Department of Systems Engineering

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SYST 302: Systems Methodology and Design II #2

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Outline

- Economic Models for Evaluation
- Economic Equivalence
- Evaluating Alternatives under Uncertainty
- Break-Even Analysis

Economic Models

- **Economic Consideration is Essential**
  - **Feasibility**
  - **Time value of money**
  - **Equivalent basis**
- **Interest Formulas**
  - **Single payment compound amount** $F = P(1+i)^n$
  - **Single payment present amount** $P = F/(1+i)^n$
  - **Equal payment series compound amount** $F = A[((1+i)^n - 1)/i]$
  - **Equal payment series sinking fund** $A = F[i/((1+i)^n - 1)]$
  - **Equal payment series present amount** $P = A[((1+i)^n - 1)/i(1+i)^n]$
  - **Equal payment series capital recovery** $A = P[i(1+i)^n/((1+i)^n - 1)]$

P**: Present capital amount,** F**:Future compound amount,** A**: Annual payment**
n**: number of periods (year),** i**: interest rate (annual)**

Equal Payment Series

- **Equal Payment Compound/Sinking**

$$F = A + A(1+i) + A(1+i)^2 + \cdots + A(1+i)^{n-1}$$

$$= A\left[\frac{(1+i)^n - 1}{(1+i) - 1}\right] = A[((1+i)^n - 1)/i]$$

$$A = F[i/((1+i)^n - 1)]$$

- **Equal Payment Capital Recovery/Present**

$$A = F[i/((1+i)^n - 1)] = P(1+i)^n[i/((1+i)^n - 1)]$$

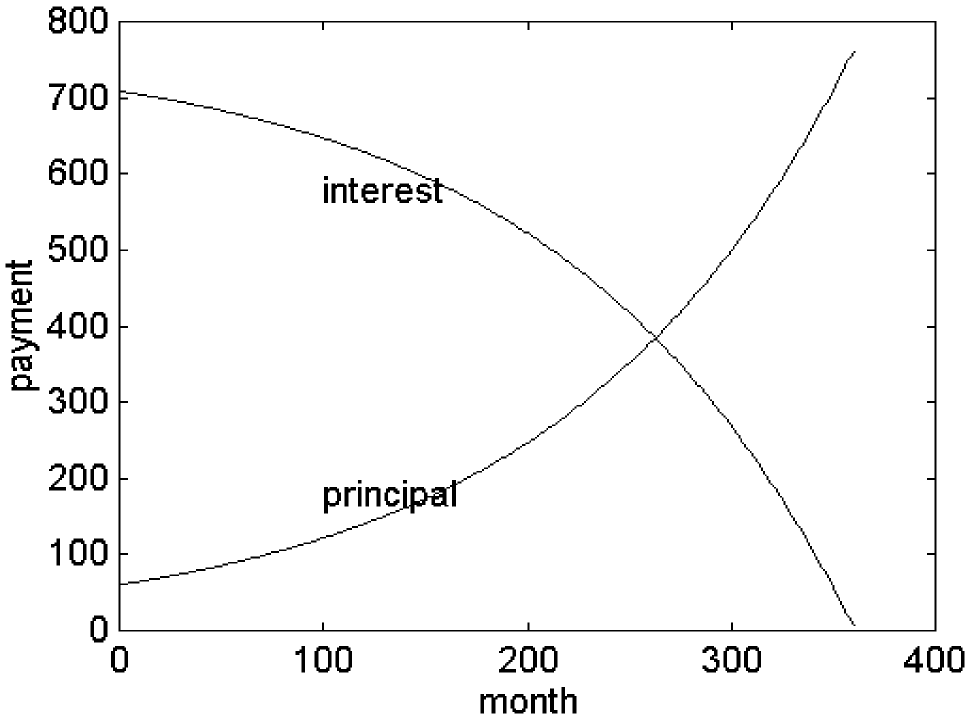
$$= P[i(1+i)^n/((1+i)^n - 1)]$$

$$P = A[((1+i)^n - 1)/i(1+i)^n]$$

***P*: Present capital amount, *F*:Future compound amount, *A*: Annual payment**
***n*: number of periods (year), *i*: interest rate (annual)**

