CHAPTER 34

Optical Images

1* · Can a virtual image be photographed?

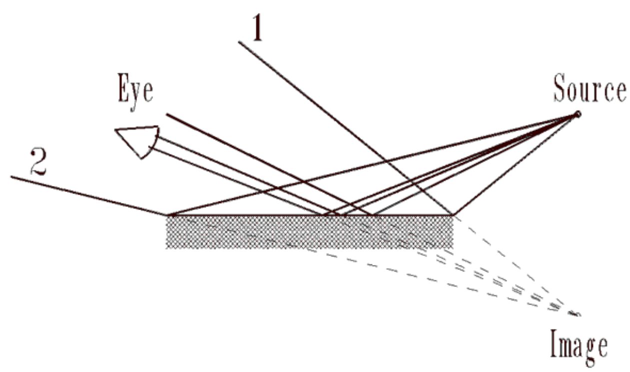
Yes. Note that a virtual image is "seen" because the eye focuses the diverging rays to form a real image on the retina. Similarly, the camera lens can focus the diverging rays onto the film.

2 · Suppose each axis of a coordinate system like the one in Figure 34-4 is painted a different color. One photograph is taken of the coordinate system and another is taken of its image in a plane mirror. Is it possible to tell that one of the photographs is of a mirror image rather than both being photographs of the real coordinate system from different angles?

Yes; the mirror image is a left-handed coordinate system.

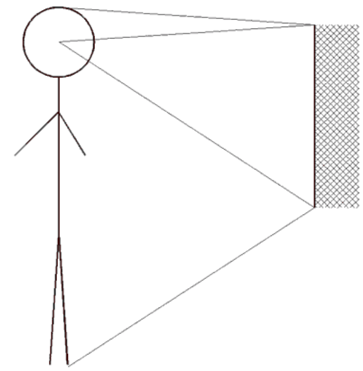
3 The image of the point object *P* in Figure 34-52 is viewed by an eye as shown. Draw a bundle of rays from the object that reflect from the mirror and enter the eye. For this object position and mirror, indicate the region of space in which the eye can see the image.

Rays from the source and reflected by the mirror are shown. The reflected rays appear to diverge from the image. The eye can see the image if it is in the region between rays 1 and 2.



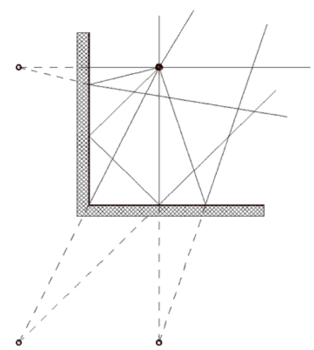
4 · A person 1.62 m tall wants to be able to see her full image in a plane mirror. (*a*) What must be the minimum height of the mirror? (*b*) How far above the floor should it be placed, assuming that the top of the person's head is 15 cm above her eye level? Draw a ray diagram.

The mirror must be half the height of the person, i.e., 81 cm. The top of the mirror must be 7.5 cm below the top of the head, or 154.5 cm above the floor. The bottom of the mirror must be 73.5 cm above the floor. The ray diagram is shown in the figure.



5* · Two plane mirrors make an angle of 90°. Show by considering various object positions that there are three images for any position of an object. Draw appropriate bundles of rays from the object to the eye for viewing each image.

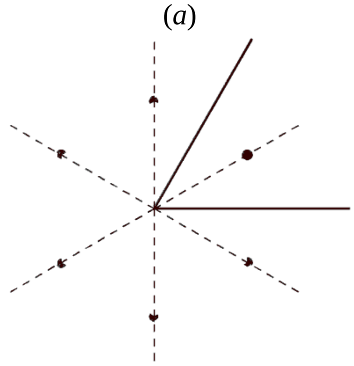
Three virtual images are formed, as shown in the adjacent figure. The eye should be to the right and above the mirrors.



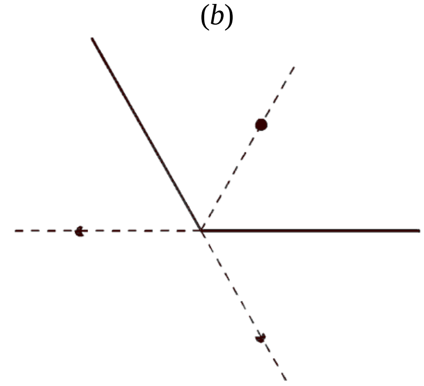
6 · (*a*) Two plane mirrors make an angle of 60° with each other. Show on a sketch the location of all the images formed of a point object on the bisector of the angle between the mirrors. (*b*) Repeat for an angle of 120°.

(*a*), (*b*) The object and its images are shown in the figures below.

(*a*)



(*b*)



7 ·· When two plane mirrors are parallel, such as on opposite walls in a barber shop, multiple images arise because each image in one mirror serves as an object for the other mirror. A point object is placed between parallel mirrors separated by 30 cm. The object is 10 cm in front of the left mirror and 20 cm in front of the right mirror. (*a*) Find the distance from the left mirror to the first four images in that mirror. (*b*) Find the distance from the right mirror to the first four images in that mirror.

(*a*) The first image in the mirror on the left is 10 cm behind the mirror. The mirror on the right forms an image 20 cm behind that mirror or 50 cm from the left mirror. This image will result in a second image 50 cm behind the left mirror. The first image in the left mirror is 40 cm from the right mirror and forms an image 40 cm behind the right mirror or 70 cm from the left mirror. That image gives an image 70 cm behind the left mirror. The fourth image behind the left mirror is 110 cm behind that mirror.

(*b*) Proceeding as in part (*a*) for the mirror on the right, one finds the location of the images to be 20 cm, 40 cm, 80 cm, and 100 cm behing the right-hand mirror.

8 ·· True or False

(*a*) The virtual image formed by a concave mirror is always smaller than the object.

(*b*) A concave mirror always forms a virtual image.

(*c*) A convex mirror never forms a real image of a real object.

(*d*) A concave mirror never forms an enlarged real image of an object.

(*a*) False (*b*) False (*c*) True (*d*) False

9* ·· Under what condition will a concave mirror produce an erect image? A virtual image? An image smaller than the object? An image larger than the object?

If $s < f$, the image is virtual, erect, and larger than the object.

If $f < s < 2f$, the image is real, inverted, and larger than the object.

If $s > 2f$, the image is real, inverted, and smaller than the object.

10 ·· Answer Problem 9 for a convex mirror.

A convex mirror always produces a virtual, erect image that is smaller than the object. It never produces an enlarged image.

11 ·· Convex mirrors are often used for rear-view mirrors on cars and trucks to give a wide-angle view. Below the mirror is written, “Warning, objects are closer than they appear.” Yet according to a ray diagram such as Figure 34-19, the image distance for distant objects is much smaller than the object distance. Why then do they appear more distant?

They appear more distant because the images are smaller than they would be in a flat mirror.

12 ·· As an object is moved from a great distance toward the focal point of a concave mirror, the image moves from

(*a*) a great distance toward the focal point and is always real.

(*b*) the focal point to a great distance from the mirror and is always real.

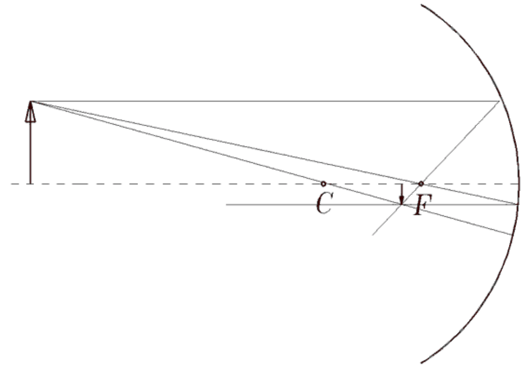
(*c*) the focal point toward the center of curvature of the mirror and is always real.

(*d*) the focal point to a great distance from the mirror and changes from a real to a virtual image.

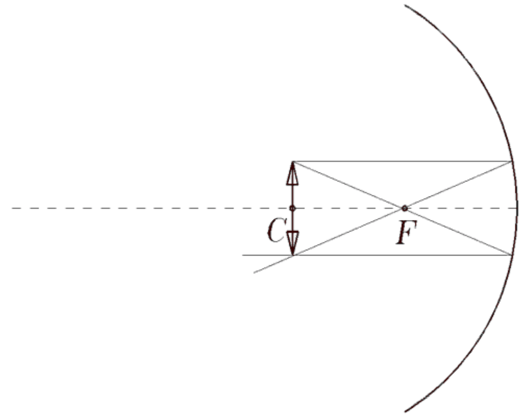
(*b*)

13* · A concave spherical mirror has a radius of curvature of 40 cm. Draw ray diagrams to locate the image (if one is formed) for an object at a distance of (*a*) 100 cm, (*b*) 40 cm, (*c*) 20 cm, and (*d*) 10 cm from the mirror. For each case, state whether the image is real or virtual; erect or inverted; and enlarged, reduced, or the same size as the object.

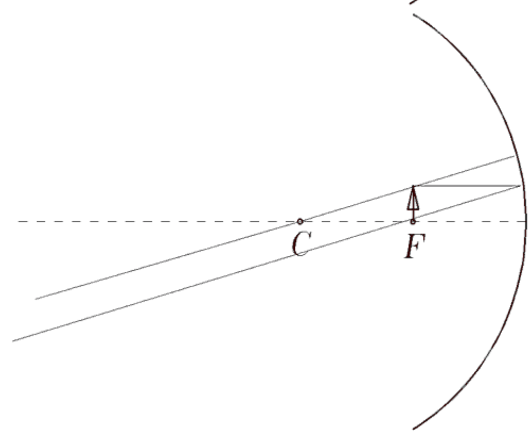
(*a*) The ray diagram is shown.
The image is real, inverted, and reduced.



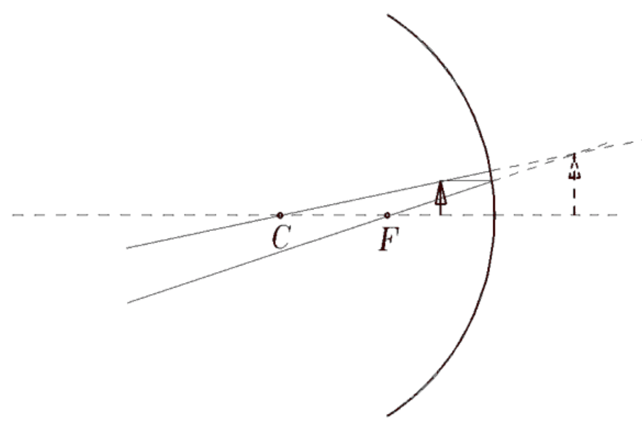
(*b*) The ray diagram is shown. The image is real, inverted, and of the same size as the object.



(*c*) The ray diagram is shown. The object is at the focal point of the mirror. In principle, the emerging rays are parallel and do not converge to form an image, real or virtual.



(*d*) The ray diagram is shown. The image is virtual, erect, and enlarged.



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14 · Use the mirror equation to locate and describe the images for the object distances and mirror of Problem 13.
(*a*), (*b*), (*c*), (*d*) Use Equ. 34-3; $s' = fs/(s - f)$ (*a*) $s' = (2000/80)$ cm = 25 cm; the image is real, inverted,

and reduced

(*b*) $s' = 40$ cm; the image is real, inverted, and of the same size as the object

(*c*) $s' = \infty$

(*d*) $s' = -20$ cm; the image is virtual, erect, and twice as large as the object

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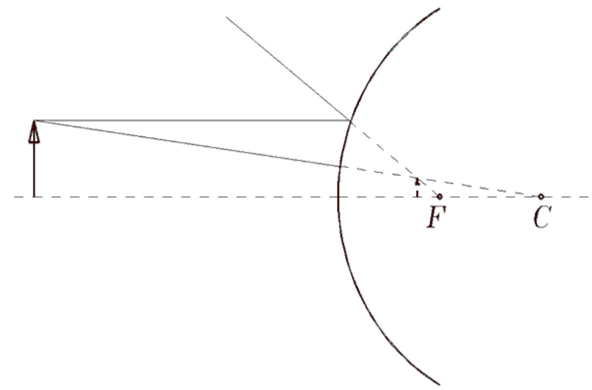
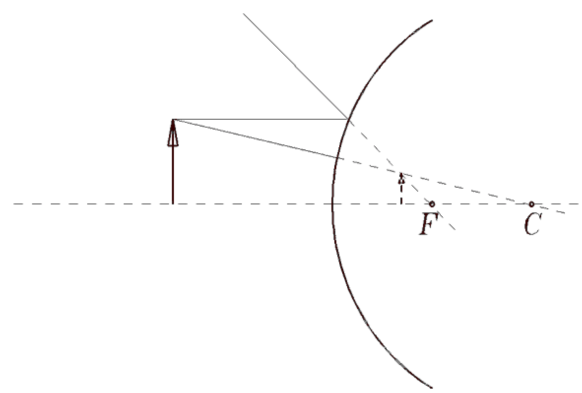
15 · Repeat Problem 13 for a convex mirror with the same radius of curvature.

