CHAPTER 28

The Magnetic Field

1* • When a cathode-ray tube is placed horizontally in a magnetic field that is directed vertically upward, the electrons emitted from the cathode follow one of the dashed paths to the face of the tube in Figure 28-30. The correct path is ____. (*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4 (*e*) 5

(*b*)

2 • Why not define ***B*** to be in the direction of ***F***, as we do for ***E***?

One cannot define the direction of the force by fiat. By experiment, ***F*** is perpendicular to ***B***.

3 • Find the magnetic force on a proton moving with velocity 4.46 Mm/s in the positive *x* direction in a magnetic field of 1.75 T in the positive *z* direction.

Use Equ. 28-1 $\boldsymbol{F} = 1.25\ \text{pN}\ \boldsymbol{i}\times\boldsymbol{k} = -1.25\ \text{pN}\ \boldsymbol{j}$

4 • A charge $q = -3.64$ nC moves with a velocity of 2.75×10^6 m/s ***i***. Find the force on the charge if the magnetic field is (*a*) $\boldsymbol{B} = 0.38\ \text{T}\ \boldsymbol{j}$, (*b*) $\boldsymbol{B} = 0.75\ \text{T}\ \boldsymbol{i} + 0.75\ \text{T}\ \boldsymbol{j}$, (*c*) $\boldsymbol{B} = 0.65\ \text{T}\ \boldsymbol{i}$, (*d*) $\boldsymbol{B} = 0.75\ \text{T}\ \boldsymbol{i} + 0.75\ \text{T}\ \boldsymbol{k}$.

(*a*), (*b*), (*c*), (*d*) Use Equ. 28-1 (*a*) $\boldsymbol{F} = -3.8\ \text{mN}\ \boldsymbol{i}\times\boldsymbol{j} = -3.8\ \text{mN}\ \boldsymbol{k}$ (*b*) $\boldsymbol{F} = -7.5\ \text{mN}\ \boldsymbol{k}$

(*c*) $\boldsymbol{F} = 0$ (*d*) $\boldsymbol{F} = 7.5\ \text{mN}\ \boldsymbol{j}$

5* • A uniform magnetic field of magnitude 1.48 T is in the positive *z* direction. Find the force exerted by the field on a proton if the proton's velocity is (*a*) $\boldsymbol{v} = 2.7\ \text{Mm/s}\ \boldsymbol{i}$, (*b*) $\boldsymbol{v} = 3.7\ \text{Mm/s}\ \boldsymbol{j}$, (*c*) $\boldsymbol{v} = 6.8\ \text{Mm/s}\ \boldsymbol{k}$, and (*d*) $\boldsymbol{v} = 4.0\ \text{Mm/s}\ \boldsymbol{i} + 3.0\ \text{Mm/s}\ \boldsymbol{j}$.

(*a*), (*b*), (*c*), (*d*) Use Equ. 28-1 (*a*) $\boldsymbol{F} = 0.639\ \text{pN}\ \boldsymbol{i}\times\boldsymbol{k} = -0.639\ \text{pN}\ \boldsymbol{j}$ (*b*) $\boldsymbol{F} = 0.876\ \text{pN}\ \boldsymbol{i}$

(*c*) $\boldsymbol{F} = 0$ (*d*) $\boldsymbol{F} = 0.71\ \text{pN}\ \boldsymbol{i} - 0.947\ \text{pN}\ \boldsymbol{j}$

6 • An electron moves with a velocity of 2.75 Mm/s in the *xy* plane at an angle of 60° to the *x* axis and 30° to the *y* axis. A magnetic field of 0.85 T is in the positive *y* direction. Find the force on the electron.

Use Equ. 28-1 $\boldsymbol{F} = -0.44(\cos 60^\circ\ \boldsymbol{i} + \cos 30^\circ\ \boldsymbol{j})\times 0.85\ \boldsymbol{j}\ \text{pN} = -0.187\ \text{pN}\ \boldsymbol{k}$

7 • A straight wire segment 2 m long makes an angle of 30° with a uniform magnetic field of 0.37 T. Find the magnitude of the force on the wire if it carries a current of 2.6 A.

Use Equ. 28-4 $F = BI\ell \sin\theta = 0.962$ N

8 • A straight wire segment $I\ell = (2.7\ \text{A})(3\ \text{cm}\ \boldsymbol{i} + 4\ \text{cm}\ \boldsymbol{j})$ is in a uniform magnetic field $\boldsymbol{B} = 1.3\ \text{T}\ \boldsymbol{i}$. Find the force on the wire.

Use Equ. 28-4 $\boldsymbol{F} = -0.140\ \mathrm{N}\ \boldsymbol{k}$

9* • What is the force (magnitude and direction) on an electron with velocity $\boldsymbol{v} = (2\boldsymbol{i} - 3\boldsymbol{j})\times10^6$ m/s in a magnetic field $\boldsymbol{B} = (0.8\boldsymbol{i} + 0.6\boldsymbol{j} - 0.4\boldsymbol{k})$ T?

Use Equ. 28-4 $\boldsymbol{F} = -0.192\ \mathrm{pN}\ \boldsymbol{i} - 0.128\ \mathrm{pN}\ \boldsymbol{j} - 0.576\ \mathrm{pN}\ \boldsymbol{k}$; $F = 0.621$ pN

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10 •• The wire segment in Figure 28-31 carries a current of 1.8 A from *a* to *b*. There is a magnetic field $\boldsymbol{B} = 1.2\ \mathrm{T}\ \boldsymbol{k}$. Find the total force on the wire and show that it is the same as if the wire were a straight segment from *a* to *b*.

Use Equ. 28-4 $\boldsymbol{F} = -0.0684\ \mathrm{N}\ \boldsymbol{j} + 0.0864\ \mathrm{N}\ \boldsymbol{i}$;

If the wire is straight from *a* to *b*, $\ell = 3\ \mathrm{cm}\ \boldsymbol{i} + 4\ \mathrm{cm}\ \boldsymbol{j}$ $\boldsymbol{F} = I\boldsymbol{\ell}\times\boldsymbol{B} = 0.0864\ \mathrm{N}\ \boldsymbol{i} - 0.0684\ \mathrm{N}\ \boldsymbol{j}$

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11 •• A straight, stiff, horizontal wire of length 25 cm and mass 50 g is connected to a source of emf by light, flexible leads. A magnetic field of 1.33 T is horizontal and perpendicular to the wire. Find the current necessary to float the wire, that is, the current such that the magnetic force balances the weight of the wire.

$\boldsymbol{F} = (I\ell B - mg)\ \boldsymbol{k} = 0$ $I = mg/\ell B = 1.48$ A

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12 •• A simple gaussmeter for measuring horizontal magnetic fields consists of a stiff 50-cm wire that hangs from a conducting pivot so that its free end makes contact with a pool of mercury in a dish below. The mercury provides an electrical contact without constraining the movement of the wire. The wire has a mass of 5 g and conducts a current downward. (*a*) What is the equilibrium angular displacement of the wire from vertical if the horizontal magnetic field is 0.04 T and the current is 0.20 A? (*b*) If the current is 20 A and a displacement from vertical of 0.5 mm can be detected for the free end, what is the horizontal magnetic field sensitivity of this gaussmeter?

(*a*) At equilibrium, $mg \sin\theta = I\ell B$ $\sin\theta = 0.08154$; $\theta = 4.68°$

(*b*) $\theta = 0.001$ rad $= \sin\theta$; solve for B $B = 4.91\ \mu$T

13* •• A current-carrying wire is bent into a semicircular loop of radius *R* that lies in the *xy* plane. There is a uniform magnetic field $\boldsymbol{B} = B\boldsymbol{k}$ perpendicular to the plane of the loop (Figure 28-32). Show that the force acting on the loop is $\boldsymbol{F} = 2IRB\boldsymbol{j}$.

With the current in the direction indicated and the magnetic field in the *z* direction, pointing out of the plane of the paper, the force is in the radial direction. On an element of length $d\ell$ the force is $dF = BIR\ d\theta$ with *x* and *y* components $dF_x = BIR\cos\theta\, d\theta$ and $dF_y = BIR\sin\theta\, d\theta$. By symmetry, the *x* component of the force is zero.

$$Fy = \int_0^{\pi} BIR\sin\theta d\theta = 2IBR.$$

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14 •• A 10-cm length of wire carries a current of 4.0 A in the positive *z* direction. The force on this wire due to a magnetic field $\boldsymbol{B}$ is $\boldsymbol{F} = (-0.2\boldsymbol{i} + 0.2\boldsymbol{j})$ N. If this wire is rotated so that the current flows in the positive *x* direction, the force on the wire is $\boldsymbol{F} = 0.2\boldsymbol{k}$ N. Find the magnetic field $\boldsymbol{B}$.

