Modeling DC Electric Motors

Introduction

A DC electric motor is a device for transforming direct current electrical power to mechanical power--usually the rotational speed and torque of a shaft. The basic relationship of an ideal motor is that mechanical (shaft) power equals the electrical power consumed by the motor armature.

$$T_m \omega_s = e_b i_a \qquad (1a)$$

where T_m is the motor torque (N-M)
ω_s is the shaft speed (rad/sec)
e_b is the back emf in the armature from interaction with the field magnet (volts)
i_a is the armature current (amps)

For a nonideal motor, an efficiency factor must be multiplied into the electrical side, since the electrical power into a real motor is always greater than the shaft power out of the motor.

The shaft speed of the motor is directly proportional to the voltage drop across the armature and inversely proportional to the current in the field coils.

$$e_b = K_e \omega_s \qquad (1b)$$

where K_e is the motor constant

From the preceding equation we note that the shaft speed is directly proportional to the voltage drop across the armature. Increasing the voltage drop across the armature increases the shaft speed.

The shaft torque is directly proportional to current.

$$T_m = K_T i \qquad (1c)$$

where K_T is the motor constant
i is the current that controls the motor torque

From the preceding equation we note that shaft torque is directly proportional to current. An increase in the shaft torque (load) produces an increase in current.

The constant of proportionality for both the shaft speed and torque relationships is the same! It is called the motor constant and is usually provided by the manufacturer.

$$K_f = \frac{pz}{2\pi a} K_\Phi \qquad (1d)$$

where p is the number of magnetic poles in the motor
z is the number of armature conductors in the motor
a is the number of parallel paths in the motor
K_Φ is a constant relating field magnetic flux per field pole (Φ) to field current (i_f)

The preceding equations are the basis for modeling the electrical to mechanical power conversion inside a DC electric motor.

DC electric motors can be constructed in a number of ways. Each type of motor has its own characteristics. Probably the most common small DC motor is the permanent magnet motor, where the field magnet is a permanent magnet. This type of motor is part of a broader class of motors called separately excited motors (referring to the field and armature magnet excitation). Another common motor is the series-wound motor in which the field and armature coils are wired in series. A less common motor configuration is the shunt-wound motor, where the field and armature coils are wired in parallel. Finally there is a hybrid configuration with both series and shunt field coils. This is called a compound motor and it is probably the most widely used large industrial DC motor today. The shaft torque vs. speed characteristic curves for the series, shunt, and compound motor configuration are shown in figure 1.

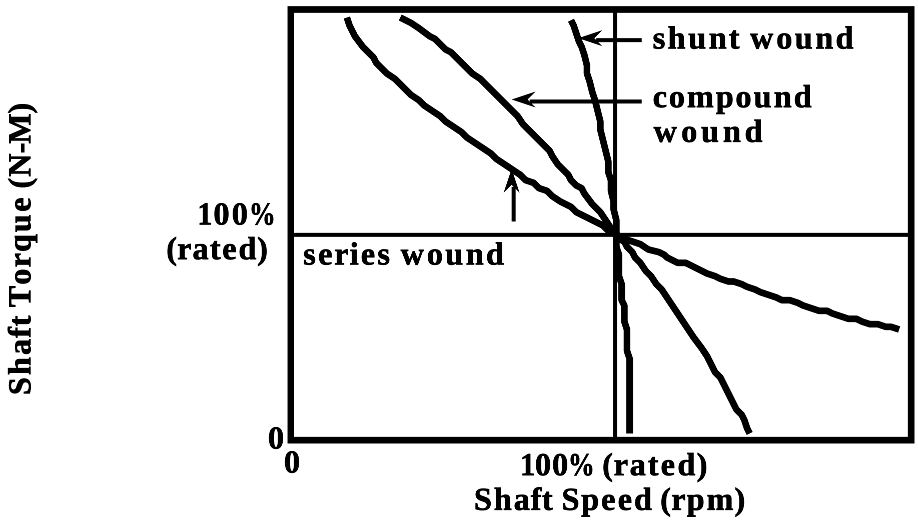

Figure 1. Generic Torque vs. Speed Curves of DC Motors

Separately Excited DC Motors

In a separately excited motor, the field magnet has a power supply that is separate from the armature electromagnet (see figure 2). That is, the motor field strength is completely independent from the armature field strength. Permanent magnet motors are included in this category since the field magnet is permanent and thus is independent from the armature. Brushless DC motors are also in this category since the armature magnet is permanent and thus is independent from the field.

