CHAPTER 26

Electric Current and Direct-Current Circuits

1* · In our study of electrostatics, we concluded that there is no electric field within a conductor in electrostatic equilibrium. How is it that we can now discuss electric fields inside a conductor?

When a current flows, the charges are not in equilibrium. In that case, the electric field provides the force needed for the charge flow.

2 · A physics professor has assembled his class at the baggage-claim carousel of the local airport to demonstrate an analog of electrical current. "Think of each suitcase on the conveyor belt as a package of electrons carrying one coulomb of charge," he says. Counting and timing the suitcases reveals that the conveyor belt represents a wire carrying a constant 2-A current (constant as long as annoyed travellers could be kept away from their baggage by some of the huskier students). (*a*) How many suitcases will go by a given point in 5.0 min? (*b*) How many electrons does that represent?

(*a*) $\Delta Q = I\,\Delta t$ — ΔQ = 600 C = 600 suitcases

(*b*) $n = \Delta Q/e$ — $n = 600/1.6\times10^{-19} = 3.75\times10^{21}$ electrons

3 · A 10–gauge copper wire carries a current of 20 A. Assuming one free electron per copper atom, calculate the drift velocity of the electrons.

$I/A = nev_\mathrm{d}$; $n = 8.47\times10^{28}$ m^{-3} (Example 26-1) — $v_\mathrm{d} = 20/(8.47\times10^{28}\times5.26\times10^{-6}\times1.6\times10^{-19})$ m/s

$A = 5.26\times10^{-6}$ m^2 (Table 26-2) — = 0.281 mm/s

4 · In a fluorescent tube of diameter 3.0 cm, 2.0×10^{18} electrons and 0.5×10^{18} positive ions (with a charge of $+e$) flow through a cross-sectional area each second. What is the current in the tube?

The positive and negative charges flow in opposite directions.

1. Find I_electron — $I_\mathrm{electron} = 2\times10^{18}\times1.6\times10^{-19}$ A = 0.32 A

2. Finbd I_ion; $I = I_\mathrm{electron} + I_\mathrm{ion}$ — I_ion = 0.08 A; I = 0.40 A

5* · In a certain electron beam, there are 5.0×10^6 electrons per cubic centimeter. Suppose the kinetic energy of each electron is 10.0 keV, and the beam is cylindrical, with a diameter of 1.00 mm. (*a*) What is the velocity of an electron in the beam? (*b*) Find the beam current.

(*a*) $v = \sqrt{2K/m_\mathrm{e}}$; $K = 10^4\times1.6\times10^{-19}$ J — $v = 5.93\times10^7$ m/s

(*b*) $I = nevA$; $A = \pi D^2/4$ — $I = 37.2\ \mu$A

6 ·· A charge $+q$ moves in a circle of radius r with speed v. (a) Express the frequency f with which the charge passes a particular point in terms of r and v. (b) Show that the average current is qf and express it in terms of v and r.

(a) $T = 2\pi r/v = 1/f$; $f = v/2\pi r$. (b) $I = \Delta Q/\Delta t = q/T = qf = qv/2\pi r$.

7 ·· A ring of radius a with a linear charge density λ rotates about its axis with angular velocity ω. Find an expression for the current.

$Q = 2\pi a\lambda$; $\omega = 2\pi f$; from Problem 6, $I = Qf = a\lambda\omega$.

8 ·· A 10-gauge copper wire and a 14-gauge copper wire are welded together end to end. The wires carry a current of 15 A. If there is one free electron per copper atom in each wire, find the drift velocity of the electrons in each wire.

1. For 10-gauge wire see Problem 3 — v_d(10-gauge) = 0.281(15/20) mm/s = 0.211 mm/s
2. For 14-gauge, $v_d = v_d(\text{10-gauge}) \times A_{10}/A_{14}$ — v_d(14-gauge) = 0.211(5.26/2.08) mm/s = 0.533 mm/s

9* ·· In a certain particle accelerator, a proton beam with a diameter of 2.0 mm constitutes a current of 1.0 mA. The kinetic energy of each proton is 20 MeV. The beam strikes a metal target and is absorbed by it. (a) What is the number n of protons per unit volume in the beam? (b) How many protons strike the target in 1.0 min? (c) If the target is initially uncharged, express the charge of the target as a function of time.

(a) 1. $I = neAv$; $v = \sqrt{2K/m_p}$ — $$v = \sqrt{\frac{40\times10^{6}\times1.6\times10^{-19}}{1.67\times10^{-27}}} = 6.19\times10^{7}\ \text{m/s};$$

2. $A = \pi D^2/4$; solve for n — $A = \pi\times10^{-6}\ \text{m}^2$; $n = 3.21\times10^{13}$

(b) $N = nAvt$ — $N = 3.75\times10^{17}$

(c) $Q = I\times t$ — $Q = (1.0\ \text{mC/s})t$

10 ·· The current in a wire varies with time according to the relation $I = 20 + 3t^2$, where I is in amperes and t is in seconds. (a) How many coulombs are transported by the wire between $t = 0$ and $t = 10$ s? (b) What constant current would transport the same charge in the same time interval?

(a) $Q = \int I\,dt$ — $$Q = \int_0^{10} (20 + 3t^2)\,dt = 1200\ \text{C}$$

(b) $I = Q/\Delta t$ — $I = 120$ A

11 ·· In a proton supercollider, the protons in a 5-mA beam move with nearly the speed of light. (a) How many protons are there per meter of the beam? (b) If the cross-sectional area of the beam is $10^{-6}\ \text{m}^2$, what is the number density of protons?

(a) $I = neAc$; $n = I/eAc$; N per meter $= nA$ — $N = 5\times10^{-3}/(1.6\times10^{-19}\times3\times10^{8})\ \text{m}^{-1} = 1.04\times10^{8}\ \text{m}^{-1}$

(b) $n = N/A$ — $n = 1.04\times10^{14}\ \text{m}^{-3}$

12 · Figure 26-8 illustrates a mechanical analog of a simple electric circuit. Devise another mechanical analog in which the current is represented by a flow of water instead of marbles.

Water, regarded as a viscous liquid, flowing from a water tower through a pipe to ground.

13* · Two wires of the same material with the same length have different diameters. Wire A has twice the diameter of wire B. If the resistance of wire B is R, then what is the resistance of wire A? (*a*) R (*b*) $2R$ (*c*) $R/2$ (*d*) $4R$ (*e*) $R/4$

(*e*)

14 ·· Discuss the difference between an emf and a potential difference.

An emf is a driving force that gives rise to a potential difference and may result in current flow if there is a conducting path.

15 ·· Name several common sources of emf. What sort of energy is converted into electrical energy in each?

Generator: mechanical to electrical energy. Dry cell and lead-acid battery: chemical to electrical energy. Photovoltaic cell: Light to electrical energy. Thermocouple: Heat to electrical energy.

16 ·· A metal bar is to be used as a resistor. Its dimensions are 2 by 4 by 10 units. To get the smallest resistance from this bar, one should attach leads to the opposite sides that have the dimensions of

(*a*) 2 by 4 units.

(*b*) 2 by 10 units.

(*c*) 4 by 10 units.

(*d*) All connections will give the same resistance.

(*e*) None of the above is correct.

(*c*)

17* ·· Two cylindrical copper wires have the same mass. Wire A is twice as long as wire B. Their resistances are related by (*a*) $R_A = 8R_B$. (*b*) $R_A = 4R_B$. (*c*) $R_A = 2R_B$. (*d*) $R_A = R_B$.

(*b*)

18 · A 10-m-long wire of resistance 0.2 Ω carries a current of 5 A. (*a*) What is the potential difference across the wire? (*b*) What is the magnitude of the electric field in the wire?

(*a*) $V = IR$ $V = 1.0$ V

(*b*) $E = V/L$ $E = 0.10$ V/m

19 · A potential difference of 100 V produces a current of 3 A in a certain resistor. (*a*) What is the resistance of the resistor? (*b*) What is the current when the potential difference is 25 V?

(*a*) $R = V/I$ $R = 33.3\ \Omega$

(*b*) $I = V/R$ $I = 0.75$ A

20 · A block of carbon is 3.0 cm long and has a square cross-sectional area with sides of 0.5 cm. A potential difference of 8.4 V is maintained across its length. (*a*) What is the resistance of the block? (*b*) What is the current in this resistor?

(*a*) $R = \rho L/A$ $R = (3.5\times10^{-5}\times0.03/0.25\times10^{-4})\ \Omega = 42\ \text{m}\Omega$

(*b*) $I = V/R$ $I = 200$ A

21* · A carbon rod with a radius of 0.1 mm is used to make a resistor. The resistivity of this material is $3.5\times10^{-5}\ \Omega\cdot$m. What length of the carbon rod will make a 10-Ω resistor?

$L = RA/\rho = \pi r^2 R/\rho$ $L = \pi\times10^{-8}\times10/3.5\times10^{-5}$ m = 8.98 mm

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22 · The third (current-carrying) rail of a subway track is made of steel and has a cross-sectional area of about 55 cm^2. What is the resistance of 10 km of this track?

$R = \rho L/A$	$R = 10^{-7} \times 10^4/55\times10^{-4}\ \Omega = 0.182\ \Omega$

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23 · What is the potential difference across one wire of a 30-m extension cord made of 16-gauge copper wire carrying a current of 5.0 A?

$R = \rho L/A$; $V = IR$	$R = 0.389\ \Omega$; $V = 1.95$ V

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24 · How long is a 14-gauge copper wire that has a resistance of 2 Ω?

$L = RA/\rho$	$L = 2\times2.08\times10^{-6}/1.7\times10^{-8}$ m = 245 m

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25* ·· A cylinder of glass 1 cm long has a resistivity of 10^{12} Ω·m. How long would a copper wire of the same cross-sectional area need to be to have the same resistance as the glass cylinder?

$L_{Cu} = L_{glass}(\rho_{glass}/\rho_{Cu})$	$L_{Cu} = 0.01(10^{12}/1.7\times10^{-8})$ m = 5.88×10^{17} m = 62.2 c·y

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26 ·· An 80.0-m copper wire 1.0 mm in diameter is joined end to end with a 49.0-m iron wire of the same diameter. The current in each is 2.0 A. (*a*) Find the electric field in each wire. (*b*) Find the potential drop across each wire.

(*b*) Find R_{Cu} and R_{Fe} and V_{Cu} and V_{Fe}	$R_{Cu} = 1.73\ \Omega$; $R_{Fe} = 6.24\ \Omega$; $V_{Cu} = 3.46$ V; $V_{Fe} = 12.48$ V
(*a*) $E = V/L$	$E_{Cu} = 3.46/80$ V/m = 43.3 mV/m; $E_{Fe} = 255$ mV/m

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27 ·· A copper wire and an iron wire with the same length and diameter carry the same current I. (*a*) Find the ratio of the potential drops across these wires. (*b*) In which wire is the electric field greater?

(*a*) $V_{Cu}/V_{Fe} = \rho_{Cu}/\rho_{Fe}$	$V_{Cu}/V_{Fe} = 0.17$
(*b*) V and therefore E is greater in the iron wire	

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28 ·· A variable resistance R is connected across a potential difference V that remains constant. When $R = R_1$, the current is 6.0 A. When R is increased to $R_2 = R_1 + 10.0\ \Omega$, the current drops to 2.0 A. Find (*a*) R_1 and (*b*) V.

Write the equations for the data given	$6R_1 = V$; $2(R_1 + 10) = V = 6R_1$
(*a*) Solve for R_1	$R_1 = 5\ \Omega$
(*b*) Find V	$V = 30$ V

29* ·· A rubber tube 1 m long with an inside diameter of 4 mm is filled with a salt solution that has a resistivity of 10^{-3} Ω·m. Metal plugs form electrodes at the ends of the tube. (*a*) What is the resistance of the filled tube? (*b*) What is the resistance of the filled tube if it is uniformly stretched to a length of 2 m?

(*a*) $R = \rho L/A$	$R = 79.6\ \Omega$
(*b*) $L' = 2L$, $A' = A/2$; $R' = 4R$	$R' = 318\ \Omega$

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30 ·· A wire of length 1 m has a resistance of 0.3 Ω. It is uniformly stretched to a length of 2 m. What is its new resistance?

The length is doubled and the area is halved. $R = 4\times0.3\ \Omega = 1.2\ \Omega$

31 ·· Currents up to 30 A can be carried by 10-gauge copper wire. (*a*) What is the resistance of 100 m of 10-gauge copper wire? (*b*) What is the electric field in the wire when the current is 30 A? (*c*) How long does it take for an electron to travel 100 m in the wire when the current is 30 A?

(*a*) $R = \rho L/A$; $A = 5.26\ \text{mm}^2$ (See Table 26-2) $R = 1.7\times10^{-8}\times100/5.26\times10^{-6}\ \Omega = 0.323\ \Omega$

(*b*) $E = IR/L$ $E = 97$ mV/m

(*c*) $v_d = I/neA$; $t = L/v_d = neAL/I$ $t = 8.47\times10^{28}\times1.6\times10^{-19}\times5.26\times10^{-4}/30\ \text{s} = 2.38\times10^{5}$ s

32 ·· A cube of copper has sides of 2.0 cm. If it is drawn out to form a 14-gauge wire, what will its resistance be?

