HALLMARK COLLEGE

Math 1314

College Algebra

LO-4

Chapters 1.3, 1.7

Skills to Master

- Identify a quadratic equation and the values of the coefficients corresponding to a, b, and c in the general form $ax^2 + bx + c = 0$
- Solve a quadratic equation by factoring
- Use the square root property to solve a quadratic equation for which the value of b is zero
- Solve a quadratic equation by completing the square
- Solve quadratic equations using the quadratic formula
- Know and use the properties of inequality
- Solve linear inequalities

Quadratic Equations

A **quadratic equation** is an equation that can be written in the form $ax^2 + bx + c = 0$, where a, b, and c are real numbers.

Since a quadratic equation in the above form has two variables that are different terms, we need a different method of solving than the strategy we employed for linear equations. You will learn three different methods to solve quadratic equations.

1. Solving by factoring
2. Solving by completing the square
3. Solving by use of the quadratic formula

Of these three methods, the first can only be used if the polynomial is factorable. The next two can be used for any quadratics (assuming that a real number solution exists), but completing the square can sometimes be more difficult to use than the quadratic formula.

Solving Quadratic Equations by Factoring

Solving by factoring is a very common method of solving in algebra. The method makes use of a property of zero:

If a and b are real numbers, and if $ab = 0$, then either $a = 0$ or $b = 0$.

Remember from a previous section that factoring a quadratic expression results in the product of two binomials. That is, a factored quadratic expression looks something like this:

$(ax + b)(cx + d)$

where a, b, c, and d are real numbers. Notice that this is a multiplication, so if the quadratic equation is equal to zero, we would have: $(ax + b)(cx + d) = 0$. Using the above property, we can conclude that either $ax + b = 0$ or $cx + d = 0$. The final step is finding the value of x in each case that makes the equation true.

Examples

Solve by factoring: $x^2 + 3x + 2 = 0$

Solve by factoring: $x^2 + x = 12$

Solve by factoring: $2x^2 - 7x - 4 = 0$

Solve by factoring: $6x^2 - 11x = -3$

Solve By Completing the Square

This method is based on the following types of quadratics:

$x^2 + 2ax + a^2 = (x + a)(x + a) = (x + a)^2$

$x^2 - 2ax + a^2 = (x - a)(x - a) = (x - a)^2$

Notice that, regardless of the value of *a*, the middle term is always two times *x* times *a*. Also, the coefficient on the x^2 term must be a 1. This provides a method by which we can *produce* a perfect square.

Steps for completing the square:

1. If the coefficient on the x^2 term is not a 1, turn it into a one by dividing through by the coefficient.
2. Take the value of the coefficient on the *x* term, divided it by two, square this result, and using the additive inverse property, add and subtract it to the left side of the equation.
3. Group in parentheses the x^2 term, the *x* term, and the positive value from above. Combine any other like terms as necessary.
4. Re-write the grouped quadratic as the square of the sum of *x* and the square root of the positive value from step 2.

Examples

$x^2 + 2x - 7 = 0$

1. Notice that step 1 is not necessary since the coefficient on the x^2 term is already a 1.
2. The value of the coefficient on the *x* term is _______. Dividing this value by 2 gives _____. Squaring this value gives_____. Add zero by changing the equation to $x^2 + 2x + \_\_ - \_\_ - 7 = 0$
3. Group with parentheses: $(x^2 + 2x + \_\_) - \_\_\_ - 7 = 0$
4. Write as a perfect square and combine like terms: $(x + \_\_\_)^2 - \_\_\_ = 0$

$b^2 - 3b = 5$

$2y^2 + 6y - 11 = 0$

Try this one on your own.

$g^2 + 2g - 2 = 0$

Solving Quadratic Equations Using the Quadratic Formula

The quadratic formula can be derived by completing the square on the general form of a quadratic equation, $ax^2 + bx + c = 0$. We won't derive it here, but it is important to keep two things in mind:

1. The values the are input into the formula come from the values of *a*, *b*, and *c* in the given quadratic equation. LEARN TO IDENTIFY THESE VARIABLES!!!
2. The equation MUST EQUAL ZERO before the formula can be used!!!

The formula is: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. (You won't have to memorize it.)

Examples

Solve using the quadratic formula: $x^2 + 2x - 7 = 0$

Identify the values: *a* = _____, *b* = _____, *c* = _____

The equation is already equal to zero, we can go straight to the formula:

Solve using the quadratic formula: $3x^2 + x + 5 = 0$

Solve using the quadratic formula: $9x^2 - 6x - 50 = 0$

Properties of Inequality

Except for one very important case, the properties of inequalities are the same as the properties of equality. This means that, except for that special case, solving inequalities proceeds in the same way as solving equalities.

Trichotomy property For any real numbers *a* and *b*, one and only one of the following must be true:

$a < b \quad a = b \quad a > b$

Transitive property For any real numbers *a*, *b*, and *c*:

If $a < b$ and $b < c$, then $a < c$

If $a > b$ and $b > c$, then $a > c$

Addition/Subtraction Properties For any real numbers *a*, *b*, and *c*:

If $a < b$, then $a + c < b + c$

If $a < b$, then $a - c < b - c$

The same properties hold for the greater than relation.

Multiplication/Division Properties For any real numbers *a*, *b*, and *c*:

If $a < b$ and $c > 0$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$

If $a < b$ and $c < 0$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$

The last property is the one to watch out for. The property tells us that, if an inequality is multiplied or divided by a NEGATIVE NUMBER, then the inequality sign is reversed. The same properties hold for the greater than relation.

Examples

Solve $3(x + 2) < 8$

Solve $-5(x - 2) \leq 20 + x$

Try the following on your own.

$3 + x \geq 3x + 1$

$5(p - 4) > 25$