1. Write the $\boldsymbol{F} = (-0.2\ \boldsymbol{i} + 0.2\ \boldsymbol{j})$ N in terms of $\boldsymbol{B}$ $(0.4\ \boldsymbol{k})\times(B_x\ \boldsymbol{i} + B_y\ \boldsymbol{j} + B_z\ \boldsymbol{k}) = -0.2\ \boldsymbol{i} + 0.2\ \boldsymbol{j}$

2. Solve for B_x, B_y, and B_z $B_y = 0.5$ T, $B_x = 0.5$ T, B_z is undetermined

3. Repeat for $\boldsymbol{F} = 0.2\,\boldsymbol{k}$ N — $(0.4\,\boldsymbol{i})\times(B_x\,\boldsymbol{i} + B_y\,\boldsymbol{j} + B_z\,\boldsymbol{k}) = 0.2\,\boldsymbol{k}$; $B_z = 0$

4. Write $\boldsymbol{B}$ — $\boldsymbol{B} = (0.5\,\boldsymbol{i} + 0.5\,\boldsymbol{j})$ T

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15 ·· A 10-cm length of wire carries a current of 2.0 A in the positive x direction. The force on this wire due to the presence of a magnetic field $\boldsymbol{B}$ is $\boldsymbol{F} = (3.0\boldsymbol{j} + 2.0\boldsymbol{k})$ N. If this wire is now rotated so that the current flows in the positive y direction, the force on the wire is $\boldsymbol{F} = (-3.0\boldsymbol{i} - 2.0\boldsymbol{k})$ N. Determine the magnetic field $\boldsymbol{B}$.

1. Write $\boldsymbol{F} = (3.0\,\boldsymbol{j} + 2.0\,\boldsymbol{k})$ N in terms of $\boldsymbol{B}$ — $(0.2\,\boldsymbol{i})\times(B_x\,\boldsymbol{i} + B_y\,\boldsymbol{j} + B_z\,\boldsymbol{k}) = 3\,\boldsymbol{j} + 2\,\boldsymbol{k}$
2. Solve for the components of $\boldsymbol{B}$ — B_x is undetermined; $B_y = 10$ T, $B_z = -15$ T
3. Repeat for $\boldsymbol{F} = (-3.0\,\boldsymbol{i} - 2.0\,\boldsymbol{k})$ N — $(0.2\,\boldsymbol{j})\times(B_x\,\boldsymbol{i} + B_y\,\boldsymbol{j} + B_z\,\boldsymbol{k}) = -3.0\,\boldsymbol{i} - 2.0\,\boldsymbol{k}$; $B_x = 10$ T, $B_z = -15$ T
4. Write $\boldsymbol{B}$ — $\boldsymbol{B} = (10\,\boldsymbol{i} + 10\,\boldsymbol{j} - 15\,\boldsymbol{k})$ T

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16 ··· A wire bent in some arbitrary shape carries a current I in a uniform magnetic field $\boldsymbol{B}$. Show explicitly that the total force on the part of the wire from some point a to some point b is $\boldsymbol{F} = I\,\boldsymbol{\ell}\times\boldsymbol{B}$, where $\boldsymbol{\ell}$ is the vector from a to b.

From Equ. 28-5 we have $F = \int_a^b dF = \int_a^b I d\ell \times B$. But I and $\boldsymbol{B}$ are constant.Thus, $\mathrm{F} = \mathrm{I}\ell\times\mathrm{B}$, where $\ell = \int_a^b d\ell$ is just the vector from a to b.

17* · True or false: The magnetic force does not accelerate a particle because the force is perpendicular to the velocity of the particle.

False

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18 · A moving charged particle enters a region in which it is suddenly deflected perpendicular to its motion. How can you tell if the deflection was caused by a magnetic field or an electric field?

1. Determine if the deflecting force depends on the particle's speed. If so, it is due to a magnetic field.
2. Examine the path of the particle. If it is circular, the deflection is due to a magnetic field; if it is parabolic, it is due to an electric field.

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19 · A proton moves in a circular orbit of radius 65 cm perpendicular to a uniform magnetic field of magnitude 0.75 T. (a) What is the period for this motion? (b) Find the speed of the proton. (c) Find the kinetic energy of the proton.

(a) Use Equ. 28-7 — $T = (2\pi\times1.67\times10^{-27}/1.6\times10^{-19}\times0.75)$ s = 87.4 ns

(b) $v = 2\pi r/T$ — $v = 4.67\times10^{7}$ m/s

(c) $K = 1/2mv^2$ — $K = 1.82\times10^{-12}$ J = 11.4 MeV

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20 · An electron of kinetic energy 45 keV moves in a circular orbit perpendicular to a magnetic field of 0.325 T. (a) Find the radius of the orbit. (b) Find the frequency and period of the motion.

(a) Use Equ. 28-6 and $v = \sqrt{2K/m}$; $r = \dfrac{\sqrt{2Km}}{qB}$ — $r = 2.20$ mm

(b) Use Equs. 28-7 and 28-8 — $T = 0.11$ ns; $f = 1/T = 9.08$ GHz

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21* · An electron from the sun with a speed of 1×10^{7} m/s enters the earth's magnetic field high above the equator where the magnetic field is 4×10^{-7} T. The electron moves nearly in a circle except for a small drift along the direction of the earth's magnetic field that will take it toward the north pole. (*a*) What is the radius of the circular motion? (*b*) What is the radius of the circular motion near the north pole where the magnetic field is 2×10^{-5} T?

(*a*), (*b*) Use Equ. 28-6 (*a*) $r = 142$ m (*b*) $r = 2.85$ m

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22 ·· Protons and deuterons (each with charge $+e$) and alpha particles (with charge $+2e$) of the same kinetic energy enter a uniform magnetic field $\boldsymbol{B}$ that is perpendicular to their velocities. Let r_p, r_d, and r_α be the radii of their circular orbits. Find the ratios r_d/r_p and r_α/r_p. Assume that $m_\alpha = 2m_d = 4m_p$.

$r = \sqrt{2Km}/qB$ (see Problem 20). With K and B the same for the three particles, $r \propto m^{1/2}/q$. Consequently, $r_d/r_p = \sqrt{2}$ and $r_\alpha/r_p = 1$.

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23 ·· A proton and an alpha particle move in a uniform magnetic field in circles of the same radii. Compare (*a*) their velocities, (*b*) their kinetic energies, and (*c*) their angular momenta. (See Problem 22.)

(*a*) From Equ. 28-6, $v = qBr/m$. With r and B constant, $v \propto q/m$. Consequently, $v_\alpha = v_p/2$.

(*b*) $K \propto mv^2 = q^2/m$. Consequently, $K_\alpha = K_p$.

(*c*) $L = mvr \propto mv$ for constant r. Consequently, $L_\alpha = 2L_p$.

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24 ·· A particle of charge q and mass m has momentum $p = mv$ and kinetic energy $K = \frac{1}{2}mv^2 = p2/2m$. If the particle moves in a circular orbit of radius r perpendicular to a uniform magnetic field B, show that (*a*) $p = Bqr$ and (*b*) $K = B^2q^2r^2/2m$.

(*a*) The centripetal force is qBv and this must equal mv^2/r. So $qB = p/r$ and $p = qBr$.

(*b*) $K = p^2/2m = q^2B^2r^2/2m$.

25* ·· A beam of particles with velocity $\boldsymbol{v}$ enters a region of uniform magnetic field $\boldsymbol{B}$ that makes a small angle θ with $\boldsymbol{v}$. Show that after a particle moves a distance $2\pi(m/qB)v\cos\theta$ measured along the direction of $\boldsymbol{B}$, the velocity of the particle is in the same direction as it was when it entered the field.

The particle's velocity has a component $\boldsymbol{v}_1$ parallel to $\boldsymbol{B}$ and a component $\boldsymbol{v}_2$ normal to $\boldsymbol{B}$. $v_1 = v\cos\theta$ and is constant. $v_2 = v\sin\theta$; the magnetic force due to this velocity component is qBv_2 and results in a circular motion perpendicular to $\boldsymbol{B}$. The period of that circular motion is given by Equ. 28-7. At the end of one period, $\boldsymbol{v}_2$ is the same as at the start of the period. In that time, the particle has moved a distance $v_1T = (v\cos\theta)(2\pi m/qB)$ in the direction of $\boldsymbol{B}$.

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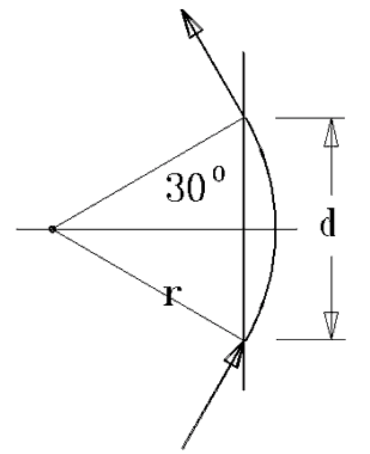
26 ·· A proton with velocity $v = 10^7$ m/s enters a region of uniform magnetic field $B = 0.8$ T, which is into the page, as shown in Figure 28-33. The angle $\theta = 60°$. Find the angle ϕ and the distance d.