1. Find the length of the wire; $AL = 0.02^3$ $L = 8\times10^{-6}/2.08\times10^{-6}$ m = 3.85 m

2. $R = \rho L/A$ $R = 1.7\times10^{-8}\times3.85/2.08\times10^{-6}\ \Omega = 0.0314\ \Omega$

33* ··· A semiconducting diode is a nonlinear device whose current I is related to the voltage V across the diode by $I = I_0(e^{eV/kT} - 1)$, where k is Boltzmann's constant, e is the magnitude of the charge on an electron, and T is the absolute temperature. If $I_0 = 10^{-9}$ A and $T = 293$ K, (*a*) what is the resistance of the diode for $V = 0.5$ V? (*b*) What is the resistance for $V = 0.6$ V?

(*a*), (*b*) $R = V/I = V/[I_0(e^{eV/kT} - 1)]$ For $V = 0.5$ V, $eV/kT = 19.785$; $R = 1.28\ \Omega$

For $V = 0.6$ V, $R = 0.0293\ \Omega$

34 ··· Find the resistance between the ends of the half ring shown in Figure 26-45. The resistivity of the material of the ring is ρ.

The element of resistance we use is a semicircular strip of width t, radius r, and thickness dr. Thus $dR = \pi r\rho/tdr$.

The strips are connected in parallel, so $\dfrac{1}{R} = \dfrac{t}{\rho\pi}\displaystyle\int_a^b \frac{dr}{r} = \frac{t}{\rho\pi}\ln(b/a)$; $R = \dfrac{\rho\pi}{t\ln(b/a)}$

35 ··· The radius of a wire of length L increases linearly along its length according to $r = a + [(b - a)/L]x$, where x is the distance from the small end of radius a. What is the resistance of this wire in terms of its resistivity ρ, length L, radius a, and radius b?

The element of resistance we use is a segment of length dx and cross-sectional area $\pi[a + (b - a)x/L]^2$. Since these resistance elements are in series, we have $R = \dfrac{\rho}{\pi}\displaystyle\int_0^L \frac{dx}{[a+(b-a)x/L]^2} = \frac{\rho L}{\pi(b-a)}\left(\frac{1}{a} - \frac{1}{a+(b-a)}\right) = \frac{\rho L}{\pi ab}$

36 ··· The space between two concentric spherical-shell conductors is filled with a material that has a resistivity of $10^9\ \Omega\cdot$m. If the inner shell has a radius of 1.5 cm and the outer shell has a radius of 5 cm, what is the resistance between the conductors? (*Hint:* Find the resistance of a spherical-shell element of the material of area $4\pi r^2$ and length dr, and integrate to find the total resistance of the set of shells in series.)

Using the *Hint* we write $R = \dfrac{\rho}{4\pi}\displaystyle\int_a^b \frac{dr}{r^2} = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$, where a and b are the inner and outer radii of the shell.

Inserting numerical values for ρ, a, and b one obtains $R = 3.71\times10^{9}\ \Omega$.

37* ··· The space between two metallic coaxial cylinders of length L and radii a and b is completely filled with a material having a resistivity ρ. (*a*) What is the resistance between the two cylinders? (See the hint in Problem 36.) (*b*) Find the current between the two cylinders ρ=30 Ω·m, a = 1.5 cm, b = 2.5 cm, L = 50 cm, and a potential difference of 10 V is maintained between the two cylinders if.

(*a*) Here the element of resistance is a cylindrical shell of thickness dr and cross-sectional area $2\pi rL$. The elements of resistance are in series. Thus $R = \frac{\rho}{2\rho L}\int_a^b \frac{dr}{r} = \frac{\rho}{2\pi L}\ln(b/a)$

(*b*) Evaluate R and $I = V/R$ — R = 4.88 Ω; I = 2.05 A

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38 · A tungsten rod is 50 cm long and has a square cross-sectional area with sides of 1.0 mm. (*a*) What is its resistance at 20°C? (*b*) What is its resistance at 40°C?

Note: For Problems 38-44 we shall neglect the effects of thermal expansion.

(*a*) $R = \rho L/A$ — $R = 5.5\times10^{-8}\times0.5/10^{-6}$ Ω = 27.5 mΩ

(*b*) $R_{40} = R_{20}(1 + \alpha\times20)$ — R_{40} = 27.5(1 + 0.09) mΩ = 30 mΩ

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39 · At what temperature will the resistance of a copper wire be 10% greater than it is at 20°C?

$\alpha(t_C - 20) = 0.1$; solve for t_C — t_C = 45.6°C

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40 ·· A toaster with a Nichrome heating element has a resistance of 80 Ω at 20°C and an initial current of 1.5 A. When the heating element reaches its final temperature, the current is 1.3 A. What is the final temperature of the heating element?

1. At constant voltage, $I_1R_1 = I_2R_2$ — $R_2 = R_1(1.5/1.3) = 1.154R_1$

2. $R_2 = R_1[1 + \alpha(t_C - 20)]$; solve for t_C — $t_C = 20 + (1.154 - 1)/\alpha = 405^{\circ}$C

41* ·· An electric space heater has a Nichrome heating element with a resistance of 8 Ω at 20°C. When 120 V are applied, the electric current heats the Nichrome wire to 1000°C. (*a*) What is the initial current drawn by the cold heating element? (*b*) What is the resistance of the heating element at 1000°C? (*c*) What is the operating wattage of this heater?

(*a*) $I = V/R$ — I = 120/8 A = 15 A

(*b*) $R_{1000} = R_{20}[1 + \alpha(1000 - 20)]$ — R_{1000} = 8(1 + 0.392) Ω = 11.14 Ω

(*c*) At $t_C = 1000^{\circ}$C, $P = V^2/R_{1000}$ — P = 1.29 kW

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42 ·· A 10-Ω Nichrome resistor is wired into an electronic circuit using copper leads (wires) of diameter 0.6 mm with a total length of 50 cm. (*a*) What additional resistance is due to the copper leads? (*b*) What percentage error in the total added resistance is produced by neglecting the resistance of the copper leads? (*c*) What change in temperature would produce a change in resistance of the Nichrome-wire equal to the resistance of the copper leads?

(*a*) $R_{Cu} = \rho_{Cu}L/A$ — R_{Cu} = 0.03 Ω

(*b*) % error = $100(R_{Cu}/R_{Nich})$ — % error = 0.3%

(*c*) At t_C, $R_{Cu} = 0.03[1 + \alpha_{Cu}(t_C - 20)]$
$= \Delta R_W = 10\alpha_W(t_C - 20)$; solve for t_C — t_C = 320°C

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43 ·· The filament of a certain lamp has a resistance that increases linearly with temperature. When a constant voltage is switched on, the initial current decreases until the filament reaches its steady-state temperature. The temperature coefficient of resistivity of the filament is 4×10^{-3} K^{-1}. The final current through the filament is one-eighth the initial current. What is the change in temperature of the filament?

$R_t = R_{20}(1 + \alpha\Delta T)] = 8R_{20}$; solve for ΔT $\qquad$ $\Delta T = 7/\alpha = 1750$ K

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44 ··· A wire of cross-sectional area A, length L_1, resistivity ρ_1, and temperature coefficient α_1 is connected end to end to a second wire of the same cross-sectional area, length L_2, resistivity ρ_2, and temperature coefficient α_2, so that the wires carry the same current. (*a*) Show that if $\rho_1 L_1\alpha_1 + \rho_2 L_2\alpha_2 = 0$, the total resistance R is independent of temperature for small temperature changes. (*b*) If one wire is made of carbon and the other is copper, find the ratio of their lengths for which R is approximately independent of temperature.

(*a*) The total resistance is $R = (1/A)[\rho_1 L_1(1 + \alpha_1\Delta T) + \rho_2 L_2(1 + \alpha_2\Delta T)]$. For the change in resistance to be zero we must have $\rho_1\alpha_1 L_1 + \rho_2\alpha_2 L_2 = 0$.

(*b*) For carbon, $\rho_C\alpha_C = -1.75\times10^{-8}$ $\Omega\cdot$m/K; for copper, $\rho_{Cu}\alpha_{Cu} = 6.63\times10^{-11}$ $\Omega\cdot$m/K. Using the result of (*a*) we find $L_{Cu}/L_C = 264$.

45* · A resistor carries a current I. The power dissipated in the resistor is P. What is the power dissipated if the same resistor carries current $3I$? (Assume no change in resistance.) (*a*) P (*b*) $3P$ (*c*) $P/3$ (*d*) $9P$ (*e*) $P/9$

(*d*)

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46 · The power dissipated in a resistor is P when the voltage drop across it is V. If the voltage drop is increased to 2 V (with no change in resistance), what is the power dissipated? (*a*) P (*b*) $2P$ (*c*) $4P$ (*d*) $P/2$ (*e*) $P/4$

(*c*)

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47 · A heater consists of a variable resistance connected across a constant voltage supply. To increase the heat output, should you decrease the resistance or increase it?

$P \propto 1/R$; decrease the resistance.

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48 ·· Two resistors dissipate the same amount of power. The potential drop across resistor A is twice that across resistor B. If the resistance of resistor B is R, what is the resistance of A? (*a*) R (*b*) $2R$ (*c*) $R/2$ (*d*) $4R$ (*e*) $R/4$

(*d*)

49* · Find the power dissipated in a resistor connected across a constant potential difference of 120 V if its resistance is (*a*) 5 Ω and (*b*) 10 Ω.

(*a*), (*b*) $P = V^2/R$ $\qquad$ (*a*) $P = 2.88$ kW; (*b*) $P = 1.44$ kW

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50 · A 10,000-Ω carbon resistor used in electronic circuits is rated at 0.25 W. (*a*) What maximum current can this resistor carry? (*b*) What maximum voltage can be placed across this resistor?

(*a*) $I_{max}^2 R = P_{max}$ $\qquad$ $I_{max} = 5$ mA

(*b*) $V = IR$ $\qquad$ $V_{max} = 50$ V

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51 · A 1-kW heater is designed to operate at 240 V. (*a*) What is its resistance, and what current does it draw? (*b*) What is the power dissipated in this resistor if it operates at 120 V? Assume that its resistance is constant.

(*a*) $P = V^2/R$; $R = V^2/P$; $I = V/R$ $\qquad$ $R = 57.6\ \Omega$; $I = 4.17$ A

(*b*) $P_{120} = P_{240}/4$ $\qquad$ $P = 250$ W

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52 · A battery has an emf of 12.0 V. How much work does it do in 5 s if it delivers a current of 3 A?

$P = \mathcal{E} I$; $W = P\Delta t$ — $W = 12\times3\times5$ J = 180 J

53* · A battery with 12-V emf has a terminal voltage of 11.4 V when it delivers a current of 20 A to the starter of a car. What is the internal resistance r of the battery?

$r = V_r/I$ — $r = 0.6/20\ \Omega = 0.03\ \Omega$

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54 · (*a*) How much power is delivered by the emf of the battery in Problem 53 when it delivers a current of 20 A? (*b*) How much of this power is delivered to the starter? (*c*) By how much does the chemical energy of the battery decrease when it delivers a current of 20 A to the starter for 3 min? (*d*) How much heat is developed in the battery when it delivers a current of 20 A for 3 min?

(*a*) $P = \mathcal{E} I$ — $P = 240$ W

(*b*) $P_S = V_S I$ — $P_S = 11.4\times20$ W = 228 W

(*c*) $\Delta E = P\Delta t$ — $\Delta E = 240\times180 = 43.2$ kJ

(*d*) $Q = (P - P_S)\Delta t$ — $Q = 2.16$ kJ

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55 · A physics student runs a 1200-W electric heater constantly in her basement bedroom during the winter time. If electric energy costs 9 cents per kilowatt-hour, how much does this electric heating cost per 30-day month?

Cost = P(kW)$\times\Delta t$(h)$\times$\$/kW·h — Cost = \$(1.2×30×24×0.09) = \$77.76

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56 · A battery with an emf of 6 V and an internal resistance of 0.3 Ω is connected to a variable resistance R. Find the current and power delivered by the battery when R is (*a*) 0, (*b*) 5 Ω, (*c*) 10 Ω, and (*d*) infinite.

(*a*), (*b*), (*c*), (*d*) $I = \mathcal{E}/(r + R)$; $P = I^2R$ — (*a*) $I = 6/0.3$ A = 20 A; $P = 0$; (*b*) $I = 1.13$ A; $P = 6.41$ W

(*c*) $I = 0.583$ A; $P = 1.70$ W; (*d*) $I = P = 0$

57* ·· Staying up late to study, and having no stove to heat water, you use a 200-W heater from the lab to make coffee throughout the night. If 90% of the energy produced by the heater goes toward heating the water in your cup, (*a*) how long does it take to heat 0.25 kg of water from 15 to 100°C? (*b*) If you fall asleep while the water is heating, how long will it take to boil away after it reaches 100°C?

(*a*) $0.9P\Delta t = mc\Delta T$; $\Delta t = mc\Delta T/0.9P$ — $\Delta t = (0.25\times85\times4.18\times10^3/180)$ s = 491 s $\cong$ 8.2 min

(*b*) $0.9P\Delta t = mL$; $\Delta t = mL/0.9P$ — $\Delta t = (0.25\times2257\times10^3/180)$ s = 3135 s $\cong$ 52.2 min

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58 ·· Suppose the bulb in a two-cell flashlight draws 4 W of power. The batteries go dead in 45 min and cost \$7.99. (*a*) How many kilowatt-hours of energy can be supplied by the two batteries? (*b*) What is the cost per kilowatt-hour of energy if the batteries cannot be recharged? (*c*) If the batteries can be recharged at a cost of 9 cents per kilowatt-hour, what is the cost of recharging them?

(*a*) $W = Pt$ — $W = 4\times3/4$ W·h = 0.003 kW·h

(*b*) Cost/kW·h = \$(kW·h)/W(kW·h) — Cost per kW·h = \$2663

(*c*) Recharging cost = \$(kW·h)$\times U$(kW·h) — Recharging cost = 0.027 cents

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59 ·· A 12-V automobile battery with negligible internal resistance can deliver a total charge of 160 A·h. (*a*) What is the total stored energy in the battery? (*b*) How long could this battery provide 150 W to a pair of headlights?