Example: Mortgage Payment
100k, 30 years, 8.5%

$$Monthly\ Payment = 100{,}000\left[\tfrac{0.085}{12}\left(1+\tfrac{0.085}{12}\right)^{360} \Big/ \left(1+\tfrac{0.085}{12}\right)^{360} - 1\right] = 768.91$$



Economic Equivalence

- **Equivalent Value**
  - **Key Factors: Amount, Time, Interest Rate**

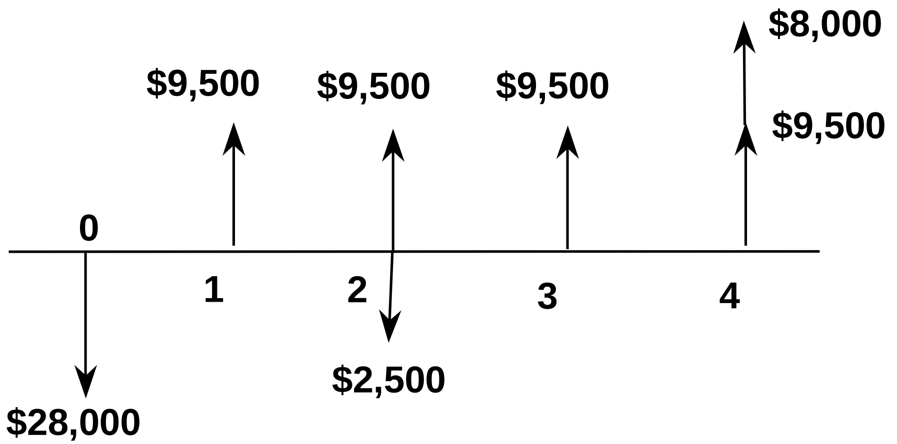
    $E = f(F_i, i, n)$
- **Examples**
  - **Single payment compound: One time investment (antique, paint, precious metal, etc)**
  - **Single payment present: Short term inter-bank loan**
  - **Equal payment compound: Continuous investment, one time return (ship, building construction)**
  - **Equal payment sinking: Reverse mortgage**
  - **Equal payment recovery: Purchase of plane, oil tanker.**
  - **Equal payment present: Investment in existing business building, warehouse, land, etc.**

Evaluating Alternatives

- **Option Selection**
  - Single alternative under certainty
  - Multiple alternatives under certainty
  - Multiple alternatives with multiple futures
- **Economic Bases for Comparison**
  - Present equivalent evaluation
  - Annual equivalent evaluation
  - Future equivalent evaluation
  - Rate-of-return evaluation
  - Payout evaluation
  - Annual equivalent cost of an asset

An Example

- **Money Flow**



With *i* = interest rate=12%

$$PE(i) = \sum_{t=0}^{n} F_t (1+i)^{-t} = -28,000 + 9500(\tfrac{1}{1.12}) + 7000(\tfrac{1}{1.12})^2 + 9500(\tfrac{1}{1.12})^3 + 17500(\tfrac{1}{1.12})^4 = 3,946$$

