HALLMARK COLLEGE

Math 1314

College Algebra

LO-3

Chapter 0.5, 1.1

Skills to Master

- Factor out a common monomial
- Factor by grouping
- Factor the difference of two squares
- Factor trinomials
- Know and use the properties of equality to solve equations
- Identify and solve linear equations
- Solve for given variables in formulas

Factoring Out a Common Monomial

If the terms of a polynomial all have a common factor, that factor can be "removed" out in what is basically the opposite of distribution.

Examples

$8x - 12y =$

$-7a^3 + 4a =$

$b^5 + 4b^3 - 6b^2 =$

$11n^3m^3 - 2nm^2 + 6n^2m =$

Factoring By Grouping

In some cases, though the terms of a polynomial may not have any common factors, pairs or more of particular terms may be grouped together and each factored separately. Then each of the terms may again have factors in common.

Examples

$ax + bx + a + b =$

$x^2 + xy + 2x + 2y =$

$15a^2b^2 + 6ab - 5abc - 2c =$

Factoring the Difference of Squares

If a binomial happens to be a difference where each term is a perfect square, it factors as the sum of the square roots of the terms times the difference of the square roots of the terms.

Examples

$x^2 - y^2 =$

$25a^6 - 16b^4 =$

$x^4y^2 - 9z^8 =$

Factoring Trinomials

Trinomials that are the product of two binomials (i.e., formed by using FOIL) can be factored. Generally, factoring trinomials is not easy. There exist various methods for doing so. We'll look first at an easy type of trinomial to factor, then at a method for factoring more difficult trinomials.

Factoring a trinomial where the coefficient on the square term is a 1

A trinomial of the type we are considering has the following form: $ax^2 + bx + c$, where *a*, *b*, and *c* are integers. We are concerned here with trinomials where $a = 1$.

To factor such a trinomial:

1. Find the factors of the *c* term
2. Choose the factors that combine (by addition or subtraction) to give the *b* term
3. Use these factors as the second terms of each binomial

Examples

Factor $x^2 + 5x + 6 =$

Factor $a^2 - a - 12 =$

Factor $y^2 + 2y - 35 =$

Factor $c^2 - 9c + 20 =$

Factoring a trinomial where the coefficient on the square term is not a 1

There are various methods by which to factor a trinomial where the value of *a* is not 1. The method I'll be showing you is called "triple play" (you'll see why).

To use the triple play method:

1. Find the factors of the *c* term
2. Choose the factors that combine (by addition or subtraction) to give the *b* term
3. Write down the *a* term three times as shown below
   $$\frac{ax \qquad\qquad ax}{a}$$
4. Form two binomials by placing the factors found in step two next to each *ax* on top (let's say that the factors are *m* and *n* for illustration purposes)
   $$\frac{(ax + m)(ax + n)}{a}$$

5. Now one of two things must be true:
    a. *a* divides evenly into both terms of *one* of the two binomials
    b. *a* does not divide evenly into either terms of the two binomials, but *a* can be factored such that one of its factors divides into both terms of one of the binomials, and the second factor divides into the two terms of the other binomial

Examples

Let's look first at an example like part a. of number 5:

Factor $2x^2 + 15x + 7$

Now let's examine an example like part b. of number 5:

Factor $6x^2 + 7x - 20$

Try these on your own:

Factor $3x^2 - 5x + 12$ (answer: $(3x + 4)(x - 3)$)

Factor $8x^2 + 2x - 15$ (answer: $(4x - 5)(2x + 3)$)

Equations—Properties of Equality

An equation is a statement indicating that two quantities are equal. If one or more variables are part of an equation, then either there are one or more real numbers that cause the equation to be true (in which case they are called the **solutions to the equation)**, or there are no such numbers. When no such numbers exist, we say that the equation has **no solution** or we can say that the solution set is the empty set.

Our goal in this section is to learn how to find the solution set (if it exists) for an equation with a single variable. Since the value of the variable is initially unknown, we will use **properties of equality** to form successively simpler equivalent equations until we reach something like "x = ", where the solution is written in the space to the right of the equal sign.

Addition/Subtraction Properties of Equality	Multiplication/Division Properties of Equality
If $a = b$, then:	If $a = b$, then:
$a + c = b + c$	$ac = bc$
$a - c = b - c$	$\frac{a}{c} = \frac{b}{c}$

Solving One-Step Equations

Examples

Solve $x + 4 = 9$

Solve $3a = 18$

Solve $y - 3 = -7$

Solve $\frac{b}{3} = -2$

Solving Two-Step Equations

Examples

Solve $2x - 3 = 17$

Solve $17 = 3x + 2$

Solve $4 - 5x = -5$

Solve $1 = -\frac{d}{3} - 4$

Solve $\frac{3}{4}c - 1 = 8$

Solving Multi-step Equations

Examples

Solve $4x + 3 = 9x - 2$

Solve $2x + 2(3x - 5) = 8x - 10$

Solve $4t + 2(t - 1) = 3t + 7$

Solve $7y - 1 = y + 3(2y + 5)$

Solve $5 - (2x + 3) = 3(x + 7)$

Solving Formulas

An equation that contains two or more *different* variables cannot be solved to produce a numerical value. But they *can* be solved for individual variables using the same properties of equality.

Examples

Solve for C: $F = \frac{9}{5}C + 32$

Solve for r: $A = p(1 + rt)$