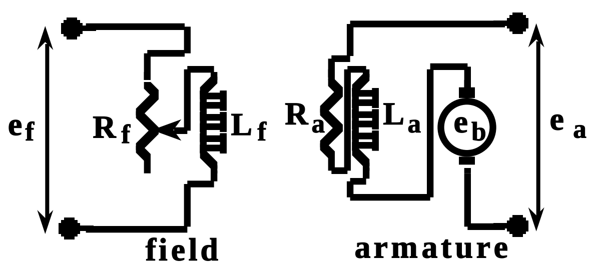

Figure 2. Schematic Diagram of a Separately Excited DC Motor

Speed control of a separately excited motor can be accomplished either by changing the voltage drop across the armature (see equation 1b) or by changing the field strength (e.g. varying the field current).

The electrical loop equation for the stator (field) circuit is as follows...

$$e_f = L_f \frac{di_f}{dt} + R_f i_f \tag{2a}$$

where e_f is the voltage drop across the stator (field) circuit (volts)
L_f is the field coil inductance
R_f is the resistance in the stator circuit, including the field windings

Notice that the voltage drop across the stator circuit is solely a function of field circuit resistance and inductance. In order to maximize the field current, and consequently the field magnetic flux, for a given stator supply voltage it is necessary to minimize the field circuit resistance and let the current come to steady state.

The electrical loop equation for the rotor (armature) circuit is as follows...

$$e_a = e_b + L\frac{di_a}{dt} + R_a i_a \tag{2b}$$

where e_a is the voltage drop across the armature circuit (volts)
L_a is the armature coil inductance
R_a is the resistance in the armature circuit

Notice that for a fixed rotor voltage, the way to maximize the rotor speed is to maximize the back emf of the armature, which requires that we reduce the resistance in the rotor circuit and to let the armature current reach steady state. The way to maximize the shaft torque is to maximize the armature current, which means minimizing the rotor circuit resistance. Incidentally, increasing the armature current also increases the power dissipated and therefore the heat generated by the motor. Excessive heat is the principal cause of reduced life in an electric motor.

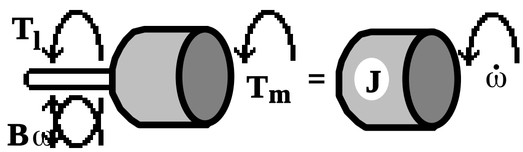

Figure 3. Dynamic Free-Body Diagram of the Motor Rotor

The inertial equation for the rotor comes from Newton's Second Law applied to figure 3 and appears as follows...

$$T_l = -T_m + J\dot{\omega} + B\omega \tag{2c}$$

where T_l is the load torque driven by the motor
J is the motor mass moment of inertia about the shaft
B is the system rotational damping (bearings, friction, etc.)

Notice that the shaft torque produced by the motor must overcome the load torque, the system rotational damping, and the rotational inertia of accelerating the motor.

<u>An Armature Controlled DC Motor</u> -- is a separately excited DC motor where the field current is usually constant and the armature current controls the motor torque. The armature magnetic field is switched to cause the motor to rotate. We rewrite equations 2c and 2b as follows...

$$T_l = -K_T i_a + J\dot{\omega} + B\omega \tag{3a}$$

$$e_a = K_e\omega + L_a\frac{di_a}{dt} + R_a i_a \tag{3b}$$

Taking the Laplace transform of the equations and setting the initial conditions to zero, we get...

$$T_l(s) = -K_T i_a(s) + (Js + B)\omega(s) \tag{4a}$$

$$e_a(s) = K_e\omega(s) + (L_a s + R_a)i_a(s) \tag{4b}$$

or

$$\begin{bmatrix} T_l(s) \\ e_a(s) \end{bmatrix} = \begin{bmatrix} (Js + b) & -K_T \\ K_e & (L_a s + R_a) \end{bmatrix} \begin{bmatrix} \omega(s) \\ i_a(s) \end{bmatrix} \tag{4c}$$

Inverting this relationship, we get...

$$\begin{bmatrix}\omega(s)\\ e_a(s)\end{bmatrix}=\frac{1}{(Js+b)(L_a s+R_a)+K_T K_e}\begin{bmatrix}(L_a s+R_a) & K_T\\ -K_e & (Js+b)\end{bmatrix}\begin{bmatrix}T_l(s)\\ e_a(s)\end{bmatrix} \tag{5}$$

This is the model for a permanent magnet motor. The block diagram of the motor is shown in figure 4.

