CHAPTER 33

Properties of Light

1* · Why is helium needed in a helium–neon laser? Why not just use neon?

The population inversion between the state $E_{2,\mathrm{Ne}}$ and the state 1.96 eV below it (see Figure 33-9) is achieved by inelastic collisions between neon atoms and helium atoms excited to the state $E_{2,\mathrm{He}}$.

2 ·· When a beam of visible white light passes through a gas of atomic hydrogen and is viewed with a spectroscope, dark lines are observed at the wavelengths of the emission series. The atoms that participate in the resonance absorption then emit this same wavelength light as they return to the ground state. Explain why the observed spectrum nevertheless exhibits pronounced dark lines.

Although the excited atoms emit the light of the same frequency on returning to the ground state, the light is emitted in a random direction, not exclusively in the direction of the incident beam. Consequently, the beam intensity is greatly diminished.

3 · A pulse from a ruby laser has an average power of 10 MW and lasts 1.5 ns. (*a*) What is the total energy of the pulse? (*b*) How many photons are emitted in this pulse?

(*a*) $E = P\Delta t$ $\quad$ $E = 15$ mJ

(*b*) $E_{\mathrm{photon}} = hc/\lambda$ (Equ. 17.1); $N = E\lambda/hc$ $\quad$ $N = 5.24\times10^{16}$

4 · A helium–neon laser emits light of wavelength 632.8 nm and has a power output of 4 mW. How many photons are emitted per second by this laser?

$E_{\mathrm{photon}} = hc/\lambda$; $n = P\lambda/hc$ $\quad$ $n = 1.27\times10^{16}\ \mathrm{s}^{-1}$

5* · The first excited state of an atom of a gas is 2.85 eV above the ground state. (*a*) What is the wavelength of radiation for resonance absorption? (*b*) If the gas is irradiated with monochromatic light of 320 nm wavelength, what is the wavelength of the Raman scattered light?

(*a*) Use Equ. 33-2 $\quad$ $\lambda = 1240/2.85$ nm $= 435$ nm

(*b*) $E_{\mathrm{Raman}} = E_{\mathrm{inc}} - \Delta E$; $\lambda_{\mathrm{Raman}} = 1240/E_{\mathrm{Raman}}$ $\quad$ $E_{\mathrm{Raman}} = (1240/320 - 2.85)$ eV $= 1.025$ eV; $\lambda_{\mathrm{Raman}} = 1210$ nm

6 ·· A gas is irradiated with monochromatic ultraviolet light of 368 nm wavelength. Scattered light of the same wavelength and of 658 nm wavelength is observed. Assuming that the gas atoms were in their ground state prior to irradiation, find the energy difference between the ground state and the atomic state excited by the irradiation.

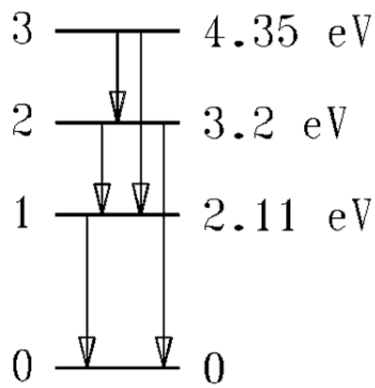
$\Delta E = hc/\lambda$; use Equs. 17-1 and 17-5 $\quad$ $\Delta E = 1240/368$ eV $= 3.37$ eV

7 ·· Sodium has excited states 2.11 eV, 3.2 eV, and 4.35 eV above the ground state. (*a*) What is the maximum wavelength of radiation that will result in resonance fluorescence? What is the wavelength of the fluorescent radiation? (*b*) What wavelength will result in excitation of the state 4.35 eV above the ground state? If that state is excited, what are the possible wavelengths of resonance fluorescence that might be observed?

(*a*) Excite to 3.2 eV level; decay to 2.11 eV level and ground state; use Equs. 17-1 and 17-5

For excitation, λ_{max} = (1240/3.2) nm = 387.5 nm; fluorescence wavelengths: λ = 1138 nm and 587.7 nm

(b) See the energy level diagram below; $\lambda = hc/\Delta E$



For excitation, λ = 1240/4.35 nm = 285 nm

The fluorescent wavelengths are as follows:

λ_{32} = 1078 nm

λ_{21} = 1138 nm

λ_{10} = 587.7 nm

λ_{31} = 553.6 nm

λ_{20} = 387.5 nm

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8 ·· Singly ionized helium is a hydrogen-like atom with a nuclear charge of 2*e*. Its energy levels are given by $E_n = -4E_0/n^2$, where E_0 = 13.6 eV. If a beam of visible white light is sent through a gas of singly ionized helium, at what wavelengths will dark lines be found in the spectrum of the transmitted radiation?

The energy difference between the ground state and the first excited state is $3E_0$ = 40.8 eV, corresponding to a wavelength of 30.4 nm. This is in the far uv, well outside the visible range of wavelengths. There will be no dark lines in the transmitted radiation.

9* · Estimate the time required for light to make the round trip in Galileo's experiment to determine the speed of light.

$\Delta t = D/c$; D = 6 km (see Problem 14)

$\Delta t = 6\times10^3/3\times10^8$ s = 20 μs

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10 · Mission Control sends a brief wake-up call to astronauts in a far away spaceship. Five seconds after the call is sent, Mission Control can hear the groans of the astronauts. How far away (at most) from the earth is the spaceship? (*a*) 7.5×10^8 m (*b*) 15×10^8 m (*c*) 30×10^8 m (*d*) 45×10^8 m (*e*) The spaceship is on the moon.

(*a*)

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11 · The spiral galaxy in the Andromeda constellation is about 2×10^{19} km away from us. How many light-years is this?

1 *c*·y = 9.46×10^{15} m (see p. EP-3)

D = 2.11 *c*·y

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12 · On a spacecraft sent to Mars to take pictures, the camera is triggered by radio waves, which like all electromagnetic waves travel with the speed of light. What is the time delay between sending the signal from the earth and receiving it on Mars? (Take the distance to Mars to be 9.7×10^{10} m.)

$\Delta t = D/c$

Δt = 323 s = 5 min 23 s

13*· The distance from a point on the surface of the earth to one on the surface of the moon is measured by aiming a laser light beam at a reflector on the surface of the moon and measuring the time required for the light to make a round trip. The uncertainty in the measured distance Δx is related to the uncertainty in the time Δt by $\Delta x = c\,\Delta t$. If the time intervals can be measured to ±1.0 ns, find the uncertainty of the distance in meters.

$\Delta x = \pm c\,\Delta t$ $\Delta x = \pm 3\times 10^{8}\times 10^{-9}$ m = ±30 cm

14 ·· In Galileo's attempt to determine the speed of light, he and his assistant were located on hilltops about 3 km apart. Galileo flashed a light and received a return flash from his assistant. (*a*) If his assistant had an instant reaction, what time difference would Galileo need to be able to measure for this method to be successful? (*b*) How does this time compare with human reaction time, which is about 0.2 s?

(*a*) $\Delta t = D/c$ $\Delta t = (2\times 3\times 10^{3}/c)$ s = 20 μs

(*b*) The human reaction time is about $10{,}000\times\Delta t$.

15 · How does a thin layer of water on the road affect the light you see reflected off the road from your own headlights? How does it affect the light you see reflected from the headlights of an oncoming car?

The layer of water greatly reduces the light reflected back from the car's headlights, but increases the light reflected by the road of light from the headlights of oncoming cars.

16 · A ray of light passes from air into water, striking the surface of the water with an angle of incidence of 45°. Which of the following four quantities change as the light enters the water: (1) wavelength, (2) frequency, (3) speed of propagation, (4) direction of propagation. (*a*) 1 and 2 only (*b*) 2, 3, and 4 only (*c*) 1, 3, and 4 only (*d*) 3 and 4 only (*e*) 1, 2, 3, and 4

(*c*)

17*·· The density of the atmosphere decreases with height, as does the index of refraction. Explain how one can see the sun after it has set. Why does the setting sun appear flattened?