$$AE(i) = PE(i)\frac{A}{P} = PE(i)[i(1+i)^n \big/ (1+i)^n - 1] = 1,299$$

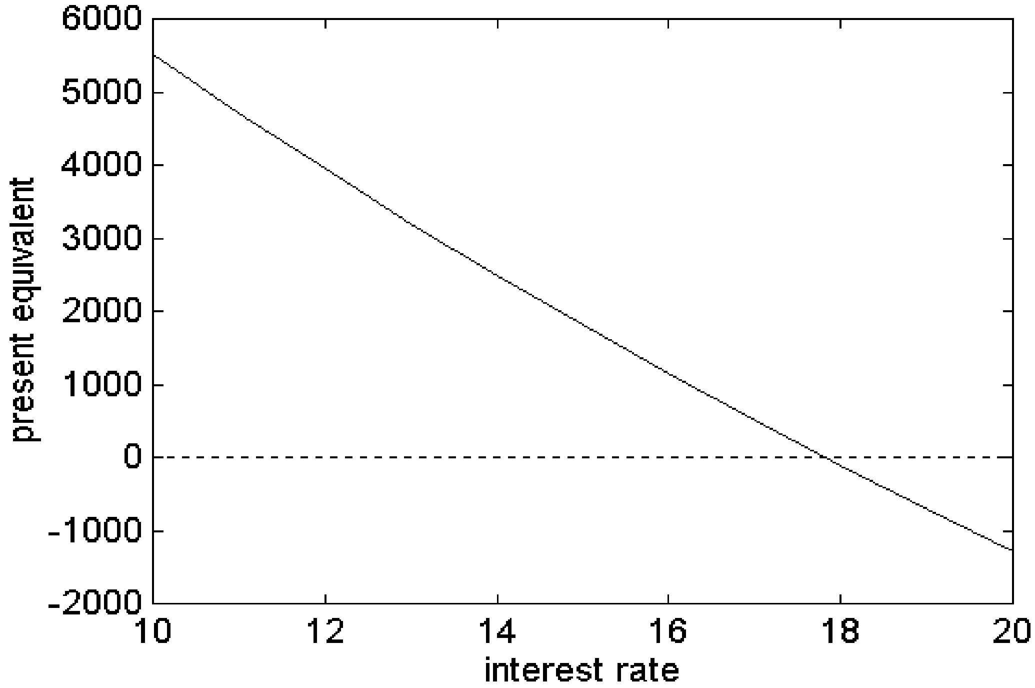
$$FE(i) = PE(i)\frac{F}{P} = PE(i)(1+i)^n = 6,211$$

PE: present equivalent, AE: annual equivalent, FE: future equivalent

Rate-of-Return Evaluation

- **Rate-of-return**: the interest rate that causes the profit equal to zero (best method for single alternative evaluation)

$$PE(i) = \sum_{t=0}^{n} F_t (1+i)^{-t} = -28{,}000 + 9500(\tfrac{1}{1+i}) + 7000(\tfrac{1}{1+i})^2 + 9500(\tfrac{1}{1+i})^3 + 17500(\tfrac{1}{1+i})^4$$



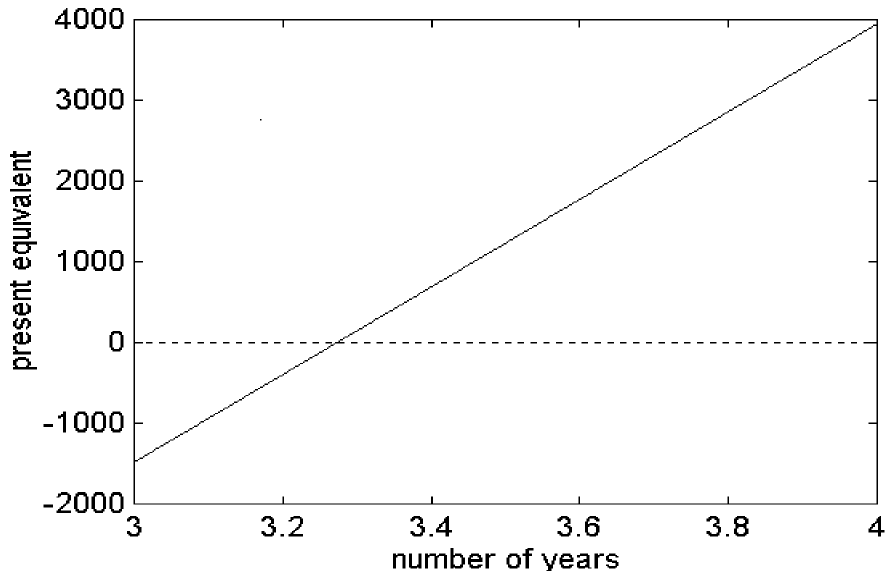
Payout Evaluation

- **Payout Period: amount of time required for the system to pay for itself (i.e., costs = savings)**

$$n = 4:\ \ PE(i) = \sum_{t=0}^{4} F_t (1+i)^{-t} = 3{,}946;$$

$$n = 3:\ \ PE(i) = \sum_{t=0}^{3} F_t (1+i)^{-t} = -1{,}482 \ \ \text{(assuming the salvage value is still 8,000)}$$

$$\Rightarrow n^* = 3.3\ years$$



Annual Equivalent Cost of an Asset

- Annual Equivalent Cost:

(1) cost of depreciation (2) cost of interest on the un-depreciated balance

$$AE = PE\frac{A}{P} - FE\frac{A}{F} = PE\frac{A}{P} - FE\left(\frac{A}{P} - i\right) = (PE - FE)\frac{A}{P} + FE \cdot i$$

$$= (PE - FE)[i(1+i)^n / (1+i)^n - 1] + FE \cdot i$$

PE: initial cost, FE: salvage value, i: interest rate

- Example: proposal costing on equipment

(1) equipment cost = $3,250,000 (2) four year contract

(3) salvage value = $750,000 (4) interest rate = 10%

$$(3{,}250{,}000 - 750{,}000)\frac{0.1(1.1)^4}{(1.1)^4 - 1} + 750{,}000 * 0.1 = \$863{,}750$$

Multiple Alternatives with Multiple Futures

- Example: New product investment

Cost \ Demand	*Low*	*Medium*	*High*
Initial Investment	500	1,300	2,000
Annual Income (3 yrs)	400	700	900

(1) Financing interest rates(%)

Bank	*Low*	*Medium*	*High*
A	15	13	7
B	14	12	8
C	20	11	6

(2) Present equivalent values*

Bank	*Low*	*Medium*	*High*
A	413	353	362
B	429	382	320
C	343	411	406

*** PE = - initial investment + annual income (P/A(i,n))**

Selecting an Alternative

- **Decision under risk**
  Assuming P(L)=.3, P(M)=.2, P(H)=.5
  then with expected value criterion, decision = C (388)

- **Decision under uncertainty**
  Laplace: Assuming P(L) = P(M) = P(H) = 1/3
  then decision = C (386)
  Maximin: decision = A (353)
  Maximax: decision = B (429)

	Exp.Val	*Laplace*	*Maxi* min	*Maxi* max
A			*X*	
B				*X*
C	*X*	*X*		

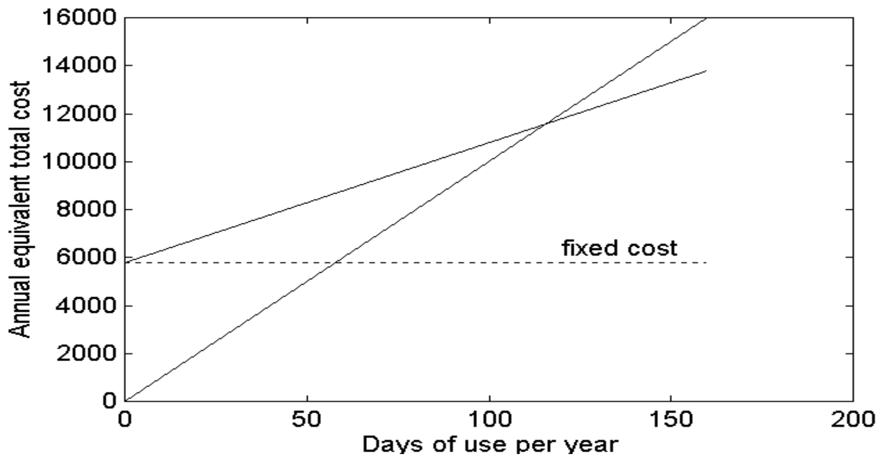
Break Even Analysis

- **Break Even:**
  (1) comparing the costs of two alternatives
  (2) evaluating profitability
- **Example: Lease or Buy**
  Buy: $25,000, 15yrs life, $4,000 salvage, annual maintenance $2,800, interest 9%, operating $50/day
  Lease: $50/day, operating $50/day

$$AE_B = (25{,}000 - 4{,}000)\frac{A}{P} + 4{,}000(0.09) + 2800 + 50N = 5766 + 50N$$