The trajectory of the particle is shown. From symmetry, it is evident that the angle ϕ in Figure 28-33 equals the angle $\theta = 60°$. From the adjoining figure we see that $d/2 = r \sin 30°$, or $d = r$. From Equ. 28-6, $r = d = 0.1305$ m.



27 ·· Suppose that in Figure 28-33 $B = 0.6$ T, the distance $d = 0.4$ m, and $\theta = 24°$. Find the speed v and the angle ϕ if the particles are (*a*) protons and (*b*) deuterons.

(*a*) $r = d/(2 \cos \theta)$; $\phi = \theta$ (see Problem 26)	$r = 0.2/\cos 24°$ m $= 0.219$ m $= m_p v_p/q_p B$
Solve for v_p	$v_p = 1.26\times10^7$ m/s; $\phi = 24°$
(*b*) Solve for v_d; $m_d = 2m_p$, $q_d = q_p$	$v_d = 6.29\times10^6$ m/s; $\phi = 24°$

28 · A beam of positively charged particles passes undeflected from left to right through a velocity selector in which the electric field is up. The beam is then reversed so that it travels from right to left. Will the beam now be deflected in the velocity selector? If so, in which direction?

Yes; it will be deflected up.

29* · A velocity selector has a magnetic field of magnitude 0.28 T perpendicular to an electric field of magnitude 0.46 MV/m. (*a*) What must the speed of a particle be for it to pass through undeflected? What energy must (*b*) protons and (*c*) electrons have to pass through undeflected?

(*a*) Use Equ. 28-9	$v = (0.46\times10^6/0.28)$ m/s $= 1.64\times10^6$ m/s
(*b*) $K = 1/2 m_p v^2$	$K = 2.25\times10^{-15}$ J $= 14.1$ keV
(*c*) $K = 1/2 m_e v^2$	$K = 7.68$ eV

30 · A beam of protons moves along the x axis in the positive x direction with a speed of 12.4 km/s through a region of crossed fields balanced for zero deflection. (*a*) If there is a magnetic field of magnitude 0.85 T in the positive y direction, find the magnitude and direction of the electric field. (*b*) Would electrons of the same velocity be deflected by these fields? If so, in what direction?

(*a*) Use Equ. 28-9	$E = vB = 12.4\times10^3\times0.85$ V/m $= 10.5$ kV/m
$q\mathbf{v}\times\mathbf{B}$ is in the z direction	$\mathbf{E} = -10.5$ kV/m $\mathbf{k}$
(*b*) For electrons, both $\mathbf{F}_B$ and $\mathbf{F}_E$ are reversed	Electrons are not deflected

31 ·· The plates of a Thomson q/m apparatus are 6.0 cm long and are separated by 1.2 cm. The end of the plates is 30.0 cm from the tube screen. The kinetic energy of the electrons is 2.8 keV. (*a*) If a potential of 25.0 V is applied across the deflection plates, by how much will the beam deflect? (*b*) Find the magnitude of the crossed magnetic field that will allow the beam to pass through undeflected.

(*a*) 1. Find the speed of the electrons; $v = \sqrt{2K/m}$ $v = 3.14\times10^7$ m/s

2. Find $E = V/d$ $E = 2083$ V/m

3. $\Delta y = \frac{qE}{2m}\left(\frac{x_1}{v}\right)^2 + \frac{qEx_1x_2}{mv^2}$ (see Equ. 28-10) $v_x = 3.14\times10^7$ m/s; $v_y = eEt/m_e = 6.99\times10^5$ m/s

$\Delta y = 7.35$ mm

(*b*) $B = E/v$ $B = 25/(0.012\times3.14\times10^7)$ T $= 6.63\times10^{-5}$ T

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32 ·· Chlorine has two stable isotopes, ^{35}Cl and ^{37}Cl, whose natural abundances are about 76% and 24%, respectively. Singly ionized chlorine gas is to be separated into its isotopic components using a mass spectrometer. The magnetic field in the spectrometer is 1.2 T. What is the minimum value of the potential through which these ions must be accelerated so that the separation between them is 1.4 cm?

From Equ. 28–12, $r = \sqrt{\frac{2m\,\Delta V}{q\,B^2}}$; write $\Delta r = \Delta s/2$ 0.007 m $= 1.20\times10^{-4}(\sqrt{37} - \sqrt{35})\sqrt{\Delta V}$

Solve for ΔV $\Delta V = 122$ kV

33* ·· A singly ionized ^{24}Mg ion (mass 3.983×10^{-26} kg) is accelerated through a 2.5-kV potential difference and deflected in a magnetic field of 557 G in a mass spectrometer. (*a*) Find the radius of curvature of the orbit for the ion. (*b*) What is the difference in radius for ^{26}Mg and ^{24}Mg ions? (Assume that their mass ratio is 26/24.)

(*a*) $r = \sqrt{\frac{2m\,\Delta V}{q\,B^2}}$; evaluate for ^{24}Mg $r_{24} = 63.5$ cm

(*b*) $r_{26} = r_{24}\sqrt{26/24}$; evaluate $\Delta r = r_{26} - r_{24}$ $\Delta r = 63.5(\sqrt{26/24} - 1)$ cm $= 2.59$ cm

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34 ·· A beam of ^6Li and ^7Li ions passes through a velocity selector and enters a magnetic spectrometer. If the diameter of the orbit of the ^6Li ions is 15 cm, what is the diameter of that for ^7Li ions?

For constant v, $r \propto m$ $D_7 = D_6(7/6) = 17.5$ cm

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35 ·· In Example 28-6, determine the time required for a ^{58}Ni ion and a ^{60}Ni ion to complete the semicircular path.

1. $v = \sqrt{2q\,\Delta V/m}$; find v_{58} and v_{60} $v_{58} = 9.96\times10^4$ m/s; $v_{60} = 9.79\times10^4$ m/s

2. $t = \pi r/v$; find t_{58} and t_{60} $t_{58} = 0.501\pi/9.96\times10^4$ s $= 15.8\ \mu$s; $t_{60} = 16.4\ \mu$s

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36 ·· Before entering a mass spectrometer, ions pass through a velocity selector consisting of parallel plates separated by 2.0 mm and having a potential difference of 160 V. The magnetic field between the plates is 0.42 T. The magnetic field in the mass spectrometer is 1.2 T. Find (*a*) the speed of the ions entering the mass spectrometer and (*b*) the difference in the diameters of the orbits of singly ionized ^{238}U and ^{235}U. (The mass of a ^{235}U ion is 3.903×10^{-25} kg.)

(*a*) Use Equ. 28-9 $v = 8\times10^4/0.42$ m/s $= 1.90\times10^5$ m/s

(*b*) $r_{235} = m_{235}v/qB$; find $\Delta D = 2r_{235}(238/235 - 1)$ $\Delta D = 9.89$ mm

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37* ·· A cyclotron for accelerating protons has a magnetic field of 1.4 T and a radius of 0.7 m. (*a*) What is the cyclotron frequency? (*b*) Find the maximum energy of the protons when they emerge. (*c*) How will your answers change if deuterons, which have the same charge but twice the mass, are used instead of protons?

(*a*) Use Equ. 28-8	$f = 21.3$ MHz
(*b*) Use Equ. 28-13	$K = 7.36\times10^{-12}$ J = 46 MeV
(*c*) From Equs. 28-8 and 28-13, K and $f \propto 1/m$	$f_d = 10.7$ MHz; $K_d = 23$ MeV

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38 ·· A certain cyclotron with magnetic field of 1.8 T is designed to accelerate protons to 25 MeV. (*a*) What is the cyclotron frequency? (*b*) What must the minimum radius of the magnet be to achieve a 25-MeV emergence energy? (*c*) If the alternating potential applied to the dees has a maximum value of 50 kV, how many revolutions must the protons make before emerging with an energy of 25 MeV?

(*a*) Use Equ. 28-8	$f = 27.4$ MHz
(*b*) From Equ. 28-13, $r = \sqrt{2Km}/qB$; find r	$r = 0.401$ m; this is the minimum radius
(*c*) Energy gain/revolution = 100 keV	Number of revolutions = $25\times10^6/10^5 = 250$

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39 ·· Show that the cyclotron frequencies of deuterons and alpha particles are the same and are half that of a proton in the same magnetic field. (See Problem 22.)

From Equ. 28-8, $f \propto q/m$. Since $q_d = e$ and $q_\alpha = 2e$, and $m_d = 1/2 m_\alpha$, $q_d/m_d = q_\alpha/m_\alpha$. Consequently, $f_d = f_\alpha$. Similarly, $f_d = f_\alpha = 1/2 f_p$.