(*a*) $U = \mathcal{E}It$ — $U = 12\times160$ W·h = 1.92 kW·h = 6.91 MJ

(*b*) $t = U/P$ — $t = 1920/150 = 12.8$ h

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60 ·· A space heater in an old home draws a 12.5-A current. A pair of 12-gauge copper wires carries the current from the fuse box to the wall outlet, a distance of 30 m. The voltage at the fuse box is exactly 120 V. (*a*) What is the voltage delivered to the space heater? (*b*) If the fuse will blow at a current of 20 A, how many 60-W bulbs can be supplied by this line when the space heater is on? (Assume that the wires from the wall to the space heater and to the light fixtures have negligible resistance.)

(*a*) Find $R_{Cu} = \rho L/A$, $L = 60$ m; $V = \mathcal{E} - IR_{Cu}$ — $R_{Cu} = 0.308\ \Omega$; $V = (120 - 0.308\times12.5)$ V = 116 V

(*b*) Find $R_{bulb} = \mathcal{E}^2/P = 240\ \Omega$; $I_{bulb} = V/R_{bulb}$ — $I_{bulb} = 116/240$ A = 0.483 A

$N = (I_{max} - 12.5)/I_{bulb}$ — $N = 15$ bulbs

61* ·· A lightweight electric car is powered by ten 12-V batteries. At a speed of 80 km/h, the average frictional force is 1200 N. (*a*) What must be the power of the electric motor if the car is to travel at a speed of 80 km/h? (*b*) If each battery can deliver a total charge of 160 A·h before recharging, what is the total charge in coulombs that can be delivered by the 10 batteries before charging? (*c*) What is the total electrical energy delivered by the 10 batteries before recharging? (*d*) How far can the car travel at 80 km/h before the batteries must be recharged? (*e*) What is the cost per kilometer if the cost of recharging the batteries is 9 cents per kilowatt-hour?

(*a*) $P = Fv$ — $P = (1200\times22.2)$ W = 26.7 kW

(*b*) $Q = It$ — $Q = (160\times10\times3600)$ C = 5.76 MC

(*c*) $W = Q\mathcal{E}$ — $W = 69.1$ MJ

(*d*) $W = Fd$ — $d = 57.6$ km

(*e*) Cost = \$0.09($\mathcal{E}It$)/1000 — Cost/km = $(0.09\times120\times160/10^3)/57.6$ = \$0.03/km

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62 ··· A 100-W heater is designed to operate with an emf of 120 V. (*a*) What is its resistance, and what current does it draw? (*b*) Show that if the potential difference across the heater changes by a small amount ΔV, the power changes by a small amount ΔP, where $\Delta P/P \approx 2\,\Delta V/V$. (*Hint:* Approximate the changes with differentials.) (*c*) Find the approximate power dissipated in the heater if the potential difference is decreased to 115 V.

(*a*) $I = P/V$; $R = V/I$ — $I = 100/120$ A = 0.833 A; $R = 144\ \Omega$

(*b*) $P = V^2/R$; $\Delta P = (dP/dV)\Delta V$ — $\Delta P = (2V/R)\Delta V = 2P(\Delta V/V)$; $\Delta P/P = 2\Delta V/V$

(*c*) Find ΔP; $P = P_0 + \Delta P$ — $\Delta P \cong -8.33$ W; $P \cong 91.7$ W

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63 · Two resistors are connected in parallel across a potential difference. The resistance of resistor A is twice that of resistor B. If the current carried by resistor A is *I*, then what is the current carried by B? (*a*) *I* (*b*) 2*I* (*c*) *I*/2 (*d*) 4*I* (*e*) I/4

(*b*)

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64 · Two resistors are connected in series across a potential difference. Resistor A has twice the resistance of resistor B. If the current carried by resistor A is *I*, then what is the current carried by B? (*a*) *I* (*b*) 2*I* (*c*) *I*/2 (*d*) 4*I* (*e*) I/4

(*a*)

65* ·· When two identical resistors are connected in series across the terminals of a battery, the power delivered by the battery is 20 W. If these resistors are connected in parallel across the terminals of the same battery, what is the power delivered by the battery? (*a*) 5 W (*b*) 10 W (*c*) 20 W (*d*) 40 W (*e*) 80 W

(*e*)

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66 · (*a*) Find the equivalent resistance between points *a* and *b* in Figure 26-46. (*b*) If the potential drop between *a* and *b* is 12 V, find the current in each resistor.

(*b*) $I = V/R$	$I_4 = 3$ A; $I_3 = 4$ A; $I_2 = 2$ A
(*a*) $R_{equ} = V/I_{tot}$	$R_{equ} = 12/9\ \Omega = 1.33\ \Omega$

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67 · Repeat Problem 66 for the resistor network shown in Figure 26-47.

(*a*) $R_{equ} = [R_2R_6/(R_2 + R_6)] + R_3$	$R_{equ} = 4.5\ \Omega$
(*b*) $I_3 = V/R_{equ}$; $I_2 = (6/8)I_3$; $I_6 = (2/8)I_3$	$I_3 = 2.67$ A; $I_2 = 2$ A; $I_6 = 0.667$ A

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68 · Repeat Problem 66 for the resistor network shown in Figure 26-48.

(*a*) $R_{equ} = [(R_3 + R_5)R_4/(R_3 + R_5 + R_4)]$	$R_{equ} = 2.67\ \Omega$
(*b*) $I_4 = V/R_4$; $I_3 = I_5 = V/(R_3 + R_5)$	$I_4 = 3$ A; $I_3 = I_5 = 1.5$ A

69* · In Figure 26-48, the current in the 4-Ω resistor is 4 A. (*a*) What is the potential drop between *a* and *b*? (*b*) What is the current in the 3-Ω resistor?

(*a*) $V = IR$	$V = 16$ V
(*b*) $I_3 = V/(R_3 + R_5)$	$I_3 = 2$ A

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70 · (*a*) Show that the equivalent resistance between points *a* and *b* in Figure 26-49 is *R*. (*b*) What would be the effect of adding a resistance *R* between points *c* and *d*?

(*a*) Here we have two branches in parallel, each branch consisting of two resistors *R* in series. Therefore each branch has a resistance 2*R* and the parallel combination gives an equivalent resistance of $4R^2/4R = R$.

(*b*) Since the potential difference between points *c* and *d* is zero, no current would flow through the resistor connected between these two points, and the effect of adding that resistor is zero.

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71 ·· The battery in Figure 26-50 has negligible internal resistance. Find (*a*) the current in each resistor and (*b*) the power delivered by the battery.

(*a*) Find R_{equ} and I_3	$R_{equ} = (4/5 + 3)\ \Omega = 3.8\ \Omega$; $I_3 = 6/3.8$ A $= 1.58$ A
Find *V* across parallel combination and currents	$V_{par} = (6 - 4.74)$ V $= 1.26$ V; $I_2 = 0.63$ A; $I_4 = 0.32$ A
(*b*) $P = \mathcal{E}I$	$P = 6\times1.58$ W $= 9.48$ W

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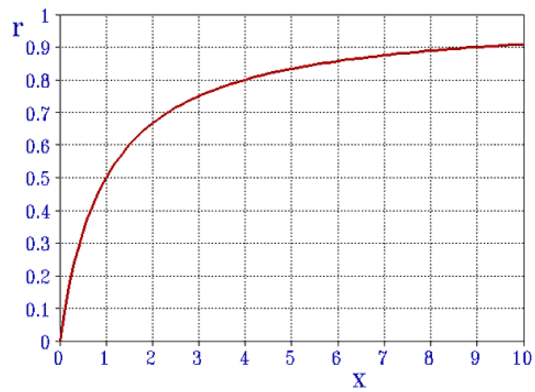
72 ·· A battery has an emf $\mathcal{E}$ and an internal resistance *r*. When a 5.0-Ω resistor is connected across the terminals, the current is 0.5 A. When this resistor is replaced by an 11.0-Ω resistor, the current is 0.25 A. Find (*a*) the emf $\mathcal{E}$ and (*b*) the internal resistance *r*.

(*a*), (*b*) Write the equations for the stated conditions	$\mathcal{E}/(r + 5\ \Omega) = 0.5$ A; $\mathcal{E}/(r + 11\ \Omega) = 0.25$ A
Solve for *r* and $\mathcal{E}$	$r = 1\ \Omega$; $\mathcal{E} = 3$ V

73* ·· Consider the equivalent resistance of two resistors R_1 and R_2 connected in parallel as a function of the ratio *x*

$= R_2/R_1$. (*a*) Show that $R_{eq} = R_1x/(1 + x)$. (*b*) Sketch a plot of R_{eq} as a function of x.

(*a*) $R_2 = xR_1$; then for parallel combination
$R_{equ} = xR_1^2/(R_1 + xR_1) = xR_1/(1 + x)$
(*b*) $r = R_{equ}/R_1$ versus x is shown in the figure



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74 ·· Repeat Problem 66 for the resistor network shown in Figure 26-51.

(*a*) The two 6-Ω resistors in parallel are equivalent to 3 Ω; find R_{equ} of this circuit — $R_{equ} = [(6 + 12)(3 + 6)/(6 + 12 + 3 + 6)]\ \Omega = 6\ \Omega$

(*b*) In upper branch, $I_{12} = I_6 = V/[(12 + 6)\ \Omega]$; in lower branch, $I = V/(9\ \Omega)$, and $I_{par} = I/2$ is the current in each of the parallel 6 Ω resistors. — Upper branch: $I_{12} = I_6 = 12/18$ A = (2/3) A
Lower branch: $I_{6,series} = 12/9 =$ (4/3) A, $I_{6,par} =$ (2/3) A
Note $I_{upper} + I_{lower} = 2\text{ A} = V/R_{equ}$

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75 ·· Repeat Problem 66 for the resistor network shown in Figure 26-52.

(*a*) Find the equivalent R for the two parallel portions — $R_{equ,8+8} = 4\ \Omega$; $R_{equ,2+4,4} = (6\times4/10)\ \Omega = 2.4\ \Omega$

Find R_{equ} of the entire circuit — $R_{equ} = (8.4\times8/16.4)\ \Omega = 4.1\ \Omega$

(*b*) $I_{upper} = V/R_{upper} = I_6$; $I_4 = 0.6I_6$; $I_{2+4} = 0.4I_6$ — $I_6 = 12/8.4$ A = 1.43 A; $I_4 = 0.858$ A; $I_{2+4} = 0.572$ A

$I_{lower} = V/R_{lower} = I_4$; $I_8 = I_4/2$ — $I_4 = 12/8 = 1.5$ A; $I_8 = 0.75$ A

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76 ·· A length of wire has a resistance of 120 Ω. The wire is cut into N identical pieces which are then connected in parallel. The resistance of the parallel arrangement is 1.875 Ω. Find N.

1. Find R of one of the N pieces — $R = N\times1.875\ \Omega$
2. Express R of the wire in terms of R — $R_{wire} = 120\ \Omega = NR = N^2\times1.875\ \Omega$
3. Solve for N — $N = 8$

77* ·· A parallel combination of an 8-Ω resistor and an unknown resistor R is connected in series with a 16-Ω resistor and a battery. This circuit is then disassembled and the three resistors are then connected in series with each other and the same battery. In both arrangements, the current through the 8-Ω resistor is the same. What is the unknown resistance R?

1. For the first arrangement, find R_{equ} — $R_{equ,1} = [16 + 8R/(R + 8)]\ \Omega = (24R + 128)/(R + 8)\ \Omega$
2. Write $I_{tot,1}$ and I_8 — $I_{tot,1} = (R + 8)V/(24R + 128)$; $I_{8,1} = I_{tot,1}R/(R + 8)$
3. For the series arrangement, find R_{equ} and $I_{8,2}$ — $I_{8,2} = V/(R + 24)$
4. Set $I_{8,1} = I_{8,2}$ and solve for R — $R = \sqrt{128}\ \Omega = 11.3\ \Omega$

78 ·· For the resistance network shown in Figure 26-53, find (*a*) R_3 such that $R_{ab} = R_1$; (*b*) R_2 such that $R_{ab} = R_3$; and (*c*) R_1 such that $R_{ab} = R_1$.

(*a*) $R_1 = R_1R_2/(R_1 + R_2) + R_3$; solve for R_3: $R_3 = R_1^2/(R_1 + R_2)$

(*b*) $R_3 = R_1R_2/(R_1 + R_2) + R_3$; solve for R_2: $R_2 = 0$

(*c*) $R_1 = R_1R_2/(R_1 + R_2) + R_3$; solve for R_1 : $R_1 = \dfrac{R_3 + \sqrt{R_3^2 + 4R_2R_3}}{2}$

79 ·· Check your results for Problem 78 using (*a*) $R_1 = 4\ \Omega$, $R_2 = 6\ \Omega$; (*b*) $R_1 = 4\ \Omega$, $R_3 = 3\ \Omega$; and (*c*) $R_2 = 6\ \Omega$, $R_3 = 3\ \Omega$.

