CHAPTER 31

Alternating-Current Circuits

Note: Unless otherwise indicated, the symbols I, V, $\mathcal{E}$, and P denote the rms values of I, V, and $\mathcal{E}$ and the average power.

1* · A 200-turn coil has an area of 4 cm^2 and rotates in a magnetic field of 0.5 T. (*a*) What frequency will generate a maximum emf of 10 V? (*b*) If the coil rotates at 60 Hz, what is the maximum emf?

(*a*) $\mathcal{E} = NBA\omega \cos \omega t$ (see Problem 30-8-5) $\omega = \mathcal{E}_{max}/NBA = 250\ s^{-1}$; $f = \omega/2\pi = 39.8$ Hz

(*b*) $\mathcal{E}_{max} = NBA\omega = 2\pi NBAf$ $\mathcal{E}_{max} = 15.1$ V

2 · In what magnetic field must the coil of Problem 1 be rotating to generate a maximum emf of 10 V at 60 Hz?

Use Equ. 31–4; solve for B $B = 0.332$ T

3 · A 2-cm by 1.5-cm rectangular coil has 300 turns and rotates in a magnetic field of 4000 G. (*a*) What is the maximum emf generated when the coil rotates at 60 Hz? (*b*) What must its frequency be to generate a maximum emf of 110 V?

(*a*) Use Equ. 31-4 $\mathcal{E}_{max} = 13.6$ V

(*b*) Use Equ. 31-4; solve for $f = \omega/2\pi$ $f = 486$ Hz

4 · The coil of Problem 3 rotates at 60 Hz in a magnetic field B. What value of B will generate a maximum emf of 24 V?

Use Equ. 31-4; solve for B $B = 0.707$ T

5* · As the frequency in the simple ac circuit in Figure 31-26 increases, the rms current through the resistor (*a*) increases. (*b*) does not change. (*c*) may increase or decrease depending on the magnitude of the original frequency. (*d*) may increase or decrease depending on the magnitude of the resistance. (*e*) decreases.

(*b*)

6 · If the rms voltage in an ac circuit is doubled, the peak voltage is (*a*) increased by a factor of 2. (*b*) decreased by a factor of 2. (*c*) increased by a factor of $\sqrt{2}$. (*d*) decreased by a factor of $\sqrt{2}$. (*e*) not changed.

(*a*)

7 · A 100-W light bulb is plugged into a standard 120-V (rms) outlet. Find (*a*) I_{rms}, (*b*) I_{max}, and (*c*) the maximum power.

(*a*) Use Equ. 31-14 $I_{rms} = 0.833$ A

(*b*) Use Equ. 31-12 $I_{max} = 1.18$ A

(*c*) $P_{max} = I_{max}\mathcal{E}_{max} = 2I_{rms}\mathcal{E}_{rms} = 2P_{av}$ $P_{max} = 200$ W

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8 · A 3-Ω resistor is placed across a generator having a frequency of 60 Hz and a maximum emf of 12.0 V. (*a*) What is the angular frequency ω of the current? (*b*) Find I_{max} and I_{rms}. What is (*c*) the maximum power into the resistor, (*d*) the minimum power, and (*e*) the average power?

(*a*) $\omega = 2\pi f$ $\omega = 377$ rad/s

(*b*) Use Equs. 31-8 and 31-12 $I_{max} = 4$ A; $I_{rms} = 2.83$ A

(*c*) $P_{max} = I_{max}^2 R$ $P_{max} = 48$ W

(*d*) $P_{min} = (|I|_{min})^2 R$ $P_{min} = 0$

(*e*) $P_{av} = 1/2 P_{max}$ $P_{av} = 24$ W

9* · A circuit breaker is rated for a current of 15 A rms at a voltage of 120 V rms. (*a*) What is the largest value of I_{max} that the breaker can carry? (*b*) What average power can be supplied by this circuit?

(*a*) $I_{max} = \sqrt{2}\, I_{rms}$ $I_{max} = 21.2$ A

(*b*) $P = I_{rms}V_{rms}$ $P = 1.8$ kW

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10 · If the frequency in the circuit shown in Figure 31-27 is doubled, the inductance of the inductor will (*a*) increase by a factor of 2. (*b*) not change. (*c*) decrease by a factor of 2. (*d*) increase by a factor of 4. (*e*) decrease by a factor of 4.

(*b*)

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11 · If the frequency in the circuit shown in Figure 31-27 is doubled, the inductive reactance of the inductor will (*a*) increase by a factor of 2. (*b*) not change. (*c*) decrease by a factor of 2. (*d*) increase by a factor of 4. (*e*) decrease by a factor of 4.

(*a*)

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12 · If the frequency in the circuit in Figure 31-28 is doubled, the capacitative reactance of the circuit will (*a*) increase by a factor of 2. (*b*) not change. (*c*) decrease by a factor of 2. (*d*) increase by a factor of 4. (*e*) decrease by a factor of 4.

(*c*)

13* · In a circuit consisting of a generator and an inductor, are there any times when the inductor absorbs power from the generator? Are there any times when the inductor supplies power to the generator?

Yes, Yes

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14 · In a circuit consisting of a generator and a capacitor, are there any times when the capacitor absorbs power from the generator? Are there any times when the capacitor supplies power to the generator?

Yes to both questions.

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15 · What is the reactance of a 1.0-mH inductor at (*a*) 60 Hz, (*b*) 600 Hz, and (*c*) 6 kHz?

(*a*), (*b*), (*c*) Use Equ. 31-25 (*a*) $X_L = 0.377$ Ω (*b*) $X_L = 3.77$ Ω (*c*) $X_L = 37.7$ Ω

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16 · An inductor has a reactance of 100 Ω at 80 Hz. (*a*) What is its inductance? (*b*) What is its reactance at 160 Hz?

(*a*), (*b*) Use Equ. 31-25; solve for L (*a*) $L = 0.199$ H (*b*) $X_L = 200$ Ω

17* · At what frequency would the reactance of a 10.0-μF capacitor equal that of a 1.0-mH inductor?

$f = (1/2\pi)(1/\sqrt{LC})$ $f = 1.59$ kHz

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18 · What is the reactance of a 1.0-nF capacitor at (*a*) 60 Hz, (*b*) 6 kHz, and (*c*) 6 MHz?

(*a*), (*b*), (*c*) Use Equ. 31-31 (*a*) $X_C = 2.65$ MΩ (*b*) $X_C = 26.5$ kΩ (*c*) $X_C = 26.5$ Ω

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19 · An emf of 10.0 V maximum and frequency 20 Hz is applied to a 20-μF capacitor. Find (*a*) I_{max} and (*b*) I_{rms}.

(*a*) 1. Find X_C using Equ. 31-31 $X_C = 398$ Ω

2. $I_{max} = \mathcal{E}_{max}/X_C$ $I_{max} = 25.1$ mA

(*b*) Use Equ. 31-12 $I_{rms} = 17.8$ mA

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20 · At what frequency is the reactance of a 10-μF capacitor (*a*) 1 Ω, (*b*) 100 Ω, and (*c*) 0.01 Ω?

(*a*), (*b*), (*c*) Use Equ. 31-31; solve for *f* (*a*) $f = 15.9$ kHz (*b*) $f = 159$ kHz (*c*) $f = 1.59$ MHz

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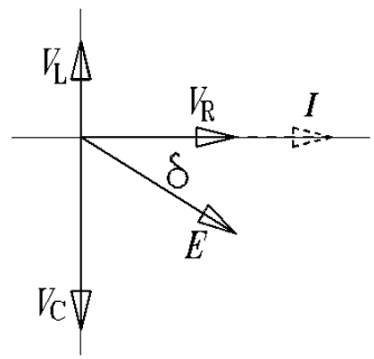
21* · Draw the resultant phasor diagram for a series *RLC* circuit when $V_L < V_C$. Show on your diagram that the emf will lag the current by the phase angle δ given by

$$\tan\delta = \frac{V_C - V_L}{V_R}$$

The phasor diagram is shown at the right. The voltages V_R, V_L, and V_C are indicated as well as the resultant voltage *E*. The current is in phase with V_R and its phasor is shown by the dashed arrow. The voltage *E* lags the current by the angle δ where $\delta = \tan^{-1}[(V_C - V_L)/V_R]$.



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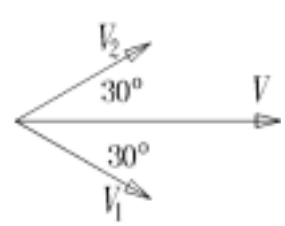
22 ·· Two ac voltage sources are connected in series with a resistor $R = 25$ Ω. One source is given by $V_1 = (5.0\text{ V})\cos(\omega t - \alpha)$, and the other source is $V_2 = (5.0\text{ V})\cos(\omega t + \alpha)$, with $\alpha = \pi/6$. (*a*) Find the current in *R* using a trigonometric identity for the sum of two cosines. (*b*) Use phasor diagrams to find the current in *R*. (*c*) Find the current in *R* if $\alpha = \pi/4$ and the amplitude of V_2 is increased from 5.0 V to 7.0 V.

(*a*) 1. Find $V = V_1 + V_2$ using $V = (8.66\cos\omega t)$ V

$\cos\delta + \cos\gamma = 2\cos 1/2(\delta + \gamma)\cos 1/2(\delta - \gamma)$

2. $I = V/R$ $I = (0.346\cos\omega t)$ A

(b) The phasor diagram for the voltages is shown in the adjacent figure. By vector addition,



$|V| = 2|V_1|\cos 30° = 8.66$ V; $|I| = |V|/R$ $I = (0.346\cos\omega t)$ A

(c) Note that the phase angle between V_1 and V_2 is 90°; so $|V| = \sqrt{|V_1|^2 + |V_2|^2}$; $|I| = |V|/R$. $|V| = 8.60$ V; $|I| = 0.344$ A

$I = [0.344\cos(\omega t + \phi)]$ A

The phase angle is $\phi = \tan^{-1}(7/5) - 45°$. $\phi = 9.46° = 0.165$ rad

23 · The SI units of inductance times capacitance are (*a*) seconds squared. (*b*) hertz. (*c*) volts. (*d*) amperes. (*e*) ohms.

(*a*)

24 ·· Making *LC* circuits with oscillation frequencies of thousands of hertz or more is easy, but making *LC* circuits that have small frequencies is difficult. Why?

To make an *LC* circuit with a small resonance frequency requires a large inductance and large capacitance. Neither is easy to construct.

25* · Show from the definitions of the henry and the farad that $1/\sqrt{LC}$ 0 has the unit s^{-1}.

The dimension of *C* is [Q]/[V]. From $V = L(dI/dt)$ and [I] = [Q]/[T] it follows that $[L] = [V][T]^2/[Q]$. Thus $[L][C] = [T]^2$, and $1/\sqrt{LC}$ 0 has the dimension of $[T]^{-1}$, i.e., units of s^{-1}.

26 · (*a*) What is the period of oscillation of an *LC* circuit consisting of a 2-mH coil and a 20-μF capacitor? (*b*) What inductance is needed with an 80-μF capacitor to construct an *LC* circuit that oscillates with a frequency of 60 Hz?

(*a*) Use Equ. 31-41; $T = 2\pi/\omega$ — $T = 1.26$ ms

(*b*) Use Equ. 31-41; solve for *L* — $L = 1/4\pi^2 f^2 C = 88$ mH

27 ·· An *LC* circuit has capacitance C_1 and inductance L_1. A second circuit has $C_2 = \frac{1}{2}C_1$ and $L_2 = 2L_1$, and a third circuit has $C_3 = 2C_1$ and $L_3 = \frac{1}{2}L_1$. (*a*) Show that each circuit oscillates with the same frequency. (*b*) In which circuit would the maximum current be greatest if the capacitor in each were charged to the same potential *V*?

(*a*) Since $L_1C_1 = L_2C_2 = L_3C_3$, the resonance frequencies of the three circuits are the same.

(*b*) From Equ. 31-43, $I_{max} = \omega Q_0 = \omega CV$. Therefore the circuit with $C = C_3$ has the greatest I_{max}.

28 ·· A 5-μF capacitor is charged to 30 V and is then connected across a 10-mH inductor. (*a*) How much energy is stored in the system? (*b*) What is the frequency of oscillation of the circuit? (*c*) What is the maximum current in the circuit?

(*a*) $U = 1/2CV^2$ — $U = 2.25$ mJ

(*b*) Use Equ. 31-41 — $f = 712$ Hz

(*c*) $I_{max} = \omega CV$ — $I_{max} = 0.671$ A

29* · A coil can be considered to be a resistance and an inductance in series. Assume that $R = 100\ \Omega$ and $L = 0.4$ H. The coil is connected across a 120-V-rms, 60-Hz line. Find (*a*) the power factor, (*b*) the rms current, and (*c*) the average power supplied.