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40 ·· Show that the radius of the orbit of a charged particle in a cyclotron is proportional to the square root of the number of orbits completed.

Since the energy gain per revolution is constant, $K \propto N$, where N is the number of complete revolutions. The radius of an orbit is proportional to $K^{1/2}$, so $r \propto N^{1/2}$.

41* · What orientation of a current loop gives maximum torque?

The normal to the plane of the loop should be perpendicular to $\boldsymbol{B}$.

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42 · A small circular coil of 20 turns of wire lies in a uniform magnetic field of 0.5 T such that the normal to the plane of the coil makes an angle of 60° with the direction of $\boldsymbol{B}$. The radius of the coil is 4 cm, and it carries a current of 3 A. (*a*) What is the magnitude of the magnetic moment of the coil? (*b*) What is the magnitude of the torque exerted on the coil?

(*a*) $\mu = NIA$	$\mu = 0.302$ A·m^2
(*b*) $\tau = \mu B \sin\theta$	$\tau = 0.131$ N·m

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43 · What is the maximum torque on a 400-turn circular coil of radius 0.75 cm that carries a current of 1.6 mA and resides in a uniform magnetic field of 0.25 T?

$\tau_{max} = \mu B$; $\mu = NIA$; $_{max} = NIAB$	$\tau_{max} = 2.83\times10^{-5}$ N·m

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44 · A current-carrying wire is bent into the shape of a square of sides $L = 6$ cm and is placed in the xy plane. It carries a current $I = 2.5$ A. What is the torque on the wire if there is a uniform magnetic field of 0.3 T (*a*) in the z direction, and (*b*) in the x direction?

(*a*) $\boldsymbol{\tau} = \boldsymbol{\mu}\times\boldsymbol{B}$; $\boldsymbol{\mu} = \pm IA\,\boldsymbol{k}$; $\boldsymbol{B} = B\,\boldsymbol{k}$	$\boldsymbol{\tau} = 0$
(*b*) $\boldsymbol{B} = B\,\boldsymbol{I}$	$\boldsymbol{\tau} = \pm2.7\times10^{-3}$ N·m $\boldsymbol{j}$

45* · Repeat Problem 44 if the wire is bent into an equilateral triangle of sides 8 cm.

(*a*) $\boldsymbol{\tau} = \boldsymbol{\mu}\times\boldsymbol{B}$; $\boldsymbol{\mu} = \pm IA\,\boldsymbol{k}$; $\boldsymbol{B} = B\,\boldsymbol{k}$ $\boldsymbol{\tau} = 0$

(*b*) $\boldsymbol{B} = B\,\boldsymbol{I}$ $\boldsymbol{\tau} = \pm 2.08\times10^{-3}$ N·m $\boldsymbol{j}$

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46 ·· A rigid, circular loop of radius *R* and mass *M* carries a current *I* and lies in the *xy* plane on a rough, flat table. There is a horizontal magnetic field of magnitude *B*. What is the minimum value of *B* such that one edge of the loop will lift off the table?

$B_{min} = mg/\pi RI$ (see Example 28-9).

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47 ·· A rectangular, 50-turn coil has sides 6.0 and 8.0 cm long and carries a current of 1.75 A. It is oriented as shown in Figure 28-34 and pivoted about the *z* axis. (*a*) If the wire in the *xy* plane makes an angle $\theta = 37°$ with the *y* axis as shown, what angle does the unit normal $\boldsymbol{n}$ make with the *x* axis? (*b*) Write an expression for $\boldsymbol{n}$ in terms of the unit vectors $\boldsymbol{i}$ and $\boldsymbol{j}$. (*c*) What is the magnetic moment of the coil? (*d*) Find the torque on the coil when there is a uniform magnetic field $\boldsymbol{B} = 1.5$ T $\boldsymbol{j}$. (*e*) Find the potential energy of the coil in this field.

(*a*) From the figure it is evident that $\boldsymbol{n}$ makes an angle of –37° with the *x* axis.

(*b*) $\boldsymbol{n} = \cos 37°\,\boldsymbol{i} - \sin 37°\,\boldsymbol{j} = 0.8\,\boldsymbol{i} - 0.6\,\boldsymbol{j}$

(*c*) $\boldsymbol{\mu} = NIA\,\boldsymbol{n}$ $\boldsymbol{\mu} = 0.336$ A·m^2 $\boldsymbol{i}$ – 0.252 A·m^2 $\boldsymbol{j}$

(*d*) $\boldsymbol{\tau} = \boldsymbol{\mu}\times\boldsymbol{B}$ $\boldsymbol{\tau} = 0.504$ N·m $\boldsymbol{k}$

(*e*) $U = -\boldsymbol{\mu}\cdot\boldsymbol{B}$ $U = 0.378$ J

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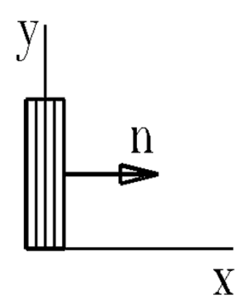
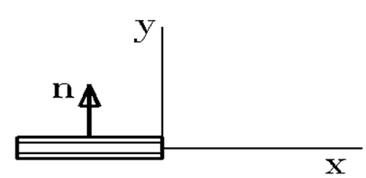
48 ·· The coil in Problem 47 is pivoted about the *z* axis and held at various positions in a uniform magnetic field $\boldsymbol{B} = 2.0$ T $\boldsymbol{j}$. Sketch the position of the coil and find the torque exerted when the unit normal is (*a*) $\boldsymbol{n} = \boldsymbol{i}$, (*b*) $\boldsymbol{n} = \boldsymbol{j}$, (*c*) $\boldsymbol{n} = -\boldsymbol{j}$, and (*d*) $\boldsymbol{n} = (\mathrm{i} + \mathrm{j})/\sqrt{2}$.

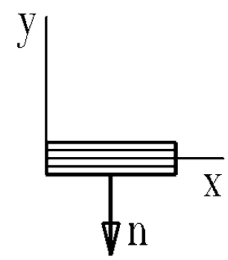
(*a*) The configuration of the coil is shown.
Since $\boldsymbol{B} = 2.0$ T $\boldsymbol{j}$, $\boldsymbol{\tau} = \boldsymbol{\mu}\times\boldsymbol{B} = \mu B\,\boldsymbol{k} = 0.84$ N·m $\boldsymbol{k}$



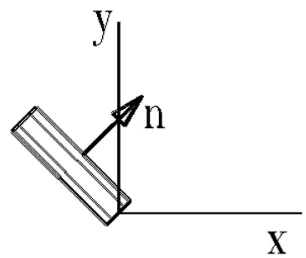
(*b*) The configuration of the coil is shown.
Since $\boldsymbol{B} = 2.0$ T $\boldsymbol{j}$ and $\boldsymbol{\mu} = \mu\,\boldsymbol{j}$, $\boldsymbol{\tau} = \boldsymbol{\mu}\times\boldsymbol{B} = 0$



(*c*) The configuration of the coil is shown.
As in part (*b*), $\tau = 0$.



(*d*) The configuration of the coil is shown.
In this case, $\boldsymbol{\tau} = \boldsymbol{\mu} \times \boldsymbol{B}$ = 0.594 N·m $\boldsymbol{k}$



49* · The SI unit for the magnetic moment of a current loop is A·m^2. Use this to show that 1 T = 1 N/A·m.
Since $[\tau] = [\mu][B]$, $[B] = [\tau]/[\mu]$ = N·m/A·m^2 = N/A·m

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50 ·· A small magnet of length 6.8 cm is placed at an angle of 60° to the direction of a uniform magnetic field of magnitude 0.04 T. The observed torque has a magnitude of 0.10 N.m. Find the magnetic moment of the magnet.
$\tau = \mu B \sin\theta$; $\mu = \tau/(B \sin\theta)$ μ = 2.89 A·m^2

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51 ·· A wire loop consists of two semicircles connected by straight segments (Figure 28-35). The inner and outer radii are 0.3 and 0.5 m, respectively. A current of 1.5 A flows in this loop with the current in the outer semicircle in the clockwise direction. What is the magnetic moment of this current loop?
$\mu = IA$; use right–hand rule to determine $\boldsymbol{n}$ $\mu = 1.5\pi(0.5^2 - 0.3^2)/2$ A·m^2 = 0.377 A·m^2;
$\boldsymbol{\mu}$ points into the paper.

---

52 ·· A wire of length L is wound into a circular coil of N loops. Show that when this coil carries a current I, its magnetic moment has the magnitude $IL^2/4\pi N$.
The circumference of each loop is $L/N = 2\pi R$ and the area of each loop is $\pi R^2 = L^2/4\pi N^2$. The magnetic moment is $\mu = NIA = IL^2/4\pi N$.

