HALLMARK COLLEGE

Math 1314

College Algebra

LO-2

Chapter 0.3, 0.4

Skills to Master

- Use and interpret rational exponents with 1 in the numerator
- Use and interpret rational exponents whose numerators are not 1
- Use and interpret radical expressions and convert between radical expressions and rational exponents
- Simplify and combine radical expressions
- Name a polynomial and identify its degree
- Add, subtract, and multiply polynomial expressions

Rational Exponents Whose Numerators Are 1

Suppose you were asked to determine the value of the numerical expression $25^{\frac{1}{2}}$. Not knowing the meaning of rational (or fractional) exponents, you might try applying the power-to-power rule: $\left(25^{\frac{1}{2}}\right)^2 = 25^{\frac{1}{2}\cdot 2} = 25^1 = 25$. Hence you conclude that if $a \geq 0$ and n is a natural number, then $a^{\frac{1}{n}}$ is the non-negative real number b such that $b^n = a$. That is, b is the nth root of a. For the above example, if $b = 25^{\frac{1}{2}}$, then $b^2 = 25$. That is, b is the number that, when multiplied by itself, produces 25. Clearly, $b = 5$. We see, then, that <u>fractional exponents produce the roots of numbers.</u>

Examples

$16^{\frac{1}{2}} = 4$ since $4^2 = 16$. The square root of 16 is 4

$8^{\frac{1}{3}} = 2$ since $2^3 = 8$. The cube root of 8 is 2

Remember that the square root of a number produces two values: one positive and one negative. But with a rational exponent, we use the **principal root**, which is always positive.

$(81x^4)^{\frac{1}{4}} = 3|x|$ since $3^4 = 81$. The 4th root of 81 is 3 and the 4th root of x^4 is *x*. But since both *x* and –*x* produce the same answer, we use take the absolute value to guarantee that the result is positive.

Also, if n is even and b is negative, then there is no real number answer. However, if n is odd and b is negative, then the answer is the negative *n*th root of b.

Examples

$(-9)^{\frac{1}{2}}$ has no real number solution since, if b is positive, there is no real number such that $-b \cdot -b = b \cdot b = -9$ (Why?)

$(-64)^{\frac{1}{3}} = -4$ since $-4 \cdot -4 \cdot -4 = (-4)^3 = -64$

Rational Exponents Whose Numerators Are Not 1

Remember that fractions are multiplied by multiplying the numerators together and the denominators together. That is, $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bc}, b \neq 0, c \neq 0$. We can use this to factor fractions as well. For example, $\frac{2}{3} = \frac{2}{1} \cdot \frac{1}{3} = 2 \cdot \frac{1}{3}$. Keeping this in mind and using the power-to-power rule for exponents, it is easy to see that, for example, $64^{\frac{2}{3}} = 64^{\frac{2}{1} \cdot \frac{1}{3}} = 64^{2 \cdot \frac{1}{3}} = (64^2)^{\frac{1}{3}}$. Or we can write $64^{\frac{2}{3}} = 64^{\frac{1}{3} \cdot \frac{2}{1}} = 64^{\frac{1}{3} \cdot 2} = \left(64^{\frac{1}{3}}\right)^2$. So for a rational expression $\frac{m}{n}$, as an exponent it is either the *n*th root raised to the power of *m*, or as the power of *m* raised to the *n*th root. (Note that this is the result of the commutative property of multiplication. Why?)

In general, for real number *b* and whole numbers *m* and *n*, $b^{\frac{m}{n}} = \left(b^{\frac{1}{n}}\right)^m = (b^m)^{\frac{1}{n}}$. We can use the same line or reasoning to interpret $b^{-\frac{m}{n}}$:

$$b^{-\frac{m}{n}} = b^{-m \cdot \frac{1}{n}} = (b^{-m})^{\frac{1}{n}} = \left(\frac{1}{b^m}\right)^{\frac{1}{n}} \text{ OR } b^{-\frac{m}{n}} = b^{-\frac{1}{n} \cdot m} = \left(b^{-\frac{1}{n}}\right)^m = \left(\frac{1}{b^{\frac{1}{n}}}\right)^m$$

Example

$$256^{-\frac{3}{4}} = \left(256^{-\frac{1}{4}}\right)^3 = \left(\frac{1}{256^{\frac{1}{4}}}\right)^3 = \left(\frac{1}{4}\right)^3 = \frac{1^3}{4^3} = \frac{1}{64}$$

Radical Expressions

Another way to express the root of a number is to use the **radical sign**. The radical sign is $\sqrt{\ }$. In general, $\sqrt[n]{b} = b^{\frac{1}{n}}$ and $\sqrt[n]{b^m} = b^{\frac{m}{n}}$. Here, *b* is called the **radicand** and *n* is the **index** of the expression. For *n* = 2, the index is understood. That is, $\sqrt[2]{b} = \sqrt{b}$.

