Abstract
The article presents a classroom-suited version of the equilibrium exchange rate model of Stockman (1987) that features Cobb-Douglas functional forms for both production and utility, and considers foreign exchange intervention explicitly. (JEL classification: F31, F41)

1. Introduction
This article presents a simplified version of the equilibrium exchange rate model of Stockman (1987). It aims to present a useful classroom teaching model of a complicated model. Stockman (1987) model has been sometimes presented as the equilibrium exchange rate model in textbooks for undergraduates (e.g. Gärtner, 1993, p. 204-210). Actually, Stockman (1987) discusses informally several models that are particular cases of a more general (but messier) one introduced earlier in Stockman (1980) (e.g. Stockman, 1987, p. 14; also Stockman, 1980, p. 689-692). A novelty in this presentation is the use of Cobb-Douglas functional forms for both production and utility, together with an explicit modelling of foreign exchange intervention. This procedure simplifies enormously the solution to the equilibrium model.

The equilibrium approach to exchange rates is identified not only with the models presented in Stockman (1980 and 1987), but also with Lucas (1982), and Svensson (1985), among others. Its concept of equilibrium refers to the assumption that markets clear through price adjustments, so equilibrium models typically assume that all prices are fully flexible. In a sense, equilibrium models are the offshoot of the macro literature on real business cycles (Taylor, 1995, p. 24, n. 21). Also, they are arguably equilibrium versions of the old-fashioned elasticity approach to the foreign exchange market (Stockman, 1980, p. 674, 693).

The model of Stockman (1987), in particular, is based on simple microeconomic principles within an intertemporal optimizing framework. Preference shifts and productivity shocks can change the real exchange
rate. Accordingly, purchasing power parity (PPP) holds neither permanently as in flexible-price ‘monetarist’-type models nor in the long run, as in the sticky-price Dornbusch (1976) model.

The major argument of the model of Stockman (1987) is the following. From microeconomics we know that real disturbances to demands for goods or supplies of goods such as preference shifts or productivity shocks cause relative prices to change. These encompass the relative price of foreign goods in terms of domestic goods, i.e. the real exchange rate defined as the terms of trade. Nominal exchange rate changes need not lead to real changes, as in the Dornbusch model; rather, real exchange rate changes are an equilibrium response to underlying changes in real factors. For that reason, neither nominal nor real exchange rate volatility are ‘good’ or ‘bad’.

As a consequence, there is a credible case to be made that governments should not invoke protectionist restrictions of trade in goods or financial assets as a response to changes in the exchange rates. Foreign exchange intervention cannot affect the real exchange rate by altering the nominal rate through monetary policy. Intervention affects the real rate only if it signals the willingness to pursue policies that affect it (Stockman, 1987, p. 29). Also, the choice of fixed versus flexible exchange rate on its own is not important for the real exchange rate, the trade balance, and the current account (Stockman, 1987, p. 12, 29). Trade deficits do not cause currency depreciation, nor does currency depreciation on its own help to reduce a trade deficit (Stockman, 1987, p. 13). Thus, one cannot blame decreased competitiveness on the exchange rate.

Why should one be interested in the equilibrium exchange rate model? One good reason is that deviations of real exchange rates from PPP appear to be persistent. As observed, ‘the central puzzle in international business cycle is that real exchange rates are volatile and persistent’ (Chari, Kehoe and McGrattan, 2000, p. 1). In practice PPP does not seem to hold. The equilibrium approach illustrates why this might not be so surprising. Another reason is related to modelling. A recent attempt to provide the traditional sticky-price Dornbusch model with microfoundations is the so-called ‘new open economy macroeconomics’ (e.g. Obstfeld and Rogoff, 1996, chapter 10). A drawback with such an approach, however, is that models are complicated, though that in itself is not a
reason to reject the approach. An attempt to simplify these models is the model of Corsetti and Pesenti (2001). They use Cobb-Douglas functions and solve their model graphically. Corsetti and Pesenti draw on some of the insights which are present in the Stockman’s (1987) model being discussed here. So a further simplification of the model of Stockman using precisely Cobb-Douglas functions will be the objective of this paper.