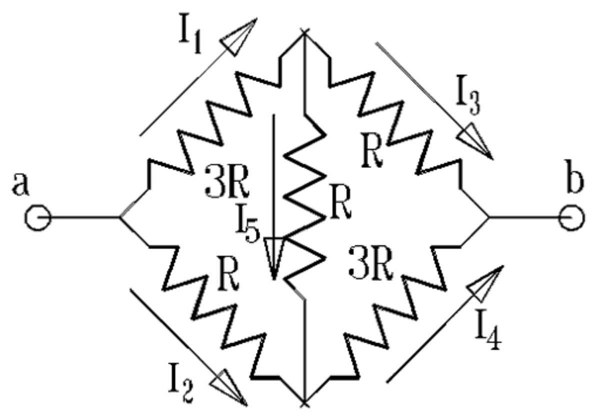
Using the results of Problem 78 we find:

(*a*) $R_3 = 16/10\ \Omega = 1.6\ \Omega$; then $R_{\text{equ}} = [(24/10) + 1.6]\ \Omega = 4\ \Omega = R_1$

(*b*) $R_2 = 0$; $R_{\text{equ}} = R_3 = 3\ \Omega$

(*c*) $R_1 = 1/2[3 + (9 + 72)^{1/2}]\ \Omega = 1/2\times 12\ \Omega = 6\ \Omega$; $R_{\text{equ}} = [(36/12) + 3]\ \Omega = 6\ \Omega = R_1$

80 ··· Nine 10-Ω resistors are connected as shown in Figure 26-54, and a potential difference of 20 V is applied between points *a* and *b*. (*a*) What is the equivalent resistance of this network? (*b*) Find the current in each of the nine resistors.



The circuit shown in Figure 26-54 is equivalent to the one shown in the drawing. We shall begin by finding the currents indicated in the circuit diagram, applying Kirchhoff's rules. We asume the circuit is completed by an emf giving a potential difference between *a* and *b* of $V = 20$ V.

$I_1 = I_5 + I_3$; $I_2 + I_5 = I_4$; $I_1 + I_2 = I_3 + I_4$.

$3RI_1 + RI_3 = 3RI_4 + RI_2 = V$; $3RI_1 + RI_5 = RI_2$.

With $R = 10\ \Omega$ and $V = 20$ V, these five simultaneous equations in five unknowns can be solved by standard methods. However, using symmetry arguments one can see that $I_1 = I_4$ and $I_2 = I_3$, thus reducing the number of unknown currents to three. The results are:

(*b*) $I_1 = I_4 = 0.4$ A, $I_2 = I_3 = 0.8$ A, $I_5 = -0.4$ A (i.e., I_5 flows up, not down as shown).

(*a*) The equivalent resistance of the cirucit is $V/(I_1 + I_2) = 20/1.2\ \Omega = 16.7\ \Omega$.

81* · Kirchhoff's loop rule follows from (*a*) conservation of charge. (*b*) conservation of energy. (*c*) Newton's laws. (*d*) Coulomb's law. (*e*) quantization of charge.

(*b*)

82 · In Figure 26-55, the emf is 6 V and $R = 0.5\ \Omega$. The rate of Joule heating in *R* is 8 W. (*a*) What is the current in the circuit? (*b*) What is the potential difference across *R*? (*c*) What is *r*?

(*a*) Find *I* using $P = I^2R$ (*b*) $V = IR$

(*c*) $Ir = E - IR$; solve for r | $I = 4$ A
$V = 2$ V
$r = 1\ \Omega$

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83 · For the circuit in Figure 26-56, find (*a*) the current, (*b*) the power delivered or absorbed by each emf, and (*c*) the rate of Joule heating in each resistor. (Assume that the batteries have negligible internal resistance.)

(*a*) $I = (E_1 - E_2)/(R_1 + R_2)$ — $I = 1$ A

(*b*) $P = IE$ — $P_{12} = 12$ W; $P_6 = -6$ W (this battery absorbs power)

(*c*) $P = IV$ — $P_2 = 2$ W; $P_4 = 4$ W

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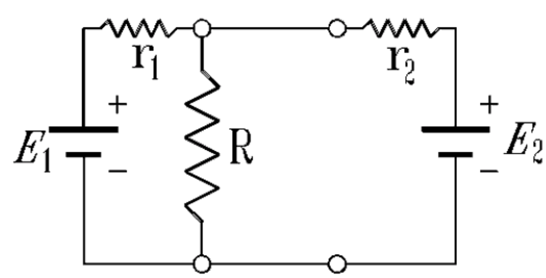
84 ·· A sick car battery with an emf of 11.4 V and an internal resistance of 0.01 Ω is connected to a load of 2.0 Ω. To help the ailing battery, a second battery with an emf of 12.6 V and an internal resistance of 0.01 Ω is connected by jumper cables to the terminals of the first battery. (*a*) Draw a diagram of this circuit. (*b*) Find the current in each part of the circuit. (*c*) Find the power delivered by the second battery and discuss where this power goes, assuming that the emfs and internal resistances of both batteries remain constant.

(*a*) The diagram is shown in the figure. Here E_1 and r_1 denote the emf of the "sick" battery and its internal resistance, E_2 and r_2 the emf of the second battery and its internal resistance, and R is the load resistance.

Let I_1, I_2, and I_R be the currents. We can now use Kirchhoff's rules to determine the unknown currents.



b) 1. Use the junction rule — $I_1 + I_2 = I_R$

2. Use the loop rule — $11.4 - 0.01I_1 - 2.0I_R = 0$; $12.6 - 0.01I_2 - 2.0I_R = 0$

3. Solve for the currents — $I_1 = -57.0$ A, $I_2 = 63.0$ A, $I_R = 6.0$ A

(*c*) $P_2 = E_2I_2$ — $P_2 = 794$ W

Of the 794 W, $11.4\times57 = 650$ W are used to recharge the weak battery, 72 W are dissipated in the load resistor, 39.7 W are dissipated in the internal resistance r_2, and 32.5 W in the internal resistance r_1.

85* ·· In the circuit in Figure 26-57, the reading of the ammeter is the same with both switches open and both closed. Find the resistance R.

1. Find the current with both switches open — $I = 1.5/450$ A $= 3.33$ mA

2. With both switches closed, 50 Ω is shorted; find R_{equ}, I_{tot}, and I_{100}; set $I_{100} = 3.33$ mA — $R_{equ} = [100R/(100 + R) + 300]\ \Omega$; $I_{tot} = 1.5/R_{equ}$; $I_{100} = 3.33\times10^{-3} = (1.5/R_{equ})[R/(100 + R)]$

3. Solve for R — $R = 600\ \Omega$

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86 ·· In the circuit in Figure 26-58, the batteries have negligible internal resistance, and the ammeter has negligible resistance. (*a*) Find the current through the ammeter. (*b*) Find the energy delivered by the 12-V battery in 3 s. (*c*) Find the total Joule heat produced in 3 s. (*d*) Account for the difference in your answers to parts (*b*) and (*c*).

Let I_1, directed up, be the current through the 12-V emf; let I_2, directed up, be the current through the ammeter; let I_3, directed down, be the current through the 2-Ω resistor at the right.

(*a*) Write the junction equation — $I_1 + I_2 = I_3$
Write the loop equations — $12 - 2I_1 + 2I_3 = 0$; $2 + 2I_3 - 2I_2 = 0$
Solve for I_1, $I_2 = I_A$, and I_3 — $I_1 = 3.67$ A, $I_2 = I_A = -1.33$ A, $I_3 = 2.33$ A
(*b*) $E = Pt = \mathcal{E}It$ — $E = 12\times3.67\times3$ J $= 132$ J
(*c*) $Q = (\Sigma I_i^2 R_i)t$ — $Q = (2\times3)(3.67^2 + 1.33^2 + 2.33^2) = 124$ J
(*d*) Difference = charging energy to 2-V battery — 2-V battery receives 8 J of energy

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87 ·· In the circuit in Figure 26-59, the batteries have negligible internal resistance. Find (*a*) the current in each resistor, (*b*) the potential difference between points *a* and *b*, and (*c*) the power supplied by each battery.

Let I_1 be the current deliver by the left battery, I_2 the current delivered by the right battery, and I_3 the current through the 6-Ω resistor, directed down.

(*a*) 1. Write the junction equations — $I_1 + I_2 = I_3$
2. Write the loop equations — $12 - 4I_1 - 6I_3 = 0$; $12 - 3I_2 - 6I_3 = 0$
3. Solve for the currents — $I_1 = 0.667$ A, $I_2 = 0.889$ A, $I_3 = 1.55$ A
(*b*) $V_{ab} = I_3R$ — $V_{ab} = 9.33$ V
(*c*) $P = \mathcal{E} I$ — $P_{\rm left} = 12\times0.667$ W $= 8$ W; $P_{\rm right} = 10.67$ W

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88 ·· Repeat Problem 87 for the circuit in Figure 26-60.

Let I_1 be the current delivered by the 7-V battery, I_2 the current delivered by the 5-V battery, and I_3, directed up, the current through the 1-Ω resistor.

(*a*) 1. Write the junction equation — $I_1 = I_2 + I_3$
2. Write the loop equations — $7 - 2I_1 - 1I_3 = 0$; $5 + 1I_3 - 3I_2 = 0$
3. Solve for the currents — $I_1 = 3$ A, $I_2 = 2$ A, $I_3 = 1$ A
(*b*) $V_{ab} = 3I_2 - \mathcal{E}_2$ — $V_{ab} = 1$ V
(*c*) $P = \mathcal{E} I$ — $P_7 = 21$ W, $P_5 = 10$ W

89* ·· Two identical batteries, each with an emf $\mathcal{E}$ and an internal resistance *r*, can be connected across a resistance *R* either in series or in parallel. Is the power supplied to *R* greater when $R < r$ or when $R > r$?

If connected in series, $I = 2\mathcal{E}/(2r + R)$ and $P = I^2/R = 4\mathcal{E}^2R/(2r + R)^2$. If connected in parallel, the emf is $\mathcal{E}$ and the two internal resistances are in parallel and equivalent to $r/2$. Then $I = \mathcal{E}/(r/2 + R)$ and $P = \mathcal{E}^2R/(r/2 + R)^2 = 4\mathcal{E}^2R/(r + 2R)^2$. If $r = R$ both arrangements provide the same power to the load. In the parallel connection, the power is greater if $R < r$ and will be a maximum when $R = r/2$. For the series connection, the power to the load is greater if $R > r$ and is greatest when $R = 2r$.

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90 ·· For the circuit in Figure 26-61, find (*a*) the current in each resistor, (*b*) the power supplied by each emf, and (*c*) the power dissipated in each resistor.

Let I_1 be the current in the 1-Ω resistor, directed up; let I_2 be the current, directed up, in the middle branch; let I_3 be the current in the 6-Ω resistor, directed down.

(*a*) 1. Write the junction equations — $I_1 + I_2 = I_3$
2. Write two loop equations — $12 - 3I_1 - 6I_3 = 0$; $4 - 2I_2 - 6I_3 = 0$
Solve for the currents — $I_1 = 2$ A, $I_2 = -1$ A, $I_3 = 1$ A
(*b*) $P = \mathcal{E} I$ — $P_8 = 16$ W, $P_{4,\rm left} = 8$ W, $P_{4,\rm right} = -4$ W

(*c*) $P = I^2R$ $P_{1\Omega} = 4$ W, $P_{2\Omega,\text{left}} = 8$ W, $P_{2\Omega,\text{middle}} = 2$ W, $P_{6\Omega} = 6$ W

91 ·· For the circuit in Figure 26-62, find the potential difference between points *a* and *b*.

Note that the two 2-V batteries are in parallel and may be regarded as a single battery with an effective internal resistance of 1 Ω (see Problem 89). Thus the circuit is one containing a total resistance of 5 Ω and two batteries in opposition giving a total emf of 2 V. The current through the 4-Ω resistor is then 0.4 A and the potential difference between *a* and *b* is $(4 - 4\times0.4)$ V = 2.4 V, with *a* at the higher potential.

92 ·· The battery in the circuit shown in Figure 26-63 has an internal resistance of 0.01 Ω. (*a*) An ammeter with a resistance of 0.01 Ω is inserted in series with the 0.74-Ω resistor at point *a*. What is the reading of the ammeter? (*b*) By what percentage is the current changed because of the ammeter? (*c*) The ammeter is removed and a voltmeter with a resistance of 1 kΩ is connected in parallel with the 0.74-Ω resistor from *a* to *b*. What is the reading of the voltmeter? (*d*) By what percentage is the voltage drop from *a* to *b* changed by the presence of the voltmeter?

(*a*) $I = \mathcal{E}/R$	$I = (1.5/0.76)$ A = 1.974 A
(*b*) Without meter, $I = 2$ A; with meter, $I = 1.974$ A	percent change = –1.32%
(*c*) Find R_{equ} of the circuit and I_{tot}	$R_{\text{equ}} = 0.01 + 740/1000.74 = 0.7495$ Ω; $I_{\text{tot}} = 2.00146$ A
Determine V_{ab}	$V_{ab} = (1.5 - 0.01\times2.00146)$ V = 1.47999 V
(*d*) Determine percent change	Without meter, $V_{ab} = 1.48000$ V; change = –0.00146%

93* ·· You have two batteries, one with $\mathcal{E} = 9.0$ V and $r = 0.8$ Ω and the other with $\mathcal{E} = 3.0$ V and $r = 0.4$ Ω. (*a*) Show how you would connect the batteries to give the largest current through a resistor *R*. Find the current for (*b*) $R = 0.2$ Ω, (*c*) $R = 0.6$ Ω, (*d*) $R = 1.0$ Ω, and (*e*) $R = 1.5$ Ω.

Let $\mathcal{E}_1$ be the 9-V battery and r_1 its internal resistance of 0.8 Ω, $\mathcal{E}_2$ be the 3-V battery and r_2 its internal resistance of 0.4 Ω. If the two batteries are connected in series and then connected to the load resistance *R*, the current through *R* is $I_s = (\mathcal{E}_1 + \mathcal{E}_2)/(r_1 + r_2 + R) = 12/(1.2 + R)$ A.