The change in atmospheric density results in refraction of the light from the sun, bending it toward the earth. Consequently, the sun can be seen even after it is just below the horizon. Also, the light from lower portion of the sun is refracted more than that from the upper portion, so the lower part appears to be slightly higher in the sky. The effect is an apparent flattening of the disk into an ellipse.

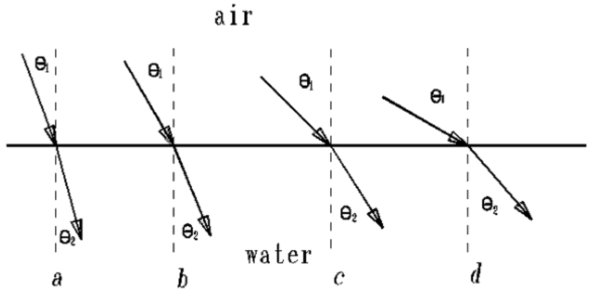
18 · Calculate the fraction of light energy reflected from an air–water interface at normal incidence.

Use Equ. 33-11; $n_1 = 1$, $n_2 = 1.33$ $I/I_0 = 0.02 = 2\%$

19 · Find the angle of refraction of a beam of light in air that hits a water surface at an angle of incidence of (*a*) 20°, (*b*) 30°, (*c*) 45°, and (*d*) 60°. Show these rays on a diagram.

(*a*), (*b*), (*c*), (*d*) Use Equ. 33-9*b* (*a*) $\theta_2 = 14.9°$ (*b*) $\theta_2 = 22.1°$ (*c*) $\theta_2 = 32.1°$

The rays are shown below. (*d*) $\theta_2 = 40.6°$



20 · Repeat Problem 18 for a beam of light initially in water that is incident on a water–air interface.

From Equ. 33-11 it follows that I/I_0 is the same as in Problem 18; i.e., $I/I_0 = 2.0\%$

21* · Find the speed of light in water and in glass.

Use Equ. 33-7 $\quad v_{\text{water}} = c/1.333 = 2.25\times10^8$ m/s; $v_{\text{glass}} = c/1.5 = 2\times10^8$ m/s

22 · The index of refraction for silicate flint glass is 1.66 for light with a wavelength of 400 nm and 1.61 for light with a wavelength of 700 nm. Find the angles of refraction for light of these wavelengths that is incident on this glass at an angle of 45°.

Use Equ. 33-9*b* $\quad$ For $\lambda = 400$ nm, $\theta_2 = 25.2°$; for $\lambda = 700$ nm, $\theta_2 = 26.1°$

23 ·· A slab of glass with an index of refraction of 1.5 is submerged in water with an index of refraction of 1.33. Light in the water is incident on the glass. Find the angle of refraction if the angle of incidence is (*a*) 60°, (*b*) 45°, and (*c*) 30°.

(*a*), (*b*), (*c*) Use Equ. 33-9*b*; $n_1 = 1.33$, $n_2 = 1.5$ $\quad$ (*a*) $\theta_2 = 50.2°$ (*b*) $\theta_2 = 38.8°$ (*c*) $\theta_2 = 26.3°$

24 ·· Repeat Problem 23 for a beam of light initially in the glass that is incident on the glass–water interface at the same angles.

(*a*), (*b*), (*c*) Use Equ. 33-9*b*; $n_1 = 1.5$, $n_2 = 1.33$ $\quad$ (*a*) $\theta_2 = 77.6°$ (*b*) $\theta_2 = 52.9°$ (*c*) $\theta_2 = 34.3°$

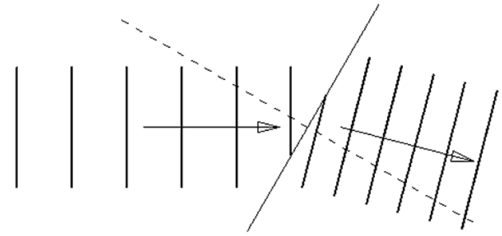
25* ·· Light is incident normally on a slab of glass with an index of refraction $n = 1.5$. Reflection occurs at both surfaces of the slab. About what percentage of the incident light energy is transmitted by the slab?

We shall neglect multiple reflections at glass-air interfaces.

1. Use Equ. 33-11 to find I in glass $\quad I_{\text{glass}} = I_0(1 - 0.5^2/2.5^2) = 0.96I_0$
2. Use Equ. 33-11 to find I transmitted $\quad I_{\text{transm}} = I_{\text{glass}}(1 - 0.5^2/2.5^2) = 0.96^2I_0 = 92.2\%\ I_0$

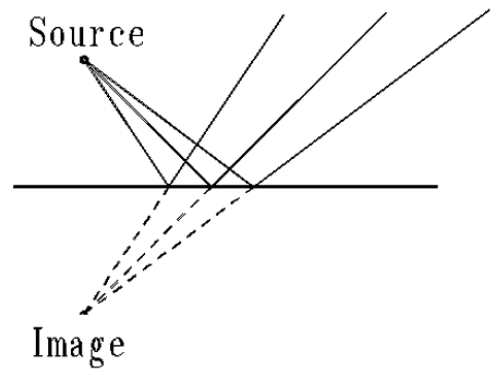
26 ·· This problem is a refraction analogy. A band is marching down a football field with a constant speed v_1. About midfield, the band comes to a section of muddy ground that has a sharp boundary making an angle of 30° with the 50-yd line as shown in Figure 33-50. In the mud, the marchers move with speed $v_2 = \frac{1}{2}0v_1$.

Diagram how each line of marchers is bent as it encounters the muddy section of the field so that the band is eventually marching in a different direction. Indicate the original direction by a ray, the final direction by a second ray, and find the angles between the rays and the line perpendicular to the boundary. Is their direction of motion bent toward the perpendicular to the boundary or away from it? As the line enters the muddy field, its speed is reduced by half and the direction of the forward motion of the line is changed. In this case, the forward motion in the muddy field makes an angle of 14.5° with respect to the normal of the boundary line. Note that the separation between successive lines in the muddy field is half that in the dry field.



27 ·· A point source of light is 5 cm above a plane reflecting surface (such as a mirror). Draw a ray from the source that strikes the surface at an angle of incidence of 45° and two more rays that strike the surface at angles slightly less than 45°, and draw the reflected ray for each. The reflected rays appear to diverge from a point called the image of the light source. Draw dotted lines extending the reflected rays back until they meet at a point behind the surface to locate the image point.

For each ray shown in the adjacent figure, the angle of incidence equals the angle of reflection at the mirror surface. When the reflected rays are extended into the region behind the mirror they converge at a single point, the image point. Note that the distance between the source and mirror is equal to the distance between the image and the mirror.



28 ·· In Figure 33-51, light is initially in a medium (such as air) of index of refraction n_1. It is incident at angle θ_1 on the surface of a liquid (such as water) of index of refraction n_2. The light passes through the layer of water and enters glass of index of refraction n_3. If θ_3 is the angle of refraction in the glass, show that $n_1 \sin \theta_1 = n_3 \sin \theta_3$. That is, show that the second medium can be neglected when finding the angle of refraction in the third medium.

We apply Snell's law consecutively, first to the n_1-n_2 interface and then to the n_2-n_3 interface. This gives $n_1 \sin \theta_1 = n_2 \sin \theta_2$ and $n_2 \sin \theta_2 = n_3 \sin \theta_3$, or $n_1 \sin \theta_1 = n_3 \sin \theta_3$.

29* ··· Figure 33-52 shows a beam of light incident on a glass plate of thickness d and index of refraction n. (*a*) Find the angle of incidence such that the perpendicular separation between the ray reflected from the top surface and that reflected from the bottom surface and exiting the top surface is a maximum. (*b*) What is this angle of incidence if the index of refraction of the glass is 1.60? What is the separation of the two beams if the thickness of the glass plate is 4.0 cm?

