HALLMARK COLLEGE

Math 1314

College Algebra

LO-10

Chapters 6.3

Skills to Master

- Multiply matrices of various dimensions by a constant (scalar)
- Add and subtract matrices of various dimensions
- Multiply 2x2 matrices
- Use the properties of matrix algebra

Matrix Algebra

In mathematics, a matrix is simply an array of numbers in rows and columns. Matrices can be used in various ways, but we are interested in how matrices relate to systems of equations. However, for this brief introduction we will only be concerned with learning what a matrix is, some properties of matrices, and the math operations that can performed on matrices.

A **matrix** is a rectangular array of *mn* numbers arranged in *m* rows and *n* columns. We say that a matrix is of size $m \times n$.

Matrices are usually represented by capital letters, but the following notations are also common:

$$[a_{ij}] \text{ or } \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

In the second representation, the matrix is an $m \times n$ matrix and the dotted lines represent omitted values. In an actual matrix with numbers, all numbers would be shown. The first representation is more compact and the values of *i* and *j* are all numbers for *i* from 1 to *m* and *j* from 1 to *n*.

Two matrices *A* and *B* are equal if both are of the same size and if the values of the corresponding elements are the same. For example,

$\begin{bmatrix} 3 & 4 & 0 \\ 1 & -3 & 7 \end{bmatrix}$ is equal to $\begin{bmatrix} 3 & 4 & 0 \\ 1 & -3 & 7 \end{bmatrix}$, but $\begin{bmatrix} 3 & 4 & 0 \\ 1 & -3 & 7 \end{bmatrix}$ is not equal to $\begin{bmatrix} 1 & 4 & 0 \\ 1 & -3 & 7 \end{bmatrix}$ since the values in position a_{11} are not the same.

Addition/Subtraction of Matrices

Adding or subtracting matrices is a simple matter of adding/subtracting the corresponding elements. But only matrices of the same dimensions may be added or subtracted.

Example

$$\begin{bmatrix} 2 & -4 \\ 5 & 3 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ 1 & 9 \end{bmatrix} =$$

$$\begin{bmatrix} 4 & 0 & -1 \\ -5 & 6 & 0 \end{bmatrix} + \begin{bmatrix} 5 & 3 & 6 \\ 2 & -1 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & 5 \\ 0.5 & 8 & -4 \end{bmatrix} + \begin{bmatrix} -4 & 1 & 0 \\ 2 & 6 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 4 & 5 \\ -6 & -1 \\ 3 & 0 \end{bmatrix} - \begin{bmatrix} 6 & -1 \\ 1 & 2 \\ 0 & 2 \end{bmatrix} =$$

$$\begin{bmatrix} 0 & 1 \\ 9 & 5 \end{bmatrix} + \begin{bmatrix} 5 & 3 \\ -7 & -1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & -1 & 3 \\ 5 & 0 & -2 \\ 11 & 9 & 8 \end{bmatrix} - \begin{bmatrix} 1 & -3 & 2 \\ 6 & 2 & 7 \\ 0 & -3 & 15 \end{bmatrix} =$$

Scalar Multiplication

A **scalar** is a real number. Multiplying a matrix by a scalar is similar to distributing.

Example

$$2\begin{bmatrix} 5 & 3 \\ -1 & 0 \end{bmatrix} =$$

$$3\begin{bmatrix} 4 & -3 \\ 7 & 2 \\ -5 & 0 \end{bmatrix} =$$

$$-5\begin{bmatrix} 2 & 1 & 4 \\ 3 & -2 & -1 \end{bmatrix} =$$

$$-1\begin{bmatrix} 5 & 4 & 0 \\ 3 & -1 & 7 \\ 10 & 6 & -9 \end{bmatrix} =$$

Multiplying Matrices

Matrices may only be multiplied if the number of columns of the first matrix is equal to the number of rows of the second. It is possible to multiply, for example, a 4x3 matrix and a 3x2 matrix in this order. The result is a matrix with the same rows as the first matrix and the same columns as the second. For the above example, the resulting matrix is of size 4x2. But note that these same two matrices cannot be multiplied in the reverse order. That is, it is not possible to multiply a 3x2 matrix by a 4x3 matrix since the number of columns of the first is not equal to the number of rows of the second. What this means is that **matrix multiplication is not commutative.** Changing the order of a matrix multiplication may either result in a different answer or in no answer at all.

To multiply two matrices:

1. Determine if the matrix multiplication is possible by looking at the number of columns of the first matrix and the number of rows of the second matrix.
2. If multiplication is possible, determine the size of the resulting matrix. For example, if A is a 4x5 matrix and B is a 5x3 matrix, then the resulting matrix C will be a 4x3 matrix.
3. Set up the resulting matrix with empty spaces.

4. Each entry c_{ij} in the resulting matrix C is the sum of the products of the corresponding entries in the *i*th row of A and the *j*th column of B.

Example

$$\begin{bmatrix} 1 & 2 & -2 \\ 4 & 3 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 0 \\ -2 & 4 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 3 & -4 \end{bmatrix} =$$

$$\begin{bmatrix} 3 & 1 \\ -1 & 0 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & -2 \\ 4 & 3 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 2 & -1 & 0 \\ -3 & 7 & 3 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix} =$$

$$\begin{bmatrix} 2 & 0 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} =$$

$$\begin{bmatrix} 0 & 3 & 10 \\ 4 & 2 & 3 \\ 6 & -1 & -5 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

Note that a matrix of size *n* x *n* is called **a square matrix.** It is always possible to multiply two square matrices of the same size, but reversing the order of the multiplication may result in a different answer. Also notice that in the last example, the second matrix (with ones in the diagonal) results in a matrix identical to the multiplier. This matrix is called the **identity matrix.** It works like 1 when multiplying. Also, in matrix addition, a matrix with zeros in all rows and columns is called the **zero matrix**. Adding a zero matrix to any other matrix of the same dimensions results in the identical matrix. Other matrix properties are as follows:

For matrices A, B, and C, and for scalars *a* and *b*:

- $A + B = B + A$
- $A + (B + C) = (A + B) + C$
- $a(bA) = (ab)A$ and $a(AB) = (aA)B$
- $(a + b)A = aA + bA$ and $a(A + B) = aA + aB$
- $A(BC) = (AB)C$
- $A(B + C) = AB + AC$ and $(A + B)C = AC + BC$