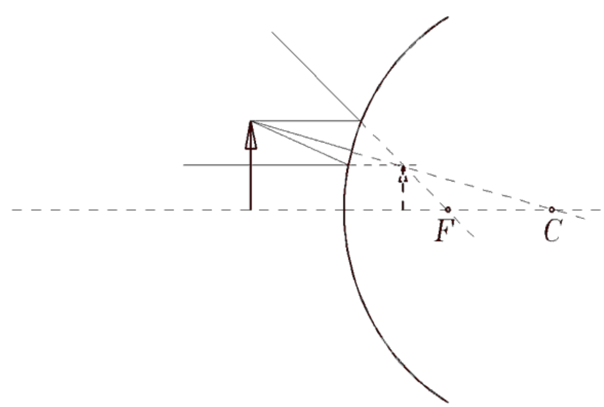
(*a*) The ray diagram is shown.
The image is virtual, erect, and reduced.



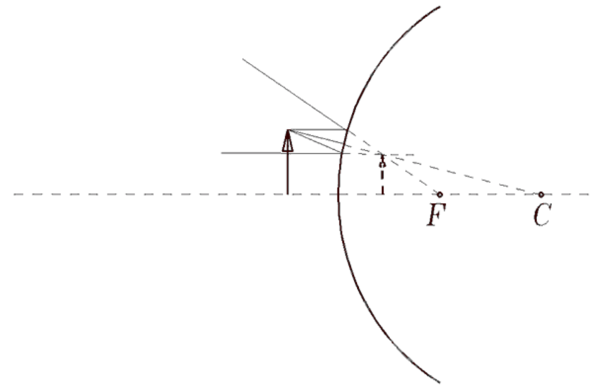
(*b*) The ray diagram is shown.
The image is virtual, erect, and reduced.



(*c*) The ray diagram is shown.
The image is virtual, erect, and reduced.



(*d*) The ray diagram is shown.
The image is virtual, erect, and reduced.



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16 · Use the mirror equation to locate and describe the images for the object distances and convex mirror of Problem 15.

(*a*), (*b*), (*c*), (*d*) Use Equ. 34-3; $s' = sf/(s - f)$; $f < 0$

(*a*) $s' = -16.7$ cm; the image is virtual, erect, and reduced
(*b*) $s' = -13.3$ cm; the image is virtual, erect, and reduced
(*c*) $s' = -10$ cm; the image is virtual, erect, and reduced
(*d*) $s' = -6.67$ cm; the image is virtual, erect, and reduced

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17*· Show that a convex mirror cannot form a real image of a real object, no matter where the object is placed, by showing that s' is always negative for a positive s.

From Equ. 34-3, $1/s' = 1/f - 1/s = (s - f)/sf$. For a convex mirror, $f < 0$. With $s > 0$, the numerator is positive and the denominator negative. Consequently, $1/s'$ is negative and so is s'.

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18 · A dentist wants a small mirror that will produce an upright image with a magnification of 5.5 when the mirror is located 2.1 cm from a tooth. (*a*) What should the radius of curvature of the mirror be? (*b*) Should it be concave or convex?

(*b*) The mirror must be concave; a convex mirror always produces a diminished virtual image.

(*a*) The image is upright, therefore virtual; use Equs. 34-3 and 34-4 to find f; $R = 2f = 2ms/(m - 1)$ $R = (11\times2.1/4.5)$ cm = 5.13 cm

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19 ·· Convex mirrors are used in stores to provide a wide angle of surveillance for a reasonable mirror size. The mirror shown in Figure 34-53 allows a clerk 5 m away from the mirror to survey the entire store. It has a radius of curvature of 1.2 m. (*a*) If a customer is 10 m from the mirror, how far from the mirror surface is his image? (*b*) Is the image in front of or behind the mirror? (*c*) If the customer is 2 m tall, how high is his image?

(*a*), (*b*) Use Equs. 34-2 and 34-3 to find s' $f = -0.6$ m; $s' = -56.5$ cm (56.6 cm behind the mirror)

(*c*) Use Equ. 34-4 $h' = (2\times56.6/10)$ m = 11.3 cm

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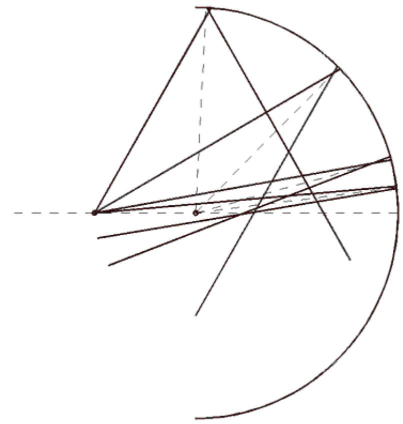
20 ·· A certain telescope uses a concave spherical mirror of radius 8 m. Find the location and diameter of the image of the moon formed by this mirror. The moon has a diameter of 3.5×10^6 m and is 3.8×10^8 m from the earth.

1. Use Equs. 34-2 and 34-3 to find s' and m $f = 4.0$ m; $s' = 4.0$ m; $m = -1.05\times10^{-8}$

2. $d' = |m|d$ $d' = 36.8$ cm

21*·· A concave spherical mirror has a radius of curvature of 6.0 cm. A point object is on the axis 9 cm from the mirror. Construct a precise ray diagram showing rays from the object that make angles of 5°, 10°, 30°, and 60° with the axis, strike the mirror, and are reflected back across the axis. (Use a compass to draw the mirror, and use a protractor to measure the angles needed to find the reflected rays.) What is the spread δx of the points where these rays cross the axis?

The rays from the point object are shown. Note that the rays that reflect from the mirror far from the axis do not converge at the same point as those that reflect from the mirror close to the mirror axis. For the small-angle rays, the point of convergence is 4.5 cm from the mirror. The 60° ray crosses the axis at 3 cm from the mirror. Consequently, the image extends from 4.5 cm to 3.0 cm, or about 1.5 cm along the axis.



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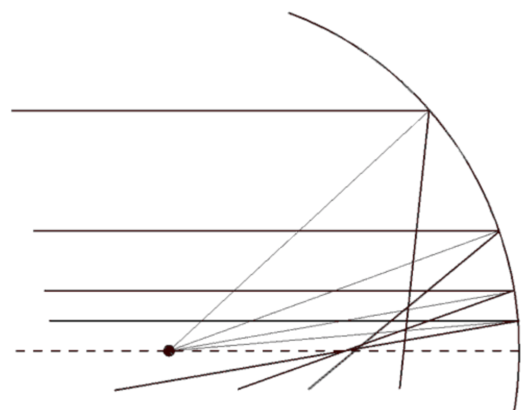
22 ·· A concave mirror has a radius of curvature 6.0 cm. Draw rays parallel to the axis at 0.5, 1.0, 2.0, and 4.0 cm above the axis and find the points at which the reflected rays cross the axis. (Use a compass to draw the mirror and a protractor to find the angle of reflection for each ray.) (*a*) What is the spread δx of the points where these rays cross the axis? (*b*) By what percentage could this spread be reduced if the edge of the mirror were blocked off so that parallel rays more than 2.0 cm from the axis could not strike the mirror?

(a) The figure to the right shows the mirror and the four rays drawn to scale. Using a calibrated ruler, the spread of the crossing points is $\delta x \approx 1.0$ cm. Note that the triangles formed by the center of curvature, the point of reflection on the mirror, and the point of intersection of the reflected ray and the mirror axis are isosceles triangles. The two equal angles are $\theta_r = \sin^{-1}(y/R)$ where y is the distance of the incoming ray from the mirror axis. Using the law of cosines, the distance between the point of intersection and the mirror is given by $d = R\{1 - [2\cos(\sin^{-1}(y/R))]^{-1}\}$. Thus for $y/R = 2/3$, $d = 1.974$ cm, and for $y/R = 1/12$, $d = 2.988$ cm. The spread $\delta x = 1.014$ cm, in agreement with the above value.



(*b*) For $y/R = 1/6$, $d = 2.82$ cm, so the spread $\delta x = 0.168$ cm. By blocking off the edges of the mirror so that only paraxial rays within 2 cm of the mirror axis are reflected, the spread is reduced by 83.4%.

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23 ·· An object placed 8 cm from a concave spherical mirror produces a virtual image 10 cm behind the mirror. (*a*) If the object is moved back to 25 cm from the mirror, where is the image located? (*b*) Is it real or virtual?

(*a*) 1. Use Equ. 34-3 to find f	$f = ss'/(s + s')$; $s' = -10$ cm; $f = 40$ cm
2. Use Equ. 34-3 to find the new s'	$s' = -66.7$ cm; the image is 66.7 cm behind the mirror
(*b*) The image is virtual	

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24 ·· An object located 100 cm from a concave mirror forms a real image 75 cm from the mirror. The mirror is then turned around so that its convex side faces the object. The mirror is moved so that the image is now 35 cm behind the mirror. How far was the mirror moved? Was it moved toward or away from the object?

1. Find the focal length of the mirror; $f = ss'/(s + s')$	$f = 42.86$ cm
2. Find s for $f = -42.86$ cm and $s' = -35$ cm	$s = fs'/(s' - f)$; $s = 191$ cm

3. Find the distance the mirror must be moved $d = 91$ cm

25*·· Parallel light from a distant object strikes the large mirror in Figure 34-54 ($r = 5$ m) and is reflected by the small mirror that is 2 m from the large mirror. The small mirror is actually spherical, not planar as shown. The light is focused at the vertex of the large mirror. (*a*) What is the radius of curvature of the small mirror? (*b*) Is it convex or concave?

(*a*) 1. Locate the image produced by the large mirror $1/s' = (1/2.5)$; $s' = 2.5$ m

2. This serves as a virtual image for the small mirror at $s = -0.5$ m. Find f_{small} and r_{small} $1/f_{small} = (1/2 - 1/0.5)\ \text{m}^{-1}$; $f_{small} = -0.667$ m

$r_{small} = 2f_{small} = -1.33$ m

(*b*) $f < 0$ The small mirror is convex

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26 ·· A woman uses a concave makeup mirror with a radius of curvature of 1.5 m. How far from the mirror should her face be for the image to be 80 cm from her face?

Since the image should be upright, it must be a virtual image behind the mirror.

Given: $s - s' = 0.8$ m, $f = 0.75$ m; find s $1/s + 1/(s - 80\ \text{cm}) = 1/(75\ \text{cm})$; $s = 30$ cm

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27 · A bird above the water is viewed by a scuba diver submerged beneath the water's surface directly below the bird. Does the bird appear to the diver to be closer to or farther from the surface than it actually is?

The bird appears to be farther from the surface than it actually is (see Equ. 34-7).

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28 · A sheet of paper with writing on it is protected by a thick glass plate having an index of refraction of 1.5. If the plate is 2 cm thick, at what distance beneath the top of the plate does the writing appear when it is viewed from directly overhead?

Use Equ. 34-7 $|s'| = (2/1.5)$ cm $= 1.33$ cm below the glass surface

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29* · A fish is 10 cm from the front surface of a fish bowl of radius 20 cm. (*a*) Where does the fish appear to be to someone in air viewing it from in front of the bowl? (*b*) Where does the fish appear to be when it is 30 cm from the front surface of the bowl?

(*a*) 1. The object is inside the bowl; determine r $r = -20$ cm

2. Use Equ. 34-5; $n_1 = 1.33$, $n_2 = 1.0$, $s = 10$ cm $s' = -8.58$ cm; 8.58 cm from the front surface of the bowl.

(*b*) Repeat (*a*) for $s = 30$ cm $s' = -35.9$ cm; 35.9 cm from the front surface of the bowl.

