prediction that 1-year rates will be 7.75% in the future. This is called the IMPLIED 1-YEAR FORWARD Rate for 1-YEAR Bonds
- In General, the Pure Expectations Theory of Term Structure of Interest Rates asserts that Spot Rates and Forward Rates are related according to the following equation: (with 1X per year Compounding)

$$(1+r_{m+n})^{m+n} = (1+r_m)^m(1+{}_mf_n)^n$$

where

${}_mf_n$ = the m-year forward rate on a bond that matures in n years (the yield on n-year bonds that is expected to prevail m years in the future)

r_{m+n} = the currently observable yield on a bond that matures m+n years from now

r_m = the currently observable yield on a bond that matures m years from now

For Example: for the same currently observable yield curve as in the example above, what should the 2 Year forward rate be for 3-year bonds

Answer: The bond that will be a 3-year bond 2 years from now is currently a five-year bond. Hence:

$(1+r_{m+n})^{m+n} = (1+r_m)^m(1+{}_mf_n)^n$

$(1+r_5)^5 = (1+r_2)^2(1+{}_2f_3)^3$

$(1.077)^5=(1.079)^2(1+{}_2f_3)^3$

${}_2f_3=7.57\%$

- If the bond pays periodic interest (2X per year) then, the relationship between spot rates and forward rates on a PERIODIC BASIS is as follows:

  $(1+r_{p(mp+np)})^{mp+np} = (1+r_{p(mp)})^{mp}(1+{}_{mp}f_{np})^{np}$

  ${}_{m}f_{n} = \#({}_{mp}f_{np})$

  where

  ${}_{mp}f_{np}$ = the m-period forward rate on a bond maturing in n_p periods

  r_{mp+np} = the currently observable periodic spot rate applicable to a zero coupon bond maturing in m_p+n_p periods

  $r_{p(mp)}$ = the currently observable periodic spot rate applicable to a zero coupon bond that matures in m_p periods

  # = the number of coupon payments per year

For Example; for the following spot yield curve, what is the implied 3-year rate in 2 years, assuming the bonds pay interest semi-annually?

Year	Period	Spot Yield (annualized)
1	1	3.0%
	2	3.4%
2	3	3.8%
	4	4.2%
3	5	4.8%
	6	5.4%
4	7	5.8%
	8	6.4%
5	9	6.8%
	10	7.2%

Answer: We are trying to find the 2-year forward rate on a 3-year bond (${}_{m}f_{n} = {}_{2}f_{3}$)

$r_m = r_2 = 4.2\% \rightarrow r_{pm} = (4.2/2) = 2.1\%$

$r_{m+n} = r_{3+2} = r_5 = 7.2\% \rightarrow r_{P(m+n)} = (7.2/2) = 3.6\%$

$(1.036)^{10} = (1.021)^4(1+{}_{4}f_{6})^6$

${}_{4}f_{6} = 4.61\%$

${}_{2}f_{3} = 2(4.61\%) = 9.22\%$

b) Spot Rates as Geometric Averages of Forward Rates

- If the Pure Expectations Theory of the Term Structure of Interest Rates is correct, then the m-period forward rate for an n-period zero-coupon bond is an UNBIASED ESTIMATE for what an n-period zero-coupon bond's spot yield will be m-periods in the future. This analysis can be used to determine the relationship between spot rates and forward rates
- In general, any observable n-year spot rate, which is the discount rate that should be used to price an n-year zero coupon bond, can be explained by the currently observable 1-year rate and a series of future expectations about what 1-year rates will be 1, 2, 3 up to (n-1) years hence:

  $(1+r_n)^n = (1+r_1)(1+{}_{1}f_1)(1+{}_{2}f_1)\ldots(1+{}_{n-1}f_1)$

  This means that the observable n-year spot rate is really the geometric average of the observable 1-year spot rate and a series of future 1-year forward rates

  $r_n = [(1+r_1)(1+{}_{1}f_1)(1+{}_{2}f_1)(1+{}_{3}f_1)\ldots(1+{}_{n-1}f_1)]^{1/n} - 1$

  where

  r_n = the observable n-year spot rate

  r_1 = the observable 1-year spot rate

  ${}_{m}f_1$ = the implied 1-year spot rate m years hence (it is the m-year forward rate on a 1-year bond)

- This means that the SPOT YIELD CURVE is Really the Observable result of the current 1-year rate and the market's expectations about a whole series of future 1-year rate Expectations
- Thus, the Observable spot yield that should exist for any maturity can be calculated if an analyst has a series of expectations about what various spot rates re likely to be in the future, using various relationships
- NOTE: SHORT TERM Rates are Set by the FED, LONG TERM Rates are determined by the Market's Expectations
- Thus, short-term yields fluctuate more than long-term yields; being essentially an average of a series of current and expected future yields, the yield on long-term bonds exhibit greater stability because its volatility (measured by σ) behaves according to the Law of Averages:

$\sigma_{\text{Long Term Rate}} = [\sigma_{\text{Short Term Rate}} / (n)^{1/2}]$

For Example: The 1-year spot rate is currently 5%. The consensus expectation is that 1-year interest rates will fall 100 basis points in 1 year and then rise 100 basis points in each of the next 2 years. You expect interest rates to stay unchanged for 2 years and then rise 100 basis points. What should a 4-year, zero coupon bond be trading for in the market? Would you buy it if it had to be held until maturity?
Answer: The 4-year spot rate represents the yield at which a zero coupon bond should trade. This spot rate, in turn, should depend upon the market's expectations about future interest rates. From the given data, the current 1-year rate and the market expectations about future 1-year interest rates are:

$r_1 = 5\%$ $\quad {}_1f_1 = 4\%$ $\quad {}_2f_1 = 5\%$ $\quad {}_3f_1 = 6\%$

Based upon the relationship between spot rates and forward rates, the 4-year spot rate should currently be:

$(1+r_4)^4 = (1+r_1)(1+{}_1f_1)(1+{}_2f_1)(1+{}_3f_1)$
$= (1.05)(1.04)(1.05)(1.06)$
$= 1.215396$

Thus, a 4-year zero-coupon bond should currently be trading in the market at:

$P_4 = [1000/(1+r_4)^4] = [1000/1.215396] = \$822.7771$

This Represents and effective Spot rate of:

PV = 822.7771
FV = 1000
n = 4
i = 4.9976

Since you expect the 4-year zero-coupon to be worth:

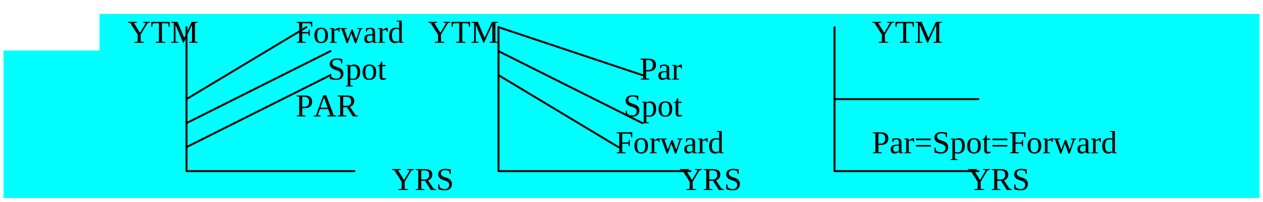
$(1+r_4)^4 = (1+r_1)(1+{}_1f_1)(1+{}_2f_1)(1+{}_3f_1)$
$= (1.05)(1.05)(1.05)(1.06)$
$= 1.2271$

$P_4 = [1000/(1+r_4)^4] = [1000/1.2271] = \$814.9295$

You would not buy the bond because, based on your expectations, the market is overpricing it.

c) The Relationship Between Par Yields, Spot Rates, and Forward Rates

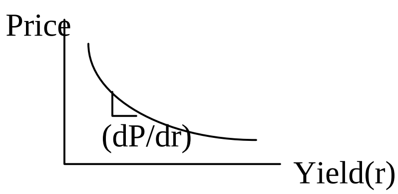
- Based upon the Concepts presented above, the relationships between PAR, SPOT and FORWARD Rates are illustrated below for positively sloped and Inverted Yield Curves:



4. ***Duration & Convexity***

- There are TWO Principal Risks in any bond investment
  1. **Credit Risk** (the risk of default)
  2. **Interest Rate Risk** (the risk that interest rates might rise. When interest rates rise, bond returns can be affected in 2 ways:
     a. The Value of the bonds declines; and
     b. The coupon interest can be re-invested at higher rates

  Note, these 2 effects affect bond returns differently. (a) is a negative factor while (b) is positive. But, there is no guarantee that they will net out to Zero
- Credit Risk can usually be controlled. Investors can choose only high quality bonds or treasury issues to reduce risk to an acceptable level. Even a portfolio of low quality bonds can have its credit risk controlled by diversification.
- Interest Rate Risk, however, cannot be reduced by diversification. It can only be controlled by buying bonds that are LESS sensitive to fluctuations in interest rates (short-term, high coupon)
- Using Modern Portfolio Theory terminology, credit risk may be viewed as being unsystematic, whereas interest rate risk can be viewed as being systematic
- The Market Risk of a stock is Measured by its β, usually estimated by regressing the historic changes in a stock's price on the corresponding changes in the value of the stock market index. Usually, this relationship is STOCHASTIC
- Bond volatility, need not be measured by such a stochastic process due to the fact the price of an individual bond is PRECISELY related to its market (the interest rate level) by a purely mathematical function (the present value equation) that is based upon the parameters of the bond (coupon, maturity, quality). Hence, the movement of bond prices that corresponds to a given change in interest rates can be precisely calculated. Thus, interest rate risk for a bond may be determined
- The Price of a Bond moves inversely with changes in its yield (see below)



- Two Parameters, known as **Modified Duration** and **Convexity** describe how the price of a bond responds to changes in its yield.

  Modified Duration = $D^* = [(-dP/dr)/P]$ → Measures Interest rate risk
  Convexity = $C = [(d^2P/dr^2)/P]$

  Therefore, the percentage change in the price of a bond is related to changes in bond yields (interest rates) by the relationship:

  $\%\Delta\text{Bond Price} = [\Delta P/P] = -D^*\Delta r + (1/2)C\Delta r^2$
- The Volatility of a Bond (%change in its price for a given basis point change in its yield), therefore, can be determined from this relationship if the bond's modified duration and convexity can be defined by the characteristics of the bond (coupon, maturity, and price).