2. The model

Building on Stockman (1987), a two country model is developed. The countries are identical except for the differences described below. In particular, the countries have equal populations, and households in each country have the same tastes. There are two normal perishable goods, say oranges and apples. The domestic country say the US produces only oranges, whereas the foreign country say the UK produces only apples. The number of units of the domestic good (oranges) produced each time period is denoted by \( Y_t \), and the quantity of the foreign good (apples) produced each time period is denoted by \( Y_t^f \). There is perfect competition among producers. The two countries trade with no barriers, transportation or transaction costs so that households can consume the same amounts of both goods as they have the same tastes and resources. The model describes the world in each one of a series of time periods. The intertemporal model is described by repeating the model (Table 1) at each time period (Stockman, 1987, p. 22). The requirements needed for that setting to work are explained below.

Instead of modelling production as a pure endowment, production of oranges and apples at each date is governed by a Cobb-Douglas technology, i.e.

\[ Y_t = K_t^\alpha N_t^{1-\alpha} \]  

(1)

and

\[ Y_t^f = (K_t^f)^\alpha N_t^{1-\alpha} \]  

(2)

where \( K_t \) is the domestic capital at time period \( t \); \( K_t^f \) is the foreign capital at \( t \); \( N_t \) stands for the labour existing at \( t \), and both countries are assumed to have equal labor \( N_t \) each period since they have the same
population each period; and \( \alpha \in (0, 1) \) gives the capital share of income, assumed to be the same in the two countries. (Given that production functions (1) and (2) are linearly homogeneous, the assumption that both countries have an equal number of residents could be relaxed, the model could be formulated in per capita terms, and \( K \) could be interpreted as the capital labour ratio.)

As both representative households in the two countries have the same tastes, they are assumed to have the same utility function \( U \) given by the Cobb-Douglas functional form as well, i.e.

\[
U_c = Y_c^\beta \left( \frac{Y_c}{Y_f} \right)^{1-\beta}
\]

where \( \beta \in (0, 1) \) stands for the share of the domestic good to be consumed regardless of changes in the relative price (the real exchange rate) (Stockman, 1987, p. 17, n. 13). The assumption of only one utility function for the two countries is implicit in most part of the informal analysis presented by Stockman (1987). Gärtner (1993, p. 206), for instance, explicitly uses a unique utility function in his textbook, although not a Cobb-Douglas one. Stockman (1987, p. 17, n. 13) himself comments on a possible usage of a Cobb-Douglas functional form for utility.

According to (3), both domestic and foreign households always spend at each date some fixed fraction of their incomes on each good, no matter what the real exchange rate is. The domestic household consumption \( C \) is given by

\[
C_c = \beta Y_c
\]

and the foreign household consumption \( C^f \) is

\[
C^f = (1 - \beta) Y^f
\]

Equation (4), for instance, says that the representative household in the US can, and wants to, allocate the same budget share to oranges at time period \( t \), regardless of the relative price of apples in terms of oranges. Since the domestic and foreign households have the same tastes, \( C_c = (1 - \beta) Y^f \) and \( C^f = \beta Y_c \) are also implied. A Cobb-Douglas utility function implies no wealth redistribution effects following either changes in tastes (occurring in
both countries equally) or productivity shocks (Stockman, 1987, p. 17, n. 13).

As in Stockman (1987, p. 22, n. 18), households are assumed to discount future utility at the same rate, and (3) is a time invariant instantaneous utility function, in which lifetime utility is additively separable in first and second period consumption. These assumptions enable equation (3) to hold each period.

Another feature of the time separable Cobb-Douglas utility function is that it is a borderline case in which the elasticity of substitution between foreign and domestic goods is equal to one; as a consequence, trade is balanced each period (Stockman, 1987, p. 26, n. 20). This will be clarified shortly. The elasticity of substitution between foreign and domestic goods $\mu$ is defined (Stockman, 1987, p. 19, n. 15) as

$$\mu = -\frac{(1/E_t) d(Y/Y_f)}{(Y_t/Y_f) d(1/E_t)}$$

where $E_t$ is the real exchange rate at time period $t$, as defined below. Considering (1) and (2) together with (14) (to be presented below) as well as their appropriate derivatives in (6), it can be verified that actually $\mu = 1$ in the Cobb-Douglas utility function.