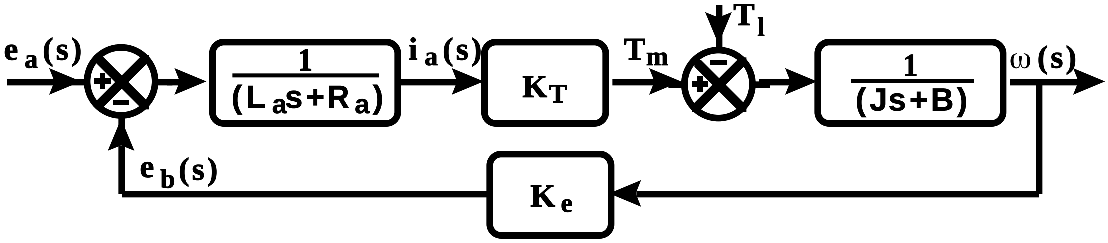

Figure 4. Block Diagram of an Armature Controlled DC Motor

<u>A Field Controlled DC Motor</u> -- is a separately excited DC motor where the field current controls the motor torque. The stator magnetic field is switched to cause the motor to rotate. We rewrite equations 2c and 2a as follows...

$$T_l = -K_T i_a + J\dot{\omega} + B\omega \tag{6a}$$

$$e_f = L_f \frac{di_f}{dt} + R_f i_f \tag{6b}$$

Taking the Laplace transform of the equations and setting the initial conditions to zero, we get...

$$T_l(s) = -K_T i_f(s) + (Js + B)\omega(s) \tag{7a}$$

$$e_f(s) = (L_f s + R_f) i_f(s) \tag{7b}$$

or

$$\begin{bmatrix}T_l(s)\\ e_f(s)\end{bmatrix}=\begin{bmatrix}(Js+b) & -K_T\\ 0 & (L_f s+R_f)\end{bmatrix}\begin{bmatrix}\omega(s)\\ i_f(s)\end{bmatrix} \tag{7c}$$

Inverting this relationship, we get...

$$\begin{bmatrix}\omega(s)\\ i_f(s)\end{bmatrix}=\frac{1}{(Js+b)(L_f s+R_f)}\begin{bmatrix}(L_f s+R_f) & K_T\\ 0 & (Js+b)\end{bmatrix}\begin{bmatrix}T_l(s)\\ e_f(s)\end{bmatrix} \tag{8}$$

This is the model for a brushless DC motor. The block diagram of the motor is shown in figure 5.

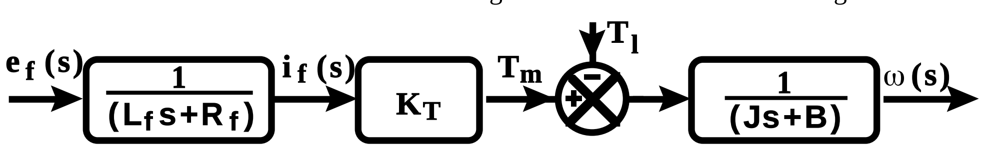

Figure 5. Block Diagram of a Field Controlled DC Motor

<u>Shunt Wound DC Motors</u>

In a shunt wound DC motor (see figure 6)) the field and armature circuits are connected in parallel across the same supply voltage.

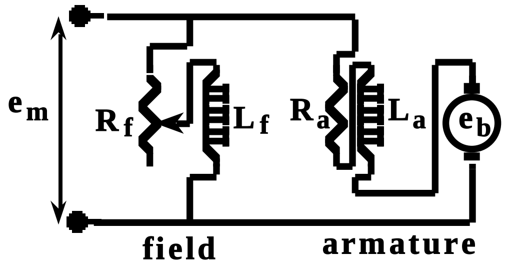

Figure 6. Schematic Diagram of a Shunt Wound DC Motor

The voltage drop across the field circuit is identical to the voltage drop across the armature circuit. The current drawn by the motor is the sum of the field and armature currents. A shunt field coil is usually wound with many turns of thin wire. This allows the field current to be small compared to the armature current. Notice from figure 1 that the shunt motor operates at nearly constant speed over a wide range of shaft loads.

