HALLMARK COLLEGE

Math 1314

College Algebra

LO-9

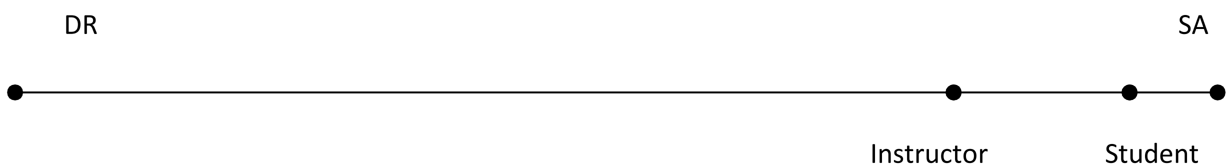
Chapter 6.1

Skills to Master

- Solve systems of equations in two variables by graphing
- Solve systems of equations in two variables by substitution
- Solve systems of equations in two variables by the addition method
- Determine when a system in two variables has infinitely many solutions and relate such a system to a graph of the equations
- Determine when a system in two variables is inconsistent and relate such a system to a graph of the equations

Systems of Equations

Suppose that your math instructor is on his way to Del Rio from San Antonio. Thirty miles outside of San Antonio, he receives an urgent call from one of his students saying that the student forgot to turn in the LOs due that day. Your instructor tells the student that he cannot stop, but that if the student drives really fast, he should be able to catch up and turn in the assignment. The instructor chugs along steadily and non-stop at 50 mph and the student, who actually lives 5 miles outside of San Antonio in the same direction of Del Rio, drives non-stop at 80 mph. How long will it take for the student to catch up with the instructor? How far from San Antonio will they be?

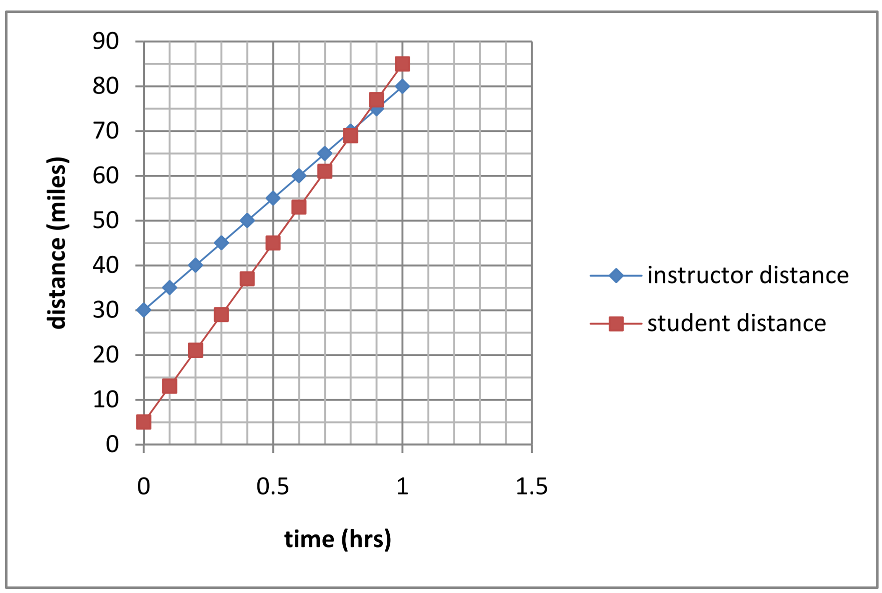


We are interested in the distance from San Antonio over time. That is, the distance each individual travels over time is a function of elapsed time. So we can write an equation for each relating distance to time, where the speed is the rate of change (i.e., the slope) and the starting point as shown above is the y-intercept. If we use *d* to stand for distance in miles and *t* to stand for time in hours, then:

Instructor: $d = 50t + 30$

Student: $d = 80t + 5$

A graph of these two linear equations is shown at right:



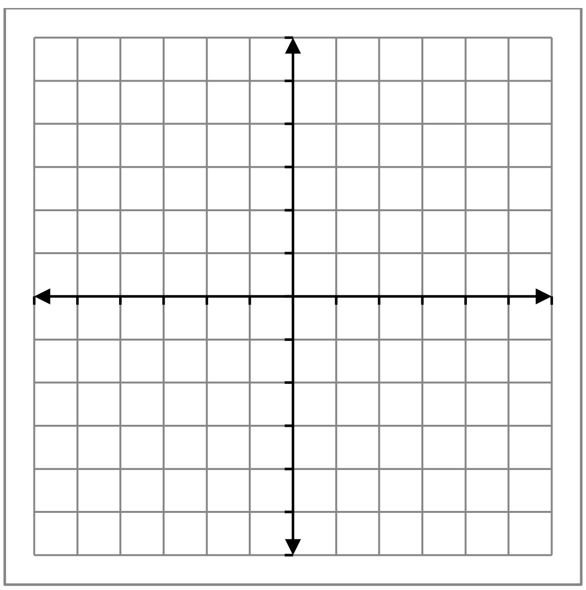
A **system of equations** is a set of equations (or functions) that may or may not have some number of points in common. Solving a system means finding the common points (if any).

Methods

- Solve by graphing
- Solve by substitution
- Solve by the addition method

Solving by Graphing

$$\begin{cases} y = 2x - 5 \\ y = -x + 4 \end{cases}$$

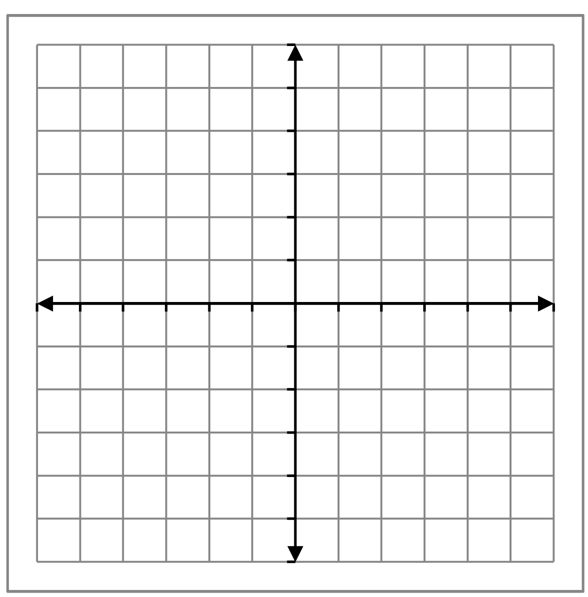


The solution is the **point of intersection of the two lines.**

Remember that there are other ways to graph a line if they are not in slope-intercept form.

Solve the system by graphing:

$$\begin{cases} y = \frac{1}{2}x + 4 \\ y = -2x - 1 \end{cases}$$



Solving by Substitution

Remember that, if two values are equal, one can be substituted in place of the other. For example, if $y = 4x + 2$ and $x = 1$, then we can substitute x in the equation, allowing us to calculate y: $y = 4(2) + 2 = 8 + 2 = 10$.

The same is true even if what is to be substituted is an expression. For example, if $y = x + 4$ and $3x + y = 7$, then we can substitute $x + 4$ in place of y in the second equation:

$3x + (x + 4) = 7$. From here we can simplify and solve to find the value of x. This is the principal behind solving by substitution.

Example

$\begin{cases} y = 3x + 1 \\ x - y = 3 \end{cases}$

$\begin{cases} x = -y + 1 \\ 5x + 2y = 1 \end{cases}$

$\begin{cases} y = x + 5 \\ x + 2y = 4 \end{cases}$

Remember that solving the system means finding a **point**. So you must find an x-coordinate value AND a y-coordinate value.

Solving by the Addition Method

Solving by the addition method involves combining like terms from a pair of equations. The goal is to adjust one or both equations such that one of the variables will cancel to zero.

Example

$\begin{cases} 3x + y = 8 \\ 2x - y = 2 \end{cases}$

$\begin{cases} 2x + 3y = 3 \\ -2x - y = 1 \end{cases}$

The previous examples were easy because the terms were already opposites of each other. What if they are not opposites to begin with?

Example

$$\begin{cases} 4x - 3y = 16 \\ -x - 2y = 7 \end{cases}$$

Example

$$\begin{cases} -2x + 5y = -3 \\ 4x - y = -3 \end{cases}$$

Suppose the equations are such that changing only one will not cancel terms?

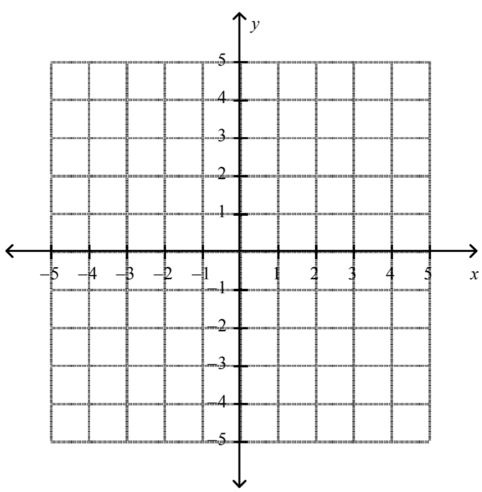
$$\begin{cases} 3x + 2y = 8 \\ -2x + 5y = -18 \end{cases}$$

$$\begin{cases} 5x + 2y = -19 \\ 3x + 7y = -23 \end{cases}$$

Systems with infinitely many solutions

Solve the system $\begin{cases} y = 3x - 5 \\ 6x - 2y = 10 \end{cases}$

Why does this happen? Look at the graphs of the two equations:

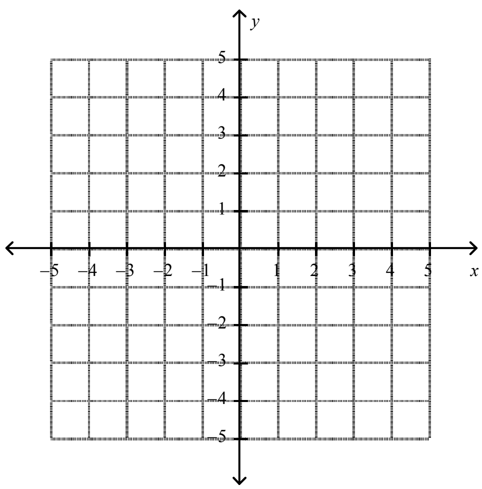


This system has an infinite number of solutions because the two equations are actually the same, one on top of the other.

Inconsistent systems

Solve the system $\begin{cases} 4x - 3y = 9 \\ -6x + 8y = 0 \end{cases}$

What does the graph of this system look like?



The system is **inconsistent** which means that there are no solutions. Graphically, this means that the lines are parallel so there is no point of intersection.