53* ·· A particle of charge q and mass m moves in a circle of radius r and with angular velocity Ω. (*a*) Show that the average current is $I = q\omega/2\pi$ and that the magnetic moment has the magnitude $\mu = \frac{1}{2}q\omega r^2$. (*b*) Show that the angular momentum of this particle has the magnitude $L = mr^2\omega$ and that the magnetic moment and angular momentum vectors are related by $\boldsymbol{\mu} = (q/2m)\boldsymbol{L}$.
(*a*) $I = \Delta q/\Delta t = q/T = qf = q\omega/2\pi$. $\mu = IA = (q\omega/2\pi)(\pi r^2) = q\omega r^2/2$
(*b*) The moment of inertia of the particle is mr^2, and so $L = mr^2\omega$. Both $\boldsymbol{\mu}$ and $\boldsymbol{L}$ point in the direction $\boldsymbol{\omega}$; so $\boldsymbol{\mu} = (q/2m)\boldsymbol{L}$.

---

54 ··· A single loop of wire is placed around the circumference of a rectangular piece of cardboard whose length and width are 70 and 20 cm, respectively. The cardboard is now folded along a line perpendicular to its length and midway between the two ends so that the two planes formed by the folded cardboard make an angle of 90°. If the wire loop carries a current of 0.2 A, what is the magnitude of the magnetic moment of this system?

The two parts can be considered as two loops; each loop carries a current *I*, and at the fold line the two loop currents cancel. The magnetic moments of the two loops make an angle of 90° with one another.

1. Find the magnetic moment of each loop $\mu = 0.014\ \mathrm{A{\cdot}m^2}$
2. Add the two moments vectorially $\mu_{tot} = 0.014\sqrt{2}\ \mathrm{A{\cdot}m^2} = 0.0198\ \mathrm{A{\cdot}m^2}$

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55 ··· Repeat Problem 54 if the line along which the cardboard is folded is 40 cm from one end.

The two parts can be considered as two loops; each loop carries a current *I*, and at the fold line the two loop currents cancel. The magnetic moments of the two loops make an angle of 90° with one another.

1. Find the magnetic moment of each loop $\mu_1 = 0.016\ \mathrm{A{\cdot}m^2}$; $\mu_2 = 0.012\ \mathrm{A{\cdot}m^2}$
2. Add the two moments vectorially $\mu_{tot} = (0.016^2 + 0.012^2)^{1/2}\ \mathrm{A{\cdot}m^2} = 0.020\ \mathrm{A{\cdot}m^2}$

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56 ··· A hollow cylinder has length L and inner and outer radii R_i and R_o, respectively (Figure 28-36). The cylinder carries a uniform charge density ρ. Derive an expression for the magnetic moment as a function of ω, the angular velocity of rotation of the cylinder about its axis.

Consider an element of charge dq in a cylinder of length L, radius r, and thickness dr. We have $dq = 2\pi L\rho\, r\, dr$. The element of current due to this rotating charge is $dI = \omega\, dq/2\pi = L\omega\rho\, r\, dr$, and the corresponding element of magnetic moment is $d\mu = A\, dI$, where $A = \pi r^2$. We now integrate:

$$\mu = L\omega\rho\pi\int_{R_i}^{R_o} r^3 dr = \frac{L\omega\rho\pi}{4}\left(R_o^4 - R_i^4\right)$$

57* ··· A nonconducting rod of mass M and length ℓ has a uniform charge per unit length λ and rotates with angular velocity $\boldsymbol{\omega}$ about an axis through one end and perpendicular to the rod. (*a*) Consider a small segment of the rod of length dx and charge $dq = \lambda\, dx$ at a distance x from the pivot (Figure 28-37). Show that the magnetic moment of this segment is $\frac{1}{2}\lambda\omega x^2\, dx$. (*b*) Integrate your result to show that the total magnetic moment of the rod is $\mu = \frac{1}{6}\lambda\omega\ell^3$. (*c*) Show that the magnetic moment $\boldsymbol{\mu}$ and angular momentum $\boldsymbol{L}$ are related by $\boldsymbol{\mu} = (Q/2M)\boldsymbol{L}$, where Q is the total charge on the rod.

(*a*) The area enclosed by the rotating element of charge is πx^2. The time required for one revolution is $1/f = 2\pi/\omega$. The average current element is then $dI = \lambda\, dx\,\omega/2\pi$ and $d\mu = A\, dI = 1/2\lambda\omega x^2 dx$.

(*b*) $$\mu = \frac{1}{2}\lambda\omega\int_0^{\ell} x^2 dx = \frac{1}{6}\lambda\omega\ell^3$$

(*c*) The angular momentum $L = I\omega$, where I is the moment of inertia of the rod, $I = (1/3)M\ell^2$. The total charge carried by the rod is $Q = \lambda\ell$. Thus $\mu = (Q/2M)L$. Moreover, since $\boldsymbol{\omega}$ and $\boldsymbol{L} = I\boldsymbol{\omega}$ point in the same direction, $\boldsymbol{\mu} = (Q/2M)\boldsymbol{L}$.

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58 ··· A nonuniform, nonconducting disk of mass M, radius R, and total charge Q has a surface charge density $\sigma = \sigma_0 r/R$ and a mass per unit area $\sigma_m = (M/Q)\sigma$. The disk rotates with angular velocity ω about its axis. (*a*) Show that the magnetic moment of the disk has a magnitude $\mu = \frac{1}{5}\pi\omega\sigma_0 R^4 = \frac{3}{10}Q\omega R^2$. (*b*) Show that the magnetic moment $\boldsymbol{\mu}$ and angular momentum $\boldsymbol{L}$ are related by $\boldsymbol{\mu} = (Q/2M)\boldsymbol{L}$.

(*a*) We are given that $\sigma = \sigma_0 r/R$ and $\sigma_m = (M/Q)\sigma$. An element of magnetic moment is then given by $d\mu = A\, dI = \pi r^2\sigma(\omega/2\pi)(2\pi r)\, dr = (\pi\omega\sigma_0/R)r^4 dr$. Integrating from $r = 0$ to $r = R$ one obtains $\mu = (1/5)\pi\omega\sigma_0 R^4$.

The total charge is the integral of $2\pi r\sigma\, dr$ from $r = 0$ to $r = R$, where $\sigma = \sigma_0 r/R$. Thus $Q = 2\pi R^2\sigma_0/3$, and $\sigma_0 = 3Q/2\pi R^2$. Substituting this expression for σ_0 into the result for μ one obtains $\mu = 3Q\omega R^2/10$.

(*b*) The moment of inertia of the disk with a surface mass density σ_m is given by

$I = \int dI = \int_0^R (M\sigma_0/Q)(r/R)r^2(2\pi r)dr = \dfrac{2\pi M\sigma_0}{5Q}R^4 = (3/5)MR^2$ and $\boldsymbol{L} = I\boldsymbol{\omega} = (3/5)MR^2\boldsymbol{\omega}$. Since $\boldsymbol{\mu}$ also points in the direction of $\boldsymbol{\omega}$ we see that $\boldsymbol{\mu} = (Q/2M)\boldsymbol{L}$.

59 ··· A spherical shell of radius R carries a surface charge density σ. The sphere rotates about its diameter with angular velocity ω. Find the magnetic moment of the rotating sphere.

For the shell, $Q = 4\pi R^2\sigma$. The angular momentum of a shell of mass M for rotation about a diameter is given by $\boldsymbol{L} = I\boldsymbol{\omega} = (2/3)MR^2\boldsymbol{\omega}$. Applying the general expression $\boldsymbol{\mu} = (Q/2M)\boldsymbol{L}$ one finds $\boldsymbol{\mu} = (4\pi/3)\sigma R^4\boldsymbol{\omega}$.

60 ··· A solid sphere of radius R carries a uniform volume charge density ρ. The sphere rotates about its diameter with angular velocity ω. Find the magnetic moment of this rotating sphere.

For the sphere, $Q = (4\pi/3)\rho R^3$ and $I = (2/5)MR^2$. Applying the general result $(Q/2M)I\boldsymbol{\omega}$, $\boldsymbol{\mu} = (4\pi/15)\rho R^5\boldsymbol{\omega}$.

61* ··· A solid cylinder of radius R and length L carries a uniform charge density $+\rho$ between $r = 0$ and $r = R_s$ and an equal charge density of opposite sign, $-\rho$, between $r = R_s$ and $r = R$. What must be the radius R_s so that on rotation of the cylinder about its axis the magnetic moment is zero?