The price of the domestic good (oranges) at time period $t$ is denoted by $P_t$, whereas the price of the foreign good (apples) at $t$ is expressed as $P_t^f$. As usual, the relative price of the foreign good in terms of the domestic good $C$ the relative price of imports $C_t$ at time period $t$ is defined as the real exchange rate, $E_t$, i.e.

$$E_t = \frac{S_t P_t^f}{P_t}$$

where $S_t$ is the nominal exchange rate at $t$ defined as the price of the foreign currency (pounds) measured in terms of the domestic currency (dollars).

If agents optimize, relative prices equal the marginal rate of substitution between goods in equilibrium. In terms of this model that means that the real exchange rate is equal to the marginal rate of substitution in consumption between foreign and domestic goods in equilibrium, i.e.
where $\partial U/\partial Y_f$ and $\partial U/\partial Y$ are the marginal utility of the foreign good and the domestic good respectively.

Equation (8) is also given in Gärtner's (1993, p. 205) presentation of the equilibrium model.

Substituting function (3) into equation (8), the latter becomes

$$E_t = \frac{1 - \beta}{\beta} \left( \frac{Y_t}{Y_f^t} \right)$$

Equation (9) shows that a shift in tastes away from the domestic good toward the foreign good occurring in both countries equally at time period $t$ (a fall in $\beta$) increases the real exchange rate $E$ in that period. That occurs because the demand for the domestic good shrinks and the demand for the foreign good goes up simultaneously. The demand for the foreign good must increase as well because any reduction in the demand for the domestic good should be accompanied by an increase in the demand for something else, given household budgets, and also because perishable goods do not allow for savings (Stockman, 1987, p. 16). Equation (9) shows, too, that factors affecting $Y$ and $Y_f$ are also able to change the real exchange rate, as discussed below.

That trade is balanced each period in this model can now be made clear. Here the balance of payments $B$ is

$$B_t = P_t (Y_t - C_t) - S_t P_t (Y_f^t - C_f^t)$$

where $Y - C$ are exports, and $Y_f - C_f$ represent imports (e.g. Mankiw, 1997, chapter 7). Considering (4), (5), and (7) in (10), equation (9) holds for $B_t = 0$.

Money market equilibria in the two countries are described in this model as

$$\frac{M_t}{P_t} \left( \frac{S_t}{S_t} \right) = Y_f^t$$

and
\[
\frac{\bar{M}_e}{\bar{M}_f} \left( \frac{S_e}{S_f} \right)^{\delta} = \left( \frac{\gamma^e}{\gamma^f} \right)^{\phi}
\]

where \( M \) and \( M_e \) are money supply targets in the domestic and foreign country respectively; \( S \) is a nominal exchange rate target; \( \delta \in (0, \infty) \) stands for the income elasticity of money demand assumed to be the same in the two countries; and \( \phi \in (-\infty, \infty) \) is a policy parameter defining the type of foreign exchange intervention.

Equations (11) and (12) are particular cases of standard LMs, to which the assumption of zero interest-elasticity of money demand is made along with the addition of a policy rule.

A critical feature of the equilibrium model is the non-existence of either a precautionary or a speculative demand for money (the implications of dropping such an assumption can be appreciated in Stockman, 1987, p. 25; p. 26, n. 22; p. 27, n. 24). Money demand depends only on income. So in (11) and (12), the real money demand in each country in terms of that country's output good is a particular function of the country's real income measured in the country's output good, as in Stockman (1987, p. 18-19). Another crucial assumption is that the demand for money in each country is expressed in terms of a different basket of goods (Stockman, 1987, p. 19). So money demands must differ across countries (Stockman, 1987, p. 19), as in (11) and (12). Such an assumption is needed because it is possible to imagine measures of real money demands in each country that are invariant to both preference shifts and productivity shocks; for instance, the nominal money demand being proportional to nominal consumption (Stockman, 1987, p. 19).