Suppose the two batteries are connected in parallel and their terminals are then connected to *R*. Let I_1 be the current delivered by $\mathcal{E}_1$, I_2 be the current delivered by $\mathcal{E}_2$, and I_p the current through the load resistor *R* in the parallel connection.

1. Write the junction equations	$I_1 + I_2 = I_p$
2. Write two loop equations	$9 - 0.8I_1 - I_pR = 0$; $3 - 0.4I_2 - I_pR = 0$
3. Solve for I_p	$I_p = 7.5/(0.4 + 1.5R)$ A
(*b*) Evaluate I_s and I_p for $R = 0.2$ Ω	$I_s = 8.57$ A; $I_p = 10.7$ A
(*c*) Evaluate I_s and I_p for $R = 0.6$ Ω	$I_s = 6.67$ A; $I_p = 5.77$ A
(*d*) Evaluate I_s and I_p for $R = 1.0$ Ω	$I_s = 5.45$ A; $I_p = 3.95$ A
(*e*) Evaluate I_s and I_p for $R = 1.5$ Ω	$I_s = 4.44$ A; $I_p = 2.83$ A

Note that for $R = 0.4$ Ω, $I_s = I_p = 7.5$ A. When $R < 0.4$ Ω, the parallel connection gives the larger current through *R*. When $R > 0.4$ Ω, the series connection gives the larger current through *R*.

94 ·· (*a*) Find the current in each part of the circuit shown in Figure 26-64. (*b*) Use your results from (*a*) to assign a potential at each indicated point assuming the potential at point *a* is zero.

Let I_1 be the current delivered by the 34-V emf, I_2 be the current through the 8-Ω resistor, I_3 be the current through the 2-Ω resistor, I_4, directed to the left, be the current through the 4-Ω resistor, I_5 be the current through the 12-Ω resistor, and I_6 be the current through the 1-Ω resistor; I_2, I_3, I_5, and I_6 are directed down.

(*a*) 1. Write the junction equations — $I_1 = I_2 + I_3$; $I_2 = I_4 + I_6$; $I_5 = I_4 + I_3$

2. Write three loop equations — $34 - 6I_1 - 2I_3 - 12I_5$; $8I_2 + 4I_4 - 2I_3 = 0$; $4I_4 + 12I_5 - 1I_6 = 0$

3. Solve for the currents — $I_1 = 3.49$ A, $I_2 = 1.29$ A, $I_3 = 2.20$ A, $I_4 = -1.48$ A, $I_5 = 0.72$ A, $I_6 = 2.77$ A

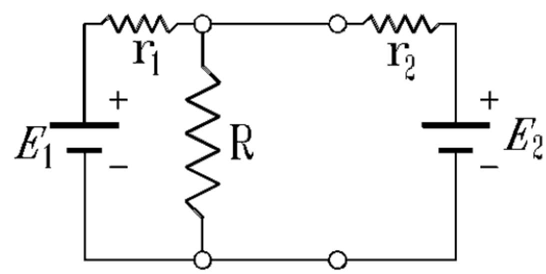
(*b*) Find the potentials at *a*, *b*, *c*, *d*, *e*, *f*, *g*, and *h* — $V_a = V_f = V_g = 0$; $V_b = 34$ V; $V_c = V_d = (34 - 3.49\times6)$ V $= 13.06$ V; $V_h = (13.06 - 2\times2.20)$ V $= 8.66$ V; $V_e = 2.74$ V

---

95 ··· In Problem 84**,** assume that the emf of the first battery increases at a constant rate of 0.2 V/h while the emf of the second battery and the internal resistances remain constant. (*a*) Find the current in each part of the circuit as a function of time. (*b*) Sketch a graph of the power delivered to the first battery as a function of time.

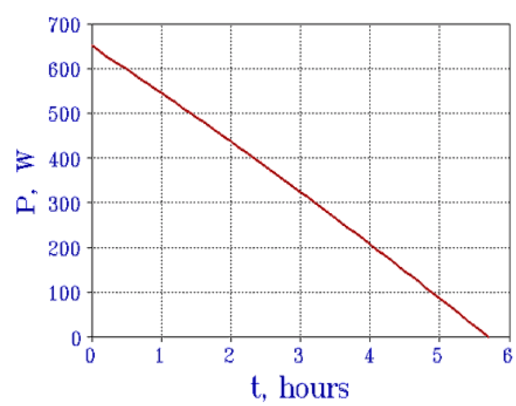
(*a*) The diagram is shown in the figure. Here E_1 and r_1 denote the emf of the "sick" battery and its internal resistance, E_2 and r_2 the emf of the second battery and its internal resistance, and R is the load resistance.

Let I_1, I_2, and I_R be the currents. We can now use Kirchhoff's rules to determine the unknown currents. Let t be the time, in hours, following the connection of the two batteries and load resistor.



1. Use the junction rule — $I_1 + I_2 = I_R$
2. Use the loop rule — $11.4 + 0.2t - 0.01I_1 - 2.0I_R = 0$; $12.6 - 0.01I_2 - 2.0I_R = 0$
3. Solve for the currents — $I_1 = (-57.0 + 10t)$ A, $I_2 = (63.0 - 10t)$ A, $I_R = 6.0$ A

(*b*) $P_1 = -E_1I_1 = [(11.4 + 0.2t)(57 - 10t)]$ W



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96 ··· (*a*) Find the current in each part of the circuit shown in Figure 26-65. (*b*) Use your results from (*a*) to assign a potential at each indicated point assuming the potential at point *a* is zero.

Let I_1 be the current, directed up, through the 2-Ω resistor; let I_2 be the current, directed down, though the 4-Ω resistor; let I_3, directed down, be the current through the 3-Ω resistor; let I_4 be the current, directed to the right, through the 24-Ω resistor; let I_5, directed down, be the current through the 8-Ω resistor; let I_6, directed up, be the current delivered by the 24-V battery.

(*a*) 1. Write the junction equations	$I_1 = I_2 + I_3$; $I_5 = I_3 + I_4$; $I_6 = I_1 + I_4$
2. Write three loop equations	$24 - 24I_4 - 8I_5 = 0$; $24 - 2I_1 - 6 - 14I_2 - 8 = 0$; $24 - 2I_1 - 6 - 3I_3 - 8I_5 = 0$
3. Solve for the currents	$I_1 = 1.5$ A, $I_2 = 0.5$ A, $I_3 = 1.0$ A, $I_4 = 0.5$ A, $I_5 = 1.5$ A, $I_6 = 2.0$ A
(*b*) Find *V* at *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*.	$V_a = V_g = 0$; $V_b = 24$ V; $V_c = (24 - 3)$ V $= 21$ V; $V_d = V_e = (21 - 6)$ V $= 15$ V; $V_f = 5$ V; $V_h = 12$ V

97* ··· Find the current in each resistor of the circuit shown in Figure 26-66.

Let I_1 be the current in the 3-Ω resistor in series with the 8-V battery; let I_2 be the current delivered by the 12-V battery; let I_3, directed down, be the current through the 2-Ω resistor; let I_4, directed down, be the current through the 4-Ω resistor; let I_5, directed down, be the current through the 3-Ω resistor; let I_6, directed to the right, be the current through the 5-Ω resistor.

1. Write the junction equations	$I_3 = I_1 + I_2$; $I_2 = I_5 + I_6$; $I_3 = I_4 + I_5$
2. Write three loop equations	$8 - 3I_1 - 2I_3 - 4I_4 = 0$; $12 - 2I_3 - 3I_5 = 0$; $12 + 3I_1 - 8 - 5I_6 = 0$
3. Solve for the currents	$I_1 = -0.0848$ A; $I_2 = 2.883$ A; $I_3 = 2.80$ A; $I_4 = 0.664$ A; $I_5 = 2.134$ A; $I_6 = 0.749$ A

---

98 ··· Suppose that the emf of the left battery in Figure 26-66 is unknown but that the current delivered by the 12-V battery is known to be 0.6 A. Find the emf of the left battery and the current delivered by it.

We use the same current convention as in Problem 97. Now $I_2 = 0.6$ A, and the unknown emf is $\mathcal{E}$.

1. Write three loop equations	$\mathcal{E} - 9I_1 - 1.2 - 4I_6 = 0$ (1); $12 - 2I_1 - 3 + 3I_6 = 0$ (2); $12 - \mathcal{E} + 3I_1 - 5I_6 = 0$ (3)
2. Add equations (1) and (3) and solve for I_1 and I_6	$I_1 = 3.083$ A; $I_6 = -0.944$ A
3. Evaluate $\mathcal{E}$	$\mathcal{E} = 25.2$ V

---

99 · The capacitor *C* in Figure 26-67 is initially uncharged. Just after the switch S is closed,

(*a*) the voltage across *C* equals $\mathcal{E}$.

(*b*) the voltage across *R* equals $\mathcal{E}$.

(*c*) the current in the circuit is zero.

(*d*) both (*a*) and (*c*) are correct.

(*b*)

---

100 ·· During the time it takes to fully charge the capacitor of Figure 26-67,

(*a*) the energy supplied by the battery is $1/2C\mathcal{E}^2$.

(*b*) the energy dissipated in the resistor is $1/2CE^2$.
(*c*) energy is dissipated in the resistor at a constant rate.
(*d*) the total charge flowing through the resistor is $1/2CE$.
(*b*)

101*·· A battery is connected to a series combination of a switch, a resistor, and an initially uncharged capacitor. The switch is closed at $t = 0$. Which of the following statements is true?
(*a*) As the charge on the capacitor increases, the current increases.
(*b*) As the charge on the capacitor increases, the voltage drop across the resistor increases.
(*c*) As the charge on the capacitor increases, the current remains constant.
(*d*) As the charge on the capacitor increases, the voltage drop across the capacitor decreases.
(*e*) As the charge on the capacitor increases, the voltage drop across the resistor decreases.
(*e*)

102 ·· A capacitor is discharging through a resistor. If it takes a time T for the charge on a capacitor to drop to half its initial value, how long does it take for the energy to drop to half its initial value?
The change in V_C is exponential, and U_C depends on V_C^2. So the energy drops to half its value in time $T/2$.

103 ·· A capacitor, resistor, and battery are connected in series. If R is doubled, how does this affect (*a*) the total energy stored, (*b*) the rate of energy storage, and (*c*) the time required to store $1/e$ of the final energy?
(*a*) No change in total energy stored. (*b*) Rate of energy storage is halved. (*c*) The time is doubled.

104 ·· A capacitor, resistor, and battery are connected in series. If C is doubled, how does this affect (*a*) the total energy stored, (*b*) the rate of energy storage, and (*c*) the time required to store $1/e$ of the final energy?
(*a*) The energy stored is doubled. (*b*) The rate of energy storage is halved. (*c*) The time is doubled.

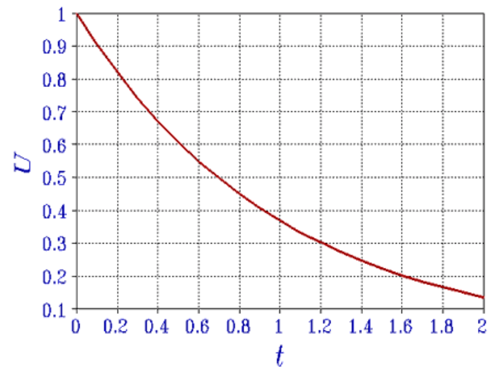
105*· A 6-μF capacitor is charged to 100 V and is then connected across a 500-Ω resistor. (*a*) What is the initial charge on the capacitor? (*b*) What is the initial current just after the capacitor is connected to the resistor? (*c*) What is the time constant of this circuit? (*d*) How much charge is on the capacitor after 6 ms?

(*a*) $Q_0 = CV_0$	$Q_0 = 600\ \mu$C
(*b*) $I_0 = V_0/R$	$I_0 = 0.2$ A
(*c*) $\tau = RC$	$\tau = 3$ ms
(*d*) $Q(t) = Q_0 e^{-t/}$	$Q = (600\, e^{-2})\ \mu\text{C} = 81.2\ \mu$C

106 · (*a*) Find the initial energy stored in the capacitor of Problem 105. (*b*) Show that the energy stored in the capacitor is given by $U = U_0 e^{-2t/}$, where U_0 is the initial energy and $\tau = RC$ is the time constant. (*c*) Sketch a plot of the energy U in the capacitor versus time t.

(*a*) $U_0 = 1/2CV_0^2$	$U_0 = 0.03$ J
(*b*) $V_C(t) = V_0 e^{-t/}$	$U(t) = 1/2CV_0^2\, e^{-2t/\tau} = U_0\, e^{-2t/}$

(*c*) A plot of U versus t is shown. Here U is in units of U_0 and t is in units of •.



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107 ·· In the circuit of Figure 26-40, emf E = 50 V and C = 2.0 μF; the capacitor is initially uncharged. At 4.0 s after switch S is closed, the voltage drop across the resistor is 20 V. Find the resistance of the resistor.

1. Use Equ. 26-36 — $20/50 = e^{-t/}$; $t/\tau = \ln(2.5) = 0.916$; $\tau = 4/0.916$ s = 4.37 s
2. $\tau = RC$ — $R = 4.36/2.0\times10^{-6}\ \Omega = 2.18\ \mathrm{M}\Omega$

---

108 ·· A 0.12-μF capacitor is given a charge Q_0. After 4 s, its charge is $\frac{1}{2}Q_0$. What is the effective resistance across this capacitor?