(*a*) Let x be the perpendicular separation between the two rays. The separation between the points of emergence of the two rays on the glass surface is $2d \tan \theta_r$, where θ_r is the angle of refraction. Then $x = 2d \tan \theta_r \cos \theta_i$, where θ_i is the angle of incidence. To find θ_i for maximum x we differentiate x with respect to θ_i and set the derivative equal to zero. $dx/d\theta_i = 2d[-\tan \theta_r \sin \theta_i + \sec^2 \theta_r \cos \theta_i (d\theta_r/d\theta_i)]$. From Snell's law, $\cos \theta_i\, d\theta_i = n \cos \theta_r\, d\theta_r$, or $d\theta_r/d\theta_i = (1/n)(\cos \theta_i / \cos \theta_r)$. Thus, $dx/d\theta_i = 2d[(1/n)(\cos^2 \theta_i / \cos^3 \theta_r) - (\sin \theta_i \sin \theta_r)/\cos \theta_r]$.

Using Snell's law and $\sin^2\theta + \cos^2\theta = 1$, and factoring out $1/(n^3\cos^3\theta_r)$, one obtains $dx/d\theta_i = (2d/n^3\cos^3\theta_r)(\sin^4\theta_i - 2n^2\sin^2\theta_i + n^2)$. Setting the quantity in the second parenthesis equal to zero and solving for $\sin\theta_i$ one obtains the result

$$\sin\theta_i = n\sqrt{1-\sqrt{1-\frac{1}{n^2}}} \quad \text{or } \theta_i = \sin^{-1}\left(n\sqrt{1-\sqrt{1-\frac{1}{n^2}}}\right).$$

(*b*) 1. Substitute $n = 1.60$ into the above expression $\theta_i = 48.5°$

2. Find θ_r using Snell's law $\theta_r = \sin^{-1}[(\sin 48.5°)/1.6] = 27.9°$

3. $x = 2d\tan\theta_r\cos\theta_i$ $x = 2.81$ cm

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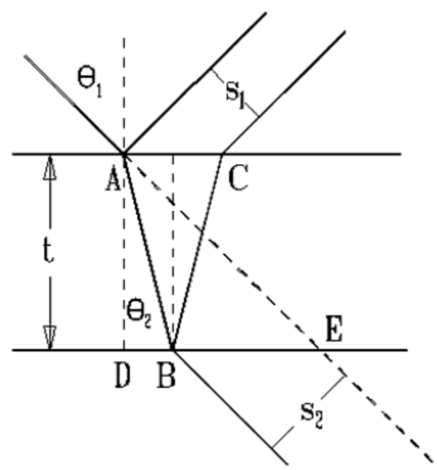
30 ··· Consider the situation shown in Figure 33-53. The index of refraction of the glass plate is *n*. Find the angle of incidence such that the perpendicular separation between the two beams emerging from the top surface is the same as the perpendicular displacement of the beam emerging from the bottom surface from the incident beam.

The situation is shown in the adjacent figure. The distance *AC* is $2t\tan\theta_2$ and the separation between the reflected rays is $s_1 = 2t\tan\theta_2\cos\theta_1$. The distance *DE* is $t\tan\theta_1$ and $DB = t\tan\theta_2$. So $BE = t(\tan\theta_1 - \tan\theta_2)$ and $s_2 = t(\tan\theta_1 - \tan\theta_2)\cos\theta_1$. It follows that $s_1 = s_2$ if $2\tan\theta_2 = \tan\theta_1 - \tan\theta_2$ or $3\tan\theta_2 = \tan\theta_1$. Using trigonometric identities, squaring both sides, and recalling that $\sin\theta_2 = (1/n)\sin\theta_1$, one arrives at the result



$$\theta_1 = \sin^{-1}\sqrt{\frac{9-n^2}{8}}$$. For glass of index of refraction $n = 1.5$, the angle of incidence, $\theta_1 = 66.7°$.

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31 · A physics student playing pocket billiards wants to strike her cue ball such that it hits a cushion and then hits the eight ball squarely. She chooses several points on the cushion and for each point measures the distance from it to the cue ball and to the eight ball. She aims at the point for which the sum of these distances is least. (*a*) Will her cue ball hit the eight ball? (*b*) How is her method related to Fermat's principle?

(*a*) Yes (*b*) Her procedure is equivalent to Fermat's principle. Since the ball presumably travels at constant speed, least time of travel equals shortest distance of travel.

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32 · A swimmer at *S* in Figure 33-54 develops a leg cramp while swimming near the shore of a calm lake and calls for help. A lifeguard at *L* hears the call. The lifeguard can run 9 m/s and swim 3 m/s. He knows physics and chooses a path that will take the least time to reach the swimmer. Which of the paths shown in Figure 33-54 does he take?

He takes the path *LES* because the time required to reach the swimmer is the least for this path.

33* · What is the critical angle for total internal reflection for light traveling initially in water that is incident on a water–air interface?

Use Equ. 33-12 $\theta_c = \sin^{-1}(1/1.333) = 48.6°$

34 ·· A glass surface (n = 1.50) has a layer of water (n = 1.33) on it. Light in the glass is incident on the glass–water interface. Find the critical angle for total internal reflection.

Use Equ. 33-12; $n_1 = 1.5$, $n_2 = 1.33$ $\theta_c = \sin^{-1}(1.33/1.5) = 62.5°$

35 ·· A point source of light is located 5 m below the surface of a large pool of water. Find the area of the largest circle on the pool's surface through which light coming directly from the source can emerge.

1. Find θ_c using Equ. 33-12 and $R = (5 \tan \theta_c)$ m $\theta_c = 48.8°$; $R = 5.7$ m
2. Find $A = \pi R^2$ $A = 102\ \text{m}^2$

36 ·· Light is incident normally on the largest face of an isosceles-right-triangle prism. What is the speed of light in this prism if the prism is just barely able to produce total internal reflection?

Here $\theta_c = 45°$; use Equ. 33-12 to find n and v $n = 1/\sin 45° = 1.414$; $v = 2.12\times10^8$ m/s

37* ·· A point source of light is located at the bottom of a steel tank, and an opaque circular card of radius 6.0 cm is placed over it. A transparent fluid is gently added to the tank such that the card floats on the surface with its center directly above the light source. No light is seen by an observer above the surface until the fluid is 5 cm deep. What is the index of refraction of the fluid?

1. Determine θ_c from the geometry $\theta_c = \tan^{-1}(6/5) = 50.2°$
2. Use Equ. 33-12 to find n $n = 1/\sin 50.2° = 1.30$

38 ·· A grain of sand is directly below center of the base of a cube of transparent material. The grain of sand is visible when viewed through the top surface but cannot be seen when looking into any of the sides of the cube. What is the minimum index of refraction of the material of which the cube is made?

Consider a ray from the sand at glancing incidence on the bottom face. The angle of refraction is then given by $\sin \theta = 1/n$. The ray then strikes the side of the cube at an angle of incidence $\theta_i = 90° - \theta$. If $\sin \theta_i \geq 1/n$ the ray will suffer total internal reflection. Using the trigonometric identity $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$ one obtains the condition $n \geq \sqrt{2}$.

Note: In the first printing of the textbook, the problem statement reads, "A grain of sand is embedded at the center of the base of a cube . . . ." In that case, the grain of sand can always be seen from the sides of the cube whatever the index of refraction. There is no minimum index of refraction for the cube.

39 ··· Light is incident normally upon one face of a prism of glass with an index of refraction n (Figure 33-55). The light is totally reflected at the right side. (*a*) What is the minimum value n can have? (*b*) When the prism is immersed in a liquid whose index of refraction is 1.15, there is still total reflection, but when it is immersed in water whose index of refraction is 1.33, there is no longer total reflection. Use this information to establish limits for possible values of n.

