HALLMARK COLLEGE

Math 1314

College Algebra

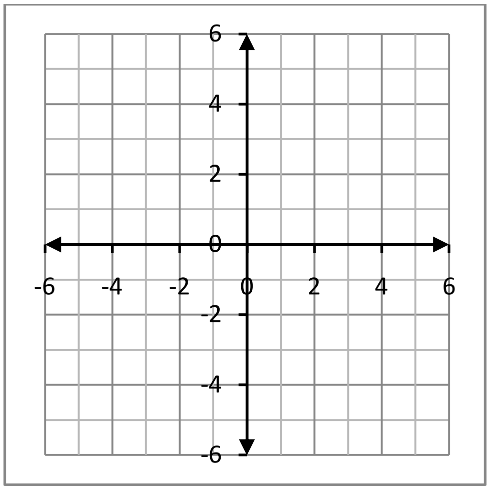
LO-5

Chapters 2.1, 2.2

Skills to Master

- Identify the parts of the rectangular (Cartesian) coordinate system, including the x-axis, y-axis, origin, and quadrants
- Plot and read points from the rectangular (Cartesian) coordinate system
- Graph a linear equation by creating a table of values
- Graph a linear equation by finding the x- and y-intercepts
- Graph a linear equation in slope-intercept form by using the values of m and b
- Graph horizontal and vertical lines in the Cartesian plane
- Use the graph of a linear equation to find the slope of the line
- Use two points found on a line to determine its slope using the slope formula
- Know the slope of vertical and horizontal lines
- Determine the slope of a line parallel or perpendicular to a second line

The Rectangular Coordinate System



Suppose that the above grid represented a neighborhood with your house at the center, the grid lines representing streets, and houses at each intersection of two grid lines. How might you give directions to someone who wanted to get from your house to any other house in the neighborhood?

The above grid is called the **coordinate plane** or the **Cartesian plane**. It is basically two numbers lines: one horizontal number line called the **x-axis**, and one vertical number line called the **y-axis**. The point at which these two axes intersect is called the **origin**

Plotting points on the plane is equivalent to giving directions. But the "directions" are in compact form: (x, y). This is called an **ordered pair** because the order is important. To plot a point on the coordinate plane, start at the **origin** (0, 0), count x spaces horizontally, then from the point at which you stop, count y spaces vertically. REMEMBER: the numbers in an ordered pair represent ONE POINT, NOT TWO POINTS! Each number in an ordered pair is called a **coordinate**. The first value is the **x-coordinate**, and the second is the **y-coordinate**.

Plot the following point on the grid above: A (4, 0), B (-1, 0), C (0, -4), D (1, -2), E (-2, 1)

Are D and E the same or different points?

In this chapter you will study equations that have the form $y = mx + b$, where the values of m and b are given. This form is called the **slope-intercept form.** Examples: $y = 2x - 1$, $y = \frac{1}{2}x + 7$, $y = -3x$

In the above examples, what are the values of m and b in each equation?

Since these equations have two different variables, it is impossible to solve them in the same way we solved before. That is, we cannot end up with a single number answer. This kind of equation allows us to determine the values of one of the variables (called the dependent variable) over all possible values of the other variable (called the independent variable). By convention, we choose *y* to be the dependent variable and *x* to be the independent variable. Since *x* is independent, we can choose any real number values for *x* and use the equation to determine the corresponding values for *y*. Examples:

$y = 2x - 1$			
x	$2x - 1$	y	(x, y)
-2	$2(-2) - 1$		
-1	$2(-1) - 1$		
0	$2(0) - 1$		
1	$2(1) - 1$		
2	$2(2) - 1$		