(*a*) $X = X_L = \omega L$; $Z = \sqrt{X^2 + R^2}$; pf = R/Z — $X_L = 150.8\ \Omega$; $Z = 181\ \Omega$; power factor = 0.552

(*b*) $I = E/Z$ — $I = 120/181$ A = 0.663 A

(*c*) $P = I^2R$ — $P = 44.0$ W

30 ·· A resistance *R* and a 1.4-H inductance are in series across a 60-Hz ac voltage. The voltage across the resistor is 30 V and the voltage across the inductor is 40 V. (*a*) What is the resistance *R*? (*b*) What is the ac input voltage?

(*a*) $I\omega L = V_L$; $IR = V_R$; $R = (V_R/V_L)\omega L$ — $R = 396\ \Omega$

(*b*) V_L leads V_R by 90°; $V = \sqrt{V_R^2 + V_L^2}$ — $V = 50$ V

31 •• A coil has a dc resistance of 80 Ω and an impedance of 200 Ω at a frequency of 1 kHz. One may neglect the wiring capacitance of the coil at this frequency. What is the inductance of the coil?

Use Equ. 31-53; $X_L = \sqrt{Z^2 - R^2} = 2\pi f L$ L = 29.2 mH

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32 •• A single transmission line carries two voltage signals given by V_1 = (10 V) cos 100t and V_2 = (10 V) cos 10,000t, where t is in seconds. A series inductor of 1 H and a shunting resistor of 1 kΩ is inserted into the transmission line as indicated in Figure 31-29. (*a*) What is the voltage signal observed at the output side of the transmission line? (*b*) What is the ratio of the low-frequency amplitude to the high-frequency amplitude?

(*a*) 1. Use Equ. 31-53 to find Z_1 and Z_2 and I_1 and I_2; Z_1 = 1005 Ω, $Z_2 = 1.005\times10^4$ Ω;
$\omega_1 = 10^2\ \text{s}^{-1}$, $\omega_2 = 10^4\ \text{s}^{-1}$ I_1 = (9.95 cos 100t) mA, I_2 = (0.995 cos 10^4t) mA

2. $V_{out} = IR$ V_{1out} = (9.95 cos 100t) V, V_{2out} = (0.995 cos 10^4t) V

(*b*) Find V_{1out}/V_{2out} V_{1out}/V_{2out} = 10

33* •• A coil with resistance and inductance is connected to a 120-V-rms, 60-Hz line. The average power supplied to the coil is 60 W, and the rms current is 1.5 A. Find (*a*) the power factor, (*b*) the resistance of the coil, and (*c*) the inductance of the coil. (*d*) Does the current lag or lead the voltage? What is the phase angle δ?

(*a*) $P = EI\times\text{pf}$ pf = cos δ = 60/180 = 0.333; δ = 70.5°

(*b*) $R = P/I^2$ R = 60/2.25 Ω = 26.7 Ω

(*c*) $X_L = R\tan\delta = \omega L$; $L = (R\tan\delta)/\omega$ L = 0.2 H

(*d*) The circuit is inductive I lags E; δ = 70.5°

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34 •• A 36-mH inductor with a resistance of 40 Ω is connected to a source whose voltage is E = (345 V) cos (150πt), where t is in seconds. Determine the maximum current in the circuit, the maximum and rms voltages across the inductor, the average power dissipation, and the maximum and average energy stored in the magnetic field of the inductor.

1. Use Equ. 31-53 to find Z; $I_{max} = E_{max}/Z$ Z = 43.45 Ω; I_{max} = 7.94 A

2. $V_{Lmax} = \omega L I_{max}$; $V_{Lrms} = V_{Lmax}/\sqrt{2}$ V_{Lmax} = 134.7 V, V_{Lrms} = 95.25 V

3. $P_{av} = 1/2 I_{max}^2 R$ P_{av} = 1.26 kW

4. $U_{Lmax} = 1/2 L I_{max}^2$, $U_{Lav} = \int P_{Lav}$, $P_{Lav} = 0$ U_{Lmax} = 1.13 J; U_{Lav} = 0

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35 •• A coil of resistance R, inductance L, and negligible capacitance has a power factor of 0.866 at a frequency of 60 Hz. What is the power factor for a frequency of 240 Hz?

1. $R/Z = \cos\delta$; find R^2/X_L^2 at f = 60 Hz $R^2/(R^2 + X_L^2) = 3/4$; $R^2 = 3X_L^2$; $X_L^2 = R^2/3$

2. At f = 240 Hz, X_L^2 is 16 times greater than at 60 Hz $X_L^2 = 16R^2/3$; $R/Z = (3/19)^{1/2} = \cos\delta = 0.397$

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36 •• A resistor and an inductor are connected in parallel across an emf $E = E_{max}$ as shown in Figure 31-30. Show that (*a*) the current in the resistor is $I_R = (E_{max}/R)\cos\omega t$, (*b*) the current in the inductor is $I_L = (E_{max}/X_L)\cos(\omega t - 90°)$, and (*c*) $I = I_R + I_L = I_{max}\cos(\omega t - \delta)$, where $\tan\delta = R/X_L$ and $I_{max} = {}_{max}/Z$ with $Z^{-2} = R^{-2} + X_L^{-2}$.

(*a*) Use Kirchhoff's law; let $E = E_{max}\cos\omega t$ $E - I_R R = 0$; $I_R = (E_{max}/R)\cos\omega t$

(*b*) Use Kirchhoff's law; I lags $\mathcal{E}$ by 90°

$I_L = (\mathcal{E}_{max}/X_L)\cos(\omega t - 90°) = (\mathcal{E}_{max}/X_L)\sin\omega t$

(*c*) 1. $I = I_R + I_L = I_{max}\cos(\omega t - \delta)$

$I = I_{max}(\cos\omega t\cos\delta + \sin\omega t\sin\delta)$

2. Compare terms of I with (*a*) and (*b*)

$I_{max}\cos\delta = \mathcal{E}_{max}/R$; $I_{max}\sin\delta = \mathcal{E}_{max}/X_L$; $\tan\delta = X_L/R$

3. Rewrite I_{max}

$I_{max}^2(\cos^2\delta + \sin^2\delta) = \mathcal{E}_{max}^2(1/R^2 + 1/X_L^2) = \mathcal{E}_{max}^2/Z^2$, where $Z^{-2} = R^{-2} + X_L^{-2}$; thus, $I_{max} = \mathcal{E}_{max}/Z$

37* •• Figure 31-31 shows a load resistor $R_L = 20\ \Omega$ connected to a high-pass filter consisting of an inductor L = 3.2 mH and a resistor $R = 4\ \Omega$. The input voltage is $\mathcal{E} = (100\text{ V})\cos(2\pi ft)$. Find the rms currents in R, L, and R_L if (*a*) f = 500 Hz and (*b*) f = 2000 Hz. (*c*) What fraction of the total power delivered by the voltage source is dissipated in the load resistor if the frequency is 500 Hz and if the frequency is 2000 Hz?

We shall do this problem for the general case and then substitute numerical values.

1. Find the resistive and inductive components of $Z_p = Z$ of the parallel combination of L and R_L

$R_p = R_L\omega^2L^2/(R_L^2 + \omega^2L^2)$; $X_p = \omega LR_L^2/(R_L^2 + \omega^2L^2)$

$Z_p = R_L\omega L/\sqrt{R_L^2 + \omega^2L^2}$

2. Find $I = I_R$ in terms of other parameters

$I = \mathcal{E}/\sqrt{(R + R_p)^2 + (X_p)^2} = I_R$

3. Write V_p, the voltage across Z_p

$V_p = IZ_p$

4. Write the currents in L and R_L

$I_L = IZ_p/\omega L$; $I_{R_L} = IZ_p/R_L$

5. Write the power dissipated in R and in R_L

$P_R = I^2R$; $P_L = I_{R_L}^2 R_L$; $P_{tot} = P_R + P_L$

(*a*) 1. For f = 500 Hz, find R_p, X_p, and Z_p

$R_p = 4.03\ \Omega$, $X_p = 8.02\ \Omega$, $Z_p = 8.98\ \Omega$

2. Find $I = I_R$

$I = I_R = 100/(8.03^2 + 8.02^2)^{1/2}$ A = 8.81 A

3. Find I_L and I_{R_L}

$I_L = 8.81\times 8.98/10.05$ A = 7.87 A; I_{R_L} = 3.96 A

(*b*) 1. For f = 2000 Hz, find R_p, X_p, and Z_p

$R_p = 16.0\ \Omega$, $X_p = 7.98\ \Omega$, $Z_p = 17.9\ \Omega$

2. Find $I = I_R$

$I = I_R = 100/(20^2 + 7.98^2)^{1/2}$ A = 4.64 A

3. Find I_L and I_{R_L}

$I_L = 4.64\times 17.9/40.2$ A = 2.07 A; I_{R_L} = 4.16 A

Note: As $f \to \infty$, $I_R = I_{R_L}$ = 5.00 A

(*c*) 1. For f = 500 Hz, find P_R, P_L, P_{tot}, and P_L/P_{tot}

P_R = 310 W, P_L = 314 W; P_{tot} = 624 W; P_L/P_{tot} = 50.3%

2. Repeat above for f = 2000 Hz

P_R = 86.1 W, P_L = 346 W; P_{tot} = 432 W; P_L/P_{tot} = 80.0%

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38 •• An ac source $\mathcal{E}_1 = (20\text{ V})\cos(2\pi ft)$ in series with a battery $\mathcal{E}_2$ = 16 V is connected to a circuit consisting of resistors $R_1 = 10\ \Omega$ and $R_2 = 8\ \Omega$ and an inductor L = 6 mH (Figure 31-32). Find the power dissipated in R_1 and R_2 if (*a*) f = 100 Hz, (*b*) f = 200 Hz, and (*c*) f = 800 Hz.

We can treat the ac and dc components separately. For the dc component, L acts like a short circuit. For convenience we let $\mathcal{E}_1$ denote the maximum value of the ac emf.

(*a*) 1. Find dc power dissipated in R_1 and R_2

$P_{1dc} = \mathcal{E}_2^2/R_1$ = 25.6 W; P_{2dc} = 32.0 W

2. Find average ac power dissipated in R_1

$P_{1ac} = 1/2\mathcal{E}_1^2/R_1$ = 20 W

3. Find $P_{2ac} = 1/2\mathcal{E}_1^2R_2/Z_2^2$; use Equ. 31-53 for Z

$X_L = 3.77\ \Omega$; $Z_2^2 = 78.2\ \Omega^2$; P_{2ac} = 20.5 W

4. Find the total power; $P = P_{dc} + P_{ac}$

P_1 = 45.6 W, P_2 = 52.5 W

(*b*) Repeat part (*a*). The only difference is that now $X_L = 7.54\ \Omega$ and $Z_2^2 = 121\ \Omega^2$. One obtains P_{2ac} = 13.2 W, and so P_1 = 45.6 W and P_2 = 45.2 W.

(*c*) Repeat part (*a*). Now $X_L = 30.2\ \Omega$ and $Z_2^2 = 974\ \Omega^2$. Then P_1 = 45.6 W, P_{2ac} =1.65 W, and P_2 = 33.65 W

39 •• A 100-V-rms voltage is applied to a series *RC* circuit. The rms voltage across the capacitor is 80 V. What is the voltage across the resistor?

Phasors V_R and V_C are 90° apart; $V_R^2 + V_C^2 = \mathcal{E}^2$ V_R = 60 V rms

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40 •• The circuit shown in Figure 31-33 is called an *RC* high-pass filter because high input frequencies are transmitted with greater amplitude than low input frequencies. (*a*) If the input voltage is $V_{in} = V_0 \cos \omega t$, show that the output voltage is

$$V_{out} = \frac{V_0}{\sqrt{(1/\omega RC)^2 + 1}}$$

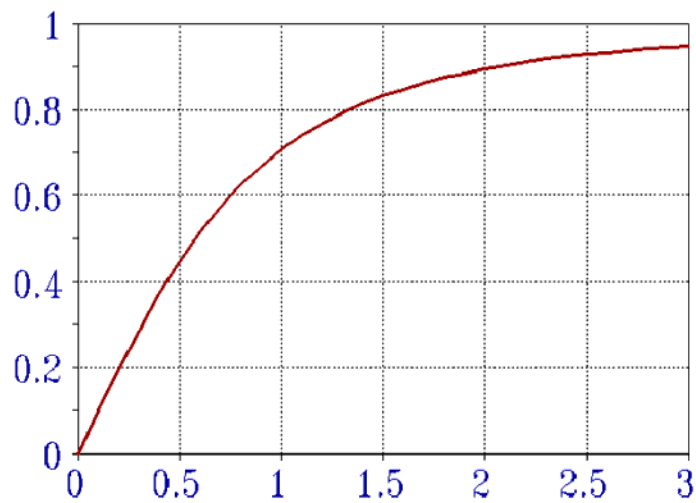
(*b*) At what angular frequency is the output voltage half the input voltage? (*c*) Sketch a graph of V_{out}/V_0 as a function of ω.

The output voltage is $V_{out} = IR$.

$$I = \frac{V_{in}}{Z} = \frac{V_{in}}{\sqrt{R^2 + 1/(\omega C)^2}}, \text{ and}$$

multiplying by *R*, $V_{out} = \dfrac{V_{in}}{\sqrt{1 + (1/\omega RC)^2}}$.