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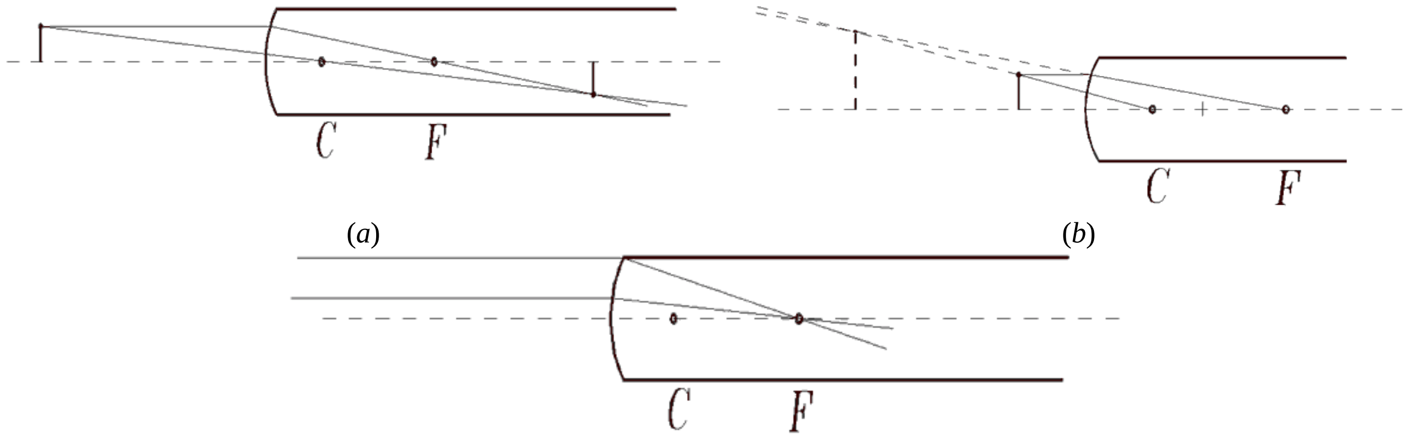
30 ·· A very long glass rod of 2-cm diameter has one end ground to a convex spherical surface of radius 5 cm. Its index of refraction is 1.5. (*a*) A point object in air is on the axis of the rod 20 cm from the surface. Find the image and state whether it is real or virtual. Repeat for (*b*) an object 5 cm from the surface and (*c*) an object very far from the surface. Draw a ray diagram for each case.

(*a*), (*b*), (*c*) Use Equ. 34-5; $n_1 = 1$, $n_2 = 1.5$; (*a*) $s' = 30$ cm; the image is real

$f = n_2r/(n_2 - 1) = 15$ cm (*b*) $s' = -15$ cm; the image is virtual

The ray diagrams are shown below (*c*) $s' = 15$ cm; the image is at F and of zero size.



(c)

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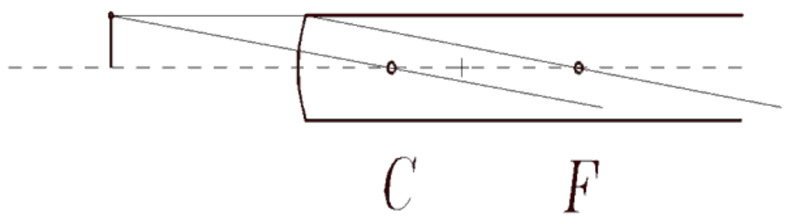
31 ·· At what distance from the rod of Problem 30 should the object be placed so that the light rays in the rod are parallel? Draw a ray diagram for this situation.

Parallel rays imply $s' = \infty$; use Equ. 34-5 to find s $s = r/0.5$; $s = 10$ cm

The ray diagram is shown



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32 ··Repeat Problem 30 for a glass rod with a concave hemispherical surface of radius –5 cm.

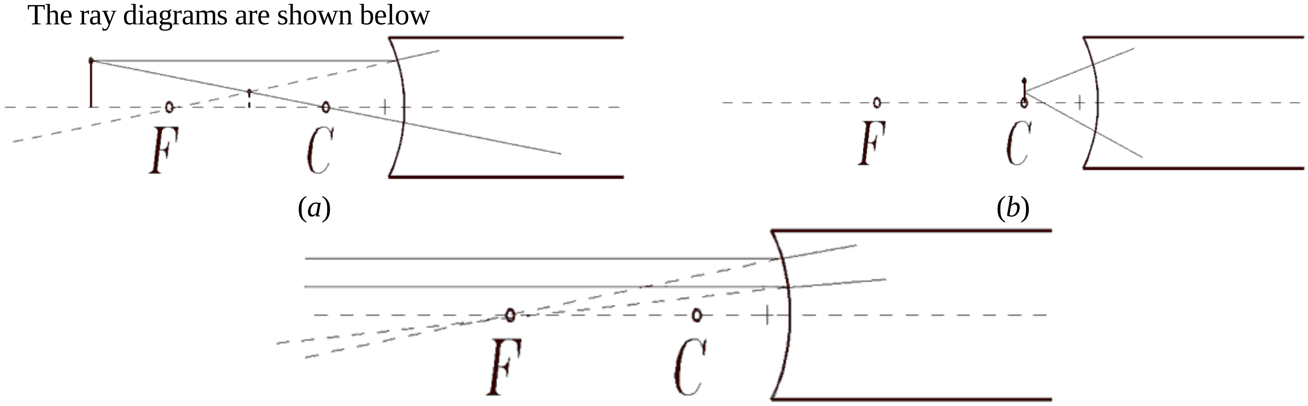
(*a*), (*b*), (*c*) Use Equ. 34-5; $r = -5$ cm; $f = -15$ cm

(*a*) $s' = -10$ cm; the image is virtual

(*b*) $s' = -5$ cm; the image is virtual and located at the object

(*c*) $s' = -15$ cm; the image is virtual and of zero size

The ray diagrams are shown below

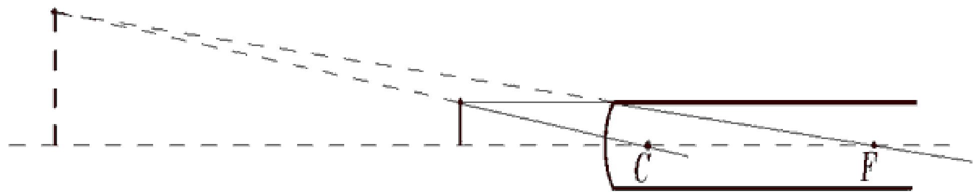


(*c*)

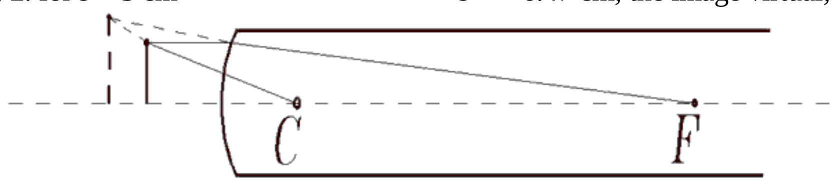
33* ·· Repeat Problem 30 when the glass rod and objects are immersed in water.

(*a*) 1. Use Equ. 34-5 with $n_1 = 1.33$, $n_2 = 1.5$ to find f; set $s = \infty$, then $s' = f$ $\quad$ $s' = f = [1.5\times5/(1.5 - 1.33)]$ cm = 44.1 cm

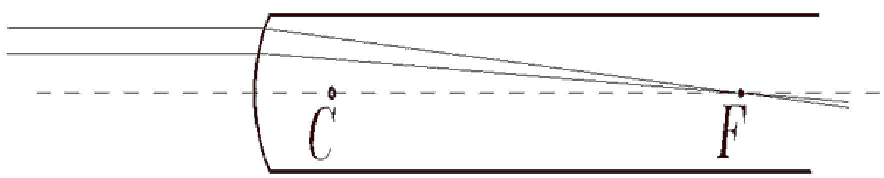
2. Use Equ. 34-5 to find s' for $s = 20$ cm $\quad$ $s' = -46.2$ cm; the image is virtual; see below.



(*b*) Repeat part (*a*) 2. for $s = 5$ cm $\quad$ $s' = -6.47$ cm; the image virtual; see below.



(*c*) For $s = \infty$, the image is real and at the focal point, i.e., $s' = 44.1$ cm; see below.



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34 ·· Repeat Problem 30 for a glass rod with a concave hemispherical surface of radius –5 cm when it and the objects are immersed in water.

(*a*), (*b*), (*c*) Use Equ. 34-5; $n_1 = 1.33$, $n_2 = 1.5$; $f = n_2r/(n_2 - n_1) = -44.1$ cm

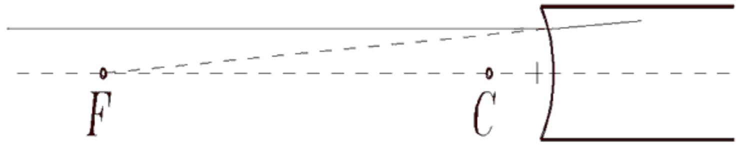
(*a*) $s' = -14.9$ cm; the image is virtual

(*b*) $s' = -5$ cm; the image is virtual located at the object

(*c*) $s' = -44.1$ cm; the image is virtual and of zero size

The ray diagrams are shown below

(*a*) (*b*)



(*c*)

35 ·· A glass rod 96 cm long with an index of refraction of 1.6 has its ends ground to convex spherical surfaces of radii 8 cm and 16 cm. A point object is in air on the axis of the rod 20 cm from the end with the 8-cm radius. (*a*) Find the image distance due to refraction at the first surface. (*b*) Find the final image due to refraction at both surfaces. (*c*) Is the final image real or virtual?

(*a*) Use Equ. 34-5; $n_1 = 1$, $n_2 = 1.6$, $r = 8$ cm — $s' = n_2rs/[(n_2 - n_1)s - n_1r]$; $s' = 64$ cm

(*b*) Use Equ. 34-5; the object is the image formed by the first surface; $r = -16$ cm, $s = 32$ cm, $n_1 = 1.6, n_2 = 1$ — $s' = -80$ cm; the image is inside the rod, 16 cm from the front

(*c*) The image is virtual

36 ·· Repeat Problem 35 for a point object in air on the axis of the rod 20 cm from the end with the 16-cm radius.

(*a*), (*b*), (*c*) Proceed as in Problem 34-35 — (*a*) $s' = -128$ cm; (*b*) $s' = 14.7$ cm (*c*) The image is real and 14.7 cm from the far end of the rod.

37* · Under what conditions will the focal length of a thin lens be positive? Negative?

The lens will be positive if its index of refraction is greater than that of the surrounding medium and the lens is thicker in the middle than at the edges. Conversely, if the index of refraction of the lens is less than that of the surrounding medium, the lens will be positive if it is thinner at its center than at the edges.

The lens will be negative if its index of refraction is greater than that of the surrounding medium and the lens is thinner at the center than at the edges. Conversely, if the index of refraction of the lens is less than that of the surrounding medium, the lens will be negative if it is thicker at the center than at the edges.

38 · The focal length of a simple lens is different for different colors of light. Why?

The focal length depends on the index of refraction, and *n* is a function of wavelength.

39 ·· An object is placed 40 cm from a lens of focal length –10 cm. The image is

(*a*) real, inverted, and diminished.
(*b*) real, inverted, and enlarged.
(*c*) virtual, inverted, and diminished.
(*d*) virtual, upright, and diminished.
(*e*) virtual, upright, and enlarged.

(*d*)

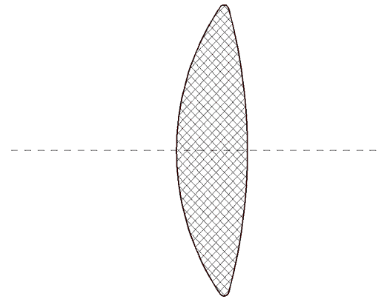
40 ·· If a real object is placed just inside the focal point of a converging lens, the image is

(*a*) real, inverted, and enlarged.
(*b*) virtual, erect, and diminished.
(*c*) virtual, erect, and enlarged.
(*d*) real, inverted, and diminished.

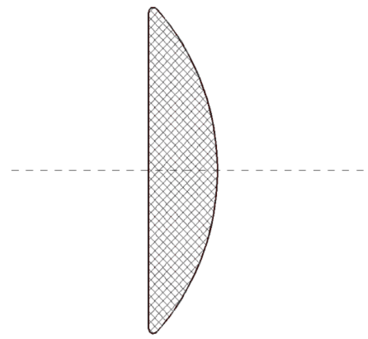
(*c*)

41* · The following thin lenses are made of glass with an index of refraction of 1.5. Make a sketch of each lens, and find its focal length in air: (*a*) double convex, r_1 = 10 cm and r_2 = –21 cm; (*b*) plano–convex, $r_1 = \infty$ and r_2 = –10 cm; (*c*) double concave, r_1 = –10 cm and r_2 = +10 cm; (*d*) plano–concave, $r_1 = \infty$ and r_2 = +20 cm.