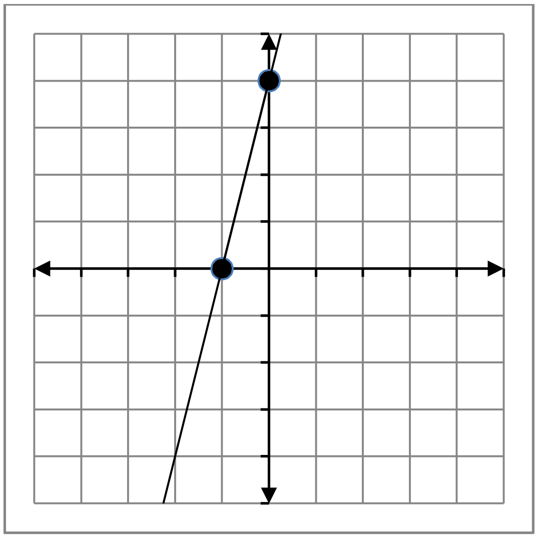
$y = -3x$			
x	$-3x$	y	(x, y)
-2	$-3(-2)$		
-1	$-3(-1)$		
0	$-3(0)$		
1	$-3(1)$		
2	$-3(2)$		

$y = \frac{1}{2}x + 7$			
x	$\frac{1}{2}x + 7$	y	(x, y)
-4	$\frac{1}{2}(-4) + 7$		
-2	$\frac{1}{2}(-2) + 7$		
0	$\frac{1}{2}(0) + 7$		
2	$\frac{1}{2}(2) + 7$		
4	$\frac{1}{2}(4) + 7$		

$y = 3$			
x	$y = 3$	y	(x, y)
-2	$y = 3$		
-1	$y = 3$		
0	$y = 3$		
1	$y = 3$		
2	$y = 3$		

Another form of a linear equation (sometimes called the **standard form**) is $Ax + By = C$, where *A*, *B*, and *C* are integer values. (NOTE: The *B* in this form is NOT the same as the *b* in the slope-intercept form!) The easiest way to graph equations in this form is to find the **x- and y-intercepts.** The x-intercept is the point at which a line intersects the x-axis; the y-intercept is the

point at which a line intersects the y-axis. The graph below shows the x- and y-intercepts for the given line. What are the coordinates of the x- intercept? Of the y-intercept?



Notice that the x-intercept is always of the form $(x, 0)$ and the y-intercept is of the form $(0, y)$. Therefore, we can find the x-intercept by substituting zero for y in a linear equation, and we can find the y-intercept by substituting zero for x in a linear equation.

Example

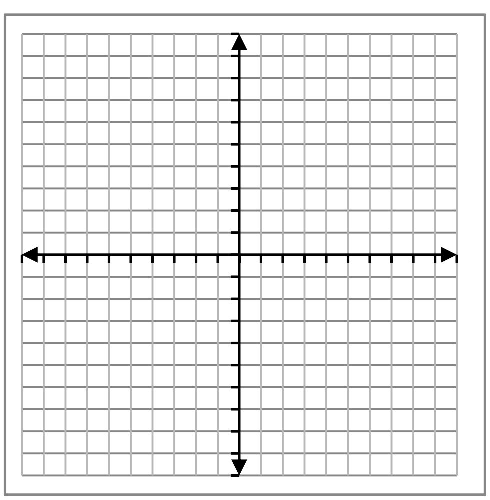
Find the x- and y-intercepts for the equation $3x + 4y = 24$.

Find the x- and y-intercepts for the equation $x - 2y = -2$.

The benefit of using this method is that once you know two points of the line, you have enough information to graph the entire line. Simply plot the points and draw the line between them.

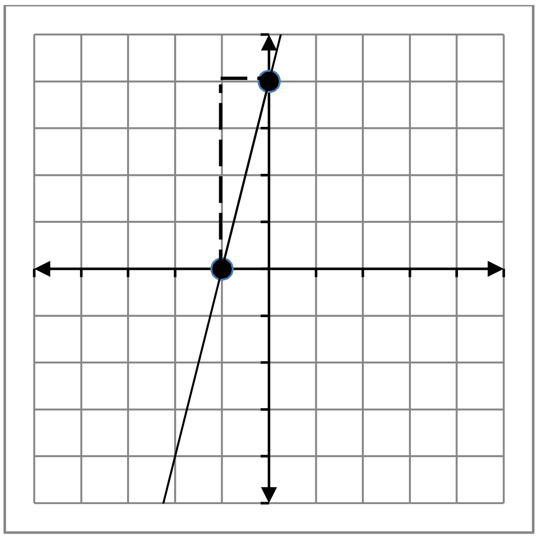
Example

Graph the two equations above on the grid below.



