



Math 1314
College Algebra

LO-6
Chapters 2.3

Skills to Master

- Know and use the point-slope equation of a line to write linear equations given either the point and slope of the line or two points on the line
- Know and use the slope-intercept equation of a line to write equations of a line given either the slope and y-intercept of the line or the slope and any point on the line
- Graph equations given in slope-intercept form
- Determine the slope and y-intercept for a line given in standard (general) form

Point-Slope form of a Linear Equation

In the previous section, we used the **slope-intercept form** of a linear equation. That form is $y = mx + b$, where m is the slope of the line and b is the y-intercept. A slightly different form is called the **point-slope form**. It is $y - y_1 = m(x - x_1)$, where as before m is the slope of the line and y_1 and x_1 are the coordinates of a point on the line. The y and the x are treated as variables. That is, whereas you would substitute actual number values for y_1 and x_1 , you would leave y and x as variables when using this equations.

This form is derived from the slope formula. If we say that x and y are any points on some line with slope m , and (x_1, y_1) is some *particular* point on the line, then we have:

$$m = \frac{y - y_1}{x - x_1}$$

We can cancel the numerator by multiplying both sides:

$$m(x - x_1) = \frac{y - y_1}{x - x_1}(x - x_1)$$

Cancelling on the right gives:

$$m(x - x_1) = y - y_1, \text{ which is the same as } y - y_1 = m(x - x_1)$$

This form allows us to write the equation of a line when the slope is known and when at least one point is known.

Example

What is the equation of the line with slope $m = 3$ and passing through the point $(4, -1)$?

What is the equation of the line with slope $m = \frac{2}{3}$ and passing through the point $(-3, 2)$?

This equation can also be used to write the equation of a line when given only two points:

Example

Write the equation of the line passing through the points $(1, -6)$ and $(-2, -9)$

Write the equation of the line passing through the points $(4, -2)$ and $(-4, 4)$

Slope-Intercept form of a Linear Equation

Yesterday you learned a bit about the slope-intercept form: $y = mx + b$. Today you learned that, in this form, the value of m is the slope of the line. Notice that when the value of x is zero, then $y = b$. That is, the point associated with $x = 0$ is $(0, b)$. But recall from yesterday that this is the special point called the **y-intercept**. Thus, b is the y-intercept of the line.

IMPORTANT: the slope-intercept form of a linear equation gives the values of the slope (m) and the y-intercept (b) **directly**.

Example

Write the equation of the line with slope $m = -\frac{2}{7}$ and y-intercept $(0, 1)$

Write the equation of the line with slope $m = 2$ and y-intercept $(0, \sqrt{7})$

We can also use this form to find the equation of a line when given the slope and a point on the line, like one of the problems above. The difference is that using this form produces only the value of b .

Example

Find the equation of the line with slope $m = 2$ and passing through $(1, 9)$

Find the equation of the line with slope $m = -1$ and passing through $(4, -4)$

Standard Form of the Equation of a Line

In the previous section you learned that another form of a linear equation is $Ax + By = C$, where A , B , and C are integers. Unlike the slope-intercept form, it is **not possible** to directly read the values of m and b (the slope and the y-intercept) from the standard form. However, we can determine from the equation how to *find* these values by solving the equation for y :

$$Ax + By = C$$

Since we want to isolate the y , we must first replace the value of Ax with zero. We do this by subtracting Ax from both sides of the equation. This gives:

$$By = -Ax + C$$

Now we want to replace the value of B with a 1. We do this by dividing. Remember to divide all terms on both sides by B ! This gives:

$$y = \frac{-A}{B}x + \frac{C}{B}$$

Compare this to $y = mx + b$. Which part is like m ? Which part is like b ?

We see, then, that the slope is $m = \frac{-A}{B}$ and the y-intercept is $b = \frac{C}{B}$.

Example

What is the slope of the line and the y-intercept for the equation $5x - y = 7$?

What is the slope of the line and the y-intercept for the equation $-3x + 4y = 0$?

To complete this section, let's look at two types of problems that require you to use several of the concepts about linear equations that you've studied so far:

Example

What is the equation of the line that passes through the point $(4, 3)$ and that is parallel to the line $y = -2x + 1$?

What is the equation of the line passing through the point $(-1, -1)$ and perpendicular to the line $y = 4x - 1$?

Try these:

What is the equation of the line passing through the point $(0, -4)$ and parallel to the line $y = 3x + 1$?

What is the equation of the line passing through the point $(2, 4)$ and perpendicular to the line $y = -\frac{1}{4}x$?