Speed control of a shunt wound motor can be accomplished either by changing the field strength (e.g. varying the resistance of the field circuit) or by changing the voltage drop across the motor.

To model a shunt wound DC motor, we notice that the stator and rotor circuits have the same voltage supply and therefore the same voltage drop. In addition, the current drawn by the motor is the sum of the field and armature currents.

$$e_m = e_r = e_f \tag{9a}$$
$$i_m = i_a + i_f \tag{9b}$$

The block diagram of this motor is shown in figure 7.

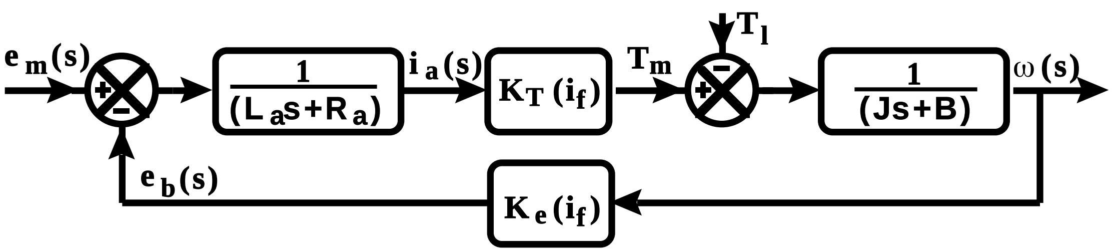

Figure 7. Block Diagram of a Shunt Wound DC Motor

Series Wound Motors

In a series wound DC motor (see figure 8) the field and armature circuits are connected in series so the same current flows through each.

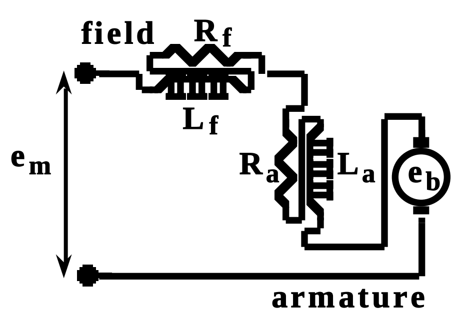

Figure 8. Schematic Diagram of a Series Wound DC Motor

The total motor voltage drop is the sum of the field and armature voltage drops, so the field coil is usually made out of a few turns of heavy wire to keep the field voltage drop to a minimum. From figure 1, we see that series motors have very large starting torques and very high no-load operating speeds. In fact it is usually very dangerous to decouple a load from a series motor in operation since the motor speed will increase quickly to a level at which the motor will disintegrate.

A series DC motor has the ideal torque-speed characteristics to power a ground vehicle (i.e. a car). It has a very large starting torque to get the vehicle moving. It has a wide speed range for operating the vehicle at various travel speeds without the need for much of a mechanical transmission.

Speed control of a series wound motor can be accomplished by changing the voltage drop across the motor.

To model a series wound DC motor, we notice that the motor supply voltage is divided across the stator and rotor circuits and the a common current flows through the field and armature coils.

$$e_m = e_r + e_f \tag{10a}$$
$$i_m = i_a = i_f \tag{10b}$$

The block diagram of this motor is shown in figure 9.

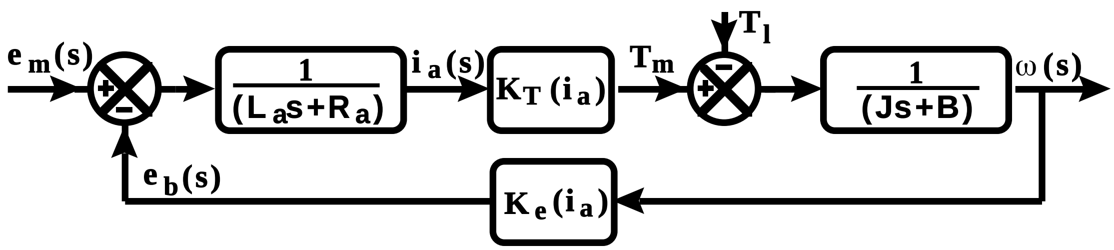

Figure 9. Block Diagram of a Series Wound DC Motor

Compound Wound DC Motors

A compound wound DC motor (see figure 10) has both series and shunt field windings so it can run as a shunt motor, a series motor, or a hybrid of the two.