The ratio V_{out}/V_{in} is shown in the figure plotted against ωRC. It is apparent that the output voltage increases and approaches the input voltage as the frequency increases.



41* •• A coil draws 15 A when connected to a 220-V 60-Hz ac line. When it is in series with a 4-Ω resistor and the combination is connected to a 100-V battery, the battery current after a long time is observed to be 10 A. (*a*) What is the resistance in the coil? (*b*) What is the inductance of the coil?

(*a*) For $t \to \infty$, $I_B = \mathcal{E}_B/(R_L + 4.0)$; solve for R_L R_L = 6.0 Ω

(*b*) $Z = \mathcal{E}/I$; $L = \sqrt{Z^2 - R_L^2}/\omega$; $\omega = 377\ \text{s}^{-1}$ Z = 14.7 Ω, L = 35.5 mH

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42 •• Figure 31-34 shows a load resistor R_L = 20 Ω connected to a low-pass filter consisting of a capacitor C = 8 μF and resistor R = 4 Ω. The input voltage is $\mathcal{E} = (100\ \text{V})\cos(2\pi ft)$. Find the rms currents in *R*, *C*, and R_L if (*a*) f = 500 Hz and (*b*) f = 2000 Hz. (*c*) What fraction of the total power delivered by the voltage source is dissipated in the load resistor if the frequency is 500 Hz and if the frequency is 2000 Hz?

We will use the complex numbers method described on pp. 980-981 of the text.

(*a*) 1. Find Z_p for the parallel R_LC group; X_C = 39.8 Ω $1/Z_p = 1/R_L + 1/-iX_C$; $Z_p = -iX_CR_L/(R_L - iX_C)$

2. Multiply numerator & denominator by $R_L + iX_C$ $Z_p = R_LX_C^2/(R_L^2 + X_C^2) - iX_CR_L^2/(R_L^2 + X_C^2)$

3. Find total $Z = R + Z_p$; use numerical values $Z_p = (15.97 - i\,8.02)$ Ω; $Z = (19.97 - i\,8.02)$ Ω,

4. Find $I_{Rrms} = \mathcal{E}_{rms}/Z$; $|Z|$ = 21.52 Ω I_{Rrms} = 3.29 A; $I_R = (4.65\ \text{A})\cos(1000\pi t + 21.9°)$

5. Find $V_{prms} = I_{Rrms} \times |Z_p|$ $|Z_p|$ = 17.87 Ω; V_{prms} = 58.8 V

6. Find $I_{Lrms} = V_{prms}/R_L$ and $I_{Crms} = V_{prms}/X_C$ I_{Lrms} = 2.94 A; I_{Crms} = 1.48 A

7. Find the total power; $P_{tot} = \mathcal{E}_{rms} I_{Rrms} \cos\delta$ $\quad P_{tot} = 216$ W

8. Find $P_L = I_{Lrms}^2 R_L$ $\quad P_L = 173$ W $= 0.80 P_{tot} = 80\%$ of total power

(*b*) Repeat part (*a*) for $f = 2000$ Hz. $X_C = 9.95\ \Omega$; $Z_p = (3.97 - i\,7.97)\ \Omega$; $Z = (7.97 - i\,7.97)\ \Omega$; $|Z| = 11.3\ \Omega$. $I_{Rrms} = 6.26$ A, $I_{Lrms} = 2.79$ A, $I_{Crms} = 1.40$ A, $P_{tot} = 313$ W, $P_L = 156$ W = 50% of total power.

---

43 •• The generator voltage in Figure 31-35 is given by $\mathcal{E} = (100\text{ V})\cos(2\pi ft)$. (*a*) For each branch, what is the amplitude of the current and what is its phase relative to the applied voltage? (*b*) What is the angular frequency ω such that the current in the generator vanishes? (*c*) At this resonance, what is the current in the inductor? What is the current in the capacitor? (*d*) Draw a phasor diagram showing the general relationships between the applied voltage, the generator current, the capacitor current, and the inductor current for the case where the inductive reactance is larger than the capacitive reactance.

(*a*) Use Equs. 31-32 and 31-33 $\quad I_{Lmax} = (25/\omega)$, current lags $\mathcal{E}$ by 90°

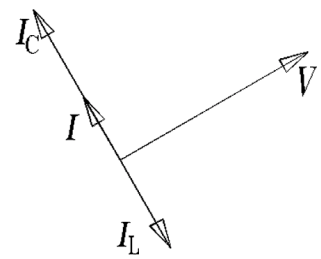
$I_{Cmax} = (2.5\times10^{-3}\omega)$, current leads $\mathcal{E}$ by 90°

(*b*) $I = 0$ if $|I_L| = |I_C|$, i.e., if $\omega = 1/\sqrt{LC}$ $\quad \omega = 100$ rad/s

(*c*) Use Equs. 31-21 and 31-28 $\quad I_L = (0.25\text{ A})\sin(100t)$; $I_C = -(0.25\text{ A})\sin(100t)$

(*d*) The phase diagram is shown on the right.

Here we have used V for the applied voltage.



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44 •• The charge on the capacitor of a series *LC* circuit is given by $Q = (15\ \mu\text{C})\cos(1250t + \pi/4)$ where t is in seconds. (*a*) Find the current as a function of time. (*b*) Find C if $L = 28$ mH. (*c*) Write expressions for the electrical energy U_e, the magnetic energy U_m, and the total energy U.

(*a*) Use the definition $I = dQ/dt$ $\quad I = -(18.75\text{ mA})\sin(1250t + \pi/4)$

(*b*) Use Equ. 31-41; $C = 1/L\omega^2$ $\quad C = 22.9\ \mu$F

(*c*) Use Equs. 29-12 and 30-16 $\quad U_e = (4.92\times10^{-6}\text{ J})\cos^2(1250t + \pi/4)$

$U = U_e + U_m$ $\quad U_m = (4.92\times10^{-6}\text{ J})\sin^2(1250t + \pi/4)$; $U = 4.92\times10^{-6}$ J

45* ••• One method for measuring the compressibility of a dielectric material uses an *LC* circuit with a parallel-plate capacitor. The dielectric is inserted between the plates and the change in resonance frequency is determined as the capacitor plates are subjected to a compressive stress. In such an arrangement, the resonance frequency is 120 MHz when a dielectric of thickness 0.1 cm and dielectric constant $\kappa = 6.8$ is placed between the capacitor plates. Under a compressive stress of 800 atm, the resonance frequency decreases to 116 MHz. Find Young's modulus of the dielectric material.

We shall do this problem for the general case and then substitute numerical values. Let t be the initial thickness of the dielectric. Then $C_0 = \kappa\varepsilon_0 A/t$ and $C_p = \kappa\varepsilon_0 A/(t - \Delta t) = C_0/(1 - \Delta t/t)$ is the capacitance under compression. We have $\omega_0 = 1/(C_0L)^{1/2}$ and $\omega_p = 1/(C_pL)^{1/2}$. $\omega_p/\omega_0 = (1 - \Delta t/t)^{1/2} \cong 1 - \Delta t/2t$ since $\omega_p/\omega_0 = 1 - \varepsilon$, where $\varepsilon << 1$. From the definition of Young's modulus we have $Y = \text{stress}/(\Delta t/t)$.

1. Find $\Delta t/t$ $\quad \Delta t/t = 2\times4/120 = 0.0667$

2. Determine Y; stress $= 808\times10^5$ N/m^2 $\quad Y = 808\times10^5/0.0667 = 1.21\times10^9$ N/m^2

46 ··· Figure 31-36 shows an inductance L and a parallel plate capacitor of width w = 20 cm and thickness 0.2 cm. A dielectric with dielectric constant κ = 4.8 that can completely fill the space between the capacitor plates can be slid between the plates. The inductor has an inductance L = 2 mH. When half the dielectric is between the capacitor plates, i.e., when $x = \frac{1}{2}w$, the resonant frequency of this LC combination is 90 MHz. (*a*) What is the capacitance of the capacitor without the dielectric? (*b*) Find the resonance frequency as a function of x. Let C_i be the initial capacitance with the dielectric and C_0 be the capacitance without the dielectric.

(*a*) 1. Use Equ. 31-41; $C_i = 1/\omega^2 L$ — C_i = 1.56 fF

2. $C_i = C_0[1 + (\kappa - 1)(x/w)]$ (see Problem 25-95) — C_0 = 0.538 fF

(*b*) Use Equ. 31-41; $C(x) = C_0(1 + 19x)$, x in m — $f = \dfrac{1}{2\pi\sqrt{1.08\times10^{-18}(1 - 19x)}}$ Hz

---

47 · True or false:

(*a*) An *RLC* circuit with a high Q factor has a narrow resonance curve.

(*b*) At resonance, the impedance of an *RLC* circuit equals the resistance R.

(*c*) At resonance, the current and generator voltage are in phase.

(*a*) True (*b*) True (*c*) True

---

48 · Does the power factor depend on the frequency?

Yes

49* · Are there any disadvantages to having a radio tuning circuit with an extremely large Q factor?

Yes; the bandwidth must be wide enough to accommodate the modulation frequency.

---

50 · What is the power factor for a circuit that has inductance and capacitance but no resistance?

The power factor is zero.

---

51 · A series *RLC* circuit in a radio receiver is tuned by a variable capacitor so that it can resonate at frequencies from 500 to 1600 kHz. If L = 1.0 μH, find the range of capacitances necessary to cover this range of frequencies.

Use Equ. 31-41; $C = 1/\omega^2 L$ — For 1600 kHz, C = 9.89 nF; for 500 kHz, C = 101 nF

---

52 · (*a*) Find the power factor for the circuit in Example 31-5 when ω = 400 rad/s. (*b*) At what angular frequency is the power factor 0.5?

(*a*) Find $X = X_L - X_C$, δ, and cos δ — $X = -450\ \Omega$; $\delta = \tan^{-1}(450/20) = 87.46°$; $\cos\delta = 0.0444$

(*b*) 1. Find $\tan\delta = X/R = (\omega L - 1/\omega C)/R$ — $\delta = \pm60°$; $\omega L - 1/\omega C = \pm34.64\ \Omega$;

2. Write the quadratic equation for ω — $4\times10^{-6}\omega^2 \pm 69.28\times10^{-6}\omega - 1 = 0$

3. Solve for ω — ω = 491 rad/s, ω = 509 rad/s

53* · An ac generator with a maximum emf of 20 V is connected in series with a 20-μF capacitor and an 80-Ω resistor. There is no inductance in the circuit. Find (*a*) the power factor, (*b*) the rms current, and (*c*) the average power if the angular frequency of the generator is 400 rad/s.

(*a*) $Z = \sqrt{R^2 + 1/\omega^2 C^2}$; power factor = R/Z — $Z = 148\ \Omega$; power factor = 0.539

(*b*) $I = E/Z$; $E = E_{max}/\sqrt{2}$ — I = 14.1/148 A = 0.0956 A

P = 0.731 W

(*c*) $P = I^2R$

---

54 •• Show that the formula $P_{av} = RE_{rms}^2/Z$ gives the correct result for a circuit containing only a generator and (*a*) a resistor, (*b*) a capacitor, and (*c*) an inductor.

(*a*) For $X = 0$, $Z = R$ and $RE_{rms}^2/Z^2 = E_{rms}^2/R = P_{av}$.

(*b*), (*c*) If $R = 0$, then $RE_{rms}^2/Z^2 = 0$, so $P_{av} = 0$, which is correct.

---

55 •• A series *RLC* circuit with $L = 10$ mH, $C = 2\ \mu$F, and $R = 5\ \Omega$ is driven by a generator with a maximum emf of 100 V and a variable angular frequency ω. Find (*a*) the resonant frequency ω_0 and (*b*) I_{rms} at resonance. When $\omega = 8000$ rad/s, find (*c*) X_C and X_L, (*d*) Z and I_{rms}, and (*e*) the phase angle δ.

(*a*) Use Equ. 31-41	$\omega_0 = 7071$ rad/s
(*b*) $I_{rms} = E_{rms}/R$ since $X = 0$ at resonance	$I_{rms} = 14.14$ A
(*c*) Use Equs. 31-25 and 31-31	$X_L = 80\ \Omega$; $X_C = 62.5\ \Omega$
(*d*) Use Equ. 31-53; $I_{rms} = E_{rms}/Z$	$Z = 18.2\ \Omega$; $I_{rms} = 3.89$ A
(*e*) Use Equ. 31-51	$\delta = \tan^{-1}(17.5/5) = 74.1°$

---

56 •• For the circuit in Problem 55, let the generator frequency be $f = \omega/2\pi = 1$ kHz. Find (*a*) the resonance frequency $f_0 = \omega_0/2\pi$, (*b*) X_C and X_L, (*c*) the total impedance Z and I_{rms}, and (*d*) the phase angle δ.