For the solid cylinder of radius R_s, $Q_+ = \pi\rho R_s^2 L$ and $\boldsymbol{L} = I\boldsymbol{\omega} = 1/2MR_s^2\boldsymbol{\omega}$. Hence $\boldsymbol{\mu}_+ = (Q_+/2M)\boldsymbol{L} = \pi\rho LR_s^4\boldsymbol{\omega}/4$. For the cylindrical shell, $Q_- = -\pi\rho L(R^2 - R_s^2)$ and $\boldsymbol{L} = I\boldsymbol{\omega} = 1/2M(R_s^2 + R^2)\boldsymbol{\omega}$. Hence $\boldsymbol{\mu}_- = -\pi\rho L(R^4 - R_s^4)\boldsymbol{\omega}/4$. Setting $\boldsymbol{\mu}_+ + \boldsymbol{\mu}_- = 0$ and solving for R_s one obtains $R_s = R/2^{1/4} = 0.841R$.

62 ··· A solid cylinder of radius R and length L carries a uniform charge density $\rho = -\rho_0$ between $r = 0$ and $r = \frac{1}{2}R$ and a positive charge density of equal magnitude, $+\rho_0$, between $r = \frac{1}{2}R$ and $r = R$ (Figure 28-38). The cylinder rotates about its axis with angular velocity $\boldsymbol{\omega}$. Derive an expression for the magnetic moment of the cylinder.

Here we can use the results of the previous problem. We now set $R_s = R/2$ in the expressions for $\boldsymbol{\mu}_+$ and $\boldsymbol{\mu}_-$, changing the signs of the magnetic moments since $\rho = -\rho_0$ for the inner cylinder and $\rho = +\rho_0$ for the cylindrical shell. The total magnetic moment is then given by $\boldsymbol{\mu} = (\pi\rho_0 L\boldsymbol{\omega}/4)[R^4 - R^4/16 - R^4/16] = 7\pi\rho_0 LR^4\boldsymbol{\omega}/32$.

63 ··· A cylindrical shell of length L with inner radius R_i and outer radius R_o carries a uniform charge density, $+\rho_0$, between R_i and radius R_s and an equal charge density of opposite sign, $-\rho_0$, between R_s and R_o. The cylinder rotates about its axis with angular velocity $\boldsymbol{\omega}$. Derive an expression for the magnetic moment of this cylinder.

For the inner cylindrical shell (see Problem 61), $\boldsymbol{\mu}_i = \pi\rho_0 L(R_s^4 - R_i^4)\boldsymbol{\omega}/4$, while for the outer shell, $\boldsymbol{\mu}_o = -\pi\rho_0 L(R_o^4 - R_s^4)\boldsymbol{\omega}/4$. The total magnetic moment is $\boldsymbol{\mu}_i + \boldsymbol{\mu}_o = -(\pi\rho_0 L\boldsymbol{\omega}/4)(R_o^4 + R_i^4 - 2R_s^4)$.

64 ··· A solid sphere of radius R carries a uniform charge density, $+\rho_0$, between $r = 0$ and $r = R_s$ and an equal charge density of opposite sign, $-\rho_0$, between $r = R_s$ and $r = R$. The sphere rotates about its diameter with angular velocity ω. Find R_s such that magnetic moment of the sphere is zero. What is the net charge carried by the sphere?

For the inner sphere of radius R_s, $Q_i = 4\pi\rho_0 R_s^3/3$ and $\boldsymbol{L} = 2MR_s^2\boldsymbol{\omega}/5$. Hence $\boldsymbol{\mu}_i = 4\pi\rho_0 R_s^5\boldsymbol{\omega}/15$. For the outer spherical shell, the charge is $Q_o = -(4\pi\rho_0/3)(R^3 - R_s^3)$. If ρ is the mass density, then $M_o = (4\pi\rho/3)(R^3 - R_s^3)$ and the moment of inertia of the spherical shell is $I = (2/5)(4\pi\rho/3)(R^5 - R_s^5)$. Using the general result $\boldsymbol{\mu} = (Q/2M)\boldsymbol{L}$ one obtains $\boldsymbol{\mu}_o = -(4\pi\rho_0/15)(R^5 - R_s^5)\boldsymbol{\omega}$. Setting $\boldsymbol{\mu}_i + \boldsymbol{\mu}_o = 0$ and solving for R_s one finds $R_s = R/2^{1/5} = 0.871R$. The net charge carried by the sphere is $Q = Q_i + Q_o$, where Q_i and Q_o are given above. Setting $R_s = R/2^{1/5}$ one obtains $Q = 2^{2/5}(4\pi\rho_0/3)R^3$.

65* ··· A solid sphere of radius R carries a uniform charge density, $+\rho_0$, between $r = 0$ and $r = \frac{1}{2}R$ and an equal charge density of opposite sign, $-\rho_0$, between $r = \frac{1}{2}0R$ and $r = R$. The sphere rotates about its diameter with angular velocity ω. Derive an expression for the magnetic moment of this rotating sphere.

For the inner sphere of radius R_i, $Q_\mathrm{i} = 4\pi\rho_0 R_\mathrm{i}^3/3$ and $\boldsymbol{L} = 2MR_\mathrm{i}^2\boldsymbol{\omega}/5$. Hence $\boldsymbol{\mu}_\mathrm{i} = 4\pi\rho_0 R_\mathrm{i}^5\boldsymbol{\omega}/15$. For the outer spherical shell, the charge is $Q_\mathrm{o} = -(4\pi\rho_0/3)(R^3 - R_\mathrm{i}^3)$. If ρ is the mass density, then $M_\mathrm{o} = (4\pi\rho/3)(R^3 - R_\mathrm{i}^3)$ and the moment of inertia of the spherical shell is $I = (2/5)(4\pi\rho/3)(R^5 - R_\mathrm{i}^5)$. Using the general result $\boldsymbol{\mu} = (Q/2M)\boldsymbol{L}$ one obtains $\boldsymbol{\mu}_\mathrm{o} = -(4\pi\rho_0/15)(R^5 - R_\mathrm{i}^5)\boldsymbol{\omega}$. We now set $R_\mathrm{i} = R/2$ and $\boldsymbol{\mu} = \boldsymbol{\mu}_\mathrm{i} + \boldsymbol{\mu}_\mathrm{o}$ and obtain $\boldsymbol{\mu} = -\pi\rho_0 R^5\boldsymbol{\omega}/4$.

66 · A metal strip 2.0 cm wide and 0.1 cm thick carries a current of 20 A in a uniform magnetic field of 2.0 T, as shown in Figure 28-39. The Hall voltage is measured to be 4.27 μV. (*a*) Calculate the drift velocity of the electrons in the strip. (*b*) Find the number density of the charge carriers in the strip.(*c*) Is point *a* or *b* at the higher potential?

(*a*) From Equ. 28-17, $v_\mathrm{d} = V_\mathrm{H}/Bw$	$v_\mathrm{d} = 0.107$ mm/s
(*b*) Use Equ. 28-18	$n = 5.85\times10^{28}\ \mathrm{m}^{-3}$
(*c*) $\boldsymbol{E} = \boldsymbol{v}_\mathrm{d}\times\boldsymbol{B}$; $\boldsymbol{v}_\mathrm{d}$ directed opposite to I	$\boldsymbol{E}$ points from b to a; $V_\mathrm{a} < V_\mathrm{b}$

67 ·· The number density of free electrons in copper is 8.47×10^{22} electrons per cubic centimeter. If the metal strip in Figure 28-39 is copper and the current is 10 A, find (*a*) the drift velocity v_d and (*b*) the Hall voltage. (Assume that the magnetic field is 2.0 T.)

(*a*) $v_\mathrm{d} = I/wten$	$v_\mathrm{d} = 3.69\times10^{-5}$ m/s
(*b*) $V_\mathrm{H} = v_\mathrm{d}Bw$	$V_\mathrm{H} = 1.48\ \mu$V

68 ·· A copper strip ($n = 8.47\times10^{22}$ electrons per cubic centimeter) 2 cm wide and 0.1 cm thick is used to measure the magnitudes of unknown magnetic fields that are perpendicular to the strip. Find the magnitude of B when $I = 20$ A and the Hall voltage is (*a*) 2.00 μV, (*b*) 5.25 μV, and (*c*) 8.00 μV.

(*a*), (*b*), (*c*) $B = V_\mathrm{H}nte/I$	(*a*) $B = 1.36$ T (*b*) $B = 3.56$ T (*c*) $B = 5.42$ T

69* ·· Because blood contains charged ions, moving blood develops a Hall voltage across the diameter of an artery. A large artery with a diameter of 0.85 cm has a flow speed of 0.6 m/s. If a section of this artery is in a magnetic field of 0.2 T, what is the potential difference across the diameter of the artery?

$V_\mathrm{H} = v_\mathrm{d}Bw$	$V_\mathrm{H} = 1.02$ mV

70 ·· The Hall coefficient R is defined as $R = E_y / J_x B_z$, where J_x is the current per unit area in the x direction in the slab, B_z is the magnetic field in the z direction, and E_y is the resulting Hall field in the y direction. Show that the Hall coefficient is $1/nq$, where q is the charge of the charge carriers, -1.6×10^{-19} C if they are electrons. (The Hall coefficients of monovalent metals, such as copper, silver, and sodium, are therefore negative.)