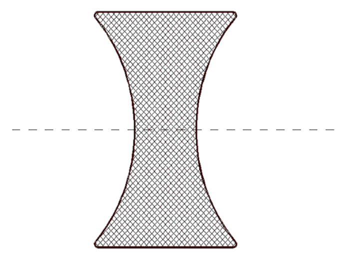
(*a*) Use Equ. 34-11 to find *f* f = 13.5 cm



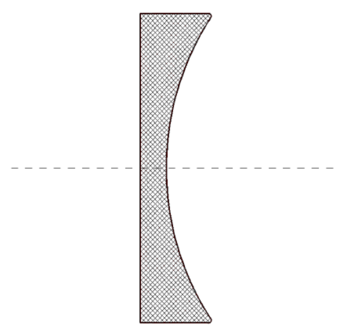
(*b*), (*c*), (*d*) Repeat as in (*a*) (*b*) f = 20 cm



(*c*) f = –10 cm



(*d*) f = –40 cm



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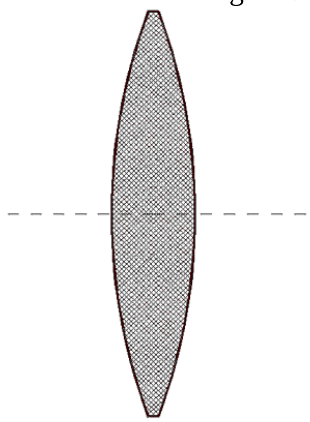
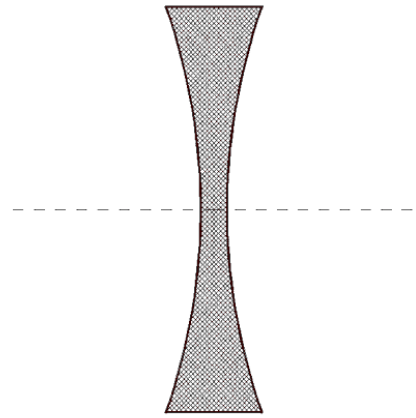
42 · Glass with an index of refraction of 1.6 is used to make a thin lens that has radii of equal magnitude. Find the radii of curvature and make a sketch of the lens if the focal length in air is (*a*) +5 cm and (*b*) –5 cm.

(*a*) Use Equ. 34-11; $r_2 = -r_1 = r = 2(n-1)f$ r = 6 cm
(*b*) Use Equ. 34-11; $r_1 = -r_2 = -|r|$ r = 6 cm

The two lenses are shown in the figures.



(a)



(b)

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43 · Find the focal length of a glass lens of index of refraction 1.62 that has a concave surface with radius of magnitude 100 cm and a convex surface with a radius of magnitude 40 cm.

Use Equ. 34-11; r_1 and r_2 have the same sign $\quad f = (0.62\times0.015)^{-1} = 108$ cm

---

44 · A double-concave lens of index of refraction 1.45 has radii of magnitudes 30 cm and 25 cm. An object is located 80 cm to the left of the lens. Find (a) the focal length of the lens, (b) the location of the image, and (c) the magnification of the image. (d) Is the image real or virtual? Upright or inverted?

(a) Use Equ. 34-11 $\quad f = -30.3$ cm

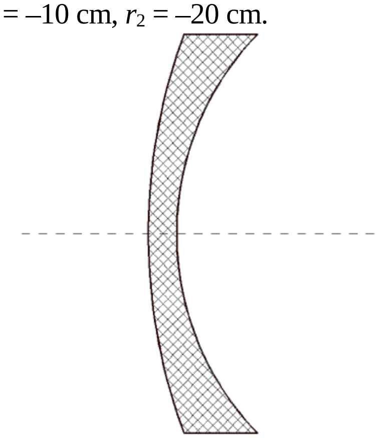
(b) Use Equ. 34-12; $s' = fs/(s - f)$ $\quad s' = -22$ cm

(c), (d) Use Equ. 34-14 $\quad m = 0.725$; the image is virtual and upright

---

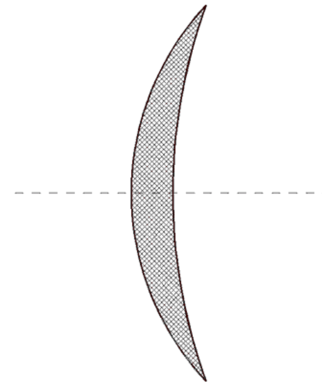
45* · The following thin lenses are made of glass of index of refraction 1.6. Make a sketch of each lens, and find its focal length in air: (a) $r_1 = 20$ cm, $r_2 = 10$ cm; (b) $r_1 = 10$ cm, $r_2 = 20$ cm; (c) $r_1 = -10$ cm, $r_2 = -20$ cm.

(a) Use Equ. 34-11 to find f $\quad f = -33.3$ cm

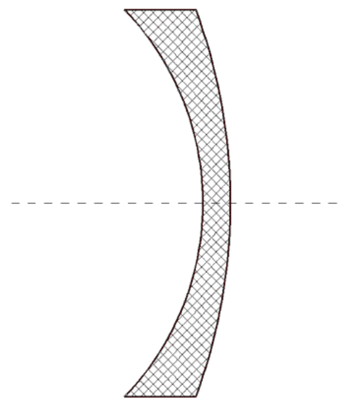


(*b*), (*c*) Repeat as in (*a*)

(*b*) f = 33.3 cm



(*c*) f = –33.3 cm



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46 · For the following object distances and focal lengths of thin lenses in air, find the image distance and the magnification and state whether the image is real or virtual and erect or inverted: (*a*) s = 40 cm, f = 20 cm; (*b*) s = 10 cm, f = 20 cm; (*c*) s = 40 cm, f = –30 cm; (*d*) s = 10 cm, f = –30 cm.

(*a*), (*b*), (*c*), (*d*) Use Equs. 34-12 and 34-14

(*a*) s' = 40 cm, m = –1; the image is real and inverted
(*b*) s' = –20 cm, m = 2; the image is virtual and erect
(*c*) s' = –17.1 cm, m = 0.43; the image is virtual and erect
(*d*) s' = –7.5 cm, m = 0.75; the image is virtual and erect

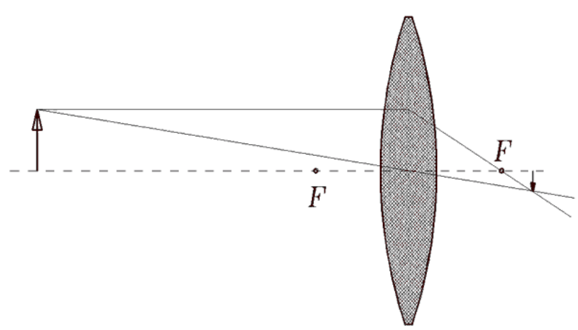
---

47 · An object 3.0 cm high is placed 20 cm in front of a thin lens of power 20 D. Draw a precise ray diagram to find the position and size of the image and check your results using the thin-lens equation.

Use Equs. 34-13 and 34-14

f = 5 cm; s' = 6.67; h' = 1.0 cm

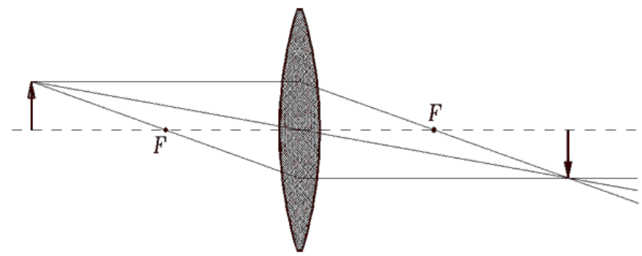
The ray diagram is shown in the figure



48 · Repeat Problem 47 for an object 1.0 cm high placed 10 cm in front of a thin lens of power 20 D.

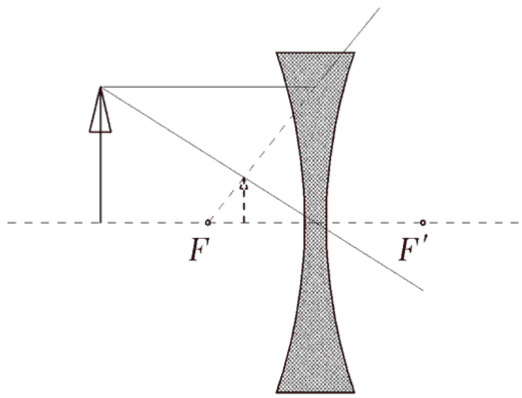
Use Equs. 34-13 and 34-14 — *f* = 5 cm; *s′* = 10 cm; *h′* = 1 cm

The ray diagram is shown in the figure



49* · Repeat Problem 47 for an object 1.0 cm high placed 10 cm in front of a thin lens whose power is –20 D.

The ray diagram is shown. Using Equ. 34-12 with *f* = 1/*D* = –5 cm, the image distance is –3.33 cm. The image is virtual, erect, and diminished. The image size is 1(3.33/10) cm = 0.33 cm. These results are in agreement with the ray diagram.

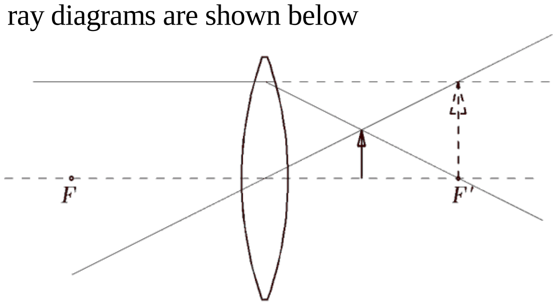


50 ·· (*a*) What is meant by a negative object distance? How can it occur? Find the image distance and magnification and state whether the image is virtual or real and erect or inverted for a thin lens in air when (*b*) *s* = –20 cm, *f* = +20 cm and (*c*) *s* = –10 cm, *f* = –30 cm. Draw a ray diagram for each of these cases.

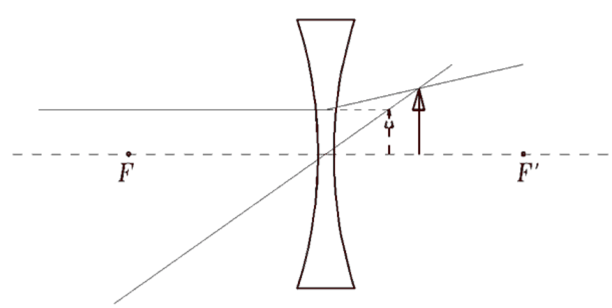
(*a*) A negative object distance implies that the object is a virtual object, i.e., that light rays converge on the object rather than diverge from the object. A virtual object can occur in a two-lens system when the first lens forms an image that is at a distance –|*s*| from the second lens.

(*b*), (*c*) Use Equs. 34-12 and 34-14 — (*b*) *s′* = 10 cm, *m* = 0.5; the image is real and erect; (*c*) *s′* = 15 cm, *m* = 1.5; the image is real and erect

The ray diagrams are shown below



(*b*)



(*c*)

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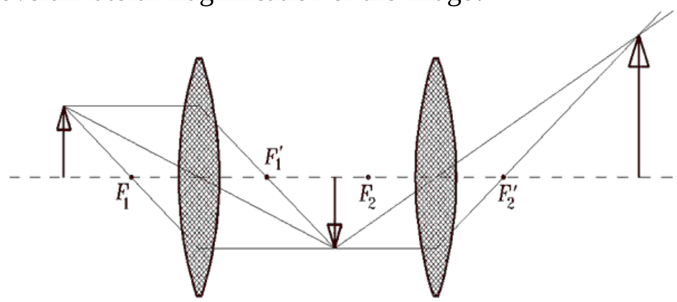
51 ·· Two converging lenses, each of focal length 10 cm, are separated by 35 cm. An object is 20 cm to the left of the first lens. (*a*) Find the position of the final image using both a ray diagram and the thin-lens equation. (*b*) Is the image real or virtual? Erect or inverted? (*c*) What is the overall lateral magnification of the image?

(*a*) The ray diagram is shown in the figure. We first locate the image formed by the first lens using Equ. 34-12 and determine the magnification. $s_1' = 20$ cm, $m_1 = -1$. This real image serves as the object for the second lens. The object distance is $s_2 = (35 - 20)$ cm = 15 cm. The image distance for this lens is $s_2' = 30$ cm, and $m_2 = -2$. The final image is 85 cm from the object.



(b) The final image is real and erect.

(*c*) The overall magnification is 2.

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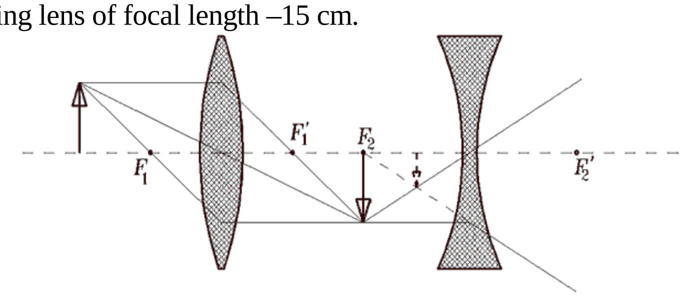
52 ·· Work Problem 51 for a second lens that is a diverging lens of focal length –15 cm.

(*a*), (*b*), (*c*) The ray diagram is shown in the figure. For the first lens, $s_1' = 20$ cm and $m_1 = -1$ as in Problem 34-51and $s_2 = 15$ cm. Now we use Equ. 34-12 with $f_2 = -15$ cm. We find $s_2' = -7.5$ cm and $m_2 = 0.5$. The final image is 47.5 cm from the object, virtual, inverted, and half as large as the object.



53* ·· A thin lens of index of refraction 1.5 has one convex side with a radius of magnitude 20 cm. When an object 1 cm in height is placed 50 cm from this lens, an upright image 2.15 cm in height is formed. (*a*) Calculate the radius of the second side of the lens. Is it concave or convex? (*b*) Draw a sketch of the lens.