The policy rule underlying equation (11) is \( M_e = \bar{M}_e \left( \frac{S_e}{\bar{S}_e} \right)^{\phi} \), where \( M \) stands for the domestic money supply. An analogous rule is implicit in (12). Intervention follows such rules whereby the money supply is varied in response to current changes in the nominal exchange rate. The polar cases of fixed and flexible exchange rates correspond to infinity and zero values, respectively, of the intervention parameter \( \phi \). Leaning-against-the-wind intervention is represented by \( \phi \in (-\infty, 0) \), whereas leaning-into-the-wind intervention is given by \( \phi \in (0, \infty) \). Such a policy rule is suggested by, for example, Marston (1985, p. 910) (see also Scarth, 1988, p. 195, and Obstfeld and Rogoff, 1996, p. 632).
Free float occurs when $\phi$ is zero because in that situation the domestic central bank focuses exclusively on the target of the domestic money supply $\bar{M}$, abstaining from any intervention in the foreign exchange market ($M_e = \bar{M}_e$ if $\phi = 0$). The fixed exchange rate regime holds when $\phi$ approaches infinity because in that case the domestic central bank focuses exclusively on its nominal exchange rate target $\bar{S}$, without thinking about the domestic money supply ($S_e = \bar{S}_e$ if $\phi \to \infty$). Despite the fact that $M$ is endogenous with a fixed exchange rate, there are no monetary consequences coming from the balance of payments because trade is balanced each period ($B_t = 0$) in this model, as was previously noted.

Leaning against the wind is the intervention operation that attempts to move the exchange rate in the opposite direction from its current trend, and leaning into the wind is motivated by the central bank's desire to support current exchange rate trends. Both leaning against the wind and leaning into the wind are here carried out by changes in $\bar{M}$. It might be noted that whether such changes are sterilized is not discussed.

Thus, if $S > \bar{S}$ for any reason, the aim of leaning against the wind is to reduce the current nominal exchange rate $S$. That can be achieved by reducing $\bar{M}$ because $\phi < 0$. If $S < \bar{S}$, the aim of the leaning-against-the-wind intervention is to increase $\bar{M}$. Since leaning into the wind signifies supporting the current nominal exchange rate trend, if $S > \bar{S}$ that sort of intervention means increasing $\bar{M}$ when $\phi > 0$. Finally, if $S < \bar{S}$, leaning into the wind implies reducing $\bar{M}$. A similar rationale applies to the foreign country.

It should be noted in (11) and (12) that central banks in the two countries are assumed to coordinate their decisions (as in Stockman, 1980, p. 682, n. 17) concerning both nominal exchange rate targets and foreign exchange intervention; accordingly, $\bar{S}$ and policy parameter $\phi$ are the same in the two countries. That assumption prevents game-theoretic aspects of the decisions to intervene in the foreign exchange market as well as international liquidity or reserve problems (Stockman, 1980, p. 682, n. 17). This completes the description of the model.

Table 1 summarises this version of the equilibrium exchange rate model, in which the seven endogenous
variables are $S$, $P$, $P^e$, $E$, $Y$, $Y^e$, and $U$.

**TABLE 1: The equilibrium exchange rate model with Cobb-Douglas utility and production functions and foreign exchange intervention**

\[
Y_t = K_t^\alpha N_t^{1-\alpha}, \quad 0 < \alpha < 1 \tag{1}
\]
\[
Y_t^e = (K_t^e)^\alpha N_t^{1-\alpha} \tag{2}
\]
\[
U_t = Y_t^\beta (Y_t^e)^{1-\beta}, \quad 0 < \beta < 1 \tag{3}
\]
\[
E_t = \frac{S_t P_t^e}{P_t} \tag{7}
\]
\[
E_t = \frac{1 - \beta}{\beta} \left( \frac{Y_t}{Y_t^e} \right) \tag{9}
\]
\[
\frac{\bar{M}_t}{\bar{P}_t} \left( \frac{S_t}{S_t^e} \right)^\phi = Y_t^e, \quad \delta > 0, \quad -\infty < \phi < \infty \tag{11}
\]
\[
\frac{\bar{M}_t^e}{\bar{P}_t^e} \left( \frac{S_t^e}{S_t} \right)^\phi = \left( Y_t^e \right)^\delta \tag{12}
\]

**3. Solution**

To find a solution for the nominal exchange rate we first insert equations (11) and (12) into (7), after using (1) and (2) in the resulting expressions; this yields

\[
S_t = E_t \left( \frac{\bar{M}_t}{\bar{M}_t^e} \right) \left( \frac{K_t}{K_t^e} \right)^\alpha \phi \tag{13}
\]

Equation (13) shows that the nominal exchange rate depends on money supply targets and capital stocks in both countries, the real exchange rate, and parameters $\alpha$ and $\delta$.