(*a*) See Problem 31-55	$f_0 = 1.125$ kHz
(*b*) Use Equs. 31-25 and 31-31	$X_L = 62.8\ \Omega$; $X_C = 79.6\ \Omega$
(*c*) Use Equ. 31-53; $I_{rms} = E_{rms}/Z$	$Z = 17.5\ \Omega$; $I_{rms} = 3.89$ A
(*d*) Use Equ. 31-51	$\delta = \tan^{-1}(-16.8/5) = -73.4°$

57* •• Find the power factor and the phase angle δ for the circuit in Problem 55 when the generator frequency is (*a*) 900 Hz, (*b*) 1.1 kHz, and (*c*) 1.3 kHz.

(*a*) Find X and Z; $X = \omega L - 1/\omega C$; $\omega = 5655\ \text{s}^{-1}$	$X = -31.9\ \Omega$; $Z = 32.3\ \Omega$; $\cos\delta = 0.155$; $\delta = -81.1°$
(*b*) Repeat part (*a*) with $\omega = 6912\ \text{s}^{-1}$	$X = -3.2\ \Omega$; $Z = 5.94\ \Omega$; $\cos\delta = 0.842$; $\delta = -32.6°$
(*c*) Repeat part (*a*) with $\omega = 8168\ \text{s}^{-1}$	$X = 20.5\ \Omega$; $Z = 21.1\ \Omega$; $\cos\delta = 0.237$; $\delta = 76.3°$

---

58 •• Find (*a*) the Q factor and (*b*) the resonance width for the circuit in Problem 55. (*c*) What is the power factor when $\omega = 8000$ rad/s?

(*a*) Use Equ. 31-59 (see Problem 31-55 for $\omega_0 L$)	$Q = 14.1$
(*b*) Use Equ. 31-60 (see Problem 31-56 for f_0)	$\Delta f = 79.6$ Hz
(*c*) Find $\cos\delta$ (see Problem 55 for δ)	$\cos\delta = 0.274$

---

59 •• FM radio stations have carrier frequencies that are separated by 0.20 MHz. When the radio is tuned to a station, such as 100.1 MHz, the resonance width of the receiver circuit should be much smaller than 0.2 MHz so that adjacent stations are not received. If $f_0 = 100.1$ MHz and $\Delta f = 0.05$ MHz, what is the Q factor for the circuit?

Use Equ. 31-60	$Q = 2002$

---

60 •• A coil is connected to a 60-Hz, 100-V ac generator. At this frequency the coil has an impedance of 10 Ω and a reactance of 8 Ω. (*a*) What is the current in the coil? (*b*) What is the phase angle between the current and

the applied voltage? (*c*) What series capacitance is required so that the current and voltage are in phase? (*d*) What then is the voltage measured across the capacitor?

(*a*) $I = V/Z$ — $I = 10.0$ A

(*b*) $\delta = \cos^{-1}(R/Z) = \sin^{-1}(X/Z)$ — $\delta = 53.1°$; the current lags the voltage

(*c*) $\delta = 0$ at resonance; $X_L = X_C$; find C — $C = 1/\omega X_L = 332\ \mu$F

(*d*) $I = V/R$; $R = Z\cos\delta$, where $Z = 10\ \Omega$; $V_C = IX_C$ — $R = 6\ \Omega$; $I = 16.7$ A; $V_C = V_L = 133$ V

61* •• A 0.25-H inductor and a capacitor C are connected in series with a 60-Hz ac generator. An ac voltmeter is used to measure the rms voltages across the inductor and capacitor separately. The rms voltage across the capacitor is 75 V and that across the inductor is 50 V. (*a*) Find the capacitance C and the rms current in the circuit. (*b*) What would be the measured rms voltage across both the capacitor and inductor together?

(*a*) 1. Find $I = V_L/\omega L$ — $I = 50/(377\times 0.25)$ A $= 0.5305$ A

2. $I/\omega C = V_C$; find C — $C = 0.5305/(75\times 377)$ F $= 18.8\ \mu$F

(*b*) Since $R = 0$, $V = |V_L - V_C|$ — $V = 25$ V

---

62 •• (*a*) Show that Equation 31-51 can be written as

$$\tan\delta = \frac{L(\omega^2 - \omega_0^2)}{\omega R}$$

Find δ approximately at (*b*) very low frequencies and (*c*) very high frequencies.

(*a*) From Equ. 31-51, $\tan\delta = \dfrac{\omega L - 1/\omega C}{R} = \dfrac{\omega^2 L - 1/C}{\omega R} = \dfrac{L(\omega^2 - \omega_0^2)}{\omega R}$.

(*b*) Rewrite $\tan\delta = \omega L/R - 1/\omega RC$. If ω is very small, $\tan\delta \approx -1/\omega RC$ and $\cot\delta = -\omega RC$. Using the expansion $\cot^{-1}x = \pm\pi/2 - x$ for small values of x and recalling that for negative values of the argument the angle approaches $-\pi/2$ as x approaches zero, we obtain $\delta = -\pi/2 + \omega RC$.

(*c*) For large values of ω, $\tan\delta \approx \omega L/R$. We then use the expansion $\tan^{-1}x = \pi/2 - 1/x$, valid for $x >> 1$, and obtain $\delta = \pi/2 - R/\omega L$.

---

63 •• (*a*) Show that in a series RC circuit with no inductance, the power factor is given by

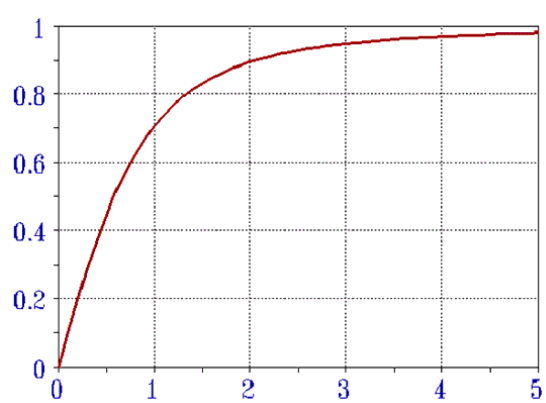
$$\cos\delta = \frac{RC\omega}{\sqrt{1 + (RC\omega)^2}}$$

(*b*) Sketch a graph of the power factor versus ω.

(*a*) Here,

$$\cos\delta = \frac{R}{\sqrt{R^2 + 1/(\omega C)^2}} = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}.$$

(*b*) The graph of $\cos\delta$ as a function of ωRC is shown in the adjacent figure. Here the ordinate is $\cos\delta$ and the abscissa is ωRC.



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64 •• In the circuit in Figure 31-37, the ac generator produces an rms voltage of 115 V when operated at 60 Hz.

What is the rms voltage across points (*a*) *AB*, (*b*) *BC*, (*c*) *CD*, (*d*) *AC*, and (*e*) *BD*?

(*a*) 1. Use Equs. 31-25, 31-25, and 31-53 and $I = \mathcal{E}/Z$	$X_L = 51.65\ \Omega$, $X_C = 106.1\ \Omega$; $Z = 7.39\ \Omega$; $I = 1.56$ A
2. $V_{AB} = IX_L$	$V_{AB} = 80.3$ V
(*b*) $V_{BC} = IR$	$V_{BC} = 77.8$ V
(*c*) $V_{CD} = IX_C$	$V_{CD} = 165$ V
(*d*) V_{AB} and V_{BC} are 90° apart; $V_{AC} = (V_{AB}^2 + V_{BC}^2)^{1/2}$	$V_{AC} = 112$ V
(*e*) $V_{BD} = (V_{BC}^2 + V_{CD}^2)^{1/2}$	$V_{BD} = 182$ V

65* •• A variable-frequency ac generator is connected to a series *RLC* circuit for which $R = 1$ kΩ, $L = 50$ mH, and $C = 2.5\ \mu$F. (*a*) What is the resonance frequency of the circuit? (*b*) What is the *Q* value? (*c*) At what frequencies is the value of the average power delivered by the generator half of its maximum value?

(*a*) $f_0 = 1/2\pi\sqrt{LC}$	$f_0 = 450$ Hz
(*b*) $Q = \omega_0 L/R = \dfrac{\sqrt{L/C}}{R}$	$Q = 0.141$

(*c*) When $\omega = \omega_0$, *P* is a maximum: $P = \mathcal{E}^2/R$. When $\omega \neq \omega_0$, *P* is given by Equ. 31-58. Set Equ. 31-58 equal to $\mathcal{E}^2/2R$. This gives $R^2\omega^2 = L^2(\omega^2 - \omega_0^2)^2$, or $L^2\omega^4 - (2L^2\omega_0^2 + R^2)\omega^2 + L^2\omega_0^2 = 0$. The quadratic equation has the solution

$$\omega^2 = \frac{(2L^2\omega_0^2 + R^2) \pm R\sqrt{4L^2\omega_0^2 + R^2}}{2L^2}.$$

Substituting appropriate numerical values one obtains $\omega^2 = 4.158\times10^8\ \text{s}^{-2}$ and $\omega^2 = 1.537\times10^5\ \text{s}^{-2}$. The corresponding (positive) frequencies are 3.25 kHz and 62.4 Hz.

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66 •• An experimental physicist wishes to design a series *RLC* circuit with a *Q* value of 10 and a resonance frequency of 33 kHz. She has a 45-mH inductor with negligible resistance. What values for the resistance *R* and capacitance *C* should she use?

1. Determine $C = 1/\omega^2 L$	$C = 0.517$ pF
2. Use Equ. 31-59 to find *R*	$R = 933\ \Omega$

---

67 •• When an *RLC* series circuit is connected to a 120-V-rms, 60-Hz line, the current is $I_{rms} = 11.0$ A and the current leads the voltage by 45°. (*a*) Find the power supplied to the circuit. (*b*) What is the resistance? (*c*) If the inductance $L = 0.05$ H, find the capacitance *C*. (*d*) What capacitance or inductance should you add to make the power factor 1?

(*a*) Use Equ. 31-57	$P = 933$ W
(*b*) Use Equ. 31-56; $R = P/I^2$	$R = 7.71\ \Omega$
(*c*) Use Equ. 31-53 and $Z^2 = \mathcal{E}^2/I^2$; note that since *I* leads $\mathcal{E}$, $X_C > X_L$.	$(X_L - X_C)^2 + 59.46\ \Omega^2 = 119\ \Omega^2$; $X_L = 18.85\ \Omega$; $X_C - 18.85\ \Omega = 7.72\ \Omega$; $X_C = 26.57\ \Omega$; $C = 99.8\ \mu$F
(*d*) $\delta = 0$ if $X_L = X_C$; add inductance (in series) add capacitance (in parallel)	$L_{tot} = 70.5$ mH; add 20.5 mH in series $C_{tot} = 140.7\ \mu$F; add 40.9 μF in parallel

---

68 •• A series *RLC* circuit is driven at a frequency of 500 Hz. The phase angle between the applied voltage and

current is determined from an oscilloscope measurement to be $\delta = 75°$. If the total resistance is known to be 35 Ω and the inductance is 0.15 H, what is the capacitance of the circuit?

Use Equs. 31-51, 31-25, and 31-31 to find X_C and C 471.2 Ω – X_C = 130.6 Ω; X_C = 340.4 Ω; C = 0935 μF

69* ·· A series *RLC* circuit with R = 400 Ω, L = 0.35 H, and C = 5 μF is driven by a generator of variable frequency f. (*a*) What is the resonance frequency f_0? Find f and f/f_0 when the phase angle δ is (*b*) 60°, and (*c*) –60°.

(*a*) $f_0 = 1/2\pi\sqrt{LC}$ f_0 = 120 Hz

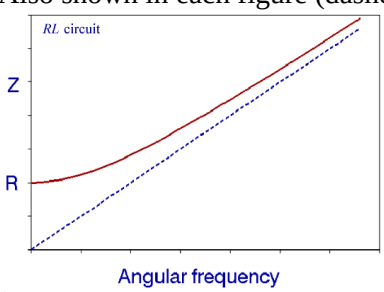
(*b*), (*c*) From Equ. 31-51, $R \tan\delta = \omega L - 1/\omega C$; solve for ω with $\delta = +60°$ and $\delta = -60°$ $0.35\omega^2 - 693\omega - 2\times10^5 = 0; \omega = 2.24\times10^3\ s^{-2}$; f = 356 Hz and for $\delta = -60°$, f = 40.7 Hz

List f/f_0 for the two cases $\delta = 60°$, f/f_0 = 2.96; $\delta = -60°$, f/f_0 = 0.338 = 1/2.96

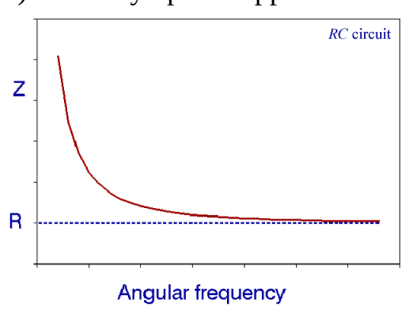
---

70 ·· Sketch the impedance Z versus ω for (*a*) a series *LR* circuit, (*b*) a series *RC* circuit, and (*c*) a series *RLC* circuit.