$V_\mathrm{H} = E_\mathrm{H}w$ and $J = I/wt$. Using the defintion of R we have $R = V_\mathrm{H}t/IB = (IB/ntq)t/IB = 1/nq$. The direction of $\boldsymbol{E}$ is $\boldsymbol{v}_\mathrm{d}\times\boldsymbol{B}$, where $\boldsymbol{v}_\mathrm{d}$ is in the direction of I if q is positive. It follows that if J is in the x direction and $\boldsymbol{B}$ points in the z direction, then $\boldsymbol{E}_\mathrm{H}$ is in the y direction for q positive. If q is negative, $\boldsymbol{v}_\mathrm{d}$ is reversed, and so is $\boldsymbol{E}_\mathrm{H}$.

71 ·· Aluminum has a density of 2.7×10^3 kg/m^3 and a molar mass of 27 g/mol. The Hall coefficient of aluminum is $R = -0.3\times10^{-10}$ m^3/C. (See Problem 70 for the definition of R.) Find the number of conduction electrons per aluminum atom.

$n = 1/Re$	$n = 2.08\times10^{29}\ \mathrm{m}^{-3}$; number of electrons/atom = 3.46

72 ·· Magnesium is a divalent metal. Its density is 1.74×10^3 kg/m^3 and its molar mass is 24.3 g/mol. Assuming that each magnesium atom contributes two conduction electrons, what should be the Hall coefficient of magnesium? How does your result compare to the measured value of -0.94×10^{-10} m^3/C?

1. Find n — $n = 2\times6.02\times10^{29}\times1.74/24.3$ m^{-3} = 8.62×10^{28} m^{-3}
2. $R = 1/ne$ — $R = -0.725\times10^{-10}$ m^3/C; thus $n_{exp} \cong 1.5$ electrons/atom

73* · True or false:

(*a*) The magnetic force on a moving charged particle is always perpendicular to the velocity of the particle.
(*b*) The torque on a magnet tends to align the magnetic moment in the direction of the magnetic field.
(*c*) A current loop in a uniform magnetic field behaves like a small magnet.
(*d*) The period of a particle moving in a circle in a magnetic field is proportional to the radius of the circle.
(*e*) The drift velocity of electrons in a wire can be determined from the Hall effect.

(*a*) True (*b*) True (*c*) True (*d*) False (*e*) True

74 · Show that the force on a current element is the same in direction and magnitude regardless of whether positive charges, negative charges, or a mixture of positive and negative charges create the current.

From Equ. 28-4, the direction of the force does not depend on the sign of the charges that carry the current.

75 · A proton with a charge $+e$ is moving with a speed v at 50° to the direction of a magnetic field $\boldsymbol{B}$. The component of the resulting force on the proton in the direction of $\boldsymbol{B}$ is (*a*) evB sin 50° cos 50°. (*b*) evB cos 50°. (*c*) zero. (*d*) evB sin 50°. (*e*) none of these.

(*c*)

76 · If the magnetic field vector is directed toward the north and a positively charged particle is moving toward the east, what is the direction of the magnetic force on the particle?

From Equ. 28-1, $\boldsymbol{F}$ points up.

77* · A positively charged particle is moving northward in a magnetic field. The magnetic force on the particle is toward the northeast. What is the direction of the magnetic field? (*a*) Up (*b*) West (*c*) South (*d*) Down (*e*) This situation cannot exist.

(*e*)

78 · A ^{7}Li nucleus with a charge of $+3e$ and a mass of 7 u (1 u = 1.66×10^{-27} kg) and a proton with charge $+e$ and mass 1 u are both moving in a plane perpendicular to a magnetic field $\boldsymbol{B}$. The two particles have the same momentum. The ratio of the radius of curvature of the path of the proton R_p to that of the ^{7}Li nucleus, R_{Li} is (*a*) $R_p/R_{Li} = 3$. (*b*) $R_p/R_{Li} = 1/3$. (*c*) $R_p/R_{Li} = 1/7$. (*d*) $R_p/R_{Li} = 3/7$. (*e*) none of these.

(*a*)

79 · An electron moving with velocity v to the right enters a region of uniform magnetic field that points out of the paper. After the electron enters this region, it will be

(*a*) deflected out of the plane of the paper.
(*b*) deflected into the plane of the paper.
(*c*) deflected upward.
(*d*) deflected downward.
(*e*) undeviated in its motion.

(*c*)

80 · How are magnetic field lines similar to electric field lines? How are they different?

Magnetic field lines are similar to electric field lines in that their density is a measure of the strength of the field; the lines point in the direction of the field; also, magnetic field lines do not cross. They differ from electric field lines in that magnetic field lines must close on themselves (there are no isolated magnetic poles), and the force on a charge depends on the velocity of the charge and is perpendicular to the magnetic field lines.

81* · A long wire parallel to the *x* axis carries a current of 6.5 A in the positive *x* direction. There is a uniform magnetic field $\boldsymbol{B}$ = 1.35 T $\boldsymbol{j}$. Find the force per unit length on the wire.

Use Equ. 28-4 $\boldsymbol{F}$ = 8.775 N/m $\boldsymbol{k}$

82 · An alpha particle (charge +2*e*) travels in a circular path of radius 0.5 m in a magnetic field of 1.0 T. Find (*a*) the period, (*b*) the speed, and (*c*) the kinetic energy (in electron volts) of the alpha particle. Take $m = 6.65\times10^{-27}$ kg for the mass of the alpha particle.

(*a*) $T = 2\pi m/qB$; $q = 2e$, $m_\alpha = 6.65\times10^{-27}$ kg $T = 0.131\ \mu$s

(*b*) $v = 2\pi r/T$ $v = 2.40\times10^7$ m/s

(*c*) $K = 1/2 m_\alpha v^2/e$ eV K = 12.0 MeV

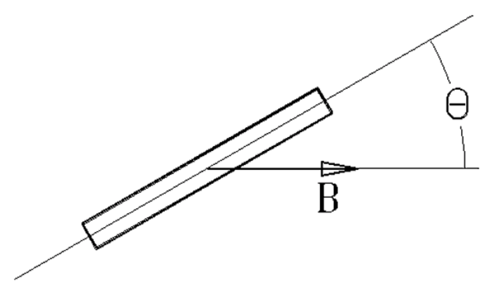
83 · If a current *I* in a given wire and a magnetic field $\boldsymbol{B}$ are known, the force $\boldsymbol{F}$ on the current is uniquely determined. Show that knowing $\boldsymbol{F}$ and *I* does not provide complete knowledge of $\boldsymbol{B}$.

If only $\boldsymbol{F}$ and *I* are known, one can only conclude that the magnetic field $\boldsymbol{B}$ is in the plane perpendicular to $\boldsymbol{F}$. The specific direction of $\boldsymbol{B}$ is undetermined.

84 ·· The pole strength q_m of a bar magnet is defined by $q_m = |\boldsymbol{\mu}|/L$, where *L* is the length of the magnet. Show that the torque exerted on a bar magnet in a uniform magnetic field $\boldsymbol{B}$ is the same as if a force $+q_m\mathrm{B}$ is exerted on the north pole and a force $-q_m\mathrm{B}$ is exerted on the south pole.

The configuration of the magnet and field are shown in the figure. Then

$\tau = (Bq_mL/2)\sin\theta + (Bq_mL/2)\sin\theta = \mu B\sin\theta = \boldsymbol{\mu}\times\boldsymbol{B}$, where $\mu = q_mL$.



85* ·· A particle of mass *m* and charge *q* enters a region where there is a uniform magnetic field *B* along the *x* axis. The initial velocity of the particle is $\boldsymbol{v} = v_{0x}\boldsymbol{i} + v_{0y}\boldsymbol{j}$ so the particle moves in a helix. (*a*) Show that the radius of the helix is $r = mv_{0y}/qB$. (*b*) Show that the particle takes a time $t = 2\pi m/qB$ to make one orbit around the helix.

(*a*) Since $\boldsymbol{B} = B\,\boldsymbol{i}$, $\boldsymbol{v}_0\times\boldsymbol{B} = v_{0y}B\,\boldsymbol{k}$; i.e., $v_x = v_{0x}$ and motion on the direction of the magnetic field is not affected by the field. In the plane perpendicular to $\boldsymbol{i}$ the motion is as described in Section 28–2, and the radius of the circular path is given by Equ. 28-6 with $v = v_{0y}$, i.e., $r = mv_{0y}/qB$.

(*b*) The time for one complete orbit is given by Equ. 28-7, i.e., $t = 2\pi m/qB$.