$$AE_L = (50 + 50)N = 100N, \qquad N: \text{ days of use per year}$$

$$\text{Break even: } 5766 + 50N = 100N \Rightarrow N = 115$$

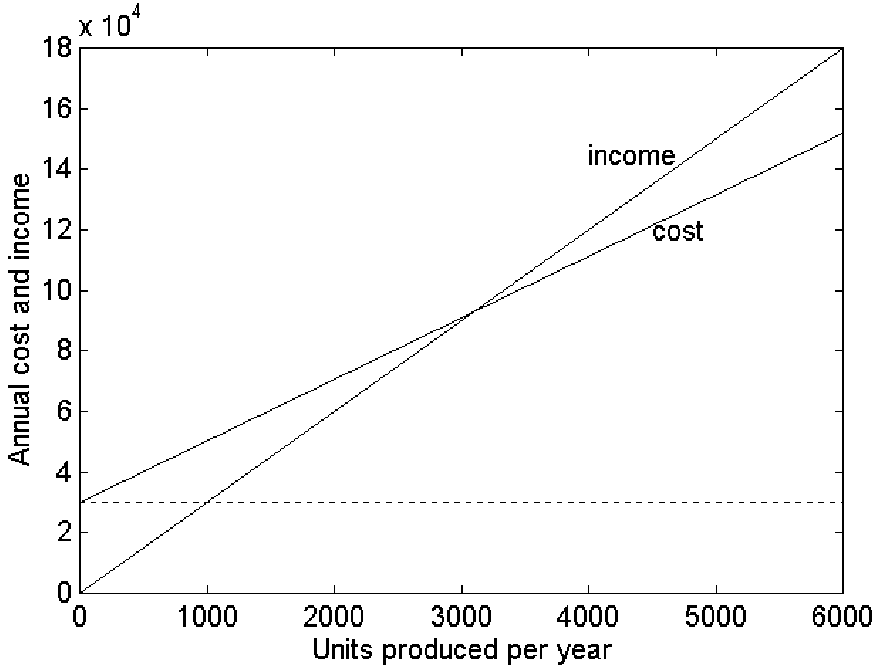


Profitability Evaluation

- Example: evaluating a manufacture proposal
  (1) Fixed cost: initial investment \$140,000, life: 8 yrs
      insurance: \$2,000/yr, maintenance: \$1,722/yr (*i* = 8%)
  (2) Variable cost: m&m: \$10/unit, labor&others \$11/unit