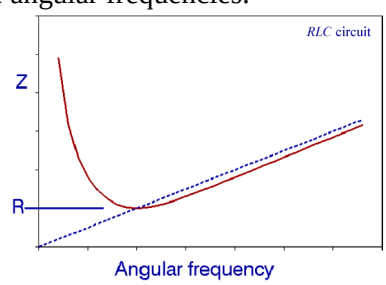
The impedance for the three circuits as functions of the angular frequency is shown in the three figures below. Also shown in each figure (dashed line) is the asymptotic approach for large angular frequencies.



RL circuit



RC circuit



RLC circuit

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71 ·· Given the circuit shown in Figure 31-38, (*a*) find the power loss in the inductor. (*b*) Find the resistance r of the inductor. (*c*) Find the inductance L.

(*a*) 1. Find the current; $I = V_R/R$ I = 1 A

2. Write the voltage across the inductor *90* $V = \sqrt{V_r^2 + V_L^2}$

3. Write total voltage drop and solve for V_r *110* $V = \sqrt{V_L^2 + (50 + V_r)^2}$; V_r = 15 V

4. Find power dissipated in r; $P_r = IV_r$ P_r = 15 W

(*b*) $r = V_r/I$ r = 15 Ω

(*c*) Find $V_L = I\omega L$ and solve for L V_L = 88.7 V; L = 0.235 H

---

72 ·· Show that Equation 31-52 can be written as

$$I_{max} = \omega E/\sqrt{L^2(\omega^2 - \omega_0^2)^2 + \omega^2 R^2}$$

From Problem 31-62 and the definition of tan δ we have $\omega X = \omega L(\omega^2 - \omega_0^2)$. The impedance of the circuit times ω is then $\omega Z = \sqrt{\omega^2 L^2(\omega^2 - \omega_0^2) + \omega^2 R^2}$ and from $I_{max} = E_{max}/Z$ we obtain the result stated in the problem.

73* ·· In a series *RLC* circuit, X_C = 16 Ω and X_L = 4 Ω at some frequency. The resonance frequency is $\omega_0 = 10^4$

rad/s. (*a*) Find L and C. If $R = 5\ \Omega$ and $\mathcal{E}_{max} = 26$ V, find (*b*) the Q factor and (*c*) the maximum current.

(*a*) 1. Write the expressions for the known data — $LC = 10^{-8}\ s^2$; $\omega L = 4\ \Omega$, $1/\omega C = 16\ \Omega$; $L/C = 64$ H/F

2. Solve for C and L — $C = 12.5\ \mu F$; $L = 0.8$ mH

(*b*) $Q = \dfrac{\sqrt{L/C}}{R}$ — $Q = 1.6$; $I_{max} = 5.2$ A

(c) $I_{max} = \mathcal{E}_{max}/Z$ — $Z = \sqrt{25 + 144}\ \Omega = 13\ \Omega$; $I_{max} = 2.0$ A

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74 •• In a series RLC circuit connected to an ac generator whose maximum emf is 200 V, the resistance is 60 Ω and the capacitance is 8.0 μF. The inductance can be varied from 8.0 mH to 40.0 mH by the insertion of an iron core in the solenoid. The angular frequency of the generator is 2500 rad/s. If the capacitor voltage is not to exceed 150 V, find (*a*) the maximum current and (*b*) the range of inductance that is safe to use.

(*a*) $I_{max} = V_{Cmax}/\omega C$ — $I_{max} = 3.00$ A

(*b*) 1. $I_{max} = \mathcal{E}_{max}/Z$; $Z^2 = \mathcal{E}_{max}^2/I_{max}^2$ — $4444\ \Omega^2 = 3600\ \Omega^2 + (2500L - 50)^2\ \Omega^2$

2. Solve for L — $L = (0.02 \pm 0.0116)$ H; $L > 31.6$ mH and $L < 8.4$ mH

3. Specify the ranges for L — 8.0 mH < L < 8.4 mH and 31.6 mH < L < 40.0 mH

---

75 •• A certain electrical device draws 10 A rms and has an average power of 720 W when connected to a 120-V-rms, 60-Hz power line. (*a*) What is the impedance of the device? (*b*) What series combination of resistance and reactance is this device equivalent to? (*c*) If the current leads the emf, is the reactance inductive or capacitive?

(*a*) $Z = \mathcal{E}/I$ — $Z = 12\ \Omega$

(*b*) Use Equs. 31-56 and 31-53 — $R = 7.2\ \Omega$, $X = 9.6\ \Omega$

(*c*) Current leads emf: see Equ. 31-33 — The reactance is capacitive

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76 •• A method for measuring inductance is to connect the inductor in series with a known capacitance, a known resistance, an ac ammeter, and a variable-frequency signal generator. The frequency of the signal generator is varied and the emf is kept constant until the current is maximum. (*a*) If $C = 10\ \mu F$, $\mathcal{E}_{max} = 10$ V, $R = 100\ \Omega$, and I is maximum at $\omega = 5000$ rad/s, what is L? (*b*) What is I_{max}?

(*a*) Use Equ. 31-41; $L = 1/\omega^2 C$ — $L = 4.0$ mH

(*b*) At resonance $X = 0$; $I = \mathcal{E}/R$ — $I_{max} = 0.10$ A

77* •• A resistor and a capacitor are connected in parallel across a sinusoidal emf $\mathcal{E} = \mathcal{E}_{max} \cos \omega t$ as shown in Figure 31-39. (*a*) Show that the current in the resistor is $I_R = (\mathcal{E}_{max}/R) \cos \omega t$. (*b*) Show that the current in the capacitor branch is $I_C = (\mathcal{E}_{max}/X_C) \cos (\omega t + 90°)$. (*c*) Show that the total current is given by $I = I_R + I_C = I_{max} \cos (\omega t + \delta)$, where $\tan \delta = R/X_C$ and $I_{max} = \mathcal{E}_{max}/Z$ with $Z^{-2} = R^{-2} + X_C^{-2}$.

(*a*) From Ohm's law, $I_R(t) = V(t)/R$. Here $V(t) = \mathcal{E}(t) = \mathcal{E}_{max} \cos \omega t$, so $I_R(t) = (\mathcal{E}_{max}/R) \cos \omega t$.

(*b*) For the capacitor, $V_C(t) = \mathcal{E}(t)$ and $V_C(t) = q(t)/C$; consequently, $d\mathcal{E}/dt = d(q(t)/C)/dt = I_C(t)/C$. $d\mathcal{E}/dt = -\mathcal{E}_{max}\, \omega \sin \omega t = \mathcal{E}_{max}\, \omega \cos (\omega t + 90^o)$. Hence, $I_C(t) = (\mathcal{E}_{max}/X_C) \cos (\omega t + 90^o)$, where $X_C = 1/\omega C$.

(*c*) From Kirchhoff's law, $I = I_R + I_C = \mathcal{E}_{max}[(1/R) \cos \omega t - (1/X_C) \sin \omega t]$. If we write $I = I_{max} \cos (\omega t + \delta)$ and use the trigonometric identity for $\cos (\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$, $I = I_{max} (\cos \omega t \cos \delta - \sin \omega t \sin \delta)$. Comparing this expression with I as given in terms of R and X_C, we see that $\tan \delta = R/X_C$. The current is given by $I_{max}^2 = I_{max}^2 \cos^2 \delta + I_{max}^2 \sin^2 \delta = \mathcal{E}_{max}^2 (1/R^2 + 1/X_C^2) = \mathcal{E}_{max}^2/Z^2$. So $Z^{-2} = R^{-2} + X_C^{-2}$.

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78 •• The impedances of motors, transformers, and electromagnets have inductive reactance. Suppose that the phase angle of the total impedance of a large industrial plant is 25° when the plant is under full operation and using 2.3 MW of power. The power is supplied to the plant from a substation 4.5 km from the plant; the 60 Hz rms line voltage at the plant is 40,000 V. The resistance of the transmission line from the substation to the plant is 5.2 Ω. The cost per kilowatt-hour is 0.07 dollars. The plant pays only for the actual energy used. (*a*) What are the resistance and inductive reactance of the plant's total load? (*b*) What is the current in the power lines and what must be the rms voltage at the substation to maintain the voltage at the plant at 40,000 V? (*c*) How much power is lost in transmission? (*d*) Suppose that the phase angle of the plant's impedance were reduced to 18° by adding a bank of capacitors in series with the load. How much money would be saved by the electric utility during one month of operation, assuming the plant operates at full capacity for 16 h each day? (*e*) What must be the capacitance of this bank of capacitors?

(*a*) 1. Use Equ. 31-57; $I = P/(E\cos\delta)$	I = 63.44 A
2. $Z = E/I$; $R = Z\cos\delta$, $X = Z\sin\delta$	R = 571 Ω, X = 266 Ω
(*b*) Find Z_{tot}; $E_{sub} = Z_{tot}I$	Z_{tot} = 634.6 Ω; I = 63.4 A, E_{sub} = 40260 V
(*c*) $P_{trans} = I^2R_{trans}$	P_{trans} = 20.9 kW
(*d*) 1. We assume P = 2.3 MW; find I	I = 60.46 A
2. Find P_{trans}	P_{trans} = 19 kW
3. Find $\Delta P_{trans}\Delta t$; $\Delta t = 30\times16$ h = 480 h	$\Delta P_{trans}\Delta t$ = 912 kWh; Money saved = $63.84
(*e*) 1. Determine X_C; assume constant R and X_L	$(266 - X_C)$ Ω = (571Ω) tan 18°; X_C = 80.5 Ω
2. Find $C = 1/\omega X_C$	C = 33 μF

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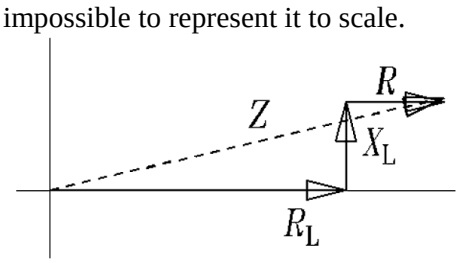
79 •• In the circuit shown in Figure 31-40, R = 10 Ω, R_L = 30 Ω, L = 150 mH, and C = 8 μF; the frequency of the ac source is 10 Hz and its amplitude is 100 V. (*a*) Using phasor diagrams, determine the impedance of the circuit when switch S is closed. (*b*) Determine the impedance of the circuit when switch S is open. (*c*) What are the voltages across the load resistor R_L when switch S is closed and when it is open? (*d*) Repeat parts (*a*), (*b*), and (*c*) with the frequency of the source changed to 1000 Hz. (*e*) Which arrangement is a better low-pass filter, S open or S closed?

(*a*) 1. Determine X_C and X_L: $X_C = 2.00\times10^3$ Ω, X_L = 9.42 Ω

2. With L shorted, X_L = 0; since $X_C >> R_L$, the impedance is very nearly equal to R_L = 30 Ω. From Problem 31-77, $\delta = \tan^{-1}(R/X_C) = 0.86^o$ and Z = 30 Ω (29.997 Ω). The total impedance of the circuit is 40 Ω and is entirely resistive. We show no phasor diagram because it is impossible to represent it to scale.

(*b*) Again, X_C>>Z for this part of the circuit, so the total impedance is effectively Z =$(40^2 + 9.42^2)^{1/2}$ Ω=41.1 Ω. The phasor diagram for this case is shown to the right.



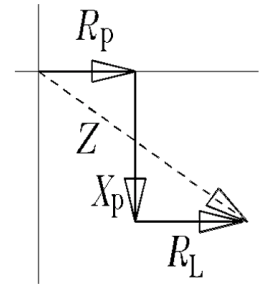
(*c*) For S open, $V_L = ER_L/(R+R_L)$ = (75 V) cos 20π*t*.

For S closed, $V_L = ER_L/Z$ = (73 V) cos (20π*t* – 13.25°)

(*d*) Now X_C = 20Ω and X_L = 942 Ω.

1. With *S* closed, X_L = 0 and the impedance of the R_L and *C* combination is given by the expression derived in Problem

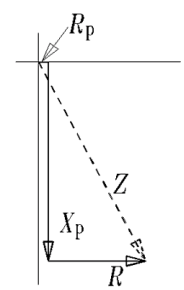
31-42: $Z_p = (9.23 - i\,13.85)\ \Omega = 16.64\ \Omega$, and $\delta_p = -56.3°$. The total impedance is then $Z = (19.23 - i\,13.85)\ \Omega = 23.7\ \Omega$, and $\delta = -35.8°$. The phasor diagram for this circuit is shown to the right.



2. With S open, we determine Z_p using the complex numbers method. Proceeding as in Problem 31-42, we find

$$Z_p = \frac{R_L X_C^2 - i X_C [R_L^2 + X_L (X_L - X_C)]}{R_L^2 + (X_L - X_C)^2} = (0.442 - i\,20.43)\ \Omega$$

or $Z_p = 20.43\ \Omega$, and $\delta_p = -88.8°$. Note that, as expected for the parallel arrangement with $X_C < X_L$, the impedance is capacitive.
The total impedance of the circuit is then
$Z = (10.44 - i\,20.43)\ \Omega = 22.95\ \Omega$, with $\delta = -62.9°$.
The phasor diagram for this circuit is shown to the right.