(*a*) See Problem 36 $n_{min} = 1.414$

(*b*) 1. Find n_{min} in liquid of n = 1.15 using Equ. 33-12 $n_{min} = 1.15/\sin 45° = 1.63$

2. Find n_{max} in water $n_{max} = 1.88$; $1.63 < n < 1.88$

40 ··· Investigate how a thin film of water on a glass surface affects the critical angle for total reflection. Take n =

1.5 for glass and n = 1.33 for water. (*a*) What is the critical angle for total internal reflection at the glass–water interface? (*b*) Is there any range of incident angles that are greater than θ_c for glass-to-air refraction, and for which light rays will leave the glass and the water and pass into the air?

(*a*) See Problem 34	$\theta_c = 62.5°$
(*b*) 1. Find θ_c for glass-to-air interface	$\theta_c = 41.8°$
2. Consider a ray at $\theta_1 = 41.8°$ at the glass-water interface. Find θ_2 and compare to θ_c for water-air interface (see Problem 35).	1.5 sin 41.8° = 1.33 sin θ_2; $\theta_2 = 48.8°$ Not that $\theta_2 = \theta_c$ for water-air interface. Therefore the ray will not leave the water for $\theta_1 \geq 41.8°$
3. Find θ in air for θ_i = 48.75	No; there are no such angles

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41* ··· A laser beam is incident on a plate of glass of thickness 3 cm. The glass has an index of refraction of 1.5 and the angle of incidence is 40°. The top and bottom surfaces of the glass are parallel and both produce reflected beams of nearly the same intensity. What is the perpendicular distance *d* between the two adjacent reflected beams?

1. Use Snell's law to find θ_r	$\theta_r = \sin^{-1}[(\sin 40°)/1.5] = 25.37°$
2. $x = 2d \tan \theta_r \cos \theta_i$ (see Problem 29); find θ_r	x = 2.18 cm

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42 ··· Figure 33-56 shows a glass prism of index of refraction n = 1.52 in the shape of an isosceles triangle with base angles of 45°. (*a*) Find the maximum angle of incidence of the beam incident on the side face so that it suffers total internal reflection at the base. (*b*) What is the maximum value of the index of refraction of the prism so that the light beam will suffer total internal reflection at the base whatever the angle of incidence?

Let θ_1 be the angle of incidence on the side face, θ_2 be the angle of refraction at the side face, and θ_3 be the angle of incidence at the base. It follows from the geometry of the system that $\theta_2 + \theta_3 = 45°$.

(*a*) 1. Find $\theta_3 = \theta_c$ and θ_2	$\theta_c = 41.14°$; $\theta_2 = 3.86°$
2. Find θ_1 using Equ. 33-9*b*	$\theta_1 = \theta_{1,max} = 5.87°$
(*b*) 1. Set $\theta_1 = 90°$ and find θ_2 and θ_3	$\theta_2 = \sin^{-1}(1/n)$; $\theta_3 = [45° - \sin^{-1}(1/n)]$
2. Use Equ. 33-12	$\sin \theta_3 = 1/n = \sin [45° - \sin^{-1}(1/n)]$
3. Solve for $n = n_{max}$	$2 \sin^{-1}(1/n_{max}) = 45°$; $n_{max} = 2.61$

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43 ·· A beam of light strikes the plane surface of silicate flint glass at an angle of incidence of 45°. The index of refraction of the glass varies with wavelength as shown in the graph in Figure 33-26. How much smaller is the angle of refraction for violet light of wavelength 400 nm than that for red light of wavelength 700 nm?

Use Equ. 33-9*b* and Fig. 33-26	$\theta_{red} = 26.1°$; $\theta_{violet} = 25.1°$; $\Delta\theta = 1.0°$

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44 ·· Repeat Problem 43 for quartz.

Use Equ. 33-9*b* and Fig. 33-26	$\theta_{red} = 27.33°$; $\theta_{violet} = 26.95°$; $\Delta\theta = 0.38°$

45*· Two polarizers have their transmission axes at an angle θ. Unpolarized light of intensity I is incident upon the first polarizer. What is the intensity of the light transmitted by the second polarizer? (*a*) $I \cos^2 \theta$ (*b*) $(I \cos^2 \theta)/2$ (*c*) $(I \cos^2 \theta)/4$ (*d*) $I \cos \theta$ (*e*) $(I \cos \theta)/4$ (*f*) None of the above.

(*b*)

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46 · Which of the following is *not* a phenomenon whereby polarized light can be produced from unpolarized light? (*a*) absorption (*b*) reflection (*c*) birefringence (*d*) diffraction (*e*) scattering

(*d*)

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47 · What is the polarizing angle for (*a*) water with $n = 1.33$ and (*b*) glass with $n = 1.5$?

Use Equ. 33-21 (*a*) $\theta_p = 53.1°$ (*b*) $\theta_p = 56.3°$

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48 · Light known to be polarized in the horizontal direction is incident on a polarizing sheet. It is observed that only 15.0% of the intensity of the incident light is transmitted through the sheet. What angle does the transmission axis of the sheet make with the horizontal ? (*a*) 8.6° (*b*) 21° (*c*) 23° (*d*) 67° (*e*) 81°

(*d*)

49*· Two polarizing sheets have their transmission axes crossed so that no light gets through. A third sheet is inserted between the first two such that its transmission axis makes an angle θ with that of the first sheet. Unpolarized light of intensity I_0 is incident on the first sheet. Find the intensity of the light transmitted through all three sheets if (*a*) $\theta = 45°$ and (*b*) $\theta = 30°$.

Let I_n be the intensity after the *n*'th polarizing sheet.

(*a*) Find I_1, I_2, and I_3; use Equ. 33-20 for I_2 and I_3 $I_1 = I_0/2$; $I_2 = I_1/2 = I_0/4$; $I_3 = I_2/2 = I_0/8$

(*b*) Repeat with $\theta_{1,2} = 30°$, $\theta_{2,3} = 60°$ $I_1 = I_0/2$; $I_2 = 3I_1/4 = 3I_0/8$; $I_3 = I_2/4 = 3I_0/32$

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50 ·· The polarizing angle for a certain substance is 60°. (*a*) What is the angle of refraction of light incident at this angle? (*b*) What is the index of refraction of this substance?

(*a*) $\theta_p + \theta_r = 90°$ $\theta_r = 30°$

(*b*) $n = \tan\theta_p$ $n = 1.73$

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51 ·· Two polarizing sheets have their transmission axes crossed and a third sheet is inserted so that its transmission axis makes an angle θ with that of the first sheet as in Problem 49. Find the intensity of the transmitted light as a function of θ. Show that the intensity transmitted through all three sheets is maximum when $\theta = 45°$.

Let I_n be the intensity after the *n*'th polarizing sheet. Since the incoming light is unpolarized, $I_1 = 1/2I_0$. Then $I_2 = I_1\cos^2\theta$ and $I_3 = I_2\cos^2(90° - \theta)$. So $I_3 = 1/2I_0\cos^2\theta\cos^2(90° - \theta) = 1/2I_0\cos^2\theta\sin^2\theta$. But $\cos\theta\sin\theta = 1/2\sin 2\theta$, so $I_3 = (1/8)I_0\sin^2 2\theta$. Since the sine is a maximum when its argument is 90°, the maximum for I_3 occurs when $\theta = 45°$.µ

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52 ·· If the middle polarizing sheet in Problem 51 is rotating at an angular velocity ω about an axis parallel to the light beam, find the intensity transmitted through all three sheets as a function of time. Assume that $\theta = 0$ at time $t = 0$.

Replace θ in the result of Problem 33-51 by ωt. The transmitted intensity is then given by $I = (I_0/8)\sin^2 2\omega t$.