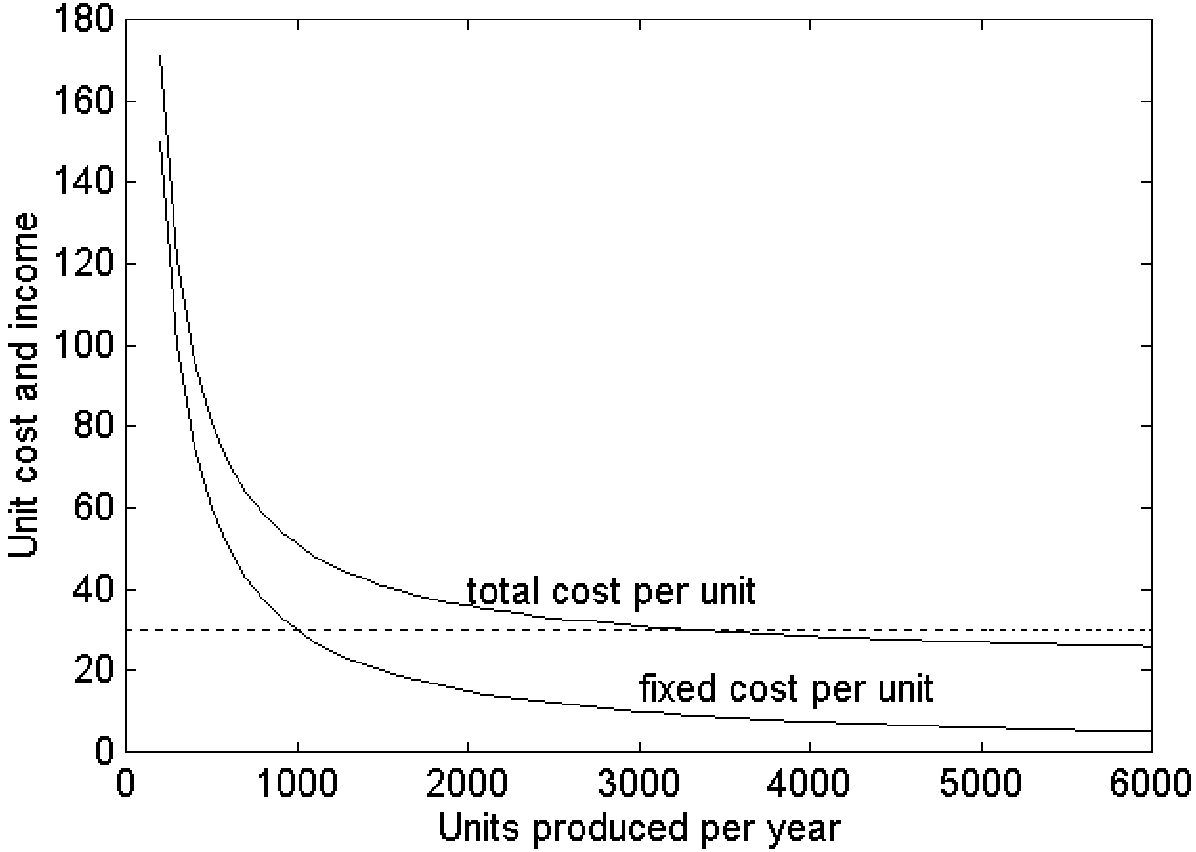
$$AE_B = 140{,}000\frac{A}{P} + 2000 + 1722 + (10+11)N = 30{,}000 + 21N$$

$$Annual\ Income = 30N \qquad \text{Break even: } 30{,}000 + 21N = 30N \Rightarrow N = 3{,}333$$

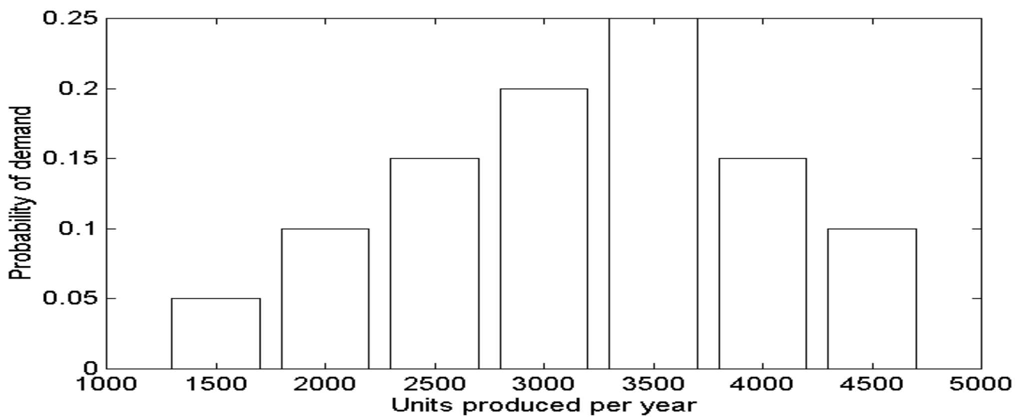


Unit Cost Break Even

Total cost per unit: $C_u = 30{,}000 / N + 21$



Break-Even Analysis under Risk



Expected demand = .05*1500+.1*2000+...+.1*4500=3,175

Question: to buy or to make?

(1) Based on expected demand:

Buy: expected cost = $30 * 3,175 = $95,250 (v)

Make: expected cost = $30,000 + $21 * 3,175 = $96,675

(2) Based on most probable demand:

Buy: cost = $30 * 3,500 = $105,000

Make: cost = $30,000 + $21 * 3,500 = $ 103,500 (v)