3. With S closed, $V_L = \mathcal{E}_{max} Z_p/Z = (70.2\text{ V}) \cos(2000t - 17.8°)$.
With S open , $V_p = \mathcal{E}_{max} Z_p/Z = (86.2\text{ V}) \cos(\omega t - 25.9°)$. The current in the RL branch has the magnitude $V_p/(R_L^2 + X_L^2)^{1/2} = 0.0915$ A and lags V_p by 88.2°. We now find that the load voltage is $V_L = (2.74\text{ V}) \cos(2000t + 62.3°)$.

(*e*) The load voltage at the higher frequency is much more attenuated with S open, while opening S does not reduce the low frequency load voltage significantly. Therefore S open is the better arrangement for a low-pass filter.

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80 •• In the circuit shown in Figure 31-41, $R_1 = 2\ \Omega$, $R_2 = 4\ \Omega$, $L = 12$ mH, $C = 30\ \mu$F, and $\mathcal{E} = (40\text{ V}) \cos(\omega t)$. (*a*) Find the resonance frequency. (*b*) At the resonance frequency, what are the rms currents in each resistor and the rms current supplied by the source emf?

(*a*) 1. Find Z^{-1} using the complex numbers method $\frac{1}{Z} = \frac{18\times10^{-10}\omega^2 + i\,3\times10^{-5}\omega}{1 + 36\times10^{-10}\omega^2} + \frac{4 - i\,1.2\times10^{-2}\omega}{16 + 1.44\times10^{-4}\omega^2}$

2. At resonance the complex part of $Z^{-1} = 0$; solve the resulting equation for ω_0. $(3\times10^{-5}\omega_0)(16 + 1.44\times10^{-4}\omega_0^2) = (1.2\times10^{-2}\omega_0)(1 + 36\times10^{-10}\omega_0^2)$; $\omega_0 = 1675$ rad/s

(*b*) 1. Find Z_C at resonance — $Z_C = (2 - i\,19.9)\ \Omega = 20\ \Omega$, $\delta = -84.3°$
2. Find $I_C = \mathcal{E}/Z_C$ — $I_{Crms} = 1.41$ A, $\delta_C = 84.3°$
3. Find Z_L at resonance — $Z_L = (4 + i\,20.1)\ \Omega = 20.5\ \Omega$, $\delta = 78.7°$
4. Find $I_L = \mathcal{E}/Z_L$ — $I_{Lrms} = 1.38$ A, $\delta_L = -78.7°$
5. Find $I_{rms} = I_{Lrms} \cos\delta_L + I_{Crms} \cos\delta_C$ — $I_{rms} = 0.411$ A

81* ·· For the circuit in Figure 31-23, derive an expression for the Q of the circuit, assuming the resonance is sharp.

Q is defined as $\omega_0/\Delta\omega$, where $\Delta\omega$ is the width of the resonance at half maximum. The currents in the three circuit elements are $I_C = V/X_C = \omega CV$, $I_L = V/\omega L$, and $I_R = V/R$, with I_C leading V and I_L lagging V by 90°. The total current is therefore $I = V\sqrt{(1/R)^2 + (\omega C - 1/\omega L)^2} = (V/R)\sqrt{1 + R^2(\omega C - 1/\omega L)^2}$. At resonance, the reactive term is zero and $I_0 = V/R$. The stored energy per cycle will be at half-maximum when $R(\omega C - 1/\omega L) = \pm 1$. This gives quadratic equations for ω with two solutions ω_+ and ω_- whose difference is $\Delta\omega = 1/RC$. Using $\omega_0 = 1/\sqrt{LC}$ and $Q = \omega_0/\Delta\omega$ one obtains $Q = R\sqrt{C/L}$.

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82 ·· For the circuit in Figure 31-23, L = 4 mH. (*a*) What capacitance C will result in a resonance frequency of 4 kHz? (*b*) When C has the value found in (*a*), what should be the resistance R so that the Q of the circuit is 8?

(*a*) Use Equ. 31-41; $C = 1/L\omega^2$ — $C = 0.396\ \mu$F

(*b*) From Problem 31-81, $Q = R\sqrt{C/L}$ — $R = 804\ \Omega$

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83 ·· If the capacitance of C in Problem 82 is reduced to half the value found in Problem 82, what then are the resonance frequency and the Q of the circuit? What should be the resistance R to give Q = 8?

1. $\omega_0 \propto 1/C^{1/2}$; $Q \propto C^{1/2}$ — f_0 = 5657 Hz; Q = 5.66
2. $R = Q\sqrt{L/C}$ — R = 1.14 kΩ

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84 ·· A series circuit consists of a 4.0-nF capacitor, a 36-mH inductor, and a 100-Ω resistor. The circuit is connected to a 20-V ac source whose frequency can be varied over a wide range. (*a*) Find the resonance frequency f_0 of the circuit. (*b*) At resonance, what is the rms current in the circuit and what are the rms voltages across the inductor and capacitor? (*c*) What is the rms current and what are the rms voltages across the inductor and capacitor at $f = f_0 + \frac{1}{2}\Delta f$, where Δf is the width of the resonance?

(*a*) Use Equ. 31-41; $f = \omega/2\pi$ — f_0 = 13.3 kHz

(*b*) 1. At resonance $Z = R$; $I = \mathcal{E}/R$ — I = 0.2 A

2. $V_L = \omega_0 LI = I(L/C)^{1/2}$, $V_C = V_L$ — $V_L = V_C$ = 600 V

(*c*) 1. Use Equs. 31-59 and 31-60; $\Delta f = R/2\pi L$ — $\Delta\omega$ = 2.78 krad/s; ω = 84.7 krad/s

2. Find Z; $X_L = \omega L$, $X_C = 1/\omega C$ — X_L = 3.05 kΩ, X_C = 2.95 kΩ; Z = 141 Ω

3. $I = \mathcal{E}/Z$; $V_L = IX_L$, $V_C = IX_C$ — I = 0.141 A; V_L = 431 V, V_C = 417 V

85* ·· Repeat Problem 84 with the 100-Ω resistor replaced by a 40-Ω resistor.

(*a*) $f_0 = (1/2\pi)\sqrt{1/LC}$ — f_0 = 13.26 kHz

(*b*) At $f = f_0$, $I = \mathcal{E}/R$; $V_L = \omega_0 LI$; $V_C = V_L$ — I = 0.5 A; $V_L = V_C$ = 1.50 kV

(*c*) $\frac{1}{2}\Delta f = f_0/2Q = f_0R/2\omega_0 L$; find $f_0 + \frac{1}{2}\Delta f$ — $f_0 + \frac{1}{2}\Delta f = f_0(1 + R/2\omega_0 L)$ = 13.26(1 + 0.0067) kHz

$X = R$, so $I = I_0/\sqrt{2}$; $V_L = IX_L$, $V_C = IX_C$ — I = 0.354 A; V_L = 1068 V, V_C = 1055 V

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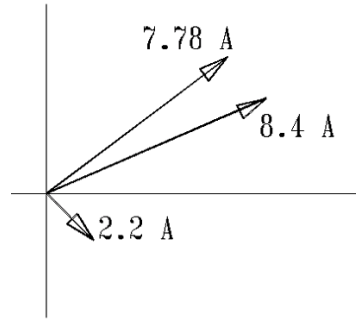
86 ··· In the parallel circuit shown in Figure 31-42, V_{max} = 110 V. (*a*) What is the impedance of each branch? (*b*) For each branch, what is the current amplitude and its phase relative to the applied voltage? (*c*) Give the current phasor diagram, and use it to find the total current and its phase relative to the applied voltage.

(*a*) Find Z_L and Z_C; use Equ. 31-53 — Z_L = 50 Ω, δ_L = 37°; Z_C = 14.1 Ω, δ_C = 45°

(*b*) $I = V/Z$ — I_L = 2.2 A, 37° lagging; I_C = 7.8 A, 45° leading

(*c*) The currents are shown on the adjoining phasor diagram. From this diagram one finds that the total current is 8.4 A and leads the applied voltage by 30°.



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87 ··· (*a*) Show that Equation 31-51 can be written as

$$\tan\delta = \frac{Q(\omega^2 - \omega_0^2)}{\omega\,\omega_0}$$

(*b*) Show that near resonance

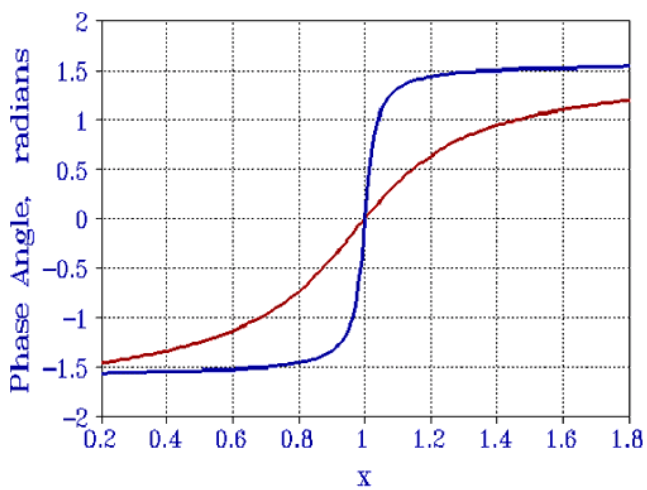
$$\tan\delta \approx \frac{2Q(\omega - \omega_0)}{\omega}$$

(*c*) Sketch a plot of δ versus x, where $x = \omega/\omega_0$, for a circuit with high Q and for one with low Q.

(*a*) From Problem 31-42, $\tan\delta = (L/\omega R)(\omega^2 - \omega_0^2)$. From Equ. 31-59 $Q/\omega_0 = L/R$ and so $\tan\delta = Q(\omega^2 - \omega_0^2)/\omega\omega_0$.

(*b*) Near resonance $\omega^2 - \omega_0^2 = (\omega + \omega_0)(\omega - \omega_0) \approx 2\omega_0\Delta\omega$ and $\tan\delta \approx 2Q\Delta\omega/\omega_0$.

A plot of δ versus $x = \omega/\omega_0$ is shown.



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88 ··· Show by direct substitution that the current given by Equation 31-50 with δ and I_{max} given by Equations 31-51 and 31-52, respectively, satisfies Equation 31-49. (*Hint*: Use trigonometric identities for the sine and cosine of the sum of two angles, and write the equation in the form

$$A \sin \omega t + B \cos \omega t = 0$$

Since this equation must hold for all times, $A = 0$ and $B = 0$.)

Begin by rewriting Equ. 31-49 in terms of the current. $L(dI/dt) + RI + (1/C)\int I dt = E_{max}\cos\omega t$. Let $I = I_{max}\cos(\omega t - \delta)$. Then $dI/dt = -\omega I_{max}\sin(\omega t - \delta)$ and $\int I dt = (I_{max}/\omega)\sin(\omega t - \delta)$. With these substitutions

the current equation reads $[-X_L \sin(\omega t - \delta) + R\cos(\omega t - \delta) + X_C \sin(\omega t - \delta)] = (\mathcal{E}_{max}/I_{max})\cos\omega t = Z\cos\omega t$, where $X_L = \omega L$, $X_C = 1/\omega C$, and $Z = \mathcal{E}_{max}/I_{max}$. Now use the identities $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ and $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ and collect the terms in $\sin\omega t$ and $\cos\omega t$. The coefficients of $\sin\omega t$ and of $\cos\omega t$ must be equal to zero. Thus, $-X_L\cos\delta + R\sin\delta + X_C\cos\delta = 0$ and $X_L\sin\delta + R\cos\delta - X_C\sin\delta = Z$. The first of these equation gives Equ. 31-51. The second equation we rewrite as $(X_L - X_C)\tan\delta + R = Z/\cos\delta$. This equation is satisfied if Z is given by Equ. 31-53.

89* ··· An ac generator is in series with a capacitor and an inductor in a circuit with negligible resistance. (*a*) Show that the charge on the capacitor obeys the equation

$$L\frac{d^2Q}{dt^2} + \frac{Q}{C} = \mathcal{E}_{max}\cos\omega t$$

(*b*) Show by direct substitution that this equation is satisfied by $Q = Q_{max}\cos\omega t$ if

$$Q_{max} = -\mathcal{E}_{max}/[L(\omega^2 - \omega_0^2)]$$

(*c*) Show that the current can be written as $I = I_{max}\cos(\omega t - \delta)$, where

$$I_{max} = \omega\mathcal{E}_{max}/(L\,|\omega^2 - \omega_0^2|) = \mathcal{E}_{max}/|X_L - X_C|$$

and $\delta = -90°$ for $\omega < \omega_0$ and $\delta = 90°$ for $\omega > \omega_0$.

(*a*) From Kirchhoff's law, $L(dI/dt) + Q/C = \mathcal{E} = \mathcal{E}_{max}\cos\omega t$. But $I = dQ/dt$, so $L\dfrac{d^2Q}{dt^2} + \dfrac{Q}{C} = \mathcal{E}_{max}\cos\omega t$.