53*·· A stack of $N + 1$ ideal polarizing sheets is arranged with each sheet rotated by an angle of $\pi/2N$ rad with respect to the preceding sheet. A plane linearly polarized light wave of intensity I_0 is incident normally on the stack. The incident light is polarized along the transmission axis of the first sheet and therefore perpendicular to the transmission axis of the last sheet in the stack. (*a*) What is the transmitted intensity through the stack? (*b*) For 3 sheets ($N = 2$), what is the transmitted intensity? (*c*) For 101 sheets, what is the transmitted intensity? (*d*) What is the direction of polarization of the transmitted beam in each case?

(*a*) 1. Find the ratio I_{n+1}/I_n $I_{n+1}/I_n = \cos^2(\pi/2N)$

2. There are N such reductions of intensity $I_{N+1}/I_1 = I_{N+1}/I_0 = \cos^{2N}(\pi/2N)$; $I_{N+1} = I_0 \cos^{2N}(\pi/2N)$

(b) Find I_3 from part (a); see also Problem 49(a) $I_3 = I_0 \cos^4(\pi/4) = I_0/4$

(c) Repeat part (b) for $N = 100$ $I_{101} = I_0 \cos^{200}(\pi/200) = 0.976\, I_0$

(d) In each case, the polarization of the transmitted beam is perpendicular to that of the incident beam.

54 ·· Show that a linearly polarized wave can be thought of as a superposition of a right and a left circularly polarized wave.

For a circularly polarized wave, the x and y components of the electric field are given by $E_x = E_0 \cos \omega t$ and $E_y = E_0 \sin \omega t$ or $E_y = -E_0 \sin \omega t$ for left and right circular polarization, respectively. Thus $\boldsymbol{E}_{\text{right}} + \boldsymbol{E}_{\text{left}} = 2E_0 \cos \omega t\ \boldsymbol{i}$, a wave polarized along the x axis.

55 ·· Suppose that in Problem 49 the middle sheet is replaced by two polarizing sheets. If the angles between the directions of polarization of adjacent sheets is 30°, what is the intensity of the transmitted light? How does this compare with the intensity obtained in Problem 49a?

Following the same procedure as in Problem 33-49 one obtains $I = 1/2 I_0 \cos^6 30° = 0.211 I_0$. For the single sheet between the two end sheets at $\theta = 45°$, $I = 0.125 I_0$. Using two sheets at relative angles of 30°increases the transmitted intensity.

56 ·· In a circularly polarized wave, the magnitude of the electric field is constant. If the wave propagates along the z axis, the angle between $\boldsymbol{E}$ and the x axis changes by 2π radians over one wavelength. Write expressions for the electric and magnetic fields of a circularly polarized wave of angular frequency ω propagating in vacuum in the positive z direction.

For the electric field, $\boldsymbol{E} = E_x\,\boldsymbol{i} + E_y\,\boldsymbol{j}$, where E_x and E_y are given by $E_x = E_0 \cos(kz - \omega t)$ and $E_y = E_0 \sin(kz - \omega t)$. Since the wave propagates in the positive z direction, $\boldsymbol{S} = S\,\boldsymbol{k}$ and $\boldsymbol{E}\times\boldsymbol{B}$ must be a vector in the $\boldsymbol{k}$ direction. So $\boldsymbol{B}$ is in the xy plane and $B_0 = E_0/c$. It follows that $\boldsymbol{B} = -B_x\,\boldsymbol{i} + B_y\,\boldsymbol{j}$, where $B_x = B_0 \sin(kz - \omega t)$ and $B_y = B_0 \cos(kz - \omega t)$. Here $k = 2\pi/\lambda$ and we have assumed that the wave is left circularly polarized.

57* ·· Show that the electric field of a circularly polarized wave propagating in the x direction can be expressed by

$$E = E_0 \sin(kx - \omega t)\,\boldsymbol{j} + E_0 \cos(kx - \omega t)\,\boldsymbol{k}$$

Note that $E_y = E_0 \sin \theta$ and $E_x = E_0 \cos \theta$, where $\theta = kx - \omega t$. Clearly, the magnitude of $\boldsymbol{E}$ is constant in time, and in a plane perpendicular to the direction of propagation, the $\boldsymbol{E}$ vector rotates with angular frequency ω.

58 ·· For the wave whose electric field is given by the expression in Problem 57, what is the corresponding expression for the magnetic field $\boldsymbol{B}$?

Proceed as in Problem 33-56 except that now $\boldsymbol{E}\times\boldsymbol{B}$ is a vector in the $\boldsymbol{i}$ direction. It follows that $\boldsymbol{B}$ is given by $\boldsymbol{B} = B_0 \sin(kx - \omega t)\,\boldsymbol{k} - B_0 \cos(kx - \omega t)\,\boldsymbol{j}$, where $B_0 = E_0/c$.

59 ·· Find expressions for the electric field $\boldsymbol{E}$ and magnetic field $\boldsymbol{B}$ for a circularly polarized wave propagating in the negative z direction. (See problems 57 and 58.)

See Problem 33-56. $\boldsymbol{E} = E_0 \cos(kz + \omega t)\,\boldsymbol{i} + E_0 \sin(kz + \omega t)\,\boldsymbol{j}$. Now $\boldsymbol{E}\times\boldsymbol{B}$ must be a vector in the $-\boldsymbol{k}$ direction, so $\boldsymbol{B} = B_0 \sin(kz + \omega t)\,\boldsymbol{i} - B_0 \cos(kz + \omega t)\,\boldsymbol{j}$, where $B_0 = E_0/c$.

60 ·· A circularly polarized wave is said to be *right circularly polarized* if the electric and magnetic fields rotate clockwise when viewed along the direction of propagation and *left circularly polarized* if the fields rotate counterclockwise. What is the sense of the circular polarization for the wave described by the expression in Problem 57? What would be the corresponding expression for a circularly polarized wave of the opposite sense?

It is right circularly polarized. For a left circularly polarized wave $\boldsymbol{E} = E_0 \sin(kx - \omega t)\,\boldsymbol{j} - E_0 \cos(kx - \omega t)\,\boldsymbol{k}$.

61*·· Vertically polarized light of intensity I_0 is incident on a stack of N ideal polarizing sheets whose angles with respect to the vertical are $\theta_n = n\pi/2N$. Determine the direction of polarization of the transmitted light and its intensity. Show that as $N \to \infty$ the direction of polarization is rotated without loss of intensity.

From Problem 53(*a*), $I_N = I_0 \cos^{2N}(\pi/2N)$, and as in Problem 53(*d*) the direction of polarization has been rotated by 90°. For $N \to \infty$ use the small angle expansion $\cos\theta = 1 - \theta^2/2 + \cdots$. Thus, $I_N = I_0(1 - \pi^2/8N^2)^{2N} = I_0(1 - \pi^2/4N)$, and as $N \to \infty$, $I_N = I_0$.

62 · True or false:

(*a*) Light and radio waves travel with the same speed through a vacuum.

(*b*) Most of the light incident normally on an air–glass interface is reflected.

(*c*) The angle of refraction of light is always less than the angle of incidence.

(*d*) The index of refraction of water is the same for all wavelengths in the visible spectrum.

(*e*) Longitudinal waves cannot be polarized.

(*a*) True (*b*) False (*c*) False (*d*) False (*e*) True

63 ·· Of the following statements about the speeds of the various colors of light in glass, which are true?

(*a*) All colors of light have the same speed in glass.

(*b*) Violet has the highest speed, red the lowest.

(*c*) Red has the highest speed, violet the lowest.

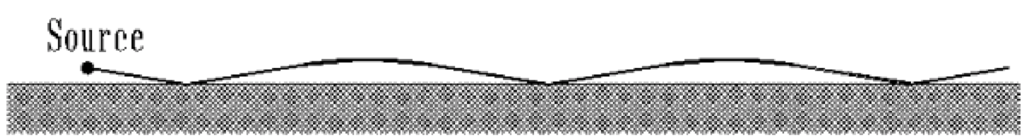
(*d*) Green has the highest speed, red and violet the lowest.