Secondly, since PPP does not hold in this model even in the long run, the real exchange rate $E$ is endogenous in (13). It can thus be determined by substituting (1) and (2) into (9) to give

\[
E_t = \frac{1 - \beta}{\beta} \left( \frac{K_t}{K_t^e} \right)^\alpha \tag{14}
\]

Equation (14) shows that the real exchange rate depends on capital stocks (and productivity shocks) in both
countries, factor shares, and preferences of consumption over both domestic and foreign goods. A shift in tastes away from the domestic good toward the foreign good occurring at time period \( t \) in the two countries equally (i.e. a fall in \( \beta \)) increases the real exchange rate in (14), as in equation (9). The real exchange rate also goes up if either domestic productivity rises or foreign productivity falls, because that implies increases in \( K \) or reductions in \( K^f \) respectively (Stockman, 1987, p. 15).

The classical dichotomy holds in this model. Nominal exchange rate changes do not cause real exchange rate changes. This might seem at odds with definition (7). However, \( S \) and \( E \) alike are endogenous variables which simultaneously respond to changing real factors, such as preferences and technology. This might explain the stylized fact that nominal and real exchange rates are highly correlated. However, an implication of this framework is that such a correlation cannot be exploited by government policy in the sense that foreign exchange intervention will fail to affect the real exchange rate by changing the nominal rate (Stockman, 1987, p. 12).

Thirdly, we insert equation (14) into (13) to produce

\[
S_c = \frac{1 - \beta}{\beta} \left( \frac{\overline{M}_c}{\overline{M}} \left( \frac{K_c}{K_c^f} \right)^{\alpha(1-\delta)} \right)
\]  

(15)

The nominal exchange rate thus depends on the same factors affecting the real rate together with money supply targets in the two countries and the income elasticity of money demand \( \delta \). Shifts in tastes affect the nominal rate in the same fashion as they alter the real rate. As in monetary models, increases in the domestic money supply target and/or falls in the foreign money supply target push the nominal exchange rate up. However, the response of the nominal rate to changes in both domestic and foreign productivities depends critically on the value of parameter \( \delta \).

Both real and nominal exchange rates should also depend on the elasticity of substitution between foreign and domestic goods \( \mu \), which in this model is equal to one. If a generic utility function rather than the specific Cobb-Douglas functional form had been adopted, the parameter \( \mu \) would have appeared explicitly in (14) and (15). For a generic utility function, Stockman (1987, p. 19, n. 15) shows that the elasticity of the real exchange
rate with respect to domestic productivity is equal to \( \frac{1}{\mu} \), and the elasticity of the nominal exchange rate with respect to domestic productivity is equal to \( \frac{1}{\mu - \delta} \). Since \( \mu = 1 \) in this model, the former elasticity is equal to one, and the latter is given by \( 1 - \delta \). Thus, the \( 1 - \delta \) term in (15) gives the elasticity of the nominal exchange rate with respect to domestic productivity.

The elasticity of the nominal exchange rate with respect to domestic productivity encompasses two effects coming from supply and demand sides, namely the ‘relative price effect’ and the ‘money demand effect’. These effects push the nominal exchange rate in opposite directions in response to increases in domestic productivity (Stockman, 1987, p. 19). A growth in domestic productivity that pushes domestic output up gives rise to a greater real exchange rate, and that causes the nominal rate to go up (equation (13)). That is the relative price effect. However, the increased output also causes the money demand to rise, and that shrinks the nominal exchange rate. That is the money demand effect.

An increase (fall) in domestic (foreign) productivity can increase or reduce the nominal exchange rate in (15). For sensible values of the income elasticity of money demand i.e. \( \delta \in (0, 1) \) increases (falls) in domestic (foreign) productivity cause the nominal exchange rate to rise (i.e. to depreciate), because the relative price effect (\( \mu = 1 \)) overshadows the money demand effect (\( \delta \)). If \( \delta \in (1, \infty) \), increases in \( K \) and/or drops in \( K_f \) provoke \( S \) to fall, because the money demand effect is dominant. When the two effects exactly compensate for each other (\( \delta = 1 \)), neither domestic nor foreign productivity shocks can alter the nominal exchange rate. In that case, the model collapses into a monetarist-type model (with tastes added up into the demand side) because only money supplies influence the nominal exchange rate.