(*a*) 1. Use Equs. 34-14 and 34-12 to find *f*

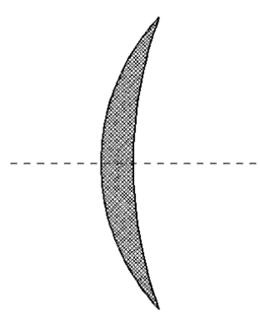
$$f = \frac{s\,s'}{s + s'};\quad s' = -2.15\,s;\quad f = \frac{-2.15\,s}{-1.15} = 93.5 \text{ cm}$$

2. Use Equ. 34-11 to find r_2;

$$r_2 = \frac{r_1 f (n - 1)}{f(n - 1) - r_1}$$

$r_2 = 35$ cm; $r_2 > 0$, side is concave

(*b*) The lens is shown in the adjacent figure.



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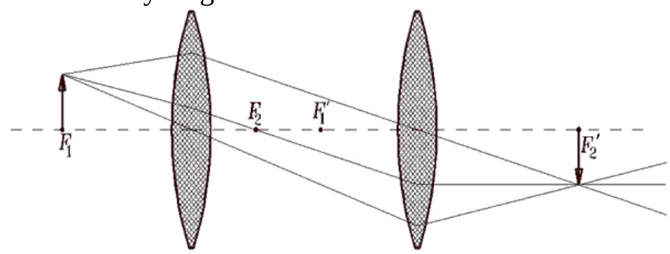
54 ·· (*a*) Show that to obtain a magnification of magnitude *m* with a converging thin lens of focal length *f*, the object distance must be given by $s = (m - 1)f/m$. (*b*) A camera lens with 50-mm focal length is used to take a picture of a person 1.75 m tall. How far from the camera should the person stand so that the image size is 24 mm?

(*a*) The condition requires that $s' = -ms$. So Equ. 34-12 becomes $f = -ms^2/(s - ms) = ms/(1 - m)$. Solving for s one obtains $s = (f/m)(1 - m)$.

(*b*) Use the above result with $m = -2.4/175$ $s = 370$ cm = 3.70 m

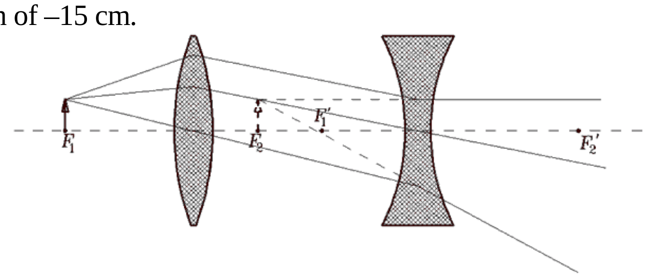
55 ·· An object is 15 cm in front of a positive lens of focal length 15 cm. A second positive lens of focal length 15 cm is 20 cm from the first lens. Find the final image and draw a ray diagram.

The ray diagram is shown in the figure. Since the object is at the focal point of the first lens, the rays entering the second lens are parallel; Equ. 34-12 shows that $s_1' = \infty$. It then follows from Equ. 34-12 that $s_2' = f_2 = 15$ cm. The final image is 50 cm from the object, real, inverted, and of the same size as the object.



56 ·· Work Problem 55 for a second lens with a focal length of –15 cm.

The ray diagram is shown in the figure. As in Problem 34-55, $s_2 = \infty$. From Equ. 34-12 we have $s_2' = -f_2$. Since $f_2 = -f_1$ the image is 20 cm from the object, virtual, erect, and of the same size as the object.

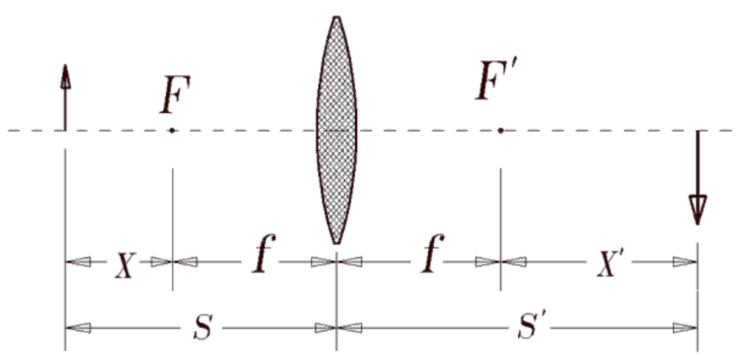


57* ··· In a convenient form of the thin-lens equation used by Newton, the object and image distances are measured from the focal points. Show that if $x = s - f$ and $x' = s' - f$, the thin-lens equation can be written as $xx' = f^2$, and the lateral magnification is given by $m = -x'/f = -f/x$. Indicate x and x' on a sketch of a lens.

From the definitions of x and x' it follows that Equ. 34-12 takes the form $\dfrac{1}{x + f} + \dfrac{1}{x' + f} = \dfrac{1}{f}$. Expanding this equation one obtains $f(x' + x + 2f) = xx' + xf + x'f + f^2$; $f^2 = xx'$.

The lateral magnification is $m = -s'/s = -(x' + f)/(x + f)$. With $x = f^2/x'$, $m = -\dfrac{x' + f}{f(f/x' + 1)} = -\dfrac{x'}{f} = -\dfrac{f}{x}$.

The variables x, x', f, s, and s' are shown in the adjacent drawing.



58 ··· An object is placed 2.4 m from a screen, and a lens of focal length f is placed between the object and the screen so that a real image of the object is formed on the screen. When the lens is moved 1.2 m toward the screen, another real image of the object is formed on the screen. (*a*) Where was the lens located before it was moved? (*b*) What is the focal length of the lens?

We shall use subscripts 1 and 2 to denote the initial and final values of s, s', and of f.

(*a*) 1. Write expressions for the data	$1/s_1 + 1/s_1' = 1/s_2 + 1/s_2'$; $s_2 = s_1 + 1.2$ m, $s_2' + s_2 = 2.4$ m,
2. The first condition is satisfied if $s_2 = s_1'$	$s_1 + s_1' = 2.4$ m
and $s_2' = s_1$; solve for s_1	$s_1 = 0.6$ m
3. Find the location of the lens	The lens is located 1.8 m from the screen
(*b*) Use Equ. 34-12	$f = [(0.6\times1.8)/2.4]$ m $= 0.45$ m

59 ··· An object is 17.5 cm to the left of a lens of focal length 8.5 cm. A second lens of focal length –30 cm is 5 cm to the right of the first lens. (*a*) Find the distance between the object and the final image formed by the second lens. (*b*) What is the overall magnification? (*c*) Is the final image real or virtual? Upright or inverted?

(*a*) 1. Find s_1' using Equ. 34-12	$s_1' = 16.53$ cm
2. Find s_2 from the data	$s_2 = -11.53$ cm
3. Find s_2' using Equ. 34-12	$s_2' = 18.7$ cm
4. Find the object-to-image distance, d	$d = (17.5 + 5 + 18.7)$ cm $= 41.2$ cm
(*b*), (*c*) $m = m_1m_2$; $m_1 = -s_1'/s_1$, $m_2 = -s_2'/s_2$	$m = -1.53$; the image is real and inverted

60 · Chromatic aberration is a common defect of (*a*) concave and convex lenses. (*b*) concave lenses only. (*c*) concave and convex mirrors. (*d*) all lenses and mirrors.

(*a*)

61* · True or false:

(*a*) Aberrations occur only for real images.

(*b*) Chromatic aberration does not occur with mirrors.

(*a*) False (*b*) True

62 · A double-convex lens of radii $r_1 = +10$ cm and $r_2 = -10$ cm is made from glass with indexes of refraction of 1.53 for blue light and 1.47 for red light. Find the focal length of this lens for (*a*) red light and (*b*) blue light.

(*a*), (*b*) Use Equ. 34-11	(*a*) $f_{red} = 10.6$ cm (*b*) $f_{blue} = 9.43$ cm

In the following problems, take the distance from the cornea–lens system of the eye to the retina to be 2.5 cm.

63 · If an object is placed 25 cm from the eye of a farsighted person who does not wear corrective lenses, a sharp image is formed

(*a*) behind the retina, and the corrective lens should be convex.

(*b*) behind the retina, and the corrective lens should be concave.

(*c*) in front of the retina, and the corrective lens should be convex.

(*d*) in front of the retina, and the corrective lens should be concave.

(*a*)

64 ·· Myopic (nearsighted) persons sometimes claim to see better under water without corrective lenses. Why?

(*a*) The accommodation of the eye's lens is better under water.

(*b*) Refraction at the water–cornea interface is less than at the air–cornea interface.

(*c*) Refraction at the water–cornea interface is greater than at the air–cornea interface.

(*d*) No reason; the effect is only an illusion and not really true.

(*b*)

65* ·· A nearsighted person who wears corrective lenses would like to examine an object at close distance. Identify the correct statement.

(*a*) The corrective lenses give an enlarged image and should be worn while examining the object.

(*b*) The corrective lenses give a reduced image of the object and should be removed.

(*c*) The corrective lenses result in a magnification of unity; it does not matter whether they are worn or removed.

(*b*)

---

66 · Suppose the eye were designed like a camera with a lens of fixed focal length $f = 2.5$ cm that could move toward or away from the retina. Approximately how far would the lens have to move to focus the image of an object 25 cm from the eye onto the retina? (*Hint:* Find the distance from the retina to the image behind it for an object at 25 cm.)

1. Use Equ. 34-12 to find s'	$s' = 2.778$ cm
2. Distance to move lens = $s' - f$	Move lens 0.278 cm toward the object

---

67 · Find the change in the focal length of the eye when an object originally at 3 m is brought to 30 cm from the eye.

Find f for the two values of s; then find Δf	$s = 3$ m, $f = 2.479$ cm; $s = 30$ cm, $f = 2.307$ cm; $\Delta f = 1.72$ mm

---

68 · Find (*a*) the focal length and (*b*) the power of a lens that will produce an image at 80 cm from the eye of a book that is 30 cm from the eye.

(*a*) Use Equ. 34-12; $s = 0.3$ m, $s' = -0.8$ m	$f = 0.48$ m
(*b*) Use Equ. 34-13	$P = 2.08$ diopters

69* · A farsighted person requires lenses with a power of 1.75 D to read comfortably from a book that is 25 cm from the eye. What is that person's near point without the lenses?

See Example 34-13; solve for $\lvert s' \rvert$ = near point	$f = 1/D = 57.1$ cm; $s' = -44.4$ cm; near point at 44.4 cm

---

70 · If two point objects close together are to be seen as two distinct objects, the images must fall on the retina on two different cones that are not adjacent. That is, there must be an unactivated cone between them. The separation of the cones is about 1 μm . (*a*) What is the smallest angle the two points can subtend? (See Figure 34-55). (*b*) How close can two points be if they are 20 m from the eye?

(*a*) $r_{eye}\theta_{min} = 2\times10^{-6}$ m	$\theta_{min} = 80\ \mu$rad
(*b*) $D = R\theta_{min}$	$D = (20\times8\times10^{-5})$ m = 1.6 mm

---

71 ·· A person with a near point of 80 cm needs to read from a computer screen that is 45 cm from her eye. (*a*) Find the focal length of the lenses in reading glasses that will produce an image of the screen at 80 cm from her eye. (*b*) What is the power of the lenses?

(*a*) Use Equ. 34-12; image must be virtual	$f = [-80\times45/(-35)]$ cm = 103 cm

(*b*) Use Equ. 34-13 $P = 0.972$ diopters

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72 ·· A nearsighted person cannot focus clearly on objects more distant than 225 cm from her eye. What power lenses are required for her to see distant objects clearly?

Use Equs. 34-12 and 34-13; image must be virtual $P = –1/2.25 = 0.444$ diopters

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73* ·· Since the index of refraction of the lens of the eye is not very different from that of the surrounding material, most of the refraction takes place at the cornea, where n changes abruptly from 1.0 in air to about 1.4. Assuming the cornea to be a homogeneous sphere with an index of refraction of 1.4, calculate its radius if it focuses parallel light on the retina a distance 2.5 cm away. Do you expect your result to be larger or smaller than the actual radius of the cornea?

1. Use Equ. 34-5 with $s = \infty$, $s' = 2.5$ cm to find r $r = 2.5(0.4/1.4)$ cm $= 0.714$ cm

2. The eye is not a homogeneous sphere. It is filled with a transparent liquid (vitreous humor) which has an index of refraction that is not known. If that index of refraction differs from 1.4, there is refraction at the inner surface of the cornea which will result in the formation of the image nearer the cornea's surface if $n > 1.4$ and farther if $n < 1.4$, where n is the index of refraction of the vitreous humor. If $n < 1.4$, then r as calculated above is too small.

---

74 ·· The near point of a certain person is 80 cm. Reading glasses are prescribed so that he can read a book at 25 cm from his eye. The glasses are 2 cm from the eye. What diopter lens should be used in the glasses?