1. $t_{1/2} = 0.693\tau$ — $\tau = 5.77$ s
2. $\tau = RC$ — $R = 48.1\ \mathrm{M}\Omega$

109*·· A 1.6-μF capacitor, initially uncharged, is connected in series with a 10-kΩ resistor and a 5.0-V battery of negligible internal resistance. (*a*) What is the charge on the capacitor after a very long time? (*b*) How long does it take the capacitor to reach 99% of its final charge?

(*a*) $Q = CV$ — $Q = 8.0\ \mu$C

(*b*) $\tau = RC$; $e^{-t/\tau} = 0.01$ — $\tau = 0.016$ s; $t/\tau = \ln(100)$; t = 73.7 ms

---

110 ·· Consider the circuit shown in Figure 26-68. From your knowledge of how capacitors behave in circuits, find (*a*) the initial current through the battery just after the switch is closed, (*b*) the steady-state current through the battery when the switch has been closed for a long time, and (*c*) the maximum voltage across the capacitor.

(*a*) $V_{C0} = 0$; $120 = 1.2\times10^6\ I_0 + 0$ — $I_0 = 0.1$ mA

(*b*) $I_{C\infty} = 0$ — $I_\infty = (120/1.8\times10^6)$ A = 0.0667 mA

(*c*) $V_{C\infty} = I_\infty\times(0.8\ \mathrm{M}\Omega)$ — $V_{C\infty} = 40$ V

---

111 ·· A 2-MΩ resistor is connected in series with a 1.5-μF capacitor and a 6.0-V battery of negligible internal resistance. The capacitor is initially uncharged. After a time $t = \tau = RC$, find (*a*) the charge on the capacitor, (*b*) the rate at which the charge is increasing, (*c*) the current, (*d*) the power supplied by the battery, (*e*) the power dissipated in the resistor, and (*f*) the rate at which the energy stored in the capacitor is increasing.

(*a*) $Q = Q_f(1 - e^{-t/})$; $Q_\tau = Q_f\times0.632$ — $Q_\tau = 0.632\times9\ \mu\mathrm{C} = 5.69\ \mu$C

(*b*), (*c*) $dQ/dt = I = I_0\, e^{-t/\tau}$	$I_\tau = 0.368\times3\ \mu A = 1.10\ \mu C/s$
(*d*) $P = IE$	$P_\tau = 6\times1.10\ \mu W = 6.62\ \mu W$
(*e*) $P_R = I^2R$	$P_R = (1.10\times10^{-6})^2\times2\times10^6$ W $= 2.42\ \mu W$
(*f*) $dU/dt = (1/2C)(dQ^2/dt) = (Q/C)I$	$dU/dt = 4.17\ \mu W = P_\tau - I^2R$

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112 ·· Repeat Problem 111 for the time $t = 2$.

In this case, $e^{-t/\tau} = 0.135$ and $(1 - e^{-2}) = 0.865$. Following the same procedure as in the preceding problem one obtains the following results for $t = 2$.

(*a*) $Q = 7.78\ \mu C$ (*b*), (*c*) $0.406\ \mu C/s = 0.406\ \mu A$ (*d*) $P = 2.44\ \mu W$ (*e*) $P_R = 0.33\ \mu W$ (*f*) $dU/dt = 2.11\ \mu W$

113*·· In the steady state, the charge on the 5-μF capacitor in the circuit in Figure 26-69 is 1000 μC. (*a*) Find the battery current. (*b*) Find the resistances R_1, R_2, and R_3.

(*a*) 1. Find the current in the 10-Ω resistor	$I_{10\Omega} = V_C/10$ A $= 200/10$ A $= 20$ A
2. $I_{bat} = I_{10\Omega} + 5$ A	$I_{bat} = 25$ A
(*b*) 1. Find $I_{5\Omega}$, I_{R3}, and I_{R1}	$I_{5\Omega} = 10$ A, $I_{R3} = 15$ A, $I_{R1} = I_{bat} = 25$ A
2. Write loop equations and solve for unknown resistors.	$310 - 25R_1 - 5\times50 - 5\times10 = 0$; $R_1 = 0.4\ \Omega$
	$310 - 10 - 200 - 15R_3 = 0$; $R_3 = 6.67\ \Omega$
	$200 + 5R_2 = 5\times50$; $R_2 = 10\ \Omega$

---

114 ·· (*a*) What is the voltage across the capacitor in the circuit in Figure 26-70? (*b*) If the battery is disconnected, give the capacitor current as a function of time. (*c*) How long does it take the capacitor to discharge until the potential difference across it is 1 V?

Note that in steady state, $I_C = 0$; in effect, we have two parallel branches.

(*a*) 1. Find I in each branch	$I_{left} = 36/90$ A $= 0.4$ A; $I_{right} = 36/60$ A $= 0.6$ A
2. Find $V_C = 40I_{right} - 10I_{left}$	$V_C = 20$ V
(*b*) 1. Find R_{equ} across C	$R_{equ} = 50\times100/150\ \Omega = 33.3\ \Omega$
2. $I_C = I_0\, e^{-t/RC}$; $I_0 = V_{C0}/R_{equ}$	$I_C = 0.6\, e^{-t/0.333}$, where t is in ms
(*c*) $V_C = V_{C0}\, e^{-t/}$	$t/\tau = \ln(20)$; $t = 0.998$ ms

---

115 ·· Show that Equation 26-34 can be written

$$\frac{dQ}{\varepsilon C - Q} = \frac{dt}{RC}$$

Integrate this equation to derive the solution given by Equation 26-35.

$$\varepsilon = R(dQ/dt) + Q/C;\ \frac{dQ}{dt} = \frac{\varepsilon C - Q}{RC};\ \frac{dt}{RC} = \frac{dQ}{\varepsilon C - Q};\ \frac{1}{RC}\int_0^t dt = \int_0^Q \frac{dQ}{\varepsilon C - Q};\ t/RC = \ln\left(\frac{\varepsilon C}{\varepsilon C - Q}\right);$$

$Q = \varepsilon C(1 - e^{-t/RC}) = Q_0(1 - e^{-t/RC})$.

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116 ··· A photojournalist's flash unit uses a 9.0-V battery pack to charge a 0.15-μF capacitor, which is then discharged through the flash lamp of 10.5-Ω resistance when a switch is closed. The minimum voltage necessary for the flash discharge is 7.0 V. The capacitor is charged through a 18-kΩ resistor. (*a*) How much time is required to charge the capacitor to the required 7.0 V? (*b*) How much energy is released when the lamp flashes? (*c*) How much energy is supplied by the battery during the charging cycle and what fraction of that energy is dissipated in the resistor?

(*a*) 1. Determine $\tau = RC$ — $\tau = 2.7$ ms

2. Determine t to charge to 7 V — $e^{-t/\tau} = 1 - 7/9 = 0.222$; $t = \tau\ln(4.505) = 4.06$ ms

(*b*) $U = 1/2CV^2$ — $U = 3.675\ \mu$J

(*c*) $E_{bat} = \mathcal{E}\int_0^t I(t)\,dt = \mathcal{E}^2/R\int_0^t e^{-t/\tau}\,dt$ — $E_{bat} = (9^2/18\times10^3)(2.7\times10^{-3})(1 - 0.222)$ J $= 9.45\ \mu$J

$E_{res} = E_{bat} - U = 5.775\ \mu$J $= 61\%$ of E_{bat}

117*··· For the circuit in Figure 26-71, (*a*) what is the initial battery current immediately after switch S is closed? (*b*) What is the battery current a long time after switch S is closed? (*c*) What is the current in the 600-Ω resistor as a function of time?

(*a*) $V_{C0} = 0$; $I_0 = \mathcal{E}/R_{200}$ — $I_0 = 50/200$ A $= 0.25$ A

(*b*) $I_\infty = \mathcal{E}/R_{tot}$ — $I_\infty = 50/800$ A $= 0.0625$ A

(*c*) 1. Let $R_1 = 200$-Ω resistor, $R_2 = 600$-Ω resistor, and I_1, I_2 be their currents, and I_3 = current into C — $I_1 = I_2 + I_3$ (1); $\mathcal{E} - R_1I_1 - Q/C = 0$ (2); $Q/C = R_2I_2$ (3)

2. Differentiate equation (2) with respect to time — $R_1(dI_1/dt) = -(1/C)(dQ/dt) = -(1/C)I_3$

3. Differentiate equation (3) with respect to time — $(1/C)I_3 = R_2(dI_2/dt)$; $dI_1/dt = -(R_2/R_1)(dI_2/dt)$

4. Use the junction equation — $dI_2/dt = (I_1 - I_2)/R_2C$

5. From equations (1) and (3) solve for I_1 — $I_1 = (\mathcal{E} - R_2I_2)/R_1$

6. Write the differential equation for I_2 — $dI_2/dt = (\mathcal{E}/R_1R_2C) - I_2(R_1 + R_2)/R_1R_2C$

7. To solve, let $I_2(t) = a + be^{-t/\tau}$; substitute for I_2 and solve for a, b, and τ using $I_2(0) = 0$ — $a = \mathcal{E}/(R_1 + R_2)$; $b = -a$; $\tau = R_1R_2C/(R_1 + R_2)$

$I_2 = 0.0625(1 - e^{-t/\tau})$, where $\tau = 0.75$ ms

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118 ··· For the circuit in Figure 26-72, (*a*) what is the initial battery current immediately after switch S is closed? (*b*) What is the battery current a long time after switch S is closed? (*c*) If the switch has been closed for a long time and is then opened, find the current through the 600-kΩ resistor as a function of time.

(*a*) $V_{C0} = 0$; $I_0 = \mathcal{E}/R_1$, where $R_1 = 1.2$ MΩ — $I_0 = 41.7\ \mu$A

(*b*) $I_\infty = \mathcal{E}/(R_1 + R_2)$, where $R_2 = 600$ kΩ — $I_\infty = 27.8\ \mu$A

(*c*) $I(t) = (V_{Ci}/R_2)e^{-t/\tau}$; $V_{Ci} = I_\infty R_2$; $\tau = R_2C$ — $I(t) = 27.8e^{-t/1.5}\ \mu$A, where t is in s

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119 ··· In the circuit shown in Figure 26-73, the capacitor has a capacitance of 2.5 μF and the resistor a resistance of 0.5 MΩ. Before the switch is closed, the potential drop across the capacitor is 12 V, as shown. Switch S is closed at $t = 0$. (*a*) What is the current in R immediately after S is closed? (*b*) At what time t is the voltage across the capacitor 24 V?

(*a*) $I_R = V_R/R$; $V_R = \mathcal{E} - V_C$ — At $t = 0$, $I_R = 24/0.5\times10^6$ A $= 48\ \mu$A

(*b*) Find t for $V_R = 12$ V — $I_R(t)R = 24e^{-t/1.25}$ V $= 12$ V; $t = 1.25\ln(2) = 8.66$ s

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120 ··· Repeat Problem 119 if the capacitor is connected with reversed polarity.

(*a*) $I_R = (\mathcal{E} + V_C)/R$ — At $t = 0$, $I_R = 48/0.5\times10^6$ A $= 96\ \mu$A

(*b*) Find t when $V_R = 12$ V — $I_RR = 48e^{-t/1.25}$ V $= 12$ V; $t = 1.25\ln(4) = 1.73$ s

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121*· A flash lamp is set off by the discharge of a capacitor that has been charged by a battery. Why not just connect the battery directly to the lamp?

The battery cannot deliver energy at the high rate required to light the flash.

122 · Which will produce more thermal energy when connected across an ideal battery, a small resistance or a large resistance?

A small resistance because $P = \mathcal{E}^2/R$.

123 · Do Kirchhoff's rules apply to circuits containing capacitors?

Yes.

124 ·· True or false:

(*a*) Ohm's law is $R = V/I$.

(*b*) Electrons drift in the direction of the current.

(*c*) A source of emf supplies power to an electrical circuit.

(*d*) When the potential drops by V in a segment of a circuit, the power supplied to that segment is IV.

(*e*) The equivalent resistance of two resistors in parallel is always less than the resistance of either resistor alone.

(*f*) The terminal voltage of a battery always equals its emf.

(*g*) The terminal voltage of a battery is always less than its emf.

(*a*) True (*b*) False (*c*) True (*d*) True (*e*) True (*f*) False (*g*) False

125*·· In Figure 26-74, all three resistors are identical. The power dissipated is

(*a*) the same in R_1 as in the parallel combination of R_2 and R_3.

(*b*) the same in R_1 and R_2.

(*c*) greatest in R_1.

(*d*) smallest in R_1.

(*c*)

126 ·· In Figure 26-74, $R_1 = 4\ \Omega$, $R_2 = 6\ \Omega$, and $R_3 = 12\ \Omega$. If we denote the currents through these resistors by I_1, I_2, and I_3, respectively, then (*a*) $I_1 > I_2 > I_3$. (*b*) $I_2 = I_3$. (*c*) $I_3 > I_2$. (*d*) none of the above is correct.

(*a*)

127 ·· A 25-W light bulb is connected in series with a 100-W light bulb and a voltage V is placed across the combination. Which bulb is brighter? Explain.

The 25-W bulb will be brighter. The resistance of the 25-W bulb is greater than that of the 100-W bulb, and in the series combination, $I^2R_{25} > I^2R_{100}$.

128 · If the battery emf in Figure 26-74 is 24 V, then (*a*) $I_2 = 4$ A. (*b*) $I_2 = 2$ A. (*c*) $I_2 = 1$ A. (*d*) none of the above is correct.

We assume that the resistances are those for Problem 126. Then (*b*) is correct.