It might be noted that, since parameter \( \phi \) does not appear in either (14) or (15), foreign exchange intervention cannot affect either the real or nominal exchange rate. It should be clear that this result depends crucially on the simplifications made in a benchmark model like this one, and does not hold in more sophisticated settings (Stockman, 1987, p. 29). This will be discussed shortly. Nevertheless, government intervention is not needed because neither high nominal exchange rate variability nor massive changes of the real exchange rate are...
'good' or ‘bad’, in the sense that they reflect the necessary equilibrium adjustment to changing tastes and productivities. Indeed, ‘variability of exchange rates is no more inherently undesirable than variability in a person's mood throughout a day, and both reflect underlying conditions and policies’ (Stockman, 1987, p. 29).

The major success of the equilibrium model is to explain why PPP should not hold, as observed at the beginning of this paper. Deviations of real exchange rates from PPP seem to be persistent. Stockman (1987, p. 28) argues that this degree of persistence appears to be too large to explain on the basis of disequilibrium models that postulate sticky nominal prices. Indeed, the length of time over which an economy usually recovers from recessions would provide a rough estimate of the time it takes the price level to adjust to its new equilibrium following a shock. This estimate would thus suggest a period of two to three years. The Dornbusch model predicts that the real and nominal exchange rates should return toward their equilibrium levels when goods prices do. But estimates suggest that nominal and real exchange rates take a much longer time (perhaps only after four to seven years) to begin returning to their original level. However, the equilibrium approach that incorporates permanent real disturbances is consistent with real exchange rate persistence. Also, the equilibrium approach is not at odds with the stylized fact that nominal and real exchange rates are highly correlated, as observed above. Here the rationale is that nominal and real exchange rates might be driven by a same common real factor, such as changing tastes and productivities.

A drawback of this simple model is the apparent ineffectiveness of foreign exchange intervention. Indeed, the volatility of real exchange rates seem to be much higher under systems of flexible exchange rates than under systems of fixed exchange rates. Stockman observes (1987, p. 29), however, that there are many conditions (not all very realistic) that an economy must meet for the nominal exchange rate system to be totally irrelevant for real exchange rates. One condition requires that all the other government policies are the same under both exchange rate systems (see Stockman, 1983). If they are not, then the behaviour of real exchange rates may differ under the two systems even if the equilibrium model is the more useful approach. Notwithstanding this fact, the equilibrium model has some other radically different other policy implications than do disequilibrium theories.
For instance, foreign exchange intervention cannot affect the real exchange rate simply by changing the nominal rate. A policy may affect the *nominal* rate only if it signals a willingness to pursue policies that affect it and correspondingly a policy may affect the *real* rate only if it signals a willingness to pursue policies that affect it. Also, ‘undervalued’ or ‘overvalued’ currencies are not the issue, since exchange rates are only endogenous reflections of underlying market conditions and government policies (Stockman, 1987, p. 29).

4. Concluding remarks

This article presents a classroom-suited version of the equilibrium exchange rate model of Stockman (1987) that features Cobb-Douglas functional forms for production and utility, and models foreign exchange intervention explicitly. The model replicates the result that volatility is not either good or bad because it is a necessary response to changes in preferences and productivity. Also, the policy implication of the equilibrium model that foreign exchange intervention is ineffective is shown in an explicit way.

A reason good enough for one to be interested in the equilibrium model is the fact that deviations of real exchange rates from PPP seem to be persistent. The equilibrium model should also be useful as an input for the everyday modelling of more complicated models, an example being its recent use by the new open economy macroeconomics literature.

*Endnote*

1. Department of Economics, University of Brasilia, 70910-900 Brasilia DF, Brazil. E-mail: SergioDaSilva@angelfire.com. I am grateful to John Fender, Ralph Bailey, Peter Sinclair, Somnath Sen, Eduardo Alencar, Derek Leslie, and an anonymous referee for comments and discussions. Remaining errors are my own.

*References*


To be published by Economic Issues 7(2), September 2002.