1. Determine s and s' $s' = –78$ cm, $s = 23$ cm

2. Use Equs. 34-12 and 34-13 $P = (1/0.23 – 1/0.78)$ D $= 3.07$ D

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75 ··· At age 45, a person is fitted for reading glasses of power 2.1 D in order to read at 25 cm. By the time she reaches 55, she discovers herself holding her newspaper at a distance of 40 cm in order to see it clearly with her glasses on. (*a*) Where was her near point at age 45? (*b*) Where is her near point at age 55? (*c*) What power is now required for the lenses of her reading glasses so that she can again read at 25 cm? (Assume the glasses are 2.2 cm from her eyes.)

(*a*) Use Equs. 34-12 and 34-13 to find s'; the uncorrected near point is at $|s'| + 2.2$ cm — $1/s' = P – 1/0.228$ m; $s' = –43.7$ cm; the near point at age 45 is at 45.9 cm

(*b*) $s = 37.8$ cm; find s' and the near point — $s' = (2.1 – 1/.378)^{–1}$ m $= 1.833$ m; near point is at 185 cm

(*c*) Find P for $s = 22.8$ cm, $s' = 183.3$ cm — $P = 3.84$ D

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76 ··· An aging physics professor discovers that he can see objects clearly only between 0.75 m and 2.5 m so he decides he needs bifocals. The upper part of the lens allows him to see objects clearly at infinity, and the lower part allows him to see objects clearly at 25 cm. Assume that the lens is 2 cm from his eye. (*a*) Calculate the power of the lens required for the upper part of his bifocals. (*b*) Calculate the power of the lens required for the lower part of his bifocals. (*c*) Is there a range of distance over which he cannot see objects clearly no matter which part of the bifocals he looks through? If so, what is that range?

(*a*) Use Equs. 34-12 and 34-13 $P = –1/2.48$ D $=– 0.403$ D

(*b*) Now $s' = –0.73$ m, $s = 0.23$ m $P = (1/0.23 – 1/0.73)$ D $= 2.98$ D

(*c*) 1. Find s_{min} for upper part; $|s'|_{min} = 73$ cm $s_{min} = (–0.403 + 1/0.73)^{–1}$ m $= 1.03$ m

2. Find s_{max} for lower part $s_{max} = (2.98 + 1/2.48)^{-1}$ m = 29.6 cm

3. Find the range for unclear vision The range for unclear vision is from 31.6 cm to 105 cm

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77* • A person with a near-point distance of 30 cm uses a simple magnifier of power 20 D. What is the magnification obtained if the final image is at infinity?

Use Equ. 34-20 $M = x_{np}/f = x_{np}D = 0.3\times20 = 6$

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78 • A person with a near point distance of 25 cm wishes to obtain a magnifying power of 5 with a simple magnifier. What should be the focal length of the lens used?

Use Equ. 34-20 $f = x_{np}/M = 5$ cm

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79 • What is the magnifying power of a lens of focal length 7 cm when the image is viewed at infinity by a person whose near point is at 35 cm?

Use Equ. 34-20 $M = 35/7 = 5$

---

80 •• A lens of focal length 6 cm is used as a simple magnifier with the image at infinity by one person whose near point is 25 cm and by another whose near point is 40 cm. What is the effective magnifying power of the lens for each person? Compare the size of the image on the retina when each looks at the same object with the magnifier.

Use Equ. 34-20; $x_{np,1} = 25$ cm, $x_{np,2} = 40$ cm $M_1 = 4.17$, $M_2 = 6.67$; $H_1/H_2 = M_1/M_2 = 0.625$

81* •• A botanist examines a leaf using a convex lens of power 12 D as a simple magnifier. What is the expected angular magnification if (*a*) the final image is at infinity, and (*b*) the final image is at 25 cm?

(*a*) Use Equ. 34-20 $M = x_{np}D = 0.25\times12 = 3$

(*b*) Use Equ. 34-12 to find s; $m = -s'/s$ $1/s = 12 + 4$; $s = 1/16$; $m = 16/4 = 4$

---

82 •• (*a*) Show that if the final image of a simple magnifier is to be at the near point of the eye rather than at infinity, the angular magnification is given by

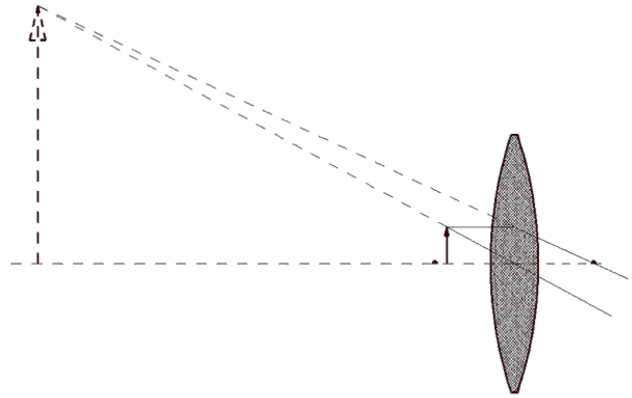
$$M = \frac{x_{np}}{f} + 1$$

(*b*) Find the magnification of a 20-D lens for a person with a near point of 30 cm if the final image is at the near point. Draw a ray diagram for this situation.

(*a*) The image is virtual and $s' = -x_{np}$. From Equ. 34-12, $s = fx_{np}/(f + x_{np})$. The angle subtended is Mh/x_{np}, where h is the height of the object. The angle subtended by the object is h/s. The angular magnification $M = (f + x_{np})/f = 1 + x_{np}/f$.

(*b*) Use the result of part (*a*); $f = 1/P = 5$ cm $M = 6 + 1 = 7$

The ray diagram is shown.



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83 ·· Show that when the image of a simple magnifier is viewed at the near point, the lateral and angular magnification of the magnifier are equal.

The lateral magnification is x_{np}/s. From the preceding problem, $x_{np}/s = 1 + x_{np}/f$ which is equal to the angular magnification obtained in Problem 34-82.

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84 ·· A microscope objective has a focal length of 0.5 cm. It forms an image at 16 cm from its second focal point. What is the magnifying power for a person whose near point is at 25 cm if the focal length of the eyepiece is 3 cm?

Use Equ. 34-22 — $M = -(16/0.5)(25/3) = -267$

85* ·· A microscope has an objective of focal length 16 mm and an eyepiece that gives an angular magnification of 5 for a person whose near point is 25 cm. The tube length is 18 cm. (*a*) What is the lateral magnification of the objective? (*b*) What is the magnifying power of the microscope?

(*a*) Use Equ. 34-21 — $m_o = -18/1.6 = -11.25$

(*b*) $M = 5m_o$ — $M = -56.25$

---

86 ·· A crude symmetric hand-held microscope consists of two converging 20-D lenses fastened in the ends of a tube 30 cm long. (*a*) What is the “tube length” of this microscope? (*b*) What is the lateral magnification of the objective? (*c*) What is the magnifying power of the microscope? (*d*) How far from the objective should the object be placed?

(*a*) $L = D - f_o - f_e$ (see Fig. 34-48) — $L = (30 - 10)$ cm = 20 cm

(*b*) Use Equ. 34-21 — $m_o = -4$

(*c*) Use Equ. 34-22 — $M = -20$

(*d*) $s_o' = f_o + L$; solve for s_o using Equ. 34-12 — $s_o = (25\times5/20)$ cm = 6.25 cm

---

87 ·· Repeat Problem 86 for the same two lenses separated by 40 cm.

(*a*), (*b*), (*c*), (*d*) Proceed as in the preceding problem — (*a*) $L = 30$ cm (*b*) $m_o = -6$ (*c*) $M = -30$ (*d*) $s_o = 5.83$ cm

---

88 ·· A compound microscope has an object with a power of 45 D and an eyepiece with a power of 80 D. The lenses are separated by 28 cm. Assuming that the final image is formed 25 cm from the eye, what is the magnifying power?

1. Determine m_e; see Problem 82; find L — $m_e = P_e x_{np} + 1 = 21$; $L = (28 - 1.25 - 2.22)$ cm = 24.53 cm

2. Determine $M = m_o m_e$; $m_o = -P_o L$ — $m_o = 11.0$; $M = -232$

89* ··· A microscope has a magnifying power of 600, and an eyepiece of angular magnification of 15. The objective lens is 22 cm from the eyepiece. Without making any approximations, calculate (*a*) the focal length of the eyepiece, (*b*) the location of the object such that it is in focus for a normal relaxed eye, and (*c*) the focal length of the objective lens.

(*a*) Use Equ. 34-20 for the eyepiece; $M_e = 15$	$f_e = 25/15 = 1.67$ cm
(*b*) Use Equ. 34-22 to find $m_o = -s'/s$ for the objective; find $s = -s'/m_o$	$m_o = -600/15 = -40$; note that $s' = (22 - 1.67)$ cm $s = 20.33/40$ cm $= 0.508$ cm
(*c*) Use Equ. 34-12 to find f_o	$f_o = (20.33\times0.508/20.84)$ cm $= 0.496$ cm

---

90 · A simple telescope has an objective with a focal length of 100 cm and an eyepiece of focal length 5 cm. It is used to look at the moon, which subtends an angle of about 0.009 rad. (*a*) What is the diameter of the image formed by the objective? (*b*) What angle is subtended by the final image at infinity? (*c*) What is the magnifying power of the telescope?

(*a*) The objective image is at $s_o' = f_o$; $D = f_o\theta$	$D = 9$ mm
(*b*), (*c*) $\theta_e = M\theta_o = M\theta$; $M = -f_o/f_e$	$M = -20$; $\theta_e = -0.18$ rad

---

91 · The objective lens of the refracting telescope at the Yerkes Observatory has a focal length of 19.5 m. When it is used to look at the moon, which subtends an angle of about 0.009 rad, what is the diameter of the image of the moon formed by the objective?

See Problem 34-90	$D = 17.6$ cm

---

92 ·· The 200-in (5.1-m) mirror of the reflecting telescope at Mt. Palomar has a focal length of 1.68 m. (*a*) By what factor is the light-gathering power increased over the 40-in (1.016-m) diameter refracting lens of the Yerkes Observatory telescope? (*b*) If the focal length of the eyepiece is 1.25 cm, what is the magnifying power of this telescope?

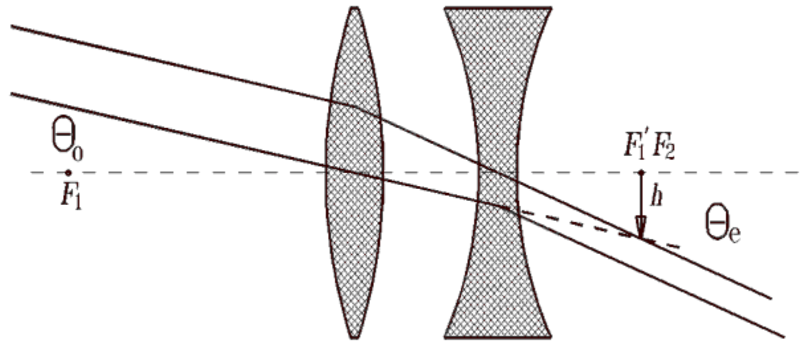
(*a*) Light-gathering power $\propto d^2$	Increase is by a factor of 25
(*b*) Use Equ. 34-23	$M = -134$

93* ·· An astronomical telescope has a magnifying power of 7. The two lenses are 32 cm apart. Find the focal length of each lens.

1. Write expressions for $M = -f_o/f_e$ and $L = f_o + f_e$	$f_o/f_e = 7$; $f_o + f_e = 32$ cm
2. Solve for f_o and f_e	$f_o = 28$ cm; $f_e = 4$ cm

---

94 ·· A disadvantage of the astronomical telescope for terrestrial use (for example, at a football game) is that the image is inverted. A Galilean telescope uses a converging lens as its objective, but a diverging lens as its eyepiece. The image formed by the objective is behind the eyepiece at its focal point so that the final image is virtual, erect, and at infinity. (*a*) Show that the magnifying power is $M = -f_o/f_e$, where f_o is the focal length of the objective and f_e is that of the eyepiece (which is negative). (*b*) Draw a ray diagram to show that the final image is indeed virtual, erect, and at infinity.



(*b*) The ray diagram is shown in the figure.

(a) The image formed by the objective lens is at the focal point, F_1', of the objective lens and therefore subtends an angle $\theta_o = h/f_o$, where we have assumed that $\theta_o << 1$ so $\tan\theta_o \approx \theta_o$. The angle subtended by the eyepiece is h/f_e, where f_e is negative. Thus the magnification $M = -f_o/f_e$ is positive. Since the object for the eyepiece is at its focal point, the image is at infinity. As is also evident from the ray diagram, the image is virtual and erect.

---

95 ·· A Galilean telescope (see Problem 94) is designed so that the final image is at the near point, which is 25 cm (rather than at infinity). The focal length of the objective is 100 cm and that of the eyepiece is –5 cm. (*a*) If the object distance is 30 m, where is the image of the objective? (*b*) What is the object distance for the eyepiece so that the final image is at the near point? (*c*) How far apart are the lenses? (*d*) If the object height is 1.5 m, what is the height of the final image? What is the angular magnification?

