$$\det\begin{bmatrix} a-\lambda & ? & ? \\ 0 & b-\lambda & ? \\ 0 & 0 & c-\lambda \end{bmatrix} = (a-\lambda)*(b-\lambda)*(c-\lambda) \quad \Rightarrow \lambda = \underline{a,b,c}$$

Upper triangle is a product of a Diagonal Triangle.

Quiz: Find a basis for N(A) if

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{cases} x = 0 \\ z = 0 \\ y = arbitrary \end{cases} \quad \text{thus, } N(A) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Ax=λx (Must be *n* x *n* matrix <u>a square matrix</u>)

Non-zero vector X and satisfies the equation = the Eigenvector.

Says the same thing $\begin{cases} (A-\lambda I)x = 0 \\ (X \in N(A-\lambda I) \end{cases}$

Describes space: N(A-λI)

Characteristic Polynomial: |A- λI| = 0

Algebraic Multiplicity of λ is the multiplicity of λ as a Zero of the Characteristic Polynomial.

N(A-λI) has certain dimension.

This dimension (includes zero to large): dim N(A-λI)

Page 177 for Definition 27.3

A(n x n) $\lambda_1, \lambda_2, ..., \lambda_k$ **eigenvalues for each eigenvector N(A-** λ_i **I)**

$A x_1 = \lambda x_1$ **and** $A x_2 = \lambda x_2$

$A(x_1 + x_2) \Rightarrow A x_1 + A x_2 \Rightarrow \lambda x_1 + \lambda x_2 \Rightarrow \lambda(x_1 + x_2)$

$A\alpha x_1 = \alpha A x_1 \Rightarrow \alpha\lambda x_1 \Rightarrow \lambda(\alpha x_1)$

N(A- λ_1 **I) is a vector space.**

β_1 **= a basis for N(A-** λ_i **I)**

$\beta_1 \cup \beta_2 \cup ..... \cup \beta_k$ **(union for all basis) This is the complete set of eigenvectors (are L.I.)**

Proof is on page 180 of text. Theorem 27.6

***** A complete set of Eigenvectors is always Linearly Independent.**

Look Back at Exercise 26.1 $\lambda=3, -2$ **or** $\lambda_1=3,\ \lambda_2=-2$

$$N(A-3I) = Sp\left\{\begin{bmatrix}1\\1\end{bmatrix}\right\} \qquad \text{Thus, } \beta_1 = \left\{\begin{bmatrix}1\\1\end{bmatrix}\right\}$$

$$N(A+2I) = Sp\left\{\begin{bmatrix}2\\3\end{bmatrix}\right\} \qquad \text{Thus, } \beta_2 = \left\{\begin{bmatrix}2\\3\end{bmatrix}\right\}$$

$$\text{Complete set of Eigenvectors} = \left\{\begin{bmatrix}1\\1\end{bmatrix},\begin{bmatrix}2\\3\end{bmatrix}\right\} \text{ for } A=\begin{bmatrix}13 & -10\\15 & -12\end{bmatrix} \ \therefore\ Sp\left\{\begin{bmatrix}1\\1\end{bmatrix},\begin{bmatrix}2\\3\end{bmatrix}\right\} = R^2$$

Given: A square matrix (n x n) is a 5 x 5 matrix where $\lambda = 7$

Then: dim N(A-7I) + r(A-7I) = 5

since n = 5 and if rank (r) = 2 $\Rightarrow$ N(A-7I)+ 2 = 5 $\Rightarrow$ N(A-7I) = 5 – 2 or 5 – 2 = 3

Therefore, dim N(A-7I) = 3 which means that there are 3 vectors.

Upper Triangular Matrix: A and B are similar if $A = PBP^{-1}$ for some (non-singular) P.

$$A = PBP^{-1} \ \Rightarrow\ (AP = PBP^{-1}P = PB) \ \Rightarrow\ P^{-1}AP = B \ \Rightarrow\ P^{-1}A(P^{-1})^{-1}$$

Theorem: Similar Matrices have the same characteristic polynomials.

A is similar to B

det $= |A-\lambda I|$ if similar $|PBP^{-1}-\lambda I|$ (remember that $I = P * P^{-1}$ and $A = PBP^{-1}$)

So $|PBP^{-1}-\lambda I| \Rightarrow |(PB-\lambda P)P^{-1}| \Rightarrow |PB-\lambda P| * |P^{-1}| \Rightarrow |P(B-\lambda)| * |P^{-1}| \Rightarrow$

$|P| * |B-\lambda I| * |P^{-1}| \Rightarrow$ if this is Zero then $|A-\lambda I| = 0$