5. ***Duration & Convexity as Measures of Interest Rate Risk***

- The mathematical relationship describing the volatility of a bond's price is:
  $\Delta P/P = -D^*\Delta r + (1/2)C\Delta r^2$

  For Example: a 10% Coupon, 5-year bond priced at 108.11 has a modified duration and convexity of 3.94 and 19.37, respectively. How much will this bond's price change (measured in percent) if the interest rates increase and decrease by 1%?
  Answer:
  When the Interest Rate rises by 1%
  $\Delta P/P = -D^*\Delta r + (1/2)C\Delta r^2$
  $= -3.94(01) + (1/2)(19.37)(.01)^2$
  $= -3.8431\%$
  When the Interest Rate falls by 1%
  $\Delta P/P = -D^*\Delta r + (1/2)C\Delta r^2$
  $= -3.94(-.01) + (1/2)(19.37)(-.01)^2$
  $= 4.0369\%$

  Note: The ASYMETRICAL Result is due to the POSITIVE CONVEXITY of the Bond
- Interest Rates Impact Bond Prices via DURATION.
- Whether interest rates rise or fall, the CONVEXITY effect tends to increase the bond's price. The greater the Convexity of a Bond, the greater is this VALUE of CONVEXITY EFFECT. Ceteris Paribus, Investors should pay a PREMIUM (low yield, high price) for highly convex bonds
- Whereas Duration factor is negative, convexity is USUALLY Positive.
- Since a negative duration factor is multiplied by the change in interest rates (which can be positive or negative), the duration effect causes bond prices to change in the OPPOSITE direction to the change in interest rates. But, the Positive CONVEXITY factor is multiplied by the change in the interest rates squared (which always produces a positive result). This leads to a positive bias in the percentage change in bond prices, regardless of the direction change in interest rates
- This Bias is called the AVERAGE GAIN from CONVEXITY

  In the above example, the Average Gain from Convexity is as follows:
  AVG. GAIN from CONVEXITY = (.5)(%ΔPrice when r increases 1%) + (.5)(% ΔPrice when r decreases 1%)
  (.5)(-3.8431%) + (.5)(4.0369) = 0.0969%
- Because Positive Convexity produces this upward bias in returns, whether interest rates move up or down, the MORE Volatile interest rates are EXPECTED to be, the MORE Attractive Positive CONVEXITY becomes
- **Strategic Implications of the Formula:** $\Delta P/P = -D^*\Delta r + (1/2)C\Delta r^2$
  1. If Interest Rates are Expected to RISE, it is Best to Hold Bonds with LOW DURATIONS whose prices will NOT be Heavily impacted by Higher Rates
  2. If Interest Rates are Expected to FALL, it is best to hold bonds with HIGH DURATIONS, because their Prices will INCREASE the MOST in a Lower-interest rate environment
  3. If interest rates are expected to become MORE VOLATILE, it is best to own bonds that have HIGH POSITIVE CONVEXITIES, because the second term in the pricing formula is high if Δr is high (squared)
  4. If interest rates are expected to become LESS VOLATILE, it is best to own bonds that have LOW CONVEXITY, because convexity is virtually IRRELEVANT if Δr is small
- Usually, since interest rates do not change very much over a short period of time, convexity is often ignored. Then, the volatility of a bond's price can simply be related to its duration through the equation: $[\Delta P/P] = -D^*\Delta r$

- In this case, modified duration is the measure of interest rate risk

6. ***Price Value of a Basis Point***

- The **PRICE VALUE of a BASIS POINT** measures how many basis points a bond's price will change if Interest rates change by one basis point

  PVBP = -.0001PD*

  For Example: What is the PVBP of a 10% Coupon, 5-year bond priced at 108.11 whose modified duration is 3.94
  Answer:
  PVBP = (-.0001)(108.11)(3.94) = -$.0426
  This means that a 1 basis point change in interest rates would cause the Dollar price of this bond to change by $0.0426 in the OPPOSITE direction to that of the interest rate change (remember that duration is technically a negative number)

- **DOLLAR DURATION** is the Price Value of a Basis Point divided by 0.0001

  Dollar Duration = -PD* = [dP/dr]

  For Example: What is the Dollar Duration of a 10% Coupon, 5-year bond priced at 108.11
  Answer:
  Dollar Duration = -(108.11)(3.94) = -426

- **DOLLAR CONVEXITY** is the dollar change in the price of a bond due to its convexity alone

  Dollar Convexity = 1/2PC = $[d^2P/dr^2]$

- **YIELD VALUE of a PRICE CHANGE**

  The PVBP can be reformulated to determine how much a bond's yield will change if its price changes by a small amount (assuming Δr^2 is insignificant)

  $\Delta r = [-\Delta P / PD^*]$

  For Example: how much will the YTM change for a 10% coupon, 5-year bond priced at 108.11 if the price of the bond increases by 1/32nd ? Assume the modified duration of the bond is 3.94. This is a YIELD VALUE of a 32nd (YV32)
  Answer
  $[\Delta P/P] = -D^*\Delta r$
  $(.03125/108.11) = -3.94\Delta r$
  $\Delta r = -.000073365 = -.0073365\%$

  NOTE: the denominator is the dollar duration of the bond

  YV32 = [1/(3200)(PVBP)]

  PVBP = [1/(3200)(YV32)]

7. ***Properties of Convexity***

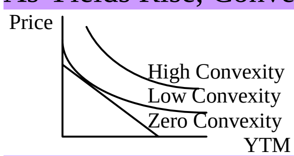
1. The CONVEXITY of a PORTFOLIO is the WEIGHTED AVERAGE of the Convexities of the bonds in the Portfolio with the Weights being the Percentage of the Portfolio that is invested in Each Bond

   For Example:

	% Portfolio	**Convexity**
A	40%	30
B	60%	85

   Portfolio Convexity = .4(30) + .6(85) - 63

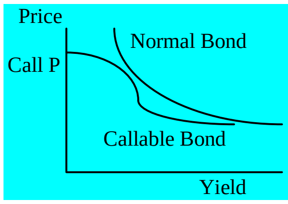
2. As Yields Rise, Convexity Falls if coupon and maturity are held constant



3. Convexity Rises with the Maturity of a bond if coupons and YTM are held constant
4. Convexity FALLS as the Coupon on a bond rises, if YTM and Maturity are held constant. Zero Coupon bonds have HIGH Convexity
5. The Convexity of a CALLABLE Bond is POSITIVE when interest rates are HGIH and the Bond is Priced well Below the Call Price. But, when Interest

rates are LOW and the bond is priced close to the call price, the Convexity of callable bonds becomes Negative

- When Interest rates are high, and the price of the bond is well-below the call price, then the price-yield relationship will be normal (convex to the origin) with the slope of the curve becoming less and less negative as yields increase. This is POSITIVE CONVEXITY and produces a gain from convexity in the pricing of bonds
- But, as Interest Rates become LOWER and the price of the bond nears its call prices, further declines in interest rates leads to PRICE COMPRESSION. Then, the price-yield relationship bends toward the call price and in the process becomes CONCAVE to the origin. This is NEGATIVE Convexity. A Region of negative convexity means that the slope of the price-yield curve becomes more negatively sloped as yields increase, causing the bond's duration to increase as interest rates rise.



$\Delta P/P = -D^*\Delta r + (1/2)C\Delta r^2$

- If the Convexity Term (C) is a Negative Value, there will be a Net LOSS from Convexity meaning that the percentage change in the price of the bond will be less positive if interest rates fall and more negative if interest rates rise, than would be indicated from its duration factor alone. In other words, when a BOND is priced so as to have NEGATIVE CONVEXITY, its price will be hurt MORE if Interest Rates Rise than it will be helped if Interest Rates Fall. This is a NEGATIVE Feature of Callable Bonds.
- NOTE: This same conclusion can be reached by observing that the value of the call option imbedded in callable bonds (which has effectively been SOLD SHORT by the Bond Holder) RISES as Interest Rates Fall. This causes the price of a CALLABLE Bond (P_{CB}) to fall farther away from the price of an equivalent NONCALLABLE BOND (P_{NCB}) as Interest rates decline according to the relationship $P_{CB} = P_{NCB} - \text{Call}$

A. Limitations of Duration Measures in Fixed Income Strategic Analysis

- *Duration measures bond price Sensitivity to SMALL Changes in Yields. If interest rate changes are LARGE, CONVEXITY must be taken into consideration*
- *Duration measures the Sensitivity of Bond Prices to INSTANTANEOUS Changes in Interest Rates. If interest rates change over a long period of time, DURATIONS themselves change*
- *Bonds with IMBEDDED OPTIONS (such as callable bonds) have price sensitivities that can be much different from those measured by conventional modified duration. This is because the cash flows paid by such bonds can actually change when INTEREST rates change (i.e., when they are called). The Duration and Convexity of bonds whose cash payment can change with interest rate movements cannot be calculated using the "MACAULAY" method. In such cases, the "EFFECTIVE" Method must be used*.

  Effective Duration = $-[(\Delta P/P)/\Delta r]$

  For Example: A bond is trading at 95 when interest rates are 8%. When interest rates fall to 7.9%, the bond trades at 95 ½. What is the Effective Duration of the Bond?
  Answer:

  $$\text{Effective Duration} = -[(\Delta P/P)/\Delta r]$$
  $$= -[(.5/95)/-.001]$$
  $$= 5.26$$

  It should be noted that when bonds have IMBEDDED Options, it is possible for the EFFECTIVE Duration of a bond to be GREATER than the MATURITY of the Bond. This can happen when the security is Highly leveraged, as may occur with certain CMO tranches. It is also possible for durations (conventionally stated) to be negative, so that a rise in interest rates causes the value of a bond to INCREASE. INTEREST-ONLY (IO) CMOs are examples of bonds whose durations can be negative because they may fall in price as interest rates decline.
- *When the Yield Curve Shifts in a NON-PARALLEL Manner, 2 bonds with the same duration before the shift can end up with different durations after the shift. Since the yields change differently along the yield curve in this case, different yield changes applied to equal durations will produce very different price changes*.

  For Example: Consider a Barbell Portfolio consisting of a 60% investment in a 4-year bond with a modified duration of 3 and 40% investment in a 15-year bond with a modified duration of 10.

  The Modified Duration of this Portfolio (weighted average) is
  $D^*_{Barbell} = (.6)(3) + (.4)(10) = 5.8$

  Compare this with a Bullet Portfolio that is 100% invested in an 8-year bond with the same modified duration of 5.8

  $D^*_{Bullet} = 5.8$

  The 2 portfolios seem to have the same interest rate risk because they both have the same duration. But, when the yield curve shifts in a non-parallel manner with 4-year yields declining 40 basis points, 8-year yields remaining unchanged and 15-year yields rising 70 basis points, the portfolios perform as follows.

$\boxed{\Delta P/P = -D^*\Delta r}$

$[\Delta P/P]_{Barbell}$= (.6)(-3)(-.004) + (.4)(-10)(+.007) =-2.08%

$[\Delta P/P]_{Bullet}$ = (1)(-5.8)(0) = 0.0%

In this case of a Steepening Yield Curve, the barbell would perform worse than the bullet, despite the fact that the 2prtfolios have the same modified duration. This is called YIELD CURVE RISK. It CANNOT be measured by traditional duration-convexity relationships. This is because the Duration of PORTFOLIOS do NOT measure Risk the same way as the Duration of INDIVDAL Bonds.

- *Like YTM, Duration is Not always the best way to Analyze BOND STRATEGIES. May need to Perform a DURATION WEIGHTED YTM Analysis instead*

For Example: Consider the Investor who examines the following bonds

Bond	Coupon	Maturity	Price	YTM	Duration	Convexity
A	8.0%	5 years	100	8.0%	4.055	21
B	9.0%	20 Years	100	9.0%	9.201	132
C	8.5%	10 Years	100	8.5%	6.231	53

The Investor wants to determine whether to buy BULLET Portfolio, consisting only of Bond C, or a BARBELL Portfolio consisting of Bonds A and B, in such proportion so as to equal the Duration of Bond C (6.231 years)

For the Duration of the BARBELL to equal that of the BULLET, the weightings are:

$D_{Barbell} = D_C = 6.231 = w_A(4.055) + (1-w_A)(9.201)$

$W_A = 57.7\%,\ W_B = 42.3\%$

Comparing the YTM of the 2 strategies:

$YTM_{Bullet(C)} = 8.5\%$

$YTM_{Barbell} = (.577)(8\%) + (.423)(9\%) = 8.42\%$

Hence, the Barbell appears Bullet is better.

But, comparing on a DURATION basis, it appears that there is the SAME Risk ($D_{Barbell}=D_{Bullet}$). Hence, one might assume that the Bullet is best (Higher YTM with an equal Risk).

But the CONVEXITIES of the 2 strategies are different

$C_{Bullet} = 53$

$C_{Barbell} = (.577)(21) + (.423)(132) = 67.95$

Higher Convexities are better than Lower Convexities

This means that the BARBELL Strategy has an AVERAGE GAIN from CONVEXITY that is Greater than that of the BULLET Strategy. That could be why there is a Lower YTM.

Examining the DURATION-WEIGHTED YTM (a close approximation of cash flow yields)

$\boxed{\text{Duration-weighted Yield} = [(w_A)(D_A)(YTM_A)+(w_B)(D_B)(YTM_B)]/[(w_A)(D_A)+(w_B)(D_B)]}$

Duration-weighted Yield_{Bullet} = 8.5%

Duration-weighted $\text{Yield}_{Barbell}$ = [(.577)(4.055)(8%) + (.423)(9.201)(9%)]/[(.577)(4.055) + (.423)(9.201)] = 8.63%

Hence, the Barbell is a better strategy with a better RISK ADJUSTED Duration Weighted Yield (this is a more complex analysis)

- ***Generalizations about This Type of Analysis***

1. *BARBELLS tend to be MORE CONVEX than BULLETS, holding Duration Constant*
2. *SPREADS (a group of more than 2 bonds with different durations) tend to have a CONVEXITY that is between that of a BARBELL and a BULLET, if the duration of the portfolios is held constant*
3. *BULLETS tend to be LESS CONVEX than BARBELLS or SPREADS, holding Duration constant*
4. *When the YIELD CURVE is VERY POSITIVELY SLOPED and Expected to FLATTEN, BARBELLS will tend to perform better than BULLETS*
5. *When the YIELD CURVE is FLAT and expected to STEEPEN, BULLET Strategies tend to Perform Better than BARBELLS*

- Also, one needs to consider the behavior of 2 portfolios over time, if interest rates change by the yield curve undergoing various types of shifts (parallel, steeping, flattening, etc.) This requires a HORIZON YIELD Analysis of the 2 strategies. Some steps one should perform in order to perform this analysis would be as follows.
  1. Assume a TIME Horizon or Holding Period for the strategy, such as 1 Year
  2. Assume a Certain type of shift in the Yield Curve (such as a parallel shift upward of 100 basis points)
  3. Calculate the Horizon return for each of the bonds under these assumptions
  4. Calculate the total value of the barbell and bullet portfolios at the end of the assumed holding period. The portfolio with the highest ending value, under the assumed terminal point conditions, would represent the best strategy
  5. Repeat the entire process under various assumptions to determine under which assumptions and conditions each of the portfolios best performs.
  - Often, when this analysis is performed, the portfolio with the HIGHEST CONVEXITY will Perform best if the Yield Curve Shifts Parallel and Substantially in either direction; else the LOWER Convexity, higher yielding strategy performs best.
  - It is difficult to generalize on non-parallel shifts
  - Main Point: Simple, summary measures of price sensitivity to yield changes, such as Duration, have limitations. Remember, DURATION measures price sensitivity to SMALL changes in interest rates over a SHORT PERIOD OF TIME. This can be misleading over longer time horizons or if yields change substantially

8. ***<u>Using Duration to Immunize Bond Portfolios Against Interest Rate Risk</u>***

- Bond Portfolios are vulnerable to INTEREST RATE Risk (when interest rates rise, bond prices fall. But, the 2 EFFECTS of this risk operate in opposite directions.
  1. **Price Effect**, which means that for an increase in interest rates will cause the price of bonds to fall; and
  2. **Coupon re-investment effect**, which means that for an increase in interest rates, there is an increase in the returns on interest that is reinvested

- As the 2 effects move in opposite directions, try to construct a bond portfolio so that the price and coupon reinvestment rate effects are equal causing the TOTAL RETURN of the portfolio at some future horizon date to be the same no matter what, thereby IMMUNIZING the portfolio against the effects of interest rate changes
- ONE Way to do this is to buy a DEDICATED portfolio of ZERO-COUPON Bonds that mature on the same date as the investor's time horizon. Since there are no coupon payments, there is no re-investment rate effect. And, since the bonds mature ate par, there is no Price effect to worry about.   If held to maturity, the yield will be the yield when purchased. Ergo, there is no interest rate risk.
- But, Zero-coupon bonds have lower yields than coupon bonds (usually). And, most portfolios are comprised of coupon bonds.
- A PORTFOLIO whose UNADJUSTED DURATION EQUALS the Investor's TIME HORIZON will be IMMUNIZED Against Interest Rate Risk. In general, the duration of a bond is less than its maturity. Thus, a portfolio of coupon bonds whose duration equals the investor's time horizon will have a maturity that is greater than the date it is to be liquidated. But, there are some LIMITATIONS to this. IT only works under certain conditions.
    1. TAXES are IGNORED, so the process only protects against interest rate risk on a PRE-TAX Basis
    2. Works only if the YIELD CURVE is FLAT and it undergoes ONLY a PARALLEL Shift. When Yield Curve reshaping occurs, the method will partially fail. This is called STOCHASTIC (Immunization) RISK. Usually, yield curves are Not Flat and do not shift in a parallel manner. Can try to use Adjusted Duration, using the FISHER-WEIL Duration formula, but then, immunization will still only work on a parallel shift. Recently, academics have tried using other multi-factor approaches, but they do not work quite as well. Still rely on the MACAULAY duration immunization technique.
    3. Works only when the DURATION of the bonds is CONTINUALLY matched to the remaining Time horizon. Ergo, not a Set it and forget it strategy. Thus, must continually rebalance the portfolio to keep the duration equal to the time horizon. As this is impractical, one can try to use bond futures contracts to accomplish the goal at a reasonable cost. This is because the duration of the portfolio can be changed by any desired amount by buying or selling the proper number of futures contract while keeping the portfolio in tact.

9. _"Corporate Credit Risk and Reward" by Bennett, Esser & Roth_

a) The Risk (Volatility)-Return Profile of Corporate v. US Treasury Bonds

- Empirical evidence indicates that Corporate Bonds have about the same volatility as US Treasuries of the same duration
- Usually, though, investors in investment-grade corporate bonds gain greater returns than those in US Treasuries with the same duration over the same period of time (for accepting the added risk)
- As ratings on corporate bonds decrease, the correlation between corporate bond returns and treasury returns decrease modestly, while the correlations

between corporate bond returns and equity returns increase modestly. Still, BBB returns maintain a fairly high correlation to treasury returns

b) ***The Diversification Benefits of Corporate Bonds in Portfolios***

- Balanced Portfolios, with corporate bonds and equities, have produced higher return/risk ratios than portfolios consisting of the same mixes of US Treasuries and Equities. This is because corporate bond returns have been higher than treasury returns, yet the volatility of the 2 types of bonds have been similar.
- Further, though junk bonds are not pure fixed-income substitutes, historical data show that junk bonds are an attractive asset class to use as a diversification vehicle to raise risk/return ratios
- Adding junk bonds to a portfolio of high grade bonds and stocks increased the returns for any given level or risk

c) ***Corporate Bond Performance and the Economic Cycle***

- History suggests that there is a tendency for corporate bonds to under-perform treasuries during times of economic distress. This is the FLIGHT to QUALITY Syndrome. But, these episodes have been brief. When the crises subside and markets return to normal, corporate bonds significantly outperform treasuries. If the future is similar to the past 20 years, investment grade corporate bonds should continue to offer attractive opportunities for the fixed income investor

EQUITY ANALYSIS

<u>"Engineered Investment Strategies: Problems & Solutions" by Robert L. Hagin</u>

- Engineered Investment approaches which rely on quantitative methods are becoming more prevalent in the investment management business. To have a good investment strategy based on quantitative factors, 3 characteristics are **necessary**
  1. Strategy should be based on SOUND THEORY
  2. Strategy should be EXPLICITY and QUANTIFIED
  3. Strategy should be WELL DOCUMENTED to prove how well it would have performed in the PAST
- **Problems** with Quantified Strategies
  1. INSUFFICIENT Rationale may exist as to why strategy worked in the past
  2. A PRIORI Reasoning or the use of BLIND ASSUMPTIONS
  3. DATA MINING – If enough regressions are performed, some correlations will be found, but these are often spurious and cannot be trusted to persist
  4. QUALITY OF DATA – Economic or financial databases which are used in investment analysis are often flawed. A common problem is SURVIVOR BIAS – a database which excludes companies that have gone out of business
  5. LOOK-AHEAD BIAS – using historical P/E Ratios based on calendar year earnings, when in fact, on a real-time basis, annual earnings results are not known for 2-3 months after the end of the year
  6. MULTIBLE FACTORS – problem of multi-colinearity that can bias results when 2 or more independent variables are correlated
  7. PAST v FUTURE – Quantitative methods usually PRESUME the future will behave like the past, which may be an unwarranted assumption
  8. Statistical ASSUMPTIONS may be WRONG – many statistical techniques assume that stock returns are NORMALLY Distributed, when many may not be
  9. LINEAR Models – Almost all models assume linearity, when actual relationships may not be linear
  10. Going from the Laboratory to the Real World is a Huge Step that not all theories can make
  11. One's own actions may impact the market in ways that are not incorporated in the empirical studies (i.e., Fidelity buying a stock may alter its price, which is not part of the study)
  12. Benchmark Portfolio may be wrong.
  13. Measurement of Skill – it takes many years to PROVE that a given strategy produces superior returns. It would take 36 years of data to prove that a strategy, which produces an average return that is 2% better than the market with a σ of +- 6% is truly superior at the 5% level of significance
  - To Resolve these problems, it is necessary to construct an equity research laboratory consisting of
    - Large Financial Database
    - Computer model that can test historical returns associated with certain factors that are common to a group of securities
    - Computer Model that can measure the affect of rebalancing for factor changes or trading costs

- Some Laboratory analysis can dispel firmly entrenched myths. For example, it is widely believed that stocks of ‘good’ companies (firms with high ROE, high profit margins, high historical growth rates, etc.) are good investments. But research finds such companies are overpriced and, produce inferior returns. The out-of-favor, lower priced “VALUE” stocks tend to produce better returns.

ECONOMICS

<u>“The Nature of Effective Forecasts” by David B. Bostian, Jr., CFA</u>

- Forecasting is part art and part science. Models breakdown due to human behavior
- How economies develop is based upon political forces rather than fixed natural economic laws
- There are several Popular Forecasting Techniques; including :
    1. ***Consensus Forecasting*** **–** which simply compiles the outlook of a large number of prominent economists and reports the average outlook of the group. PROBLEM: the consensus seldom properly predicts the turning points in the economy, and usually, when the consensus is most wrong is when the market moves are MOST PRONOUNCED
    2. ***Scenario Forecasting*** **–** a highly statistical methodology requiring the forecaster to identify several alternative sequential events, each leading to a different economic outlook, assign probabilities to each path of events, and then compile a probability distribution of economic outcomes: PROBLEMS – though useful, the main drawback is that it is very difficult to compile ALL possible scenarios, and outlier scenarios are often omitted from the analysis, and multiple scenarios often lead to decision paralysis
    3. ***Historical Forecasting*** **–** assumes the past is prologue. Though it may be true sometimes, in general, it is not. However, historical input should not be ignored. Successful forecasts do not dwell upon the past, but they try to identify what is different about current and future trends
    4. ***Rate of Change Forecasting*** **–** preferred by the author. His method of forecasting business cycle changes (Macro Economic Index) which consists of 26 variables that measure a breadth of activities. When the rate of change of the MAJORITY of these indicators change direction, a signal of economic change is given. This can forecast the TIMING of Cyclical turns, but not the AMPLITUDE of these moves

a) Most Economic Forecasts FAIL for one of Several Reasons;
   a. **Linear Perception –** this concept that the past can be linearly extrapolated into the future
   b. **Group Think –** most forecasters feel uncomfortable making predictions that are significantly different from their colleagues
   c. **Messenger Syndrome –** most forecasters do NOT like to deliver BAD NEWS, ergo, they will sugar-coat their forecasts
   d. **Poor Data Quality –** Government statistics are often revised substantially, so economic forecasts may be based upon wrong data
   e. **Faulty Theories –** Currently Keynsian, Monetarist, and Supply Side Theories exist side-by-side. Sometimes they lead to similar conclusions. Sometimes not.

b) For investors, it is not significant to merely forecast the Economic outlook, one must somehow relate it to the outlook for individual securities. This is risky. It may be easier to relate economic forecasts to an industry group. To do this, one needs to base an outlook for an **industry upon 6 factors**

   a. **Valuations given to the Industry in the Market –** Is the industry over or under-valued based upon historical benchmarks. For the Market as a whole, extreme over-valuations occur when the market value of all NYSE stocks exceeds 70% of the GDP, under-valuations occur when it reaches 40% or less of GDP
   b. **Economic factors as the impact the outlook for particular industries**
   c. **Productivity trends from various industries**
   d. **Demographic factors and how they impact the secular demand trends for individual industries**
   e. **Government policy issues and their impact on industries**
   f. **Profit outlook for individual industries, given the previous 5 factors**

<u>"Developing a Recommendation for a Global Portfolio" by Charles I. Clough, Jr., CFA</u>

- In developing a Global Portfolio, it is important to assess where fundamental changes are taking place and where the conventional valuation formulas may be wrong
- When consensus estimates are used in conjunction with consensus valuation models, the valuation results generated are consensus opinions; thus insuring that results will be sub-par
- It is better to build an international portfolio strategy for the 90s based upon the following propositions
    1) Markets are RATIONAL even when they appear to be under-, or over-valuing assets for long periods of time. Valuation formulas should NOT be the sole basis of measuring expected rates of return; LIQUIDITY TRENDS and EARNINGS MOMENTUM also play a role in determining Market outlooks
        - In recent years, the Money Supply has been growing at slow rates. But, the economies of the world have also been growing slowly, ergo there has not been a need to absorb great amounts of liquidity. Consequently, liquidity trends have been stable enough to establish positive yield curves almost everywhere: most liquidity has been parked in the short-term part of the yield curve. Soon, though, liquidity will move out along the yield curve causing stock and bond prices to rise further, despite their seemingly already over-valuation based on historical standards. Historically, FINANCIAL MARKETS perform best when the domestic money supply growth rate exceeds that of nominal GDP. In the 90s, equity markets will appear over-valued, relative to bond returns, but they will be propelled upwards by rising liquidity. However, a point will be reached when investment funds will seek better values outside of major industrial markets, and then spectacular gains will occur in the equity markets of developing nations.
    2) Some Inputs to the Valuation Process necessarily must be CONJECTURAL
    3) One must look at NATIONAL MARKETS with both a top-down and bottom-up approach. Often, one should invest in a nation because the industry that dominates its market is expected to do well, rather than because of the economic policies of the nation's government
    4) During the 90s, most of the world's economic growth will occur OUTSIDE the US
    5) There is a GLOBAL CREDIT CONTRACTION underway. Returns on household deposits will remain low. Money & Credit Expansion are slow, nominal GDP growth rates are slow, and Bank Consolidations occur
    6) The US Dollar shall be a strong currency as long as the credit contraction continues due to persistent shrinking dollar liquidity.
    7) By the mid-90s, capital will be flowing to places where it is needed: Eastern Europe, Latin America and China. As there is little demand for speculative capital in the developed world, this capital should be available at low cost. When capital is available for the development of undeveloped economies, usually interest rates are low. When rates are high, capital flows to more speculative projects
    8) DEMOGRAPHIC Trends in industrialized nations may send rates lower. During the 90s, the US labor force grew by only 0.5%, whereas it grew by 1.75% in the

80s and 2.75% in the 70s. When demand for housing, airports, office space, and shopping centers slows, there will be declining demand for borrowing, and economic growth rates will slow as well. The downsizing of employment by corporations may be a rational response to the upcoming lack of manpower and high labor costs

9) The development of large trading blocs makes ECONOMIES of SCALE important. There is also REDUNDANT CAPACITY. With slowing population growth, it will take many years for that excess capacity to be assimilated. Consequently, demand for expansion capital in developed nations will remain low and keep a downward pressure on interest rates. Even though governments borrow heavily, this borrowing is small relative to the private sector.
10) DEMOGRAPHIC trends also impact the level of savings. As the population AGES, savings rates should increase. Initially, savings will flow to traditional areas like bank accounts; but due to the low returns on these savings, this money will flow more towards equities. This will cause bond yields to be lower and P/E ratios to be higher than in the past. There is reason to believe that barring any tightening of monetary policy, stock prices could appear to be overvalued for some time.

"Is Purchasing Power Parity a Useful Guide to the Dollar?" by Craig S. Hakkio

- Purchasing Power Parity is useful for predicting movements in the dollar, relative to other currencies in the LONG RUN; it is less useful in predicting short-run exchange rate movements
- There are THREE Different CONCEPTS that relate to Purchasing Power Parity

1) **The Law of One Price**
   - The Price of a Good that is FREELY TRADABLE should be the SAME Everywhere (except for the impact of transportation costs, tariffs, and other costs).
   - If Gold Costs $400 in NYC, and the French franc is worth $0.20 (fr5/$), then gold should cost fr2000 in Paris
2) **Absolute Purchasing Power Parity**
   - The Exchange Rate Between 2 Currencies should EQUAL the RATIO of the Price INDICIES of the 2 countries
   - If the CPI is 265.4 in the US and 1,327 in France, in the same BASE YEAR, then the Exchange rates should be:
   - (1,327.0/265.4 = fr5/$) or (265.4/1327 = $.20/fr)
   - This Concept GENERALLY DOES NOT WORK for 2 Reasons
     - The LAW of ONE PRICE does not always Hold
     - Price Indexes of different nations do NOT include the Same basket of goods, nor are they calculated in the Same Manner

3) **Relative Purchasing Power Parity**
   - The Rate of change of Exchange Rates is Related to the RATE of Change of EXPECTED INFLATION between the 2 Nations
   - When the Expected Inflation rate is 4% in the US and 5% in France, then the franc will fall against the dollar by approximately 1% per year

$[S_{t(\$/fr)}/S_{0(\$/fr)}] = [(1+I_{US})/(1+I_{France})]$ OR $[S_{t(fr/\$)}/S_{0(fr/\$)}] = [(1+I_{France})/(1+I_{US})]$

$S_{t(\$/fr)} = (1.04/1.05) = .99S_{0(\$/fr)}$ OR $S_{t(fr/\$)} = (1.05/1.04) = 1.01S_{0(fr/\$)}$

- If ABSOLUTE Purchasing Power Parity holds, relative purchasing power parity will also hold.
- BUT, RELATIVE PURCHASHING POWER PARITY can hold EVEN IF Absolute Purchasing Power Parity Does NOT hold
- Thus, Relative Purchasing Power Parity is the preferred method of using the concept to predict exchange rate movements

REASONS WHY RELATIVE PURCHASING POWER PARITY WILL NOT WORK in Explaining Exchange Rate Movements over the Short Term

- Price Levels tend to be STICKY in the Short Run while exchange rates fluctuate continuously
- When Exchange rates are fixed by international agreement, they are not likely to move in line with changes in the relative inflation rates among nations

Consequently, Relative Purchasing Power Parity is more likely to explain LONG RUN changes in exchange rates rather than short-run exchange rate fluctuations. There are 3 sources which prove RELATIVE Purchasing Power Parity works over the long run

a.) Charts showing exchange rates that are predicted using Purchasing Power Parity v. Actual exchange rates suggest that Purchasing power parity holds over the LONG RUN but not at all times

b.) Statistical Tests Show a Tendency for deviations from purchasing power parity create movements back to purchasing power parity equilibrium

c.) Statistical tests also suggest it takes several years to achieve purchasing power parity. When the spread between the actual rate and the PPP predicted rate is large, it will generally revert back to the mean.