Examples

$\sqrt[3]{125} = 5$ since $5^3 = 125$

$$\sqrt[5]{32^2} = 32^{\frac{2}{5}} = \left(32^{\frac{1}{5}}\right)^2 = 2^2 = 4$$

$$\sqrt{9^3} = 9^{\frac{3}{2}} = \left(9^{\frac{1}{2}}\right)^3 = 3^3 = 27$$

An important fact to keep in mind is that $\sqrt[n]{b^n} = \left(\sqrt[n]{b}\right)^n = b$. That is, taking the *n*th root of a number raised to the *n*th power, or raising an *n*th root to the *n*th power cancels the root and

returns the radicand (or the base). Notice that this makes sense since $\sqrt[n]{b^n} = \left(b^{\frac{1}{n}}\right)^n = b^{\frac{n}{n}} = b^1 = b$. We will use this fact when solving equations that contain powers or roots.

Examples

$\sqrt{18^2} = \sqrt{324} = 18$; $\sqrt[3]{216} = \sqrt[3]{6^3} = 6$; $\sqrt[5]{x \cdot x \cdot x \cdot x \cdot x} = \sqrt[5]{x^5} = x$

Simplifying and Combining Radicals

For real numbers *a*, *b* and *n*:

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$$

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}, b \neq 0$$

This means that the multiplication of two *n*th roots can be written as the *n*th root of the product of the radicands, and the division of two *n*th roots can be written as the *n*th root of the quotient of the radicands. Note that this only works if the values of *n* are the same!!!

Example

$\sqrt{5} \cdot \sqrt{20} = \sqrt{5 \cdot 20} = \sqrt{100} = 10$

Since the roots of most numbers are irrational, we often leave a radical expression in radical form. So rather than 1.414…, we write $\sqrt{2}$. This allows us to combine the same radicals as a multiplication. For example, $\sqrt[3]{7} + \sqrt[3]{7} + \sqrt[3]{7} = 3\sqrt[3]{7}$ (i.e., three times the square root of seven). In general, for real numbers *a*, *b*, *c* and *n*, $a\sqrt[n]{c} + b\sqrt[n]{c} = (a+b)\sqrt[n]{c}$. Note that this only works when the radicals have the same value. It is not possible, for instance, to simplify $\sqrt[n]{c} + \sqrt[m]{c}$ since *n* and *m* are not equal (assuming that they are not equal). Nor can we simplify $\sqrt[n]{a} + \sqrt[n]{b}$ since *a* and *b* are not the same (assuming they are not equal). However, if *a* or *b* can be factored such that one of the factors is a perfect power of *n* and the other is the same for both *a* and *b*, then we can simplify the expression. For example, $\sqrt[3]{54} + \sqrt[3]{16} = \sqrt[3]{2 \cdot 27} + \sqrt[3]{2 \cdot 8} = \sqrt[3]{2} \cdot \sqrt[3]{27} + \sqrt[3]{2} \cdot \sqrt[3]{8} = \sqrt[3]{3^3} \cdot \sqrt[3]{2} + \sqrt[3]{2^3} \cdot \sqrt[3]{2} = 3\sqrt[3]{2} + 2\sqrt[3]{2} = 5\sqrt[3]{2}$

Examples

$\sqrt{27} - \sqrt{12} =$

$\sqrt{50} + \sqrt{200} =$

$\sqrt[5]{64z} - 2\sqrt[5]{2z^6} =$

Polynomials

A **monomial** is a number or the product of a number and one or more variables with whole-number exponents. The number is called the **coefficient** of the monomial.

Examples

$3x$ Coefficient =

ab^2 Coefficient =

$-5ab^2c^4$ Coefficient =

-12

The **degree** of a monomial is the sum of the exponents of its variables. All non-zero constants have a degree of zero. For the examples above, what is the degree of each monomial?

A monomial or a sum of monomials is called a **polynomial**. Each monomial in a polynomial is called a **term** of the polynomial.

Examples

Monomials	Binomials	Trinomials
$3x^2$	$2a + 3b$	$x^2 + 7x - 4$
$-25xy$	$4x^3 - 3x^2$	$4y^4 - 2y + 2$
a^2b^3c	$-2x^3 - 4y^2$	$12x^3y^2 - 8xy - 24$

The degree of a polynomial is the degree of the term with highest degree.

Any terms in a polynomial that have the same variables with the same exponents are called **like terms**. The important thing to remember about like terms is that **they can be added or subtracted**.

Like Terms	Unlike Terms
$x, 5x, -3x$	$2x, 3x^2, -x^3$
$ab, 2ab, -9ab$	$a^2b, 2ab^3, 4a^3b^2$
$mn^2, 3mn^2, \frac{1}{3}mn^2$	$3mn, 3mn^2, 3m^2$

Like terms are combined by **adding or subtracting their coefficients**.

Examples

$3a + 4a =$

$7xy - 2xy + 5xy =$

$-ab^2 + 2ab^2 + 5ab^2 =$

$9m + 3n - 4m + 6n =$

Multiplying Monomials

Examples

$(2abc)(-4ab^3c^2) =$

$7x^2y^4 \cdot xy =$

$-p^3q^2 \cdot 4q^3 =$

Multiplying a Monomial and a Polynomial

Use the distributive property.

Examples

$5(2x + 3y)$=

$x(3x^3 - 4x + y)$

$-2ab^2(ab - 3a^2b) =$

Multiplying Two Binomials

Use the FOIL method: First Last Inside Outside

Examples

$(a + 1)(a - 2) =$

$(3x - y)(2x + y) =$

$(n - 1)(m + 2) =$

$(4g - 3h)(g + h) =$