(*b*) If $Q = Q_{max}\cos\omega t$ then $d^2Q/dt^2 = -\omega^2 Q$. So the result of (*a*) can be written $Q(1/C - L\omega^2) = \mathcal{E}$, and dividing both sides by L and recalling that $1/LC = \omega_0^2$, one obtains $Q_{max} = \mathcal{E}_{max}/[L(\omega_0^2 - \omega^2)]$.

(*c*) $I = dQ/dt = -\omega Q_{max}\sin\omega t = [(\omega\mathcal{E}_{max}/L)/(\omega^2 - \omega_0^2)]\sin\omega t$. Let $I_{max} = [(\omega\mathcal{E}_{max}/L)/|\omega^2 - \omega_0^2|] = \mathcal{E}_{max}/|X_L - X_C|$. Then if $\omega > \omega_0$, $I = I_{max}\sin\omega t = I_{max}\cos(\omega t - \delta)$, and if $\omega < \omega_0$, $I = -I_{max}\sin\omega t = I_{max}\cos(\omega t + \delta)$, where $\delta = -90°$.

90 ··· Figure 31-19 shows a plot of average power P_{av} versus generator frequency ω for an *RLC* circuit with a generator. The average power P_{av} is given by Equation 31-58. The "full width at half-maximum" $\Delta\omega$ is the width of the resonance curve between the two points where P_{av} is one-half its maximum value. Show that, for a sharply peaked resonance, $\Delta\omega \approx R/L$ and, hence, that $Q \approx \omega_0/\Delta\omega$ in this case (Equation 31-60). [*Hint*: At resonance, the denominator of the expression on the right of Equation 31-58 is ω^2R^2. The half-power points will occur when the denominator is twice the value near resonance, that is, when $L^2(\omega^2 - \omega_0^2)^2 = \omega^2R^2 \approx \omega_0^2R^2$. Let ω_1 and ω_2 be the solutions of this equation. For a sharply peaked resonance, $\omega_1 \approx \omega_0$ and $\omega_2 \approx \omega_0$. Then, using the fact that $\omega + \omega_0 \approx 2\omega_0$, one finds that $\Delta\omega = \omega_2 - \omega_1 \approx R/L$.]

From Equ. 31-58 it follows that $P = 1/2P_{res}$ when $(L/R)^2(\omega^2 - \omega_0^2)^2 = \omega^2$. We now replace (L/R) by Q/ω_0 and write $(\omega^2 - \omega_0^2) = (\omega - \omega_0)(\omega + \omega_0) \approx \Delta\omega\,\omega_0$, where $\Delta\omega$ is the width at half maximum. We then have $Q \approx \omega_0/\Delta\omega$.

91 ··· Show by direct substitution that Equation 31-47*b* is satisfied by $Q = Q_0 e^{-Rt/2L}\cos\omega' t$ where $\omega' = \sqrt{(1/LC) - (R/2L)^2}$ and Q_0 is the charge on the capacitor at $t = 0$.

With $Q = Q_0 e^{-Rt/2L}\cos\omega' t$, the first and second derivatives of Q are

$$\frac{dQ}{dt} = -Q_0 e^{-Rt/2L}(\omega)\left(\omega'\sin\omega' t + \frac{R}{2L}\cos\omega' t\right) \text{ and } \frac{d^2Q}{dt^2} = Q_0 e^{-Rt/2L}\left[\left(\frac{R^2}{4L^2} - \omega'^2\right)\cos\omega' t + \frac{R\omega'}{L}\sin\omega' t\right].$$

If these expressions are substituted into Equ. 31–47*b*, the coefficient of $\sin\omega' t$ vanishes. To satisfy the differential equation for all values of t the coefficient of $\cos\omega' t$ must vanish. This requires that $R^2/2L^2 + \omega'^2 - 1/LC = 0$, which yields theresult for ω' given in the problem.

92 ··· (*a*) Compute the current $I = dQ/dt$ from the solution of Equation 31-47*b* given in Problem 91, and show that

$$I = -I_0\left(\sin\omega' t + \frac{R}{2L\omega'}\cos\omega' t\right)e^{-Rt/2L}$$

where $I_0 = \omega' Q_0$. (*b*) Show that this can be written

$$I = -\frac{I_0}{\cos\delta}(\cos\delta\sin\omega' t + \sin\delta\cos\omega' t)e^{-Rt/2L}$$

$$= -\frac{I_0}{\cos\delta}\sin(\omega' t + \delta)e^{-Rt/2L}$$

where $\tan\delta = R/2L\omega'$. When $R/2L\omega'$ is small, $\cos\delta \approx 1$, and $I \approx I_0 \sin(\omega' t + \delta)e^{-Rt/2L}$.

(*a*) With $Q_0 = I_0/\omega'$, $I = \frac{dQ}{dt} = -I_0 e^{-Rt/2L}\left(\sin\omega' t + \frac{R}{2L\omega'}\cos\omega' t\right)$.

(*b*) With $\tan\delta = (R/2L\omega')$ one has $I = -(I_0/\cos\delta)(\cos\delta\sin\omega' t + \sin\delta\cos\omega' t)\, e^{-Rt/2L}$, and using the trigonometric identity for the sum of two angles one obtains $I = -(I_0/\cos\delta)\sin(\omega' t + \delta)\, e^{-Rt/2L}$.

93* ··· One method for measuring the magnetic susceptibility of a sample uses an *LC* circuit consisting of an air-core solenoid and a capacitor. The resonant frequency of the circuit without the sample is determined and then measured again with the sample inserted in the solenoid. Suppose the solenoid is 4.0 cm long, 0.3 cm in diameter, and has 400 turns of fine wire. Assume that the sample that is inserted in the solenoid is also 4.0 cm long and fills the air space. Neglect end effects. (In practice, a test sample of known susceptibility of the same shape as the unknown is used to calibrate the instrument.) (*a*) What is the inductance of the empty solenoid? (*b*) What should be the capacitance of the capacitor so that the resonance frequency of the circuit without a sample is 6.0000 MHz? (*c*) When a sample is inserted in the solenoid, the resonance frequency drops to 5.9989 MHz. Determine the sample's susceptibility.

(*a*) $L = \mu_0 n^2 A\ell$ $\qquad L = (4\pi\times10^{-7})(10^4)^2(\pi\times9\times10^{-6}/4)(4\times10^{-2})$ H = 35.5 μH

(*b*) $4\pi^2 f_0^2 = 1/LC$; $C = (4\pi^2 f_0^2 L)^{-1}$ $\qquad C = 19.8$ pF

(*c*) $df_0/dL = -f_0/2L$; $\Delta f_0/f_0 = -\Delta L/2L$; $\Delta L = \chi L$ $\qquad \chi = -2\Delta f_0/f_0 = 3.67\times10^{-4}$

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94 ··· A concentric cable of cylindrical cross section has an inner conductor of 0.4 cm diameter and an outer conductor of 2.0 cm diameter. Air fills the space between the conductors. (*a*) Find the resonance frequency of a one-meter length of this conductor. (*b*) What length of conductor will result in a resonance frequency of 18 GHz?

(*a*) Use Equs. 31-41, 25-11 and Problem 30-95*b* $\qquad f = \frac{1}{2\pi\sqrt{\mu_0\varepsilon_0}\,\ell} = \frac{c}{2\pi\ell} = 47.7$ MHz /m

(*b*) Use the result from part (*a*) $\qquad \ell = (47.7/18\times10^3)$ m= 2.65 mm

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95 ··· Repeat Problem 94 if the inner and outer conductors of the cable are separated by a dielectric of dielectric constant $\kappa = 5.8$.

(*a*) In the result of Problem 94, replace ε_0 by $\kappa\varepsilon_0$ $\qquad f = (47.7\text{ MHz})/5.8^{1/2} = 19.8$ MHz

(*b*) Proceed as in Problem 94 $\qquad \ell = 1.10$ mm

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96 ··· At what frequency will the voltage across the load resistor of Problem 37 be half the source voltage? We shall use the notation of Problem 37. We first write the condition in terms of the variables: $IZ_p = E/2 = IZ/2$ or $2Z_p = Z$ and $4Z_p^2 = Z^2$. From the expressions for Z_p and Z in terms of R, R_L, and L we then require

$\frac{4R_L^2X_L^2}{R_L^2+X_L^2}=\left(R+\frac{R_LX_L^2}{R_L^2+X_L^2}\right)^2+\frac{X_L^2R_L^4}{(R_L^2+X_L^2)^2}$. Expand (see Problem 97) and collect terms in X_L^4, X_L^2 and X_L^0 using the values given for R and R_L. The resulting equation is a quadratic in X_L^2 with the solution $X_L^2 = 6.25\ \Omega^2$. Thus $X_L = 2.5\ \Omega$ and with $L = 3.2$ mH, the corresponding frequency is $f = 124$ Hz.

97* ··· At what frequency will the voltage across the load resistor of Problem 42 be half the source voltage?

1. Write $Z_p = Z$ of parallel combination of C and R_L	$Z_p = (R_p^2 + X_p^2)^{1/2}$, where $R_p = R_LX_C^2/(R_L^2 + X_C^2)$ and $X_p = -R_L^2X_C/(R_L^2 + X_C^2)$
2. Write $Z_T = Z$ of the circuit	$Z_T = [(R + R_p)^2 + X_p^2]^{1/2}$
3. If $V_p = \mathcal{E}/2$, then we must have $Z_p = Z_T/2$ or $4Z_p^2 = Z_T^2$	$R^2R_L^4 + R^2X_C^4 + R_L^2X_C^4 + 2R^2R_L^2X_C^2 + 2RR_L^3X_C^2 + 2RR_LX_C^4 + R_L^4X_C^2 = 4R_L^2X_C^4 + 4R_L^4X_C^2$
4. Substitute numerical values and solve for X_C^2	$X_C^2 = 6.25\ \Omega^2$; $X_C = 2.5\ \Omega = 1/\omega C$
5. Evaluate $f = \omega/2\pi$ with $C = 8\ \mu$F	$f = 7.96$ kHz

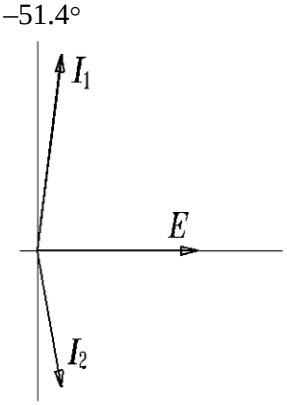
98 ··· (*a*) Find the angular frequency ω for the circuit in Problem 80 such that the magnitude of the reactance of the two parallel branches are equal. (*b*) At that frequency, what is the power dissipation in each of the two resistors?

(*a*) Set $X_L = X_C$; $\omega = (LC)^{-1/2}$	$\omega = 1667$ rad/s; $f = 265$ Hz
(*b*) 1. Find Z_C and Z_L for $\omega = 1667$ rad/s	$Z_C = (2 - i\,20)\ \Omega = 20.1\ \Omega$; $Z_L = (4 + i\,20)\ \Omega = 20.4\ \Omega$
2. $P = 1/2(\mathcal{E}_{max}/Z)^2R$	$P_1 = 3.96$ W; $P_2 = 7.69$ W

99 ··· (*a*) For the circuit of Problem 80, find the angular frequency ω for which the power dissipation in the two resistors is the same. (*b*) At that angular frequency, what is the reactance of each of the two parallel branches? (*c*) Draw a phasor diagram showing the current through each of the two parallel branches. (*d*) What is the impedance of the circuit?

(*a*) 1. $I_1^2R_1 = I_2^2R_2$ or $I_1^2/I_2^2 = R_2/R_1 = Z_L^2/Z_C^2$	$R_2^2 + X_L^2 = 2(R_1^2 + X_C^2)$; $X_L^2 = 2X_C^2 - 8\ \Omega$
2. Solve the resulting quadratic in ω^2	$\omega^2 = 3.90\times10^6$ (rad/s)^2; $\omega = 1975$ rad/s
3. $f = \omega/2\pi$	$f = 314$ Hz
(*b*) Find X_L, X_C, Z_L, and Z_C	$X_L = 23.7\ \Omega$; $Z_L = (4 + i\,23.7)\ \Omega = 24.0\ \Omega$, $\delta_L = 80.4°$
	$X_C = 16.9\ \Omega$; $Z_C = (2 - i\,16.9)\ \Omega = 17.0\ \Omega$, $\delta_C = -83.3°$
(*d*) $Z = Z_LZ_C/(Z_L + Z_C)$	$Z = 45.1\ \Omega$, $\delta = -51.4°$

(*c*) The applied voltage and the currents in the two branches are shown on the adjoining phasor diagram



100 · A transformer is used to change (*a*) capacitance. (*b*) frequency. (*c*) voltage. (*d*) power. (*e*) none of these.
(*c*)

101*· True or false: If a transformer increases the current, it must decrease the voltage.
True

102 ·· An ideal transformer has N_1 turns on its primary and N_2 turns on its secondary. The power dissipated in a load resistance R connected across the secondary is P_2 when the primary voltage is V_1. The current in the primary windings is then (*a*) P_2/V_1. (*b*) $(N_1/N_2)(P_2/V_1)$. (*c*) $(N_2/N_1)(P_2/V_1)$. (*d*) $(N_2/N_1)^2(P_2/V_1)$.
(*a*)

103 · An ac voltage of 24 V is required for a device whose impedance is 12 Ω. (*a*) What should the turn ratio of a transformer be so the device can be operated from a 120-V line? (*b*) Suppose the transformer is accidentally connected reversed, i.e., with the secondary winding across the 120-V line and the 12-Ω load across the primary. How much current will then flow in the primary winding?