The Slope of a Line

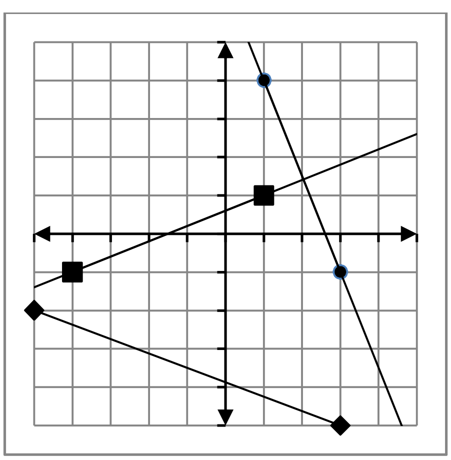
The **slope** of a line is the measure of the slant of the line. A better and more precise way to think of the slope of a line is the rate at which the y values change with respect to the x values.



Two find the slope of a line from the graph of that line, count the **rise** (the vertical distance from one point to any other point on the line); from there count the **run** (the horizontal distance from one point to any other point on the line); the slope is the ratio **rise over run**. In the above example, the rise is +5 and the run is +1, so the slope of the above line is $\frac{5}{1} = 5$. The values of both the rise and the run may be positive or negative. When counting, **the rise is positive when counting up and negative when counting down; the run is positive when counting right and negative when counting left.**

Example

Find the slopes of each of the three lines below.



How can the slope be found if the graph is not given? The slope of a line can be found when two points on the line are given by using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$, where the points are $(x_1,\ y_1)$ and $(x_2,\ y_2)$. In the above example, the marked points are (1, 4) and (3, -1) for the first line and (-4, -1) for the second line (square); (3, -5) and (-5, -2) for the third line (diamond). Use these values to find the slopes of the lines.

Slope of line one:

Slope of line two:

Slope of line three:

What is the slope of the line passing through the points (4, -9) and (-8, -9)?

For this last example, notice that the slope is zero. The line above is a horizontal line. **The slope of a horizontal (flat) line is always zero.** What about the slope of a vertical line? The points (6, 0) and (6, -3) are points on a vertical line. Use the slope formula to find the slope of this line.

The slope of a vertical line is always undefined. This is true because the run in such a line is zero. Since division by zero is undefined, there is no real number associated with the slope. Thus we say that the slope is undefined.

Parallel and Perpendicular Lines

There is an important relationship between the slopes of parallel and perpendicular lines:

1. Parallel lines have equal slopes
2. Perpendicular lines have slopes that are negative reciprocals of each other

Example

Find the slopes of the lines passing through the given points:

Line 1: (4, 3) and (-1, -1)

Line 2: (5, 6) and (-10, -6)

Line 3: (4, -5) and (8, -10)

Which two lines are parallel? Which lines are perpendicular?

When given another line in slope-intercept form, the slope of a second line can be found as follows:

1. To find the slope of another line that is parallel to the given line, simply copy the value of the slope from the equation given
2. To find the slope of a second line that is perpendicular to the given line, take the reciprocal of the slope of the given line and change its sign

Examples

Find the slope of a line that is parallel to $y = 4x - 5$

Find the slope of a line that is perpendicular to $y = -\frac{1}{2}x + 4$

Try these:

Find the slope of a line that is parallel to $y = \frac{1}{5}x - 3$

Find the slope of a line that is perpendicular to $y = 2x + 9$