(*e*) Red and violet have the highest speed, green the lowest.

(*c*)

64 ·· It is a common experience that on a calm, sunny day one can hear voices of persons in a boat over great distances. Explain this phenomenon, keeping in mind that sound is reflected from the surface of the water and that the temperature of the air just above the water's surface is usually less than that at a height of 10 or 20 m above the water.

The sound is reflected specularly from the surface of the water (we assume it is calm). It is then refracted back toward the water in the region above the water because the speed of sound depends on the temperature of the air and is greater at the higher temperature. The pattern of the sound wave is shown schematically below.



65* · A beam of monochromatic red light with a wavelength of 700 nm in air travels in water. (*a*) What is the wavelength in water? (*b*) Does a swimmer underwater observe the same color or a different color for this light?

(*a*) $\lambda_n = v/f = c/nf$; $\lambda_n = \lambda_0/n$ $\lambda_{\text{water}} = 700/1.333$ nm $= 525$ nm

(*b*) Color observed depends on frequency The swimmer observed same color in water and air

66 · As the speed of computer operations increases, computer architecture acquires greater importance; the time required to transfer a signal between the central processing unit (CPU) and memory can be a limiting factor in determining the time required for computation. What is the maximum separation between a memory chip and the CPU to allow transfer information between these units in less than 0.5 ns?

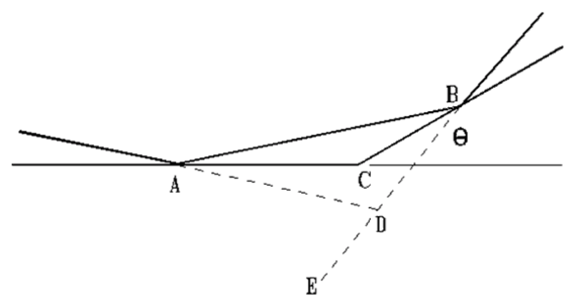
$\Delta x = c\Delta t = 15$ cm

67 ·· The critical angle for total internal reflection for a substance is 45°. What is the polarizing angle for this substance?

Use Equs. 33-12 and 33-21 $\theta_p = \tan^{-1}(1/\sin\theta_c) = 54.7°$

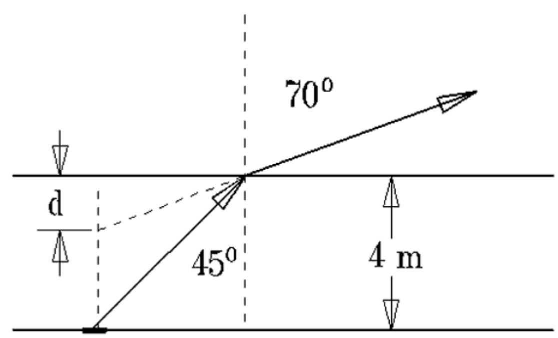
68 ·· Figure 33-57 shows two plane mirrors that make an angle θ with each other. Show that the angle between the incident and reflected rays is 2θ.

Note that the triangle *ABC* is isosceles; the angles *CAB* and *ACB* are equal and their sum equals θ. Also from the law of reflection, the angles *CAD* and *CBD* equal the angle *ABC*. Since the angle *BAD* is twice *BAC* and the angle *DBA* is twice *CBA*, the angle *ADE* is twice the angle θ. The angle *ADE* is the angle between the direction of the incoming ray and that reflected by the two mirror surfaces.



69* ·· A silver coin sits on the bottom of a swimming pool that is 4 m deep. A beam of light reflected from the coin emerges from the pool making an angle of 20° with respect to the water's surface and enters the eye of an observer. Draw a ray from the coin to the eye of the observer. Extend this ray, which goes from the water–air interface to the eye, straight back until it intersects with the vertical line drawn through the coin. What is the apparent depth of the swimming pool to this observer?

The sketch shows the ray from the coin through the water and to the eye of the observer. The angles shown are as determined below.



1. Find the angle θ_i at the water-air interface $\theta_i = \sin^{-1}[(\sin 70°)/1.33] = 45°$

2. Find d $d = (4\times\tan 45°\times\tan 20°)$ m $= 1.46$ m

70 ·· Two affluent students decide to improve on Galileo's experiment to measure the speed of light. One student reflecting electromagnetic waves from a satellite that is 37.9 Mm above the earth's surface. If the distance between London and New York is neglected, the distance traveled is twice this distance. One student claps his hands, and when the other student hears the sound over the phone, she claps her hands. The first student measures the time between his clap and his hearing the second one. Calculate this time lapse, neglecting the students' response times. Do you think this experiment would be successful? What improvements for measuring this time interval would you suggest? (Time delays in the electronic circuits that are greater than those due to the light traveling to the satellite and back make this experiment not feasible.)

$\Delta t = D_{tot}/c$ $\Delta t = (4\times37.9\times10^6/3\times10^8)$ s = 0.505 s

This experiment is bound to fail since the reaction time of about 0.2 s is comparable to the time of travel of the signals. One might initiate a return pulse by an electronic switch, thereby reducing the "reaction time" to a a few nanoseconds.

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71 ·· Fishermen always insist on silence because noise on shore will scare fish away. Suppose a fisherman cast a baited hook 20 m from the shore of a calm lake to a point where the depth is 15 m. Show that noise on shore cannot possibly be sensed by fish at that point. *Note*: The speed of sound in air is 330 m/s; the speed of sound in water is 1450 m/s.

Assume that the sound source is the voice of the fisherman. If the fisherman's mouth is 2 m above the surface of the water, the angle of incidence of the sound is $\theta_i = \cot^{-1}(2/20) = 84.3°$. The critical angle for total reflection of the sound is given by Equ. 33-12, which can be rewritten as $\sin\theta_c = v_1/v_2 = 330/1450 = 0.228$. Thus $\theta_c = 13.2°$ and since $\theta_i > \theta_c$ all of the sound is totally reflected at the air-water interface.

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72 ·· A swimmer at the bottom of a pool 3 m deep looks up and sees a circle of light. If the index of refraction of the water in the pool is 1.33, find the radius of the circle.

$R = (3 \tan \theta_c)$ m (see Problem 33-35) $R = 3.42$ m

73* ·· Show that when a mirror is rotated through an angle θ, the reflected beam of light is rotated through 2θ.

Let α be the initial angle of incidence. Since the angle of reflection with the normal to the mirror is also α, the angle between incident and reflected rays is 2α. If the mirror is now rotated by a further angle θ, the angle of incidence is increased by θ to $\alpha + \theta$, and so is the angle of reflection. Consequently, the reflected beam is rotated by 2θ relative to the incident beam.

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74 ·· Use Figure 33-26 to calculate the critical angles for total internal reflection for light initially in silicate flint glass that is incident on a glass–air interface if the light is (*a*) violet light of wavelength 400 nm, and (*b*) red light of wavelength 700 nm.

(*a*), (*b*) Use Equ. 33-12 and Figure 33-26 (*a*) $\theta_c = 36.8°$ (*b*) $\theta_c = 38.4°$

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75 ·· Show that for normally incident light, the intensity transmitted through a glass slab with an index of refraction of *n* is approximately given by

$$I_T = I_0\left[\frac{4n}{(n+1)^2}\right]^2$$

We shall neglect multiple reflections at the glass-air interfaces. At the air-glass interface, the reflected intensity is $I_r = I_0(1-n)^2/(1+n)^2$ so the intensity transmitted into the glass is $I_t = I_0 - I_r = 4nI_0/(1+n)^2$. At the glass-air interface there is again the same reduction in transmitted intensity. Consequently, the intensity of the light transmitted through the glass plate (neglecting multiple internal reflections) is $I_t = I_0[4n/(1+n)^2]^2$.