(*a*) Use Equ. 31-65 $N_2/N_1 = 1/5$

(*b*) $V_2 = (N_2/N_1)V_1$; $I_2 = V_2/Z_2$; $I_1 = (N_2/N_1)I_2$ $I_1 = 50$ A

104 · A transformer has 400 turns in the primary and 8 turns in the secondary. (*a*) Is this a step-up or step-down transformer? (*b*) If the primary is connected across 120 V rms, what is the open-circuit voltage across the secondary? (*c*) If the primary current is 0.1 A, what is the secondary current, assuming negligible magnetization current and no power loss?

(*a*) It is a step-down transformer.

(*b*) $V_2 = (N_2/N_1)V_1$ $V_2 = 2.4$ V

(*c*) Use Equ. 31-67 $I_2 = 5.0$ A

105*· The primary of a step-down transformer has 250 turns and is connected to a 120-V-rms line. The secondary is to supply 20 A at 9 V. Find (*a*) the current in the primary and (*b*) the number of turns in the secondary, assuming 100% efficiency.

(*a*) $I_1/I_2 = V_2/V_1$ $I_1 = 20(9/120) = 1.5$ A

(*b*) $N_2/N_1 = V_2/V_1$ $N_2 = 250(9/120) = 18.75 \cong 19$ turns

106 · A transformer has 500 turns in its primary, which is connected to 120 V rms. Its secondary coil is tapped at three places to give outputs of 2.5, 7.5, and 9 V. How many turns are needed for each part of the secondary coil?

Use Equ. 31-65 2.5 V, $N_2 = 10.4$; 7.5 V, $N_2 = 31.25$; 9 V, $N_2 = 37.5$

107 · The distribution circuit of a residential power line is operated at 2000 V rms. This voltage must be reduced to 240 V rms for use within the residences. If the secondary side of the transformer has 400 turns, how many turns are in the primary?

Use Equ. 31-65 $N_1 = 3333$

108 ·· An audio oscillator (ac source) with an internal resistance of 2000 Ω and an open-circuit rms output voltage of 12 V is to be used to drive a loudspeaker with a resistance of 8 Ω. What should be the ratio of primary to secondary turns of a transformer so that maximum power is transferred to the speaker? Suppose a

second identical speaker is connected in parallel with the first speaker. How much power is then supplied to the two speakers combined?

Note: In a simple circuit maximum power transfer from source to load requires that the load resistance equals the internal resistance of the source (see Problem 26-150).

1. Find the effective loudspeaker resistance at the primary of the transformer in terms of R_{sp} and N_1/N_2 — $R_{eff} = V_1/I_1 = [V_2 (N_1/N_2)]/[I_2 (N_2/N_1)] = (V_2/I_2)(N_1/N_2)^2$
2. Set $R_{eff} = R_{int}$ and solve for N_1/N_2 — $N_1/N_2 = (2000/8)^{1/2} = 15.8$
3. With $R_{sp} = 4\ \Omega$ find R_{eff}, I_1, and $I_1^2 R_{eff} = P_{sp}$ — $R_{eff} = 1000\ \Omega$; $I_1 = 4$ mA; $P_{sp} = 16$ mW

109*·· One use of a transformer is for *impedance matching*. For example, the output impedance of a stereo amplifier is matched to the impedance of a speaker by a transformer. In Equation 31-67, the currents I_1 and I_2 can be related to the impedance Z in the secondary since $I_2 = V_2/Z$. Using Equations 31-65 and 31-66, show that $I_1 = \mathcal{E}/[(N_1/N_2)^2 Z]$

and, therefore, $Z_{eff} = (N_1/N_2)^2 Z$.

$Z = V_2/I_2$. $V_2 = (N_2/N_1)\mathcal{E}$ and $I_2 = (N_1/N_2)I_1$. So $Z = (N_2/N_1)^2\mathcal{E}/I_1$ or $Z_{eff} = \mathcal{E}/I_1 = Z(N_1/N_2)^2$.

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110 · True or false:

(*a*) Alternating current in a resistance dissipates no power because the current is negative as often as it is positive.

(*b*) At very high frequencies, a capacitor acts like a short circuit.

(*a*) False (*b*) True

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111 · A 5.0-kW electric clothes dryer runs on 240 V rms. Find (*a*) I_{rms} and (*b*) I_{max}. (*c*) Find the same quantities for a dryer of the same power that operates at 120 V rms.

(*a*), (*b*) Use Equs. 31-14 and 31-12 — $I_{rms} = 20.8$ A, $I_{max} = 29.5$ A

(*c*) Multiply results of (*a*) and (*b*) by 2 — $I_{rms} = 41.7$ A, $I_{max} = 58.9$ A

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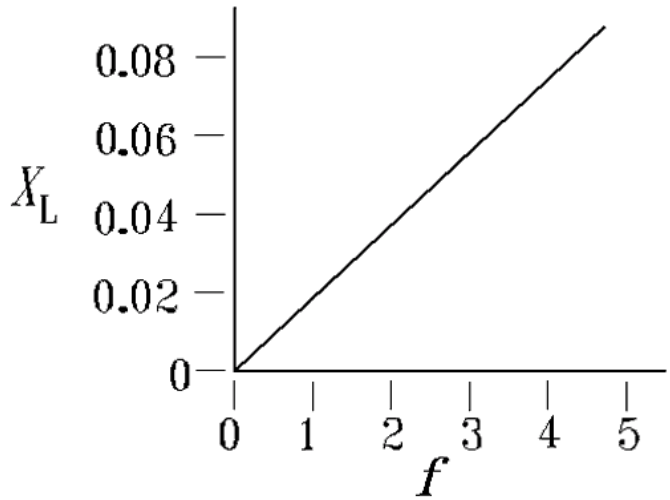
112 · Find the reactance of a 10.0-μF capacitor at (*a*) 60 Hz, (*b*) 6 kHz, and (*c*) 6 MHz.

(*a*), (*b*), (*c*) Use Equ. 31-31 — (*a*) 265 Ω (*b*) 2.65 Ω (*c*) 2.65 mΩ

113*· Sketch a graph of X_L versus f for $L = 3$ mH.

The graph is shown on the right. Here X_L is in Ω and f is in Hz.



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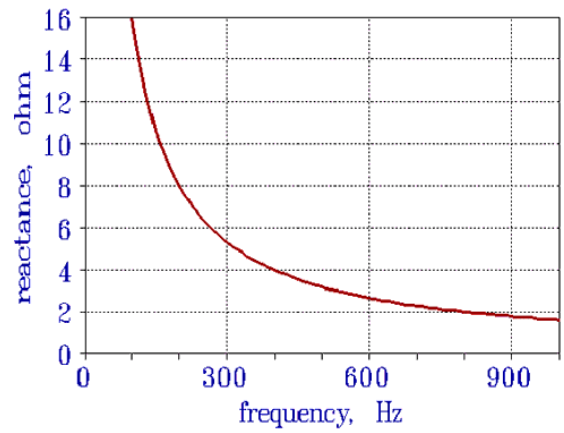
114 · Sketch a graph of X_C versus f for $C = 100\ \mu$F.

The capacitive reactance as a function of frequency is shown in the adjoining figure.



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115 ·· A resistance R carries a current I = (5.0 A) sin 120πt + (7.0 A) sin 240πt. (*a*) What is the rms current? (*b*) If the resistance R is 12 Ω, what is the power dissipated in the resistor? (*c*) What is the rms voltage across the resistor?

(*a*) 1. Find I^2 — $I^2 = [25 \sin^2 (120\pi t) + 70 \sin (120\pi t) \sin (240\pi t) + 49 \sin^2 (240\pi t)]\ \text{A}^2$

2. Determine $[(I^2)_{av}]^{1/2} = I_{rms}$ — $(I^2)_{av} = (12.5 + 24.5)\ \text{A}^2$; $I_{rms} = 6.08$ A

(*b*) Use Equ. 31-56 — $P = 444$ W

(*c*) $V = IR$ — $V_{rms} = 73$ V

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116 ·· Figure 31-43 shows the voltage V versus time t for a "square-wave" voltage. If V_0 = 12 V, (*a*) what is the rms voltage of this waveform? (*b*) If this alternating waveform is rectified by eliminating the negative voltages so that only the positive voltages remain, what now is the rms voltage of the rectified waveform?

(*a*) Note that $-V_0^2 = V_0^2$ — $V_{rms} = V_0 = 12$ V

(*b*) Find $[(V^2)_{av}]^{1/2}$ — $(V^2)_{av} = V_0^2/2$; $V_{rms} = 8.49$ V

117*·· A pulsed current has a constant value of 15 A for the first 0.1 s of each second and is then 0 for the next 0.9

s of each second. (*a*) What is the rms value for this current waveform? (*b*) Each current pulse is generated by a voltage pulse of maximum value 100 V. What is the average power delivered by the pulse generator?

(*a*) $I_{rms} = (\langle I^2 \rangle)^{1/2}$, where < > denotes the time average $\quad I_{rms} = (0.1 \times 225/1.0)^{1/2} = 4.74$ A

(*b*) $P_{av} = I_{rms} V_{rms}$; $V_{rms} = (\langle V^2 \rangle)^{1/2}$ $\quad V_{rms} = 31.6$ V; $P_{av} = 150$ W

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118 •• A circuit consists of two capacitors, a 24-V battery, and an ac voltage connected as shown in Figure 31-44. The ac voltage is given by $E = (20\text{ V}) \cos(120\pi t)$ 0 where t is in seconds. (*a*) Find the charge on each capacitor as a function of time. Assume transient effects have had sufficient time to decay. (*b*) What is the steady-state current? (*c*) What is the maximum energy stored in the capacitors? (*d*) What is the minimum energy stored in the capacitors?

(*a*) $Q = CV$ $\quad Q_1 = [72 + 60 \cos(120\pi t)]\ \mu$C

$Q_2 = [36 + 30 \cos(120\pi t)]\ \mu$C

(*b*) $I = dQ/dt$; $Q = Q_1 + Q_2$ $\quad I = -(33.9\text{ mA}) \sin(120\pi t)$

(*c*) $U_{max} = 1/2 C V_{max}^2$ $\quad V_{max} = 44$ V; $U_{max} = 4.36$ mJ

(*d*) $U_{min} = 1/2 C V_{min}^2$ $\quad V_{min} = 4$ V; $U_{min} = 36\ \mu$J

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119 •• What are the average and rms values of current for the two current waveforms shown in Figure 31-45?

(*a*) The current in the first half cycle of time interval ΔT is given by $I = 4t/(\Delta T)$, so $I^2 = 16t^2/(\Delta T)^2$. To find the mean value of I^2 we integrate I^2 from 0 to ΔT and divide by ΔT. One obtains $(I^2)_{av} = 16/3$. Thus $I_{rms} = 2.31$. The average current is 2.0.

(*b*) This is identical to Problem 31-116(*b*) except that here we are considering a current waveform and a magnitude of 4. It follows that $I_{rms} = 2.83$. The average current is 2.0.

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120 •• In the circuit shown in Figure 31-46, $E_1 = (20\text{ V}) \cos(2\pi f t)$, $f = 180$ Hz, $E_2 = 18$ V, and $R = 36\ \Omega$. Find the maximum, minimum, average, and rms values of the current through the resistor.

1. $I = (E_1 + E_2)/R$ $\quad I = [0.5 + 0.556 \cos(1131t)]$ A
2. I_{max} when $\cos \omega t = 1$; I_{min} when $\cos \omega t = -1$ $\quad I_{max} = 1.056$ A; $I_{min} = -0.056$ A
3. $(\cos \omega t)_{av} = 0$ $\quad I_{av} = 0.5$ A
4. Find $(I^2)_{av}$ and I_{rms} $\quad (I^2)_{av} = [(0.556)^2/2 + 0.25]\ \text{A}^2$; $I_{rms} = 0.636$ A

121* •• Repeat Problem 120 if the resistor R is replaced by a 2-μF capacitor.

1. Write Kirchhoff's law equation $\quad (20 \cos \omega t + 18)\text{ V} = q(t)/C$
2. Let $q(t)/C = A \cos \omega t + B$ $\quad$ This is a steady state solution for $A = 20$ V, $B = 18$ V
3. $I = dq/dt = -AC\omega \sin \omega t$ $\quad I = -(45.2 \sin \omega t)$ mA; $I_{max} = 45.2$ mA; $I_{min} = -45.2$ mA; $I_{av} = 0$; $I_{rms} = 32.0$ mA

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122 •• Repeat Problem 120 if the resistor R is replaced by a 12-mH inductor.

The inductance acts as a short circuit to the constant voltage source. The current is infinite at all times. Consequently, $I_{max} = I_{rms} = \infty$; there is no minimum current.
