HALLMARK COLLEGE

Math 1314

College Algebra

LO-1

Chapters 0.1, 0.2

Skills to Master

- Determine the real number sets to which a given number belongs
- Name and use the properties of real numbers
- Graph subsets of the real numbers
- Use inequality symbols
- Use interval notation and convert between interval notation, inequalities, and graphs for a given subset of the real numbers
- Write an expression containing natural number exponents as one without exponents
- Know and use the rules of exponents
- Know and use the order of operations
- Evaluate expressions
- Write numbers in scientific notation and convert between decimal notation and scientific notation

Number Sets

Natural Numbers {1, 2, 3, …}

Whole Numbers {0, 1, 2, …}

Integers {…, -2, -1, 0, 1, 2, …}

Rational Numbers $\{x\colon x = \frac{a}{b}, a, b \in \mathbb{N}\}$

Irrational Numbers $\{x\colon x$ is not rational$\}$

Real Numbers {*x*: *x* is rational or *x* is irrational}

PRACTICE

Classify the following numbers:

4___________________________

-7___________________________

3/5__________________________

1.4__________________________

0____________________________

π___________________________

Number Properties

The following properties are assumed to be true for all real numbers:

- Addition properties:
    - *Closure:* adding any two real numbers results in a real number
    - *Associativity*: for any real numbers *a*, *b*, and *c*, $a + (b + c) = (a + b) + c$
    - *Commutativity:* for any real numbers *a* and *b*, $a + b = b + a$
    - *Identity property of zero:* for any real number *a*, $a + 0 = 0 + a = a$
    - *Additive inverse property:* for any real number $a \neq 0$, there exists a unique $-a$ such that $a + (-a) = -a + a = 0$
- Multiplication properties:
    - *Closure:* multiplying any two real numbers results in a real number
    - *Associativity*: for any real numbers *a*, *b*, and *c*, $a(bc) = (ab)c$
    - *Commutativity:* for any real numbers *a* and *b*, $ab = ba$
    - *Identity property of one:* for any real number *a*, $a \cdot 1 = 1 \cdot a = a$
    - *Multiplicative inverse property:* for any real number $a \neq 0$, there exists a unique a^{-1} such that $(a)(a^{-1}) = (a^{-1})(a) = 1$
- Distributive property:
    - For any real numbers *a*, *b*, and *c*, $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$

Examples:

Name the number property:

$5 + (7 + 9) = (5 + 7) + 9$

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$4 \times \frac{1}{4} = 1$

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$(ab)c = c(ab)$

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$-9 + 0 = -9$

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Inequalities

Symbol	Read as	Examples	
$\neq$	"not equal to"	$5 \neq 8$	$0.25 \neq \frac{1}{3}$
$<$	"is less than"	$12 < 20$	$0.17 < 1.1$
$>$	"is greater than"	$15 > 9$	$\frac{1}{2} > 0.2$
$\leq$	"is less than or equal to	$25 \leq 25$	$1.7 \leq 2.3$
$\geq$	"is greater than or equal to"	$19 \geq 19$	$15.2 \geq 13.7$
$\approx$	"is approximately equal to"	$\sqrt{2} \approx 1.414$	$\pi \approx 3.14$

Intervals and Interval Notation

For any real numbers *a* and *b*:

(b, ∞) is equivalent to $x > b$

$[b, \infty)$ is equivalent to $x \geq b$

$(-\infty, b)$ is equivalent to $x < b$

$(-\infty, b]$ is equivalent to $x \leq b$

(a, b) is equivalent to $a < x < b$

$[a, b)$ is equivalent to $a \leq x < b$

$(a, b]$ is equivalent to $a < x \leq b$

$[a, b]$ is equivalent to $a \leq x \leq b$

Natural Number Exponents

Addition of two or more of the same number can be written as a multiplication. For example, $4 + 4 + 4 + 4 + 4 = 5 \cdot 4$. With variables: $a + a + a + a = 4a$. Exponents serve a similar purpose, but for multiplication of several of the same number. For example, $7 \cdot 7 \cdot 7 \cdot 7 = 7^4$. With variables, $b \cdot b \cdot b \cdot b \cdot b = b^5$. MAKE SURE YOU KNOW THE DIFFERENCE BETWEEN THESE FORMS OF NOTATION! Exponents can only be affected by multiplication.

In exponential notation, the b in the previous example is called the **base** and the 5 is the **power** to which the base is raised. If no exponent is given, the base is understood to have the exponent 1. That is, $x^1 = x$. We will later see that $x^0 = 1$ for all real numbers other than zero.

Rules for Exponents

For any real number x and for natural numbers m and n:

$$x^m x^n = x^{m+n}$$

$$\frac{x^m}{x^n} = x^{m-n}, x \neq 0$$

$$(x^m)^n = x^{mn}$$

$$x^{-n} = \frac{1}{x^n}$$

For any real number y:

$$(xy)^n = x^n y^n$$

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}, y \neq 0$$

$$\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$$

Examples:

Simplify.

$(a^3 b^7)(ab^3) =$

$\frac{r^6 t^4}{r^2 t^5} =$

$(3n^2 p)^3 =$

Order of Operations and Evaluating Expressions

When an expression includes several different operations, the operations must be performed in the following order:

1. Parentheses (or other grouping symbols)—evaluate all expressions within parentheses
2. Exponents—evaluate all exponent operations
3. Multiplication/Division—perform multiplication and division operations as they occur from left to right.
4. Addition/Subtraction—perform addition and subtraction operations as they occur from left to right.

Example: Evaluate $\frac{3[4-(6+10)]}{2^2-(6+7)} =$

An the variables in an expression are assigned values, the values can be substituted in the expression. The expression can then be evaluated. Example:

Evaluate the given expression for $x =$ -2, $y =$ 3, and $z =$ 4

$-x^2 + y^2z =$

$\frac{2z^3 - 3y^2}{5x^2} =$

Scientific Notation

Format: $N \times 10^n$, where $1 \leq |N| < 10$ and *n* is an integer.

Convert from decimal to scientific notation:

1. Go to the decimal and move it either left or right until there is **a single digit** to the left of the decimal point.
2. Write a $\times$ 10 after the last digit and the exponent is the same as the number of places you moved the decimal. If the movement was to the **left**, the exponent is positive; if to the **right**, the exponent is negative.
3. Examples:
   $43.2 =$
   $-53{,}235 =$
   $0.112 =$
   $0.0003441 =$
   $2{,}500{,}000 =$

Convert from scientific notation to decimal:

1. Go to the decimal point and move it either:
    a. The same number of spaces to the **right** as the exponent on the 10 if the exponent is positive
    b. The same number of spaces to the **left** as the exponent on the 10 if the exponent is negative
2. Write your answer without the $\times 10$
3. Examples
   $2.11 \times 10^{-3} =$
   $-3.1 \times 10^{3} =$
   $4 \times 10^{5} =$
   $-1.04 \times 10^{-5} =$
   $2.201 \times 10^{7} =$