(*a*) Find s_o' using Equ. 34-12	$s_o' = 103.45$ cm
(*b*) For the eyepiece, $s_e' = -25$ cm; find s_e	$s_e = -6.25$ cm (the object of the eyepiece is virtual)
(*c*) $D = s_o' + s_e$	$D = 97.2$ cm
(*d*) 1. Find m_o and m_e from Equ. 34-14; $M = m_o m_e$	$m_o = -0.0345$; $m_e = -4$; $M = 0.138$
2. $h' = Mh$	$h' = 20.7$ cm
3. Find the angle subtended by the object	$\theta_o \approx 1.5/30 = 0.05$ rad
4. Find the angle subtended by the image	$\theta_e = \tan^{-1}(20.7/6.25) = 1.28$ rad
5. Angular magnification = θ_e/θ_o	Angular magnification = 25.6

---

96 ··· A hunter lost in the mountains tries to make a telescope from two lenses of power 2.0 and 6.5 D, and a cardboard tube. (*a*) What is the maximum possible magnifying power? (*b*) How long must the tube be? (*c*) Which lens should be used as the eyepiece? Why?

(*a*) 1. Use the 6.5 D lens for the eyepiece	$M = P_e/P_o = 3.25$ with image at infinity
2. Place the virtual image at x_{np}; find s_e	$s_e = x_{np}f_e/(f_e + x_{np})$
3. Find θ_o for an object of height h	$\theta_o = h/(f_o + f_e - s_e)$
4. $M = \theta_e/\theta_o$; $\theta_e = h/s_e$	$M = [f_o + f_e - x_{np}f_e/(f_e + x_{np})]/[x_{np}f_e/(f_e + x_{np})]$
5. Evaluate M for $x_{np} = 0.25$ m	$M = 5.865$; this is the maximum magnification
(*b*) For maximum magnification $D = f_o + s_e$	$D = (50 + 9.52)$ cm = 59.5 cm

(*c*) Since $|M|$ for the image at infinity is f_o/f_e, it is evident that one should use the short focal length lens for the eyepiece.

97* ··· If you look into the wrong end of a telescope, that is, into the objective, you will see distant objects reduced in size. For a refracting telescope with an objective of focal length 2.25 m and an eyepiece of focal length 1.5 cm, by what factor is the angular size of the object reduced?

In this case, $M = -f_e/f_o$ as the role of the objective and eyepiece are reversed $M = -1.5/225 = -6.67\times10^{-3} = -1/150$

---

98 · The image of a real object formed by a convex mirror
(*a*) is always real and inverted.
(*b*) is always virtual and enlarged.
(*c*) may be real.
(*d*) is always virtual and diminished.
(*d*)

---

99 · The glass of a converging lens has an index of refraction of 1.6. When the lens is in air, its focal length is 30 cm. If immersed in water, its focal length will be
(*a*) greater than 30 cm.
(*b*) less than 30 cm.
(*c*) the same as before, 30 cm.
(*d*) negative.
(*a*)

---

100 ·· True or false:
(*a*) A virtual image cannot be displayed on a screen.
(*b*) A negative image distance implies that the image is virtual.
(*c*) All rays parallel to the axis of a spherical mirror are reflected through a single point.
(*d*) A diverging lens cannot form a real image from a real object.
(*e*) The image distance for a positive lens is always positive.
(*a*) True (*b*) True (*c*) False (*d*) True (*e*) False

101*· Show that a diverging lens can never form a real image from a real object. (*Hint:* Show that s' is always negative.)
From Equ. 34-12 it follows that $s' = sf/(s - f)$. For the diverging lens, $f < 0$ and $s > 0$ for a real object. Consequently, the denominator is positive and the numerator negative, so s' must always be negative.

---

102 · A camera uses a positive lens to focus light from an object onto a film. Unlike the eye, the camera lens has a fixed focal length, but the lens itself can be moved slightly to vary the image distance to the image on the film. A telephoto lens has a focal length of 200 mm. By how much must it move to change from focusing on an object at infinity to one at a distance of 30 m?
1. Find s' for $s = \infty$ and $s = 30$ m and Δs $s_\infty' = 200$ mm; $s_{30}' = 201.34$ mm; $\Delta s = 1.34$ mm

---

103 · A wide-angle lens of a camera has a focal length of 28 mm. By how much must it move to change from focusing on an object at infinity to one at a distance of 5 m? (See Problem 102.)
Proceed as in Problem 102 $s_\infty' = 28$ mm; $s_5' = 28.158$ mm; $\Delta s = 0.158$ mm

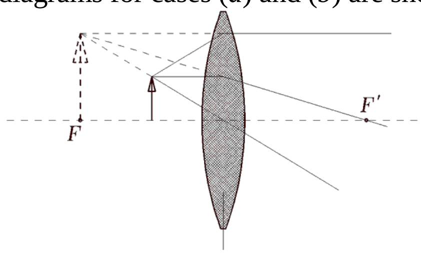
---

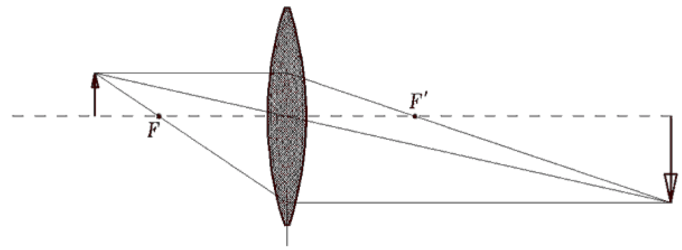
104 · A converging lens made of polystyrene (index of refraction, 1.59) has a focal length of 50 cm. One surface is convex with radius of magnitude 50 cm. Find the radius of the second surface. Is it convex or concave?
1. Use Equ. 34-11 and solve for r_2 $$r_2 = \frac{r_1 f(n-1)}{f(n-1) - r_1}$$
2. Evaluate r_2 $r_2 = -72$ cm; the surface is convex

105*· A thin converging lens of focal length 10 cm is used to obtain an image that is twice as large as a small object. Find the object and image distances if (*a*) the image is to be erect and (*b*) the image is to be inverted. Draw a ray diagram for each case.

(*a*) 1. Erect image → virtual image; $m = -s'/s$	$m = 2$; $s' = -2s$
2. Use Equ. 34-12 to find s and s'	$s = f/2 = 5$ cm; $s' = -10$ cm
(*b*) 2. Inverted image → real image	$s' = 2s$; $s = 3f/2 = 15$ cm; $s' = 30$ cm

The ray diagrams for cases (*a*) and (*b*) are shown below.





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106 ·· A scuba diver wears a diving mask with a face plate that bulges outward with a radius of curvature of 0.5 m. There is thus a convex spherical surface between the water and the air in the mask. A fish is 2.5 m in front of the diving mask. (*a*) Where does the fish appear to be? (*b*) What is the magnification of the image of the fish?

(*a*) 1. Use Equ. 34-5; solve for s'	$s' = \dfrac{n_2 r s}{(n_2 - n_1)s - n_1 r}$ 0
2. Evaluate s' for $n_1 = 1.33$, $n_2 = 1$, $s = 2.5$ m	$s' = -0.839$ m; the fish appears closer to the diver
(*b*) Use Equ. 34-6	$m = 0.446$; the fish appears to be smaller

---

107 ·· You wish to see an image of your face for applying makeup or shaving. If you want the image to be upright, virtual, and magnified 1.5 times when your face is 30 cm from the mirror, what kind of mirror should you use, convex or concave, and what should its focal length be?

The image should be upright and enlarged. Therefore one must use a concave mirror.

1. $s' = -1.5s = -45$ cm; use Equ. 34-12 to find f	$f = 90$ cm

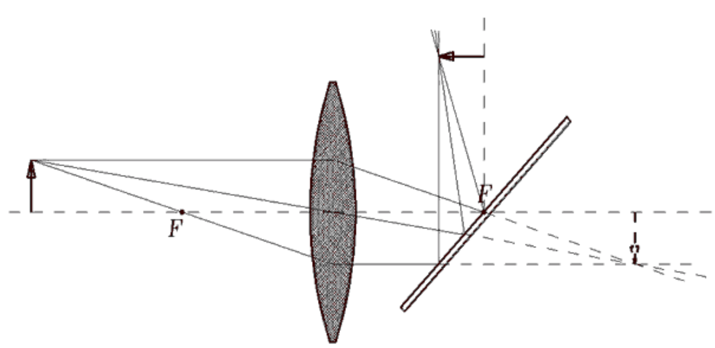
---

108 ·· A small object is 20 cm from a thin positive lens of focal length 10 cm. To the right of the lens is a plane mirror that crosses the axis at the second focal point of the lens and is tilted so that the reflected rays do not go back through the lens (Figure 34-56). (*a*) Find the position of the final image. (*b*) Is this image real or virtual? (*c*) Sketch a ray diagram showing the final image.

(*c*) The ray diagram is shown in the figure.

(*a*) From Equ. 34-12 it follows that $s' = 20$ cm. The mirror reflects the rays so that an image is formed 10 cm above the axis of the lens as shown in the ray diagram.

(*b*) The image is a real image.



109*·· A 35-mm camera has a picture size of 24 mm by 36 mm. It is used to take a picture of a person 175 cm tall so that the image just fills the height (24 mm) of the film. How far should the person stand from the camera if the focal length of the lens is 50 mm?

1. Find $m = -s'/s$ — $m = -2.4/175 = -1.37\times10^{-2}$
2. Use Equ. 34-12 to find s — $s = 0.0137s\times5/(0.0137s - 5)$; $s = 370$ cm = 3.7 m

---

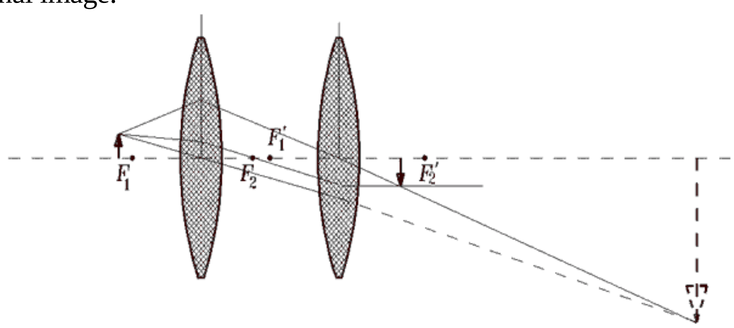
110 ·· A 35-mm camera with interchangeable lenses is used to take a picture of a hawk that has a wing span of 2 m. The hawk is 30 m away. What would be the ideal focal length of the lens used so that the image of the wings just fills the width of the film, which is 36 mm?

$s' = (36/200)s$; $s = 30$ m; use Equ. 34-12 to find f — $f = 0.018s/(1 + 0.018) = 0.530$ m = 530 mm

---

111 ·· An object is placed 12 cm to the left of a lens of focal length 10 cm. A second lens of focal length 12.5 cm is placed 20 cm to the right of the first lens. (*a*) Find the position of the final image. (*b*) What is the magnification of the image? (*c*) Sketch a ray diagram showing the final image.

(*c*) The ray diagram is shown in the figure. The enlarged, inverted image formed by the first lens serves as a virtual object for the second lens. The image formed from this virtual object is the real, inverted image shown in the ray diagram.



(*a*) 1. Use Equ. 34-12 to find s_1' — $s_1' = 60$ cm
2. Find s_2 — $s_2 = (20 - 60)$ cm = -40 cm
3. Use Equ. 34-12 to find s_2' — $s_2' = 9.52$ cm; the image is 9.52 cm from the second lens.

(*b*) $m = m_1m_2$ — $m = (-60/12)(9.52/40) = -1.19$

---

112 ·· (*a*) Show that if f is the focal length of a thin lens in air, its focal length in water is

$$f' = \frac{n_w(n - 1)}{n - n_w} f,$$

where n_w is the index of refraction of water and n is that of the lens. (*b*) Calculate the focal length in air and in water of a double-concave lens of index of refraction $n = 1.5$ that has radii of magnitude 30 and 35 cm.

In the derivation of Equ. 34-11, it was assumed that the medium outside the lens is air with index of refraction $n = 1$. Following the steps leading to Equ. 34-11, and replacing unity by n_W, Equs. 34-8 and 34-9 become

$$\frac{n_W}{s} + \frac{n}{s_1'} = \frac{n - n_W}{r_1} \text{ and } -\frac{n}{s_1'} + \frac{n_W}{s'} = \frac{n_W - n}{r_2}.$$

Adding these equations and denoting the focal length by f', one obtains $n_W\left(\frac{1}{s} + \frac{1}{s'}\right) = (n - n_W)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{n_W}{f'}$. Comparison with Equ. 34-11 yields the result cited in the problem statement.

