CHAPTER 24

Electric Potential

1*  A uniform electric field of 2 kN/C is in the x direction. A positive point charge \( Q = 3 \ \mu C \) is released from rest at the origin. (a) What is the potential difference \( V(4 \text{ m}) - V(0) \)? (b) What is the change in the potential energy of the charge from \( x = 0 \) to \( x = 4 \text{ m} \)? (c) What is the kinetic energy of the charge when it is at \( x = 4 \text{ m} \)? (d) Find the potential \( V(x) \) if \( V(x) \) is chosen to be zero at \( x = 0 \), (e) 4 kV at \( x = 0 \), and (f) zero at \( x = 1 \text{ m} \).

(a) Use Equ. 24-2b; \( \Delta V = -E \Delta x \)  
\( \Delta V = -8 \text{ kV} \)

(b) \( \Delta U = q \Delta V \)  
\( \Delta U = -24 \text{ mJ} \)

(c) Use energy conservation  
\( K = 24 \text{ mJ} \)

(d), (e), (f) Use Equ. 24-2b  
(d) \( V(x) = -(2 \text{ kV/m})x \); (e) \( V(x) = 4 \text{ kV} - (2 \text{ kV/m})x \);  
(f) \( V(x) = 2 \text{ kV} -(2 \text{ kV/m})x \)

2  An infinite plane of surface charge density \( \sigma = +2.5 \ \mu C/m^2 \) is in the \( yz \) plane. (a) What is the magnitude of the electric field in newtons per coulomb? In volts per meter? What is the direction of \( E \) for positive values of \( x \)? (b) What is the potential difference \( V_b - V_a \) when point \( b \) is at \( x = 20 \text{ cm} \) and point \( a \) is at \( x = 50 \text{ cm} \)? (c) How much work is required by an outside agent to move a test charge \( q_0 = +1.5 \text{ nC} \) from point \( a \) to point \( b \)?

(a) \( E_s = \sigma/2\varepsilon_0 \)  
\( E_s = (2.5\times10^{-6}/2 \times 8.85\times10^{-12}) \text{ N/C} = 141 \text{ kN/C} \)

(b) \( \Delta V = -E_s \Delta x \)  
\( V_b - V_a = (-141 \times -0.3) \text{ kV} = 42.4 \text{ kV} \)

(c) \( W = \Delta U = q \Delta V \)  
\( W = (42.4 \times 10^3 \times 1.5\times10^{-9}) \text{ J} = 63.6 \text{ \mu J} \)

3  Two large parallel conducting plates separated by 10 cm carry equal and opposite surface charge densities such that the electric field between them is uniform. The difference in potential between the plates is 500 V. An electron is released from rest at the negative plate. (a) What is the magnitude of the electric field between the plates? Is the positive or negative plate at the higher potential? (b) Find the work done by the electric field on the electron as the electron moves from the negative plate to the positive plate. Express your answer in both electron volts and joules. (c) What is the change in potential energy of the electron when it moves from the negative plate to the positive plate? What is its kinetic energy when it reaches the positive plate?

(a) \( E \) is uniform; \( E = \Delta V/\Delta x \)  
\( E = 500/0.1 \text{ V/m} = 5 \text{ kV/m}; \text{ high } V \text{ at the positive plate} \)

(b) \( W = q \Delta V \)  
\( W = 500 \times 1.6\times10^{-19} \text{ J} = 8\times10^{-17} \text{ J} = 500 \text{ eV} \)
4. Explain the distinction between electric potential and electrostatic potential energy.
   The difference is analogous to that between gravitational potential and gravitational potential energy.

   Electrostatic potential refers to the difference in potential energy per unit charge between the point of interest and an arbitrary reference point; electrostatic potential energy is the potential energy of a charge with respect to an arbitrary zero.

5*. A positive charge is released from rest in an electric field. Will it move toward a region of greater or smaller electric potential?
   The positive charge will move toward lower potential energy, in this case toward the lower electric potential.

6. A lithium nucleus and an α particle are at rest. The lithium nucleus has a charge of +3e and a mass of 7 u; the α particle has a charge of +2e and a mass of 4 u. Which of the methods below would accelerate them both to the same kinetic energy?
   (a) Accelerate them through the same electrical potential difference.
   (b) Accelerate the α particle through potential $V_1$ and the lithium nucleus through $(2/3)V_1$.
   (c) Accelerate the α particle through potential $V_1$ and the lithium nucleus through $(7/4)V_1$.
   (d) Accelerate the α particle through potential $V_1$ and the lithium nucleus through $(2\times7)/(3\times4)V_1$.
   (e) None of the above.

7. A positive charge of magnitude 2 µC is at the origin. (a) What is the electric potential $V$ at a point 4 m from the origin relative to $V = 0$ at infinity? (b) How much work must be done by an outside agent to bring a 3-µC charge from infinity to $r = 4$ m, assuming that the 2-µC charge is held fixed at the origin? (c) How much work must be done by an outside agent to bring the 2-µC charge from infinity to the origin if the 3-µC charge is first placed at $r = 4$ m and is then held fixed?
   (a) Use Equ. 24-8
   (b) $W = q \Delta V$
   (c) Note that $W = kq_1q_2/r$.

8. The distