HALLMARK COLLEGE

Math 1314

College Algebra

LO-7

Chapters 3.1, 3.2

Skills to Master

- Determine whether a given equation represents a function
- Determine whether a given graph represents a function (vertical line test)
- Understand and use function notation
- Graph functions by creating a table of values
- Find the vertex of a quadratic function
- Graph a quadratic function by finding the vertex and creating a table of values

Functions and Function Notation

In the previous chapter you learned about linear equations. This chapter develops an idea that is important in all areas of mathematics. To begin, consider the following sets of numbers:

{1, 2, 3, 4} and {2, 4, 6, 8}

There are numerous ways to take one number from the first set and one from the second set such that each pair is related in some way. For example, (1, 2), (2, 4), (3, 6), (4, 8). In this case, the relation is that the second number is twice the first. There can be many other ways to pair these numbers, each of which forms a relation. Another example might be (1, 8), (2, 6), (3, 4), (4, 2). How might you describe this relation?

A **function** is a specific kind of relation. Think of a function as a machine that takes an input from one set and outputs something belonging to another set (or possibly the same set). You've seen this with linear functions. For example, if our function is $y = 3x + 2$, and our input (for x) is -1, then the output is $y = 3(-1) + 2 = -3 + 2 = -1$. From this you can write an ordered pair, (-1, -1), and then you can plot it on the coordinate plane.

<u>What makes a function special is that, for every input value, there must be exactly one output value.</u>

NON-EXAMPLE

Suppose we had the following pairs of numbers for some relation: (4, 1), (3, 7), (3, 1), (2, 1)

In this case, the input values are the first elements in the ordered pairs and the output values are the second elements. Notice that for the input value 3, there are two possible output values: 7 and 1. Hence, this relation is NOT a function.

> A **function** *f* is a correspondence between a set of input values *x* (called the **domain**) and a set of output values *y* (called the **range**), where to each *x*-value in the domain there corresponds exactly one *y*-value in the range.

In the first example, the set {1, 2, 3, 4} is the domain and the set {2, 4, 6, 8} is the range.

To determine whether a given equation represents a function, ask yourself if, for any input value it is possible for there to be more than one output value.

EXAMPLE

Which of the following are functions?

$3x - 2y = 5$

If we input a 3 for x, for instance, can there be more than one value for y that makes the equation true?

$3x - y^2 = 1$

If we input a 1 for x, is can there be more than one value for y that makes the equation true?

$y = x^2 + 2x - 3$

If we input a 2 for x, can there be more than one value for y that makes the equation true?

FUNCTION NOTATION

In chapter 2 we wrote linear functions in the form $y = mx + b$, which assigns the output value directly to y. More generally, we can say that $y = f(x)$, where $f(x)$ (read "f of x") is a special notation. It indicates that for the function f (whatever it may be), the input values belong to x. So for a linear function, we would write $f(x) = mx + b$. This is exactly equivalent to the above since $y = f(x)$. The x can be replaced by any particular value or even by another variable or another function.

EXAMPLES

For the function $f(x) = 2x^2 - 3x + 9$, find

$f(2) =$ $\qquad$ $f(x + 4) =$

$f(t) =$ $\qquad$ $f(☺) =$

PRACTICE

For the function $f(x) = x^2 + x - 3$, find

$f(-1) =$ $\qquad$ $f(3x) =$

$f(r) =$ $\qquad$ $f(x - 2) =$

DOMAIN AND RANGE

For different functions it is often necessary to determine what the domain and range values can be. For our purposes, both the domain and range will come from the set of real numbers. But we will sometimes find restrictions on which values can be input and limits on which values will be output.

Determining domain is not too difficult in most cases, but finding the range can sometimes be challenging. One method is to solve an equation for x and determine the range for y in the same way that we determine the domain for x. But this method doesn't work for radical functions (you'll see why in the example below). In the case of quadratic functions, since the vertex of the parabola is the maximum (or minimum) value for the function, then the domain is less than or equal to (or greater than or equal to) the y-coordinate of the vertex (see last example below).

EXAMPLES

Find the domain and range for the given functions:

$f(x) = 5x - 3$

domain =

range =

$f(x) = \dfrac{1}{1-x}$

domain =

range =

$f(x) = \sqrt{x+1}$

domain =

range =

$f(x) = x^2 + 3x - 1$

domain =

range =

PRACTICE

$f(x) = -x - 2$

domain =

range =

$f(x) = \frac{1}{x+2}$

domain =

range =

$f(x) = \sqrt{x-3}$

domain =

range =

$f(x) = 4x^2 - 1$

domain =

range =

To determine domain:

- If the equation is linear or quadratic, then the domain is all real numbers.
- If the equation contains *x* in the denominator, determine which value(s) will result in zero in the denominator; the domain will have the form $x \neq c$, where *c* is some real number.
- If the equation contains *x* inside a square root sign, determine what values of *x* inside the radical will result in values greater than or equal to zero. The domain will have the form $x \geq c$, where *c* is some real number

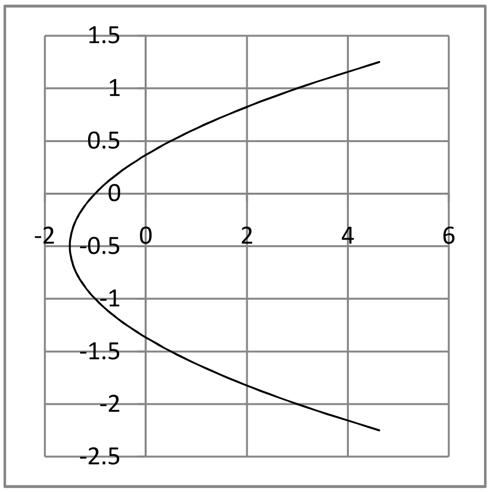
To determine the range:

- Solve the equation for *x*
- Use the steps above, substituting *y* in place of *x*
- For a radical function, solve for the radical if necessary and set the right side of the function to greater than or equal to zero and solve for y.
- For quadratic functions, find the y-coordinate of the vertex and determine whether the function opens up or down. If it opens up, then all y values will be greater than or equal to the y-coordinate value. If it opens down, all y-values will be less than or equal to the y-coordinate value

THE VERTICAL LINE TEST

A simple way to determine whether a relation is a function from its graph is by use of the vertical line test. To use this test, draw some vertical lines through the graph. If there is any place on the graph for which the vertical line touches two or more points on the graph, then the relation is NOT a function.

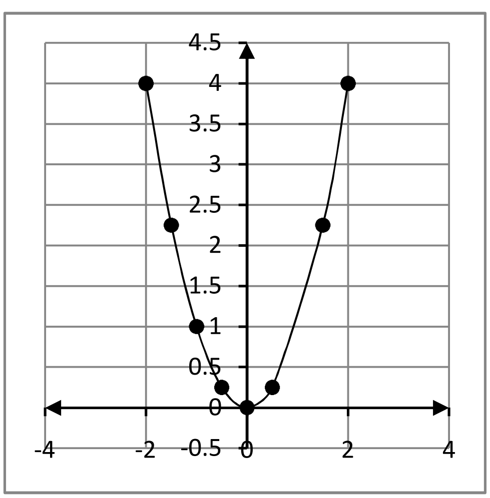
Example



Quadratic Functions

A quadratic function is a second-degree polynomial of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and a is not zero. The graph of a quadratic equation is a **parabola.**

Example: The function is $f(x) = x^2$



x	y
-2	4
-1.5	2.25
-1	1
-0.5	0.25
0	0
0.5	0.25
1.5	2.25
2	4

The easiest way to graph a quadratic function is to create a table of values and then plotting the points. The table above shows the values for the given function.

Characteristics of a Parabola

Some characteristics of parabolas can be determined from its equation:

- Vertical orientation: if the value of a is greater than zero (positive), then the parabola **opens up**; if the value of a is less than zero (negative), then the parabola **opens down**.
- Vertex: the *x*-coordinate of the **vertex** (the point at which the graph turns) is determined by the formula $x = \frac{-b}{2a}$; the *y*-coordinate can be determined by substituting the value obtained for *x* back into the original equation.
- If the graph intersects the *x*-axis, the point or points of intersection are called the **roots** or **zeros** of the function. Since the points are located on the *x*-axis, then the values of the y-coordinates are zero. That is, the zeros can be found by setting $ax^2 + bx + c = 0$ and solving. Notice that there are either 2, 1, or zero solutions.
- Line of Symmetry: the **line of symmetry** is at $x = c$, where *c* is the value of the x-coordinate of the vertex. This line divides the parabola into two mirror-image halves. Points on a horizontal line through the parabola are equidistant from the line of symmetry.
- *y*-intercept: the value of *c* is the y-intercept, the point at which the graph crosses the y-axis.

Example

Find the vertex, zeros, and line of symmetry for the given function. Also determine whether the graph opens up or down: $f(x) = \frac{2}{9}x^2 - \frac{4}{9}x - \frac{16}{9}$

Vertex:

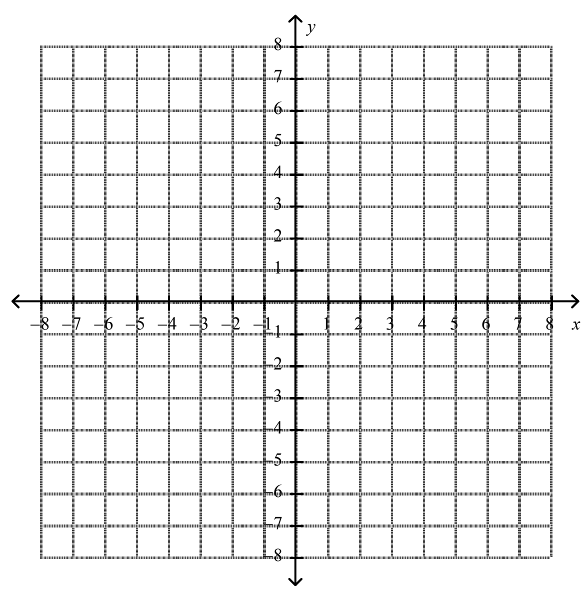
Line of symmetry:

Zeros:

y- intercept:

Orientation:

These characteristics can help us graph a parabola.

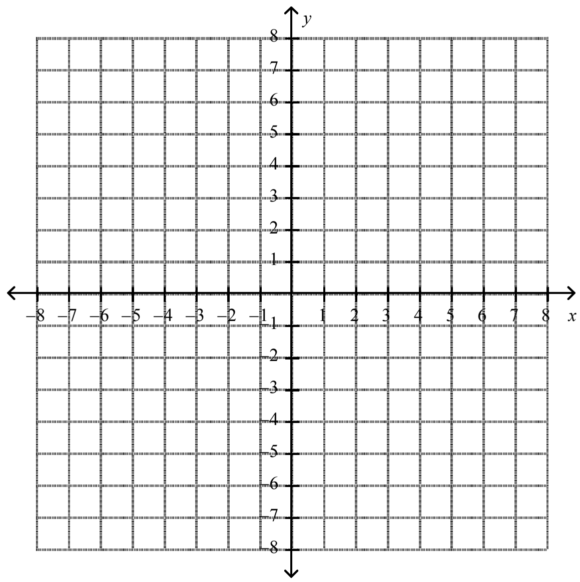


In general, it's probably easiest to first find the vertex of the parabola, then pick 2 x-values on either side of the line of symmetry and create a table of values.

Example

Graph the quadratic function $f(x) = -x^2 - 4x - 5$

Find the vertex and plot the point on the graph.



Create a table of values, choosing values for x on either side of the line of symmetry.

x	y

Plot these points and draw the graph

Try this one:

Graph the quadratic function $f(x) = x^2 + 2x - 2$