86 ·· A metal crossbar of mass *M* rides on a pair of long, horizontal conducting rails separated by a distance ℓ and connected to a device that supplies constant current *I* to the circuit, as shown in Figure 28-40. A uniform magnetic field *B* is established as shown. (*a*) If there is no friction and the bar starts from rest at *t* = 0, show that at time t the bar has velocity $v = (BI\ell/M)t$. (*b*) In which direction will the bar move? (*c*) If the coefficient of static friction is μ_s, find the minimum field B necessary to start the bar moving.

(*a*), (*b*) The force on the bar is $F = BI\ell$ and its acceleration is $F/M = BI\ell/M = a$. In the absence of friction the velocity is $v = at = BI\ell\, t/M$ and is directed to the right since $\boldsymbol{I}\ell\times\boldsymbol{B}$ is directed to the right.

(*c*) To start the bar moving, $F_{min} = \mu_s Mg = B_{min}I\ell$. So $B_{min} = \mu_s Mg/I\ell$.

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87 ·· Assume that the rails in Figure 28-40 are frictionless but tilted upward so that they make an angle θ with the horizontal. (*a*) What vertical magnetic field B is needed to keep the bar from sliding down the rails? (*b*) What is the acceleration of the bar if B has twice the value found in part (*a*)?

(*a*) Note that with the rails tilted, $\boldsymbol{F}$ still points horizontally to the right. The component of $\boldsymbol{F}$ along the rail is then

$BI\ell\cos\theta$ and the component of Mg along the rail is $-Mg\sin\theta$. To hold the bar in position $\boldsymbol{F}_{tot} = 0$. Solving for B one obtains $B = (Mg/I\ell)\tan\theta$.

(*b*) If B has twice the value found in (*a*), then the net force along the rail is $Mg\sin\theta$ directed upward. Consequently, $a = g\sin\theta$.

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88 ·· A long, narrow bar magnet that has magnetic moment $\boldsymbol{\mu}$ parallel to its long axis is suspended at its center as a frictionless compass needle. When placed in a magnetic field $\boldsymbol{B}$, the needle lines up with the field. If it is displaced by a small angle θ, show that the needle will oscillate about its equilibrium position with frequency $f = \frac{1}{2\pi}\sqrt{\mu B/I}$, where I is the moment of inertia about the point of suspension.

$\tau = I(d^2\theta/dt^2) = -\mu B\sin\theta$. For $\theta << 1$, $\sin\theta \cong \theta$. Thus $(d^2\theta/dt^2) = -(\mu B/I)\theta$. This is the differential equation for the SHO (see Equ. 14-2), and on comparison with Equ. 14-12 one obtains $f = \dfrac{\sqrt{\mu B/I}}{2\pi}$.

89* ·· A conducting wire is parallel to the y axis. It moves in the positive x direction with a speed of 20 m/s in a magnetic field $\boldsymbol{B} = 0.5$ T $\boldsymbol{k}$. (*a*) What are the magnitude and direction of the magnetic force on an electron in the conductor? (*b*) Because of this magnetic force, electrons move to one end of the wire leaving the other end positively charged, until the electric field due to this charge separation exerts a force on the electrons that balances the magnetic force. Find the magnitude and direction of this electric field in the steady state. (*c*) Suppose the moving wire is 2 m long. What is the potential difference between its two ends due to this electric field?

(*a*) Use Equ. 28–1; $q = -1.6\times10^{-19}$ C	$\boldsymbol{F} = 1.6\times10^{-18}$ N $\boldsymbol{j}$
(*b*) At steady state, $q\boldsymbol{E} + \boldsymbol{F} = 0$	$\boldsymbol{E} = 10$ V/m $\boldsymbol{j}$
(*c*) $\Delta V = E\,\Delta x$	$\Delta V = 20$ V

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90 ··· The rectangular frame in Figure 28-41 is free to rotate about the axis A-A on the horizontal shaft. The frame is 10 cm long and 6 cm wide and the rods that make up the frame have a mass per unit length of 20 g/cm. A uniform magnetic field $B = 0.2$ T is directed as shown. A current may be sent around the frame by means of the wires attached at the top. (*a*) If no current passes through the frame, what is the period of this physical pendulum for small oscillations? (*b*) If a current of 8.0 A passes through the frame in the direction indicated by the arrow, what is then the period of this physical pendulum? (*c*) Suppose the direction of the current is opposite to that shown. The frame is displaced from the vertical by some angle θ. What must be the magnitude of the current so that this frame will be in equilibrium?

(*a*) 1. Find the moment of inertia of the frame $\quad I = [2(1/3)\times0.2\times0.1^2 + 0.12\times0.1^2]$ kg·m^2 = 2.53 g·m^2

2. Find D, the distance from the pivot to the CM — $D = (0.4\times0.05 + 0.12\times0.1)/0.52$ m $= 0.0615$ m

$T = 0.565$ s

3. Use Equ. 14-31 — $\tau_{tot} = (0.52\times9.81\times0.0615 + 0.2\times8.0\times0.006)\theta$ N·m $= 0.323\theta$ N·m

(*b*) 1. With $\boldsymbol{B}$ and I as shown, $\boldsymbol{F}_m$ is downward; total restoring torque = $(MgD + BIA)\theta$ — $T = 0.556$ s

2. Use Equ. 14-31 with $MgD \rightarrow (MgD + BIA)$ — $I = MgD/BA = 261$ A

(*c*) The total torque = 0; $MgD \sin\theta = BIA \sin\theta$

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91 ··· A stiff, straight, horizontal wire of length 25 cm and mass 20 g is supported by electrical contacts at its ends, but is otherwise free to move vertically upward. The wire is in a uniform, horizontal magnetic field of magnitude
0.4 T perpendicular to the wire. A switch connecting the wire to a battery is closed and the wire is shot upward, rising to a maximum height h. The battery delivers a total charge of 2 C during the short time it makes contact with the wire. Find the height h.

1. $\Delta p = F\Delta t = BI\ell\,\Delta t = BQ\ell = mv_0$; evaluate v_0 — $v_0 = 0.4\times2\times0.25/0.02$ m/s $= 10$ m/s
2. $h = v_0^2/2g$ — $h = 5.10$ m

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92 ··· A solid sphere of radius R carries a charge density $-\rho_0$ in the region $r = 0$ to $r = R_s$ and an equal charge density of opposite sign, $+\rho_0$, between $r = R_s$ and $r = R$. The net charge carried by the sphere is zero. (*a*) What must be the ratio R/R_s? (*b*) If this sphere rotates with angular velocity ω about its diameter, what is its magnetic moment?

(*a*) $Q_+ = (4\pi/3)R_s^3\rho_0$; $Q_- = -(4\pi/3)(R^3 - R_s^3)\rho_0$; $Q_+ + Q_- = 0$; solve for R_s. — $2R_s^3 = R^3$; $R_s = R/2^{1/3} = 0.794R$

(*b*) Use the result of Problem 64 with $R_s = R/2^{1/3}$ — $\boldsymbol{\mu} = -(4\pi/15)\rho_0R^5(1 - 2^{-2/3})\boldsymbol{\omega} = -0.31\rho_0R^5\boldsymbol{\omega}$

93* ··· A circular loop of wire with mass M carries a current I in a uniform magnetic field. It is initially in equilibrium with its magnetic moment vector aligned with the magnetic field. The loop is given a small twist about a diameter and then released. What is the period of the motion? (Assume that the only torque exerted on the loop is due to the magnetic field.)

$\tau = -\mu B \sin\theta \cong -IAB\theta = -\pi R^2 IB\theta = MR^2(d^2\theta/dt^2)$. This is the differential equation for a SHO, and comparison with Equs. 14-2 and 14-12 shows that $T = 2\pi\sqrt{M/(\pi I B)}$.

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94 ··· A small bar magnet has a magnetic moment μ that makes an angle θ with the x axis and lies in a nonuniform magnetic field given by $B = B_x(x)\,i + B_y(y)\,j$. Use $F_x = -dU/dx$ and $F_y = -dU/dy$ to show that there is a net force on the magnet that is given by

$$\boldsymbol{F} \approx \mu_x \frac{\partial B_x}{\partial x}\,i + \mu_y \frac{\partial B_y}{\partial y}\,\boldsymbol{j}$$

Let $\boldsymbol{\mu} = \mu_x\,\boldsymbol{i} + \mu_y\,\boldsymbol{j} + \mu_z\,\boldsymbol{k}$. $U = -\boldsymbol{\mu}\cdot\boldsymbol{B} = -\mu_xB_x - \mu_yB_y$. Since μ is constant but $\boldsymbol{B}$ depends on x and y, $F_x = -dU/dx = \mu_x(\partial B_x/\partial x)$, $F_y = -dU/dy = \mu_y(\partial B_y/\partial y)$, and $\boldsymbol{F} = \mu_x(\partial B/\partial x)\,\boldsymbol{i} + \mu_y(\partial B_y/\partial y)\,\boldsymbol{j}$.