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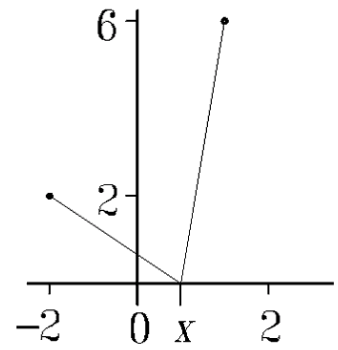
76 ·· A ray of light begins at the point *x* = -2 m, *y* = 2 m, strikes a mirror in the *xz* plane at some point *x*, and reflects through the point *x* = 2 m, *y* = 6 m. (*a*) Find the value of *x* that makes the total distance traveled by the ray a minimum. (*b*) What is the angle of incidence on the reflecting plane? What is the angle of reflection?



(*a*) The adjacent figure shows the two points and the reflecting surface. The distance traveled by the ray that strikes the surface at x is $D = [(2 + x)^2 + 4]^{1/2} + [(2 – x)^2 + 36]^{1/2}$. Set $dD/dx = 0$. $(2 + x)/[(2 + x)^2 + 4]^{1/2} – (2 – x)/[(2 – x)^2 + 36]^{1/2} = 0$. Solving for x we find $x = –1$ m.

(*b*) With $x = –1$ m, $\theta_i = \tan^{-1}(1/2) = 26.6°$ and $\theta_r = \tan^{-1}(3/6) = 26.6° = \theta_i$.

77* ·· Light passes symmetrically through a prism having an apex angle of α as shown in Figure 33-58. (*a*) Show that the angle of deviation δ is given by

$$\sin\frac{\alpha + \delta}{2} = n \sin\frac{\alpha}{2}$$

(*b*) If the refractive index for red light is 1.48 and that for violet light is 1.52, what is the angular separation of visible light for a prism with an apex angle of 60°?

(*a*) With respect to the normal to the left face of the prism, let the angle of incidence be θ_i and the angle of refraction be θ_r. From the geometry of Figure 33-58 it is evident that $\theta_r = \alpha/2$. The angle of deviation at this refracting interface is $\theta_i - \alpha/2$. By symmetry, the angle of deviation at the second refracting interface is also of this magnitude. Thus, $\delta = 2\theta_i - \alpha$ and $\theta_i = 1/2(\alpha + \delta)$. Using Snell's law, $\sin\theta_i = n \sin(\alpha/2)$, we obtain the stated expression, $\sin 1/2(\alpha + \delta) = n \sin 1/2\alpha$.

(*b*) 1. $\delta = 2\sin^{-1}[n \sin 1/2\alpha] – \alpha$. Find δ_{violet} and δ_{red} — $\delta_{violet} = 38.93°$; $\delta_{red} = 35.46°$

2. Angular separation = $\delta_{violet} – \delta_{red}$ — Angular separation = 3.47°

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78 ·· (*a*) For a light ray inside a transparent medium having a planar interface with a vacuum, show that the polarizing angle and the critical angle for internal reflection satisfy $\tan\theta_p = \sin\theta_c$. (*b*) Which angle is larger?

(*a*) From Equs. 33-12 and 33-21 it follows directly that $\tan\theta_p = \sin\theta_c$. Note that in Equs. 33-12 and 33-21 the roles of n_1 and n_2 are interchanged. Therefore, if n_2 is the index of refraction of the optically denser medium, i.e., $n_2 > n_1$, the expression for θ_c would read $\sin\theta_c = n_1/n_2$, or $\tan\theta_p = 1/\sin\theta_c$

(*b*) For any value of θ, $\tan\theta > \sin\theta$, and $\tan\theta < 1/\sin\theta$. Consequently, $\theta_p > \theta_c$.

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79 ·· Light is incident from air on a transparent substance at an angle of 58.0° with the normal. The reflected and refracted rays are observed to be mutually perpendicular. (*a*) What is the index of refraction of the transparent substance? (*b*) What is the critical angle for total internal reflection in this substance?

(*a*) From Figure 33-37 $\theta_i = \theta_p = \tan^{-1} n$ — $n = \tan 58° = 1.60$

(*b*) Use Equ. 33-12 — $\theta_c = 38.7°$

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80 ·· A light ray in dense flint glass with an index of refraction of 1.655 is incident on the glass surface. An unknown liquid condenses on the surface of the glass. Total internal reflection on the glass–liquid interface occurs for an angle of incidence on the glass–liquid interface of 53.7°. (*a*) What is the refractive index of the unknown liquid? (*b*) If the liquid is removed, what is the angle of incidence for total internal reflection? (*c*) For the angle of incidence found in part (*b*), what is the angle of refraction of the ray into the liquid film? Does a ray emerge from the liquid film into the air above? Assume the glass and liquid have perfect planar surfaces.

(*a*) Use Equ. 33-12 $n_L = 1.655 \sin 53.7° = 1.33$

(*b*) Use Equ. 33-12 $\theta_c = 37.2°$

(*c*) Use Equ. 33-9*b* $\theta_r = 48.7°$; ray does not emerge (see Problem 33-40)

81*·· Given that the index of refraction for red light in water is 1.3318 and that the index of refraction for blue light in water is 1.3435, find the angular separation of these colors in the primary rainbow. (Use the equation given in Problem 86.)

1. Find θ_{1m} for red and blue light; Red light: $\theta_{1m} = 59.48°$; blue light: $\theta_{1m} = 58.80°$

$$\cos\theta_{1m} = \sqrt{\frac{n^2-1}{3}}$$

2. Use Equ. 33-18 to find ϕ_d for red and blue light Red light: $\phi_d = 137.75°$; blue light: $\phi_d = 139.42°$

3. Angular separation = $\phi_{d,blue} - \phi_{d,red}$ Angular separation = 1.67°

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82 ·· A ray of light falls on a rectangular glass block ($n = 1.5$) that is almost completely submerged in water ($n = 1.33$) as shown in Figure 33-59. (*a*) Find the angle θ for which total internal reflection just occurs at point *P*. (*b*) Would total internal reflection occur at point *P* for the value of θ found in part (*a*) if the water were removed? Explain.

(*a*) 1. Find θ_c at glass-water interface $\theta_c = \sin^{-1}(1.33/1.5) = 62.5°$

2. Find θ; note that θ_r(glass) = 90° – θ_c = 27.5° $\theta = \sin^{-1}(1.5 \sin 27.5) = 43.9°$

(*b*) The angle of incidence at *P* is again 62.5°. If the water is removed, the critical angle for total internal reflection is $\sin^{-1}(1/1.5) = 41.8°$. Therefore, total internal reflection will definitely occur if the water is removed.

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83 ·· (*a*) Use the result for Problem 75 to find the ratio of the transmitted intensity to the incident intensity through *N* parallel slabs of glass for light of normal incidence. (*b*) Find this ratio for three slabs of glass with *n* = 1.5.

(*c*) How many slabs of glass with $n = 1.5$ will reduce the intensity to 10% of the incident intensity?

(*a*) Each slab reduces the intensity by the factor $[4n/(n+1)^2]^2$. Consequently, with *N* slabs, $I_t = I_0[4n/(n+1)^2]^{2N}$.

(*b*) Use the result in (*a*) with $N = 3$ $I_t/I_0 = (6/2.5^2)^6 = 0.783$

(*c*) Solve for *N* with $I_t/I_0 = 0.1$ $2N \log(0.96) = \log(0.1) = -1$; $N = 28.2 \approx 28$

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84 ·· Light is incident on a slab of transparent material at an angle θ_1 as shown in Figure 33-60. The slab has a thickness *t* and an index of refraction *n*. Show that

$$n = \frac{\sin\theta_1}{\sin[\arctan(d/t)]}$$

where *d* is the distance shown in the figure and arctan (*d/t*) is the angle whose tangent is *d/t*.