113*·· (*a*) Find the focal length of a *thick* double-convex lens with an index of refraction of 1.5, a thickness of 4 cm, and radii of +20 cm and –20 cm. (*b*) Find the focal length of this lens in water.

(*a*) Here we must consider refraction at each surface separately, using Equ. 34-5. To find the focal length we imagine the object at $s = \infty$, find the image from the first refracting surface at s_1'. That image serves as the object for the second refracting surface. We shall find that this is a virtual image for the second refracting surface, i.e., s_2 is negative. Using Equ. 34-5 once more, we shall locate the image formed by the second refracting surface by the virtual object at s_2. The location of that image is then the focal point of the thick lens.

1. Use Equ. 34-5 to find s_1'	$s_1' = nr_1/(n - 1) = 60$ cm
2. Find $s_2 = -(s_1' - 4$ cm)	$s_2 = -56$ cm
3. Use Equ. 34-5 to find s_2'	$1/s_2' = (1.5/56 + 0.5/20)$ cm^{-1}; $s_2' = 19.3$ cm
4. f is measured from lens's center; $f = s_2' + 2$ cm	$f = 21.3$ cm

(*b*) We proceed as in part (*a*) except that now $n_1 = 1.33$ for the first refraction and $n_2 = 1.33$ for the second refraction to determine the focal length in water, which we denote by f_w.

1. Use Equ. 34-5 to find s_1' and s_2	$s_1' = 1.5\times20/(1.5 - 1.33)$ cm = 176 cm; $s_2 = -172$ cm
2. Use Equ. 34-5 to find s_2'	$s_2' = 77.3$ cm
3. $f_w = s_2' + 2$ cm	$f_w = 79.3$ cm

Note that if we use the expression given in Problem 112 we obtain $f_w = 83.3$ cm, in only moderate agreement with the exact result given above.

114 ·· A 2-cm-thick layer of water ($n = 1.33$) floats on top of a 4-cm-thick layer of carbon tetrachloride ($n = 1.46$) in a tank. How far below the top surface of the water does the bottom of the tank appear to be to an observer looking from above at normal incidence?

1. Find the depth seen from the water at CCl_4-H_2O interface; use Equ. 34-7	$s' = (1.33/1.46)(4$ cm$) = -3.64$ cm
2. Find depth from water at water-air interface	$s = (2 + 3.64)$ cm = 5.64 cm
3. Find depth as seen from air; use Equ. 34-7	$s' = -(1/1.33)(5.64$ cm$) = -4.24$ cm

Apparent depth = 4.24 cm

115 ·· While sitting in your car, you see a jogger in your side mirror, which is convex with a radius of curvature of magnitude 2 m. The jogger is 5 m from the mirror and is approaching at 3.5 m/s. How fast does the jogger appear to be running when viewed in the mirror?

The speed of the jogger as seen in the mirror is $v' = ds'/dt$. We shall derive an expression for v' in terms of f and $v = ds/dt$. From Equ. 34-12, $s' = (1/f - 1/s)^{-1}$; differentiating we find $ds'/dt = -(1/f - 1/s)^{-2}(1/s^2)ds/dt = -(s'/s)^2v$.

1. Find s' when $s = 5$ m	$s' = -0.833$ m
2. Find $\lvert v'\rvert$ when $\lvert v\rvert = 3.5$ m/s	$v' = 0.0972$ m/s

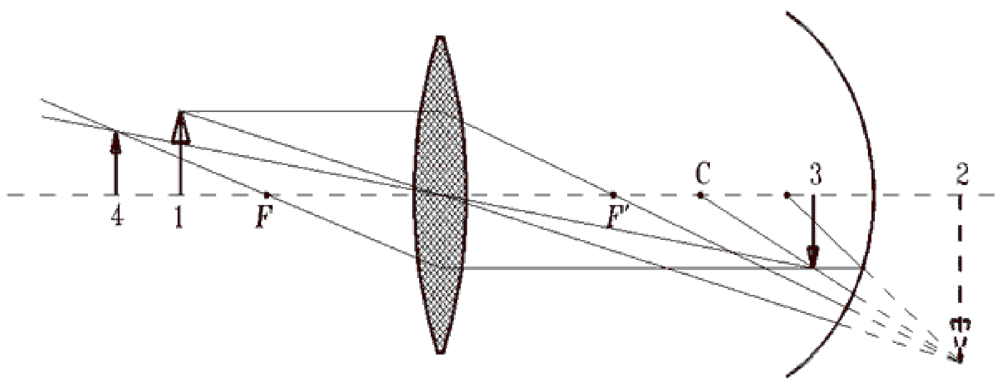
116 ·· In the seventeenth century, Antonie van Leeuwenhoek, the first great microscopist, used simple spherical lenses made first of water droplets and then of glass for his first instruments. He made staggering discoveries with these simple lenses. Consider a glass sphere of radius 2.0 mm with an index of refraction of 1.50. Find the focal length of this lens. *Hint:* Use the equation for refraction at a single spherical surface to find the image distance for an infinite object distance for the first surface. Then use this image point as the object point for the second surface.

1. Use Equ. 34-8 to find s_1' for $s_1 = \infty$ and $r_1 = r$ — $s_1' = 3r = 6$ mm
2. Find $s_2' = f$ with $s_2 = -r$ and $r_2 = -r$ — $s_2' = r/2 = 1$ mm; $f = 1$ mm

---

117*··· An object is 15 cm to the left of a thin convex lens of focal length 10 cm. A concave mirror of radius 10 cm is 25 cm to the right of the lens. (*a*) Find the position of the final image formed by the mirror and lens. (*b*) Is the image real or virtual? Erect or inverted? (*c*) Show on a diagram where your eye must be to see this image.

We begin with parts (*b*) and (*c*), the ray diagram for this situation. This is shown below. **1** represents the object. Two rays from **1** are shown; one passes through the center of the lens, the other is paraxial and then passes through the focal point F'. The two rays intersect behind the mirror, and the image formed there, identified as **2**, serves as a virtual object for the mirror. Two rays are shown emanating from this virtual image, one through the center of the mirror, the other passing through its focal point (halfway between C and the mirror surface) and then continuing as a paraxial ray. These two rays intersect in front of the mirror, forming a real image, identified as **3**. Finally, the image **3** serves as a real object for the lens; again we show two rays, a paraxial ray that then passes through the focal point F and a ray through the center of the lens. These two rays intersect to form the final real, erect, and diminished image, identified as **4**. To see this image the eye must be to the left of the image **4**.



(*a*) 1. Use Equ. 34-12 to find the location of **2** — $s' = 30$ cm; thus **2** is 5 cm behind the mirror
2. Use Equ. 34-3 to locate **3**, where $s = -5$ cm — $s' = 2.5$ cm; the image is 22.5 cm from the lens
3. Use Equ. 34-12 to locate **4**, where $s = 22.5$ cm — $s' = 18$ cm; **4** is 18 cm to the left of the lens

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118 ··· Find the final image for the situation in Problem 108 when the mirror is not tilted. Assume that the image is viewed by an eye to the left of the object looking through the lens into the mirror.

If the mirror is normal to the axis of the lens, the image formed by the mirror is at the center of the lens.

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119 ··· When a bright light source is placed 30 cm in front of a lens, there is an erect image 7.5 cm from the lens. There is also a faint inverted image 6 cm in front of the lens due to reflection from the front surface of the lens. When the lens is turned around, this weaker, inverted image is 10 cm in front of the lens. Find the index of refraction of the lens.

The mirror surfaces must be concave to create inverted images on reflection. Therefore, the lens is a diverging lens.

1. Find $|r_1|$ using Equ. 34-1 — $|r_1| = 10$ cm
2. Find $|r_2|$ using Equ. 34-1 — $|r_2| = 15$ cm
3. Find f using Equ. 34-12; $s' = -7.5$ cm — $f = -10$ cm
4. Find n using Equ. 34-11; $r_1 = -10$ cm, $r_2 = 15$ cm — $n = 1.6$

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120 ··· A horizontal concave mirror with radius of curvature of 50 cm holds a layer of water with an index of refraction of 1.33 and a maximum depth of 1 cm. At what height above the mirror must an object be placed so that its image is at the same position as the object?

We will asume that the object is very small compared to r so that all incident and reflected rays traverse 1 cm of water. The problem involves two refractions at the air-water interface and one reflection at the mirror. Let s_1 be the object distance, and s_1' be the image distance of the image due to the air-water refraction; let s_2 be the distance of that image to the mirror surface, and s_2' the image distance of the image formed by the mirror; let s_3 be the distance of that image to the water-air interface and s_3' be the image distance of the image formed due to refraction at that interface. We then require that $s_3' = s_1$.

1. Find s_1' due to air-water refraction; use Equ. 34-7	$s_1' = -ns_1$
2. Find the object distance for the mirror	$s_2 = 1 - s_1' = 1 + ns_1$
3. Find s_2'; use Equ. 34-1	$s_2' = (2/r - 1/s_2)^{-1} = [2/r - 1/(1 + ns_1)]^{-1}$
4. Find distance of s_2' from air-water interface	$s_3 = 1 - s_2' = 1 - [2/r - 1/(1 + ns_1)]^{-1}$
5. Find final image distance s_3' using Equ. 34-7	$s_3' = -s_3/n$
6. Set $s_3' = s_1$ and solve for s_1	$s_1 = n - n\times[2/r - 1/(1 + ns_1)]^{-1}$; $s_1^2 + (2 - r)s_1/n + (1 - \mathrm{r})/n^2 = 0$
7. Find the positive root of the quadratic equation for r	$s_1 = 36.8$ cm

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121* ··· A lens with one concave side with a radius of magnitude 17 cm and one convex side with a radius of magnitude 8 cm has a focal length in air of 27.5 cm. When placed in a liquid with an unknown index of refraction, the focal length increases to 109 cm. What is the index of refraction of the liquid?

From Equ. 34-11, $n = \dfrac{r_1 r_2}{f\,(r_2 - r_1)} + 1$; find n $\qquad n = 1.55$

From the result of Problem 112, $n_{\mathrm{L}} = \dfrac{f_{\mathrm{L}}\, n}{f_{\mathrm{L}} + f\,(n-1)}$ $\qquad n_{\mathrm{L}} = 1.36$

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122 ··· A glass ball of radius 10 cm has an index of refraction of 1.5. The back half of the ball is silvered so that it acts as a concave mirror (Figure 34-57). Find the position of the final image seen by an eye to the left of the object and ball for an object at (*a*) 30 cm and (*b*) 20 cm to the left of the front surface of the ball.

The problem involves two refractions and one reflection. We shall use the same notation as in Problem 120.

(*a*) 1. Find s_1' using Equ. 34-5; $n_1 = 1$, $n_2 = 1.5$	$s_1' = 90$ cm
2. Find s_2	$s_2 = (20 - 90)$ cm $= -70$ cm
3. Find s_2' using Equ. 34-1	$s_2' = 4.67$ cm
4. Find s_3	$s_3 = (20 - 4.67)$ cm $= 15.33$ cm
5. Find s_3' using Equ. 34.5; $n_1 = 1.5$, $n_2 = 1$	$r = -10$ cm; $s_3' = -20.9$ cm
6. Locate the image	The image is 0.9 cm behind the mirror surface
(*b*) Proceed as in part (*a*), with $s_1 = 20$ cm	$s_3 = -20$ cm; the image is at the mirror surface

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123 ··· (*a*) Show that a small change dn in the index of refraction of a lens material produces a small change in the focal length df given approximately by $df/f = -dn/(n-1)$. (*b*) Use this result to find the focal length of a thin lens for blue light, for which $n = 1.53$, if the focal length for red light, for which $n = 1.47$, is 20 cm.

(*a*) From Equ. 34-11 $f = C(n-1)^{-1}$, where C is a constant that depends on r_1 and r_2. Thus $df/dn = -C/(n-1)^2 = -f/(n-1)$ and $df/f = -dn/(n-1)$.

(*b*) 1. Find Δf using the result of (*a*) $\Delta f = -(20\text{ cm})[0.06/0.47] = -2.55$ cm

2. Find $f_{blue} = f_{red} + \Delta f$ $f_{blue} = 17.45$ cm

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124 ··· The lateral magnification of a spherical mirror or a thin lens is given by $m = -s'/s$. Show that for objects of small horizontal extent, the longitudinal magnification is approximately $-m^2$. (*Hint:* Show that $ds'/ds = s'^2/s^2$.)

We examine the amount by which the image distance s' changes due to a change in s. From Equ. 34-12, $s' = (1/f - 1/s)^{-1}$. Now take the derivative of s' with respect to s. We find

$$\frac{ds'}{ds} = -\frac{1}{\left(\frac{1}{f} - \frac{1}{s}\right)^2}\frac{1}{s^2} = -\frac{s'^2}{s^2} = -m^2$$

. So the image of an object of length Δs will have a length $-m^2\Delta s$.

Note: Problem 125 has been deleted in all but the first printing.