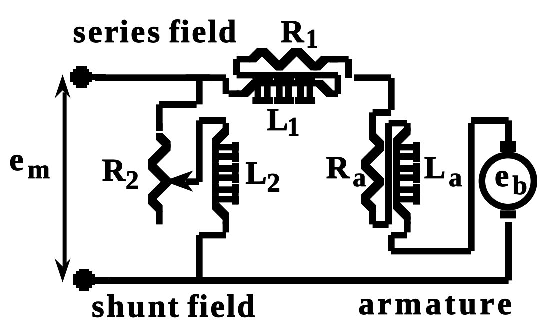

Figure 10. Schematic Diagram of a Compound Wound DC Motor

The total motor voltage drop is the sum of the series field and armature voltage drops, so the series field coil is usually made out of a few turns of heavy wire to keep the series field voltage drop to a minimum. A shunt field coil is usually wound with many turns of thin wire to minimize the shunt field current. In most compound wound DC motors the field windings have separate connections so they can be switched in or out as desired.

From figure 1, we see that the series and shunt torque vs. speed curves are the limiting cases for the compound wound DC motor. In fact the behavior of the motor can be adjusted anywhere between the two limiting cases by changing the relative strength of the two fields. By utilizing a switching speed controller, a compound DC motor can provide the high starting torque of a series wound motor as well as the nearly uniform running speed of a shunt wound motor.

Brushless DC Motors

This is a relatively new type of motor that uses a permanent magnet for the rotor (armature) and coils for the stator (field). An electronic circuit produces multiple-phase alternating current inducing a rotating magnetic field in the stator. The rotor, being a permanent magnet, simply follows the stator magnetic field around. The speed of the motor is controlled by adjusting the frequency of the stator power.

A brushless DC motor behaves just like a synchronous AC motor. It produces full torque at the synchronous speed dictated by the stator frequency and the number of stator poles. Since an electronic circuit is generating the stator frequency, it can be varied over a wide range and thus produce full torque over a wide range of shaft speeds.

Stepping Motors

Stepping motors are very commonly used in controlled positioning applications, usually in an open loop configuration (no position feedback). They are designed to have a high holding torque at the current position. They advance one step for each pulse of electrical power applied, although some controllers are designed to “micro-step” the motor by dividing each full step into several smaller steps. The running speed of a stepper motor is simply the pulse rate of the controller (up to 20,000 hz) divided by the number of steps per revolution of the motor (usually 200 or 400).

$$n = 60\,\frac{f}{steps} \tag{11}$$

where n is the motor shaft speed (rpm)
f is the pulse frequency (hz, pulses/sec)
steps is the number of steps per revolution for the motor

Stepper motors are very easy to integrate into positioning systems because they are essentially digital motors requiring digital control. The torque supplied by a stepper motor decreases as rotor speed increases (see figure 11).

Table 1. Typical Stepper Motor Characteristics

NEMA* Frame Size	Nominal Motor Diameter (in)	Typical Power (hp)	Typical Holding Torque (in-oz)	Comments
5	0.5	very very low	0.25-0.5	for small instruments only
11	1.1	very low	0.5-2.5	
15	1.5	low	5-10	
17	1.7	low	13-26	
23	2.3	0.08-0.2	60-150	most popular
34	3.4	0.1-0.3	150-450	competitive with DC servomotors
42	4.2	0.3-1.0	400-1,600	competitive with DC servomotors
65	6.5	1.6-2.5	1,600-8,000	competitive with AC servomotors

*NEMA--National Electric Manufacturers Association

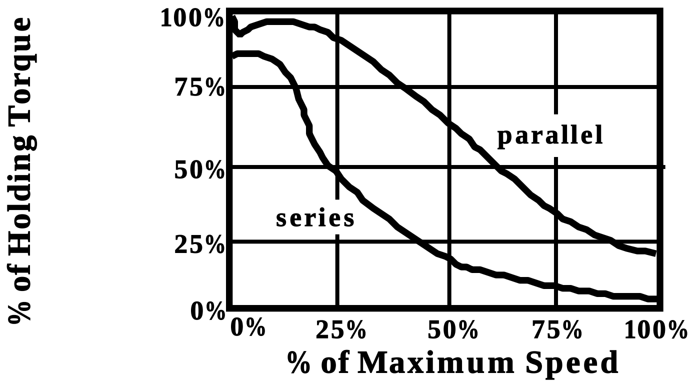

Figure 11. Generic Torque vs. Speed Characteristic of a Stepper Motor

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