129*· A 10.0-Ω resistor is rated as being capable of dissipating 5.0 W of power. (*a*) What maximum current can this resistor tolerate? (*b*) What voltage across this resistor will produce the maximum current?

(*a*) $I^2R = P$ $I_{max} = 0.707$ A

(*b*) $V = IR$ $V = 7.07$ V

130 · Margaret is economizing by turning off her space heater and warming herself with a toaster. She pushes the toaster plunger down and dozes off, but after 4 min it pops up again. Eventually the cold wakes her up, so she pushes the plunger down again and gets a little more sleep. This happens once every 15 min, with the toaster engaged for 4 min each time. It is a poor night's sleep, but she is determined to save money. Energy costs 9 cents per kilowatt-hour, and a 120-V source is used. (*a*) How much does it cost to operate an electric toaster for 4 min if its resistance is 11.0 Ω? (*b*) How much would it cost to operate a 5.0-Ω-heater connected across 120 V for 8 h?

(*a*) $E = (\mathcal{E}^2/R)t$; cost = $\$(9\times10^{-5})\times E(\mathrm{W\cdot h})$ — $E = 87.3\ \mathrm{W\cdot h}$; cost = 0.785 cent

(*b*) $E = (\mathcal{E}^2/R)t$; cost = $\$(9\times10^{-2})\times E(\mathrm{kW\cdot h})$ — $E = (2.88\ \mathrm{kW})\times(8\ \mathrm{h}) = 23.04\ \mathrm{kW\cdot h}$; cost = \$2.07

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131 · A 12-V car battery has an internal resistance of 0.4 Ω. (*a*) What is the current if the battery is shorted momentarily? (*b*) What is the terminal voltage when the battery delivers a current of 20 A to start the car?

(*a*) $I = \mathcal{E}/r$ — $I = 30$ A

(*b*) $V_{\mathrm{term}} = \mathcal{E} - Ir$ — $V_{\mathrm{term}} = 4$ V

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132 ·· The current drawn from a battery is 1.80 A when a 7.0-Ω resistor is connected across the battery terminals. If a second 12-Ω resistor is connected in parallel with the 7-Ω resistor, the battery delivers a current of 2.20 A. What are the emf and internal resistance of the battery?

1. Find R_{equ} for 7 Ω and 12 Ω in parallel — $R_{\mathrm{equ}} = 4.42\ \Omega$
2. Write the equations for the two conditions — $\mathcal{E} = 1.8(r + 7)$; $\mathcal{E} = 2.20(r + 4.42)$
3. Solve for r and $\mathcal{E}$ — $r = 7.19\ \Omega$; $\mathcal{E} = 25.5$ V

133*·· A 16-gauge copper wire insulated with rubber can safely carry a maximum current of 6 A. (*a*) How great a potential difference can be applied across 40 m of this wire? (*b*) Find the electric field in the wire when it carries a current of 6 A. (*c*) Find the power dissipated in the wire when it carries a current of 6 A.

(*a*) Find $R = \rho L/A$; see Table 26-2 for A — $R = 1.7\times10^{-8}\times40/1.31\times10^{-6}\ \Omega = 0.519\ \Omega$

$V = IR$ — $V = 3.11$ V

(*b*) $E = V/L$ — $E = 0.0779$ V/m

(*c*) $P = I^2R$ — $P = 18.7$ W

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134 ·· An automobile jumper cable 3 m long is constructed of multiple strands of copper wire that has an equivalent cross-sectional area of 10.0 mm^2. (*a*) What is the resistance of the jumper cable? (*b*) When the cable is used to start a car, it carries a current of 90 A. What is the potential drop that occurs across the jumper cable? (*c*) How much power is dissipated in the jumper cable?

(*a*) $R = \rho L/A$; $L = 6$ m (jumper cable has two leads) — $R = 0.0102\ \Omega$

(*b*) $V = IR$ — $V = 0.918$ V

(*c*) $P = IV$ — $P = 82.6$ W

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135 ·· A coil of Nichrome wire is to be used as the heating element in a water boiler that is required to generate 8.0 g of steam per second. The wire has a diameter of 1.80 mm and is connected to a 120-V power supply. Find the length of wire required.

1. $P = E/\Delta t$; $E = mL_{\mathrm{v}}$ — $P = 8 \times 2257/1 = 18.1$ kW
2. $P = \mathcal{E}^2/R$; $R = \mathcal{E}^2/P = \rho L/A$; solve for L — $L = \mathcal{E}^2A/\rho P = 2.03$ m

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136 ·· A closed box has two metal terminals *a* and *b*. The inside of the box contains an unknown emf $\mathcal{E}$ in series with a resistance R. When a potential difference of 21 V is maintained between a and b, there is a current of 1 A between the terminals a and b. If this potential difference is reversed, a current of 2 A in the reverse direction is observed. Find $\mathcal{E}$ and R.

1. Write the conditions for the data given — $\mathcal{E}$ – 21 V = (1 A)×R; $\mathcal{E}$ + 21 V =(–2 A)×R
2. Solve for R and $\mathcal{E}$ — R = 14 Ω; $\mathcal{E}$ = –7 V

The emf is 7 V; in the first case the current flows into the emf, in the second it flows out of the emf.

137*·· The capacitors in the circuit in Figure 26-75 are initially uncharged. (*a*) What is the initial value of the battery current when switch S is closed? (*b*) What is the battery current after a long time? (*c*) What are the final charges on the capacitors?

(*a*) Since V_C = 0 for both capacitors, the resistors are effectively in parallel. Find R_{equ} of circuit and I_0.	R_{equ} = (7.5×12/19.5 + 10) Ω = 14.6 Ω I_0 = 3.42 A
(*b*) Now I_C = 0, and resistors are in series. Find I_∞.	I_∞ = 50/52 A = 0.962 A
(*c*) Find V_C for 10-μF and 5-μF capacitors	For both capacitors, V_C = 27×0.962 V = 26.0 V
$Q = CV$	For C = 10 μF, Q = 260 μC; for C = 5μF, Q = 130 μC

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138 ·· The circuit in Figure 26-76 is a slide-type *Wheatstone bridge.* It is used for determining an unknown resistance R_x in terms of the known resistances R_1, R_2, and R_0. The resistances R_1 and R_2 comprise a wire 1 m long. Point a is a sliding contact that is moved along the wire to vary these resistances. Resistance R_1 is proportional to the distance from the left end of the wire (labeled 0 cm) to point a, and R_2 is proportional to the distance from point a to the right end of the wire (labeled 100 cm). The sum of R_1 and R_2 remains constant. When points a and b are at the same potential, there is no current in the galvanometer and the bridge is said to be balanced. (Since the galvanometer is used to detect the absence of a current, it is called a *null detector.*) If the fixed resistance R_0 = 200 Ω, find the unknown resistance R_x if (*a*) the bridge balances at the 18-cm mark, (*b*) the bridge balances at the 60-cm mark, and (*c*) the bridge balances at the 95-cm mark.

(*a*), (*b*), (*c*) For balance $R_1/R_2 = R_x/R_0$ — (*a*) R_x = 200(18/82) Ω = 43.9 Ω; (*b*) R_x = 300 Ω; (*c*) R_x = 3.8 kΩ

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139 ·· For the Wheatstone bridge of Problem 138, the bridge balances at the 98-cm mark when R_0 = 200 Ω. (*a*) What is the unknown resistance? (*b*) What effect would an error of 2 mm in the location of the balance point have on the measured value of the unknown resistance? (*c*) How should R_0 be changed so that the balance point for this unknown resistor will be nearer the 50-cm mark?

(*a*) $R_x = R_0(R_1/R_2)$	R_x = 9800 Ω
(*b*) An error of 2 mm gives a $\Delta R_x/R_x \cong 0.2/2$	Error ≅ 10% (the error of 2 mm in 98 cm is small)
(*c*) Use a resistor $R_0 \cong R_x$	Use R_0 = 10 kΩ

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140 ·· The wires in a house must be large enough in diameter so that they do not get hot enough to start a fire. Suppose a certain wire is to carry a current of 20 A, and it is determined that the Joule heating of the wire should not exceed 2 W/m. What diameter must a copper wire have to be safe for this current?

$P/L = I^2R/L = 4I^2\rho/\pi d^2$; evaluate d — d = 2.08 mm

141*·· You are given n identical cells, each with emf $\mathcal{E}$ and internal resistance r = 0.2 Ω. When these cells are connected in parallel to form a battery, and a resistance R is connected to the battery terminal, the current through R is the same as when the cells are connected in series and R is attached to the terminals of that battery. Find the value of the resistor R.

1. Write I for the series connection — $I_s = n\mathcal{E}/(nr + R)$
2. Write I for the parallel connection — $I_p = \mathcal{E}/(r/n + R)$
3. Set $I_s = I_p$ and solve for R with $r = 0.2\ \Omega$ — $R = 0.2\ \Omega$

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142 ·· A cyclotron produces a 3.50-μA proton beam of 60-MeV energy. The protons impinge and come to rest inside a 50-g copper target within the vacuum chamber. (*a*) Determine the number of protons that strike the target per second. (*b*) Find the energy deposited in the target per second. (*c*) How much time elapses before the target temperature rises 300°C? (Neglect cooling by radiation.)

(*a*) $I = ne$, where n = number of protons/s — $n = 3.5\times10^{-6}/1.6\times10^{-19} = 2.19\times10^{13}$
(*b*) Energy/s = power = IV — $P = 3.5\times10^{-6}\times60\times10^{6} = 210$ J/s
(*c*) $P\Delta t = C_{Cu}\Delta T$ — $\Delta t = 0.386\times50\times300/210$ s $= 27.6$ s

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143 ·· Compact fluorescent light bulbs cost \$6 each and have an expected lifetime of 8000 h. These bulbs consume 20 W of power, but produce the illumination equivalent to 75-W incandescent bulbs. Incandescent bulbs cost about \$1.50 each and have an expected lifetime of 1200 h. If the average household has, on the average, six 75-W incandescent light bulbs on constantly, and if energy costs 11.5 cents per kilowatt-hour, how much money would a consumer save each year by installing the energy-efficient fluorescent light bulbs?

1. Find the energy saving per year — $\Delta P = 6\times55$ W = 330 W; $\Delta E = 2891$ kW·h
2. Find the cost of bulbs/year — Fluoresc.: (1.095/y)×\$36 = \$39.42; Incandesc.: \$65.70
3. Find the cost saving — Δ\$ = \$(2891×0.115 + 65.70 – 39.42) = \$358.75

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144 ·· The space between the plates of a parallel-plate capacitor is filled with a dielectric of constant κ and resistivity ρ. (*a*) Show that the time constant for the decrease of charge on the plates is $\tau = \varepsilon_0\kappa\rho$. (*b*) If the dielectric is mica, for which $\kappa = 5.0$ and $\rho = 9\times10^{13}\ \Omega\cdot$m, find the time it takes for the charge to decrease to $1/e^2 \approx 14\%$ of its initial value.

(*a*) $\tau = RC = (\rho d/A)(\kappa\varepsilon_0 A/d) = \varepsilon_0\kappa\rho$
(*b*) $e^{-t/\tau} = e^{-2}$; $t = \tau\ln(e^2) = 2$; $\tau = 3.98\times10^{3}$ s; t = 2 h 13 min

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145*·· The belt of a Van de Graaff generator carries a surface charge density of 5 mC/m^2. The belt is 0.5 m wide and moves at 20 m/s. (*a*) What current does it carry? (*b*) If this charge is raised to a potential of 100 kV, what is the minimum power of the motor needed to drive the belt?

(*a*) $I = dQ/dt = \sigma w\, dx/dt = \sigma wv$ — $I = (5\times10^{-3}\times0.5\times20)$ A = 50 mA
(*b*) $P = IV$ — $P = 5$ kW

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146 ·· Conventional large electromagnets use water cooling to prevent excessive heating of the magnet coils. A large laboratory electromagnet draws 100 A when a voltage of 240 V is applied to the terminals of the energizing coils. To cool the coils, water at an initial temperature of 15°C is circulated through the coils. How many liters per second must pass through the coils if their temperature should not exceed 50°C?

1. Find the power dissipated; $P = IV$ — $P = 24$ kW

2. $P = (dm/dt)c\Delta T$; $c = 4.184$ J/g·K	$dm/dt = 164$ g/s $= 0.164$ L/s

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147 ··· We show in Figure 26-77 the basis of the sweep circuit used in an oscilloscope. S is an electronic switch that closes whenever the potential across its terminals reaches a value V_c and opens when the potential has dropped to 0.2 V. The emf E, much greater than V_c, charges the capacitor C through a resistor R_1. The resistor R_2 represents the small but finite resistance of the electronic switch. In a typical circuit, $E = 800$ V, $V_c = 4.2$ V, $R_2 = 0.001\ \Omega$, $R_1 = 0.5$ MΩ ($0.5\times10^6\ \Omega$), and $C = 0.02\ \mu$F. (*a*) What is the time constant for charging of the capacitor C? (*b*) Show that in the time required to bring the potential across S to the critical potential $V_c = 4.2$ V, the voltage across the capacitor increases almost linearly with time. (*Hint:* Use the expansion of the exponential for small values of exponent.) (*c*) What should be the value of R_1 so that C charges from 0.2 V to 4.2 V in 0.1 s? (*d*) How much time elapses during the discharge of C through switch S? (*e*) At what rate is power dissipated in the resistor R_1 and in the switch resistance?