Referring to the figure of Problem 33-30, the distance $d = DB = t \tan\theta_2$. From Equ. 33-9*b*, $n = \sin\theta_1/\sin\theta_2$. So $n = (\sin\theta_1)/[\sin(\tan^{-1} d/t)]$.

85* ·· Suppose rain falls vertically from a stationary cloud 10,000 m above a confused marathoner running in a circle with constant speed of 4 m/s. The rain has a terminal speed of 9 m/s. (*a*) What is the angle that the rain appears to make with the vertical to the marathoner? (*b*) What is the apparent motion of the cloud as observed by the marathoner? (c) A star on the axis of the earth's orbit appears to have a circular orbit of angular diameter of 41.2 seconds of arc. How is this angle related to the earth's speed in its orbit and the velocity of photons received from this distant star? (d) What is the speed of light as determined from the data in part (c)?

(a) $\tan\theta = v_{run}/v_{rain}$; $\theta = \tan^{-1}(v_{run}/v_{rain})$ — $\theta = \tan^{-1}(4/9) = 24.0°$

(b) Cloud moves in circle of radius $R = H\tan\theta$ — Radius of circle of cloud motion is $R = 4444$ m

(c) As in (a) where $v_{run} = v_{earth}$, $v_{rain} = c$ — $\tan\theta = v_{earth}/c$; $\theta = 1/2\times$(angular diameter)

(d) 1. Find $v_{earth} = 2\pi R_{earth-sun}/(1\text{ y})$ — $v_{earth} = (2\pi\times1.5\times10^{11}/3.156\times10^{7})$ m/s $= 2.986\times10^{4}$ m/s

2. $c = v_{earth}/\tan\theta$ — $c = [2.986\times10^{4}/\tan(20.6'')]$ m/s $= 2.99\times10^{8}$ m/s

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86 ··· Equation 33-18 gives the relation between the angle of deviation ϕ_d of a light ray incident on a spherical drop of water in terms of the incident angle θ_1 and the index of refraction of water. (a) Assume that $n_{air} = 1$, and differentiate ϕ_d with respect to θ_1. [*Hint*: If $y = \arcsin x$, then $dy/dx = (1 - x^2)^{-1/2}$.] (b) Set $d\phi_d/d\theta_1 = 0$ and show that the angle of incidence θ_{1m} for minimum deviation is given by

$$\cos\theta_{1m} = \sqrt{\frac{n^2-1}{3}}$$

and find θ_{1m} for water, where the index of refraction for water is 1.33.

(a) With $n_{air} = 1$, Equ. 33-18 reads $\phi_d = \pi + 2\theta_1 - 4\sin^{-1}[(\sin\theta_1)/n)]$. Use the *Hint* to take the derivative of ϕ_d with respect to θ_1 to obtain

$$\frac{d\phi_d}{d\theta_1} = 2 - \frac{4\cos\theta_1}{\sqrt{n^2 - \sin^2\theta_1}}$$

(b) Set the above expression equal to zero and solve for $\cos\theta_1$; the angle θ_1 is the angle of incidence for minimum deviation, θ_{1m}. Replacing $\sin^2\theta_1$ by $1 - \cos^2\theta_1$ one arrives at the result quoted in the problem statement.

With $n = 1.33$, $\theta_{1m} = 59.6°$.

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87 ··· (a) Show that a light ray transmitted through a glass slab emerges parallel to the incident ray but displaced from it. (b) For an incident angle of 60°, glass of index of refraction $n = 1.5$, and a slab of thickness 10 cm, find the displacement measured perpendicularly from the incident ray.

(a) Using Equ. 33-9b at both surfaces of the slab it follows that the angle of the emerging ray with the normal to the slab is equal to the angle of incidence. That is, the emerging ray and incident ray are parallel.

(b) The displacement is given by $s_2 = t(\tan\theta_1 - \tan\theta_2)\cos\theta_1$ (see Problem 33-30).

1. Find θ_2 using Equ. 33-9b — $\theta_2 = 35.26°$

2. Find s_2 — $s_2 = 5.12$ cm

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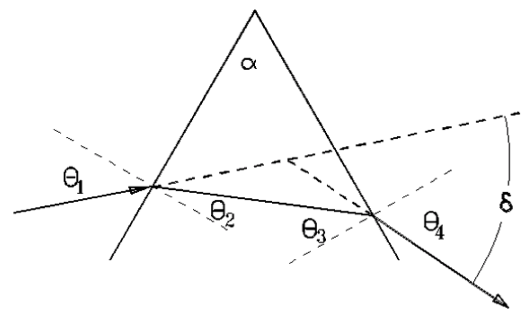
88 ··· Show that if the apex angle α of the prism of Problem 77 is small, the angle of deviation δ is given by $\delta = (n-1)\alpha$, independent of the angle of incidence.

We use the notation and the figure of Problem 33-89 (see below). The deviation angle δ is given by $\delta = \sin^{-1}[n\sin(\theta_3 - \alpha)] + \sin^{-1}(n\sin\theta_3) - \alpha$. We now take the derivative of δ with respect to α.

$$\frac{d\delta}{d\alpha} = \frac{n\cos(\alpha - \theta_3)}{\sqrt{1 - [n\sin(\alpha - \theta_3)]^2}} - 1.$$ In the limit $\alpha \to 0$, $d\delta/d\alpha \to 0$; i.e., for small α, the angle of deviation is independent of α. In Problem 33-39 we show that when the ray passes through the prism

symmetrically, the angles θ_2 and θ_3 are both equal to $\alpha/2$. Since for small α the angle δ is independent of α, we need only determine δ when $\theta_2 = \theta_3 = \alpha/2$. Using Snell's law, Equ. 33-9*b*, $\theta_1 = \theta_4 = \sin^{-1}[n \sin(\alpha/2)]$. For $\alpha << 1$, $\sin(\alpha/2) = \alpha/2$, so $\delta = \theta_1 + \theta_4 - \alpha = n\alpha - \alpha = (n-1)\alpha$.

89*··· Show that the angle of deviation δ is a minimum if the angle of incidence is such that the ray passes through the prism symmetrically as shown in Figure 33-58.



The figure to the right shows the prism and the path of the ray through it. The light dashed lines are the normals to the prism faces. The triangle formed by the interior ray and the prism faces has interior angles of α, $90° - \theta_2$, and $90° - \theta_3$. Consequently, $\theta_2 + \theta_3 = \alpha$. Next we note that the deviation angle $\delta = \theta_1 + \theta_4 - \alpha$. These are purely geometric relations. We now employ Snell's law to relate θ_3 to $_4$ and θ_1 to θ_2. We have $\sin\theta_1 = n \sin\theta_2$ and $\sin\theta_4 = n\sin\theta_3$. We can now write an expression for δ
in terms of one of the angles θ_1 and the apex angle α.
$\delta = \sin^{-1}(n\sin\theta_2) + \sin^{-1}(n\sin\theta_3) - \alpha = \sin^{-1}[n\sin(\theta_3 - \alpha)] + \sin^{-1}(n\sin\theta_3) - \alpha$. The only variable in this expression is θ_3. To determine the condition for minimizing δ we now take the derivative with respect to θ_3 and set it equal to zero.

$$\frac{d\delta}{d\theta_3} = -\frac{n\cos(\alpha - \theta_3)}{\sqrt{1-[n\sin(\alpha-\theta_3)]^2}} + \frac{n\cos\theta_3}{\sqrt{1-(n\sin\theta_3)^2}} = 0$$

. This equation is satisfied if $\alpha - \theta_3 = \theta_3$ or $\theta_3 = \alpha/2$.

But $\theta_2 = \alpha - \theta_3 = \alpha$2, which shows that the deviation angle is a minimum if the ray passes through the prism symmetrically.