(*a*) $\tau = R_1C$	$\tau = 10$ ms
(*b*) $V(t) = E(1 - e^{-t/})$; $e^{-t/\tau} = 1 - V(t)/E$; note that for	$e^{-t/\tau} = 1 - \varepsilon$; $e^{t/\tau} \cong 1 + \varepsilon$; $t/\tau = \ln(1 + \varepsilon) \cong \varepsilon$
(*c*) Since $V \propto t$, $\Delta t = \ \Delta V/E$; find τ and R_1	$t \cong \ V/E$; $V(t) = (E/\ )t$
(*d*) $V_C(t) = V_{C0}\, e^{-t/\ '}$, where $' = R_2C$; find t	$\tau = 0.1\times800/4 = 20$ s $= R_1C$; $R_1 = 10$ MΩ
(*e*) $E_1 = \int I^2 R_1\, dt = \int[(E/\ )^2/R_1]t^2 dt$;	$' = 2\times10^{-11}$ s; $t = 2\times10^{-11}\ln(4.2/.2) = 60.9$ ps
$P_1 = E_1/\Delta t$	$E_1 = \int_{0.005}^{0.105} 1.6\times10^{-4} t^2\, dt = 6.17\times10^{-8}\ J$; $P_1 = 617$ nW
$E_2 = \Delta U_C = 1/2\,C(V_i^2 - V_f^2)$; $P_2 = E_2/\Delta t'$	$E_2 = 17.6\times10^{-8}$ J; $P_2 = 2.89$ kW (while discharging C)

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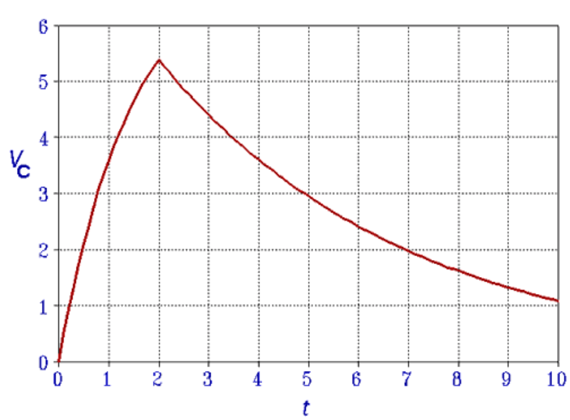
148 ··· In the circuit shown in Figure 26-78, $R_1 = 2.0$ MΩ, $R_2 = 5.0$ MΩ, and $C = 1.0\ \mu$F. At $t = 0$, switch S is closed, and at $t = 2.0$ s, switch S is opened. (*a*) Sketch the voltage across C and the current through R_2 between $t = 0$ and $t = 10$ s. (*b*) Find the voltage across the capacitor at $t = 2$ s and at $t = 8$ s.

We can use the results obtained in Problem 117. The current in the resistor R_2 at time t following the closing of the switch is given by $I_2 = [E/(R_1 + R_2)](1 - e^{-t/})$, where $\tau = R_1R_2C/(R_1 + R_2)$. The voltage across C is the same as that across R_2, namely I_2R_2. Inserting the appropriate numerical values, $\tau = 1.43$ s and $ER_2/(R_1 + R_2) = 7.14$ V.

(*a*) Find $V_C = I_2R_2$ at $t = 2$ s	$V_C(2\text{ s}) = 7.14(1 - e^{-2/1.43})$ V $= 5.377$ V
When S is opened at $t = 2$ s, C discharges through R_2 with a time constant $' = R_2C$	$' = 5$ s; let $t' = t - 2$; $V_C(t') = 5.377\, e^{-t'/5}$

The voltage across the capacitor as a function of time is shown in the figure. Here V_C is in volts and t is in seconds. The current therough the 5-Ω resistor R_2 follows the same time course, its value is, of course, $V_C/5\times10^6$ A.



(*b*) The value of V_C at $t = 2$ s has already been determined; it is 5.377 V.

$V_C(8\text{ s}) = 5.377\, e^{-6/5}$ V $= 1.62$ V,

as shown in the figure.

149*··· If the capacitor in the circuit in Figure 26-70 is replaced by a 30-Ω resistor, what currents flow through the resistors?

1. Write the junction equations; currents are down, and in the 30-Ω resistor to the right	$I_{10} + I_{40} = I_{80} + I_{20}$; $I_{10} = I_{30} + I_{80}$
2. Write the loop equations	$36 - 10I_{10} - 80I_{80} = 0$; $36 - 40I_{40} - 20I_{20} = 0$; $10I_{10} + 30I_{30} - 40I_{40} = 0$
3. Solve the set of five linear equations	$I_{10} = 0.740$ A; $I_{40} = 0.472$ A; $I_{30} = 0.383$ A; $I_{80} = 0.357$ A; $I_{20} = 0.855$ A

150··· Two batteries with emfs E_1 and E_2 and internal resistances r_1 and r_2 are connected in parallel. Prove that if a resistor is connected in parallel with this combination, the optimal load resistance (the resistance at which maximum power is delivered) is $R = r_1r_2/(r_1 + r_2)$.

Let I_1 be the current delivered by E_1, I_2 be the current delivered by E_2, and I_3 be the current through the resistor R. Then the junction equation is $I_1 + I_2 = I_3$. The two loop equations are $E_1 - r_1I_1 - RI_3 = 0$ and $E_2 - r_2I_2 - RI_3 = 0$. Solving for I_3 one finds $I_3 = (E_1r_2 + E_2r_1)/[r_1r_2 + R(r_1 + r_2)]$ and $P = I_3^2R = [(E_1r_2 + E_2r_1)/(r_1 + r_2)]^2[R/(R + A)^2]$, where $A = r_1r_2/(r_1 + r_2)$. Note that the quantity in the first square bracket is independent of R. To find the condition for which P is a maximum, differentiate P with respect to R and set the derivative equal to zero. Except for a constant factor, $dP/dR = [(R + A)^2 - 2R(R + A)]/(R + A)^4 = 0$. This is satisfied when $R = A = r_1r_2/(r_1 + r_2)$.

151··· Capacitors C_1 and C_2 are connected in parallel by a resistor and two switches as shown in Figure 26-79. Capacitor C_1 is initially charged to a voltage V_0, and capacitor C_2 is uncharged. The switches S are then closed. (*a*) What are the final charges on C_1 and C_2? (*b*) Compare the initial and final stored energies of the system. (*c*) What caused the decrease in the capacitor-stored energy?

(*a*) $Q = C_1V_0 = Q_1 + Q_2$ and $Q_1/C_1 = Q_2/C_2 = V$	$Q_1 = C_1^2V_0/(C_1 + C_2)$; $Q_2 = C_1C_2V_0/(C_1 + C_2)$
(*b*) Write expressions for U_i and U_f	$U_i = 1/2C_1V_0^2$; $U_f = 1/2Q_1^2/C_1 + 1/2Q_2^2/C_2 = 1/2C_1^2V_0^2/(C_1 + C_2$

(*c*) Since $C_1 + C_2 > C_1$, the final stored energy is less than the initial stored energy. The decrease equals the energy dissipated as Joule heat in the resistor connecting the two capacitors.

152··· (*a*) In Problem 151,find the current through R after the switches S are closed as a function of time. (*b*) Find the energy dissipated in the resistor as a function of time. (*c*) Find the total energy dissipated in the resistor and compare it with the loss of stored energy found in part (*b*) of Problem 151.

(*a*) Let q_1 and q_2 be the time-dependent charges on the two capacitors after the switches are closed. Then the circuit condition is $q_1/C_1 - IR - q_2/C_2 = 0$. But the current $I = dq_2/dt$ and $q_1 = Q - q_2 = C_1V_0 - q_2$. We therefore can write the differential equation $R(dq_2/dt) + q_2(C_1 + C_2)/C_1C_2 = V_0$. This first-order homogeneous differential equation has a solution of the form $q_2(t) = a + be^{-t/}$. Substituting this expression for q_2 we find that the equation is satisfied if $a = V_0C_{equ}$, where $C_{equ} = C_1C_2/(C_1 + C_2)$, and $\tau = RC_{equ}$. Finally, the constant b is determined from the boundary condition $q_2(0) = 0$, which requires $b = -a$. The current is $dq_2/dt = (V_0 - q_2/C_{equ})/R = (V_0/R)e^{-t/}$.

(*b*) The power dissipated in the resistor is given by $P(t) = I^2R = (V_0^2/R)e^{-2t/\tau}$

(*c*) The energy dissipated in the resistor is the integral of $P(t)$ between $t = 0$ and $t = \infty$.

$E = \frac{V_0^2}{R}\int_0^\infty e^{-2t/RC_{equ}}dt = \frac{1}{2}V_0^2C_{equ}$. This exactly the difference between the initial and final stored energies as found in the preceding problem, which confirms the statement at the end of that problem that the difference in the stored energies equals the energy dissipated in the resistor.

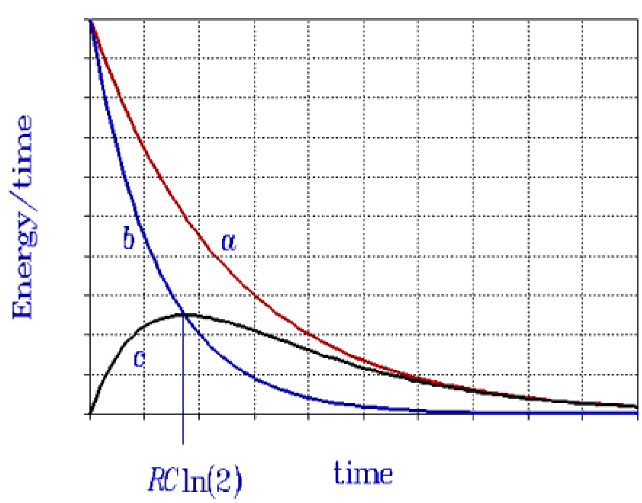
153*··· In the circuit in Figure 26-80, the capacitors are initially uncharged. Switch S_2 is closed and then switch S_1 is closed. (*a*) What is the battery current immediately after S_1 is closed? (*b*) What is the battery current a long time after both switches are closed? (*c*) What is the final voltage across C_1? (*d*) What is the final voltage across C_2? (*e*) Switch S_2 is opened again after a long time. Give the current in the 150-Ω resistor as a function of time.

(*a*) At $t = 0$ the capacitor voltages are zero. So at $t = 0$, effectively, $R = 100\ \Omega$. Find $I_{bat}(0)$.	$I_{bat} = 12/100\ A = 0.12\ A$
(*b*) For $t = \infty$, $R_{equ} = 300\ \Omega$	$I_{bat} = 12/300\ A = 0.040\ A$
(*c*) $V_{C1} = \mathcal{E} - 100I_{bat}$	$V_{C1} = (12 - 4.0)\ V = 8.0\ V$
(*d*) $V_{C2} = 150I_{bat}$	$V_{C2} = 6.0\ V$
(*e*) $I = I_0 e^{-t/}$; $\tau = RC$	$I = [0.040 \exp(-t/7.5\times10^{-3})]\ A$

154··· In the *RC* circuit in Figure 26-40*a*, the capacitor is initially uncharged and the switch is closed at time $t = 0$. (*a*) What is the power supplied by the battery as a function of time? (*b*) What is the power dissipated in the resistor as a function of time? (*c*) What is the rate at which energy is stored in the capacitor as a function of time? Plot your answers to parts (*a*), (*b*), and (*c*) versus time on the same graph. (*d*) Find the maximum rate at which energy is stored in the capacitor as a function of the battery voltage *e* and the resistance *R*. At what time does this maximum occur?

(*a*) $P_{bat} = \mathcal{E}I$, where I is given by Equ. 26-36	$P_{bat} = (\mathcal{E}^2/R)\, e^{-t/RC}$
(*b*) $P_R = I^2 R$	$P_R = (\mathcal{E}^2/R)\, e^{-2t/RC}$
(*c*) $dU_C/dt = d(1/2Q^2/C)dt = QI/C$; See Equ. 26-35 for $Q(t)$	$dU_C/dt = (\mathcal{E}^2/R)(e^{-t/RC} - e^{-2t/RC})$
(*d*) Differentiate dU_C/dt with respect to t and set = 0	$d^2U_C/dt^2 = (\mathcal{E}^2/R^2C)(2e^{-2t/RC} - e^{-t/RC}) = 0$ $t = RC \ln(2)$; then $dU_C/dt = \mathcal{E}^2/4R$

A plot of the results for (*a*), (*b*), and (*c*) are shown.



155··· A linear accelerator produces a pulsed beam of electrons. The current is 1.6 A for the 0.1-μs duration of each pulse. (*a*) How many electrons are accelerated in each pulse? (*b*) What is the average current of the beam if there are 1000 pulses per second? (*c*) If each electron acquires an energy of 400 MeV, what is the average power output of the accelerator? (*d*) What is the peak power output? (*e*) What fraction of the time is the accelerator actually accelerating electrons? (This is called the *duty factor* of the accelerator.)

(*a*) $Q = I_{\text{pulse}}\,\Delta t = ne$; $n = I_{\text{pulse}}\,\Delta t/e$ $\quad$ $n = 1.6\times10^{-7}/1.6\times10^{-19} = 10^{12}$

(*b*) $I_{\text{av}} = Q_{\text{pulse}}$/(time between pulses) $\quad$ $I_{\text{av}} = 1.6\times10^{-7}/10^{-3}$ A $= 0.16$ mA

(*c*) $P_{\text{av}} = I_{\text{av}}\,V$ $\quad$ $P_{\text{av}} = 64$ kW

(*d*) $P_{\text{peak}} = I_{\text{pulse}}\,V$ $\quad$ $P_{\text{peak}} = 640$ MW

(*e*) duty factor = Δt/(time between pulses) $\quad$ duty factor = $10^{-7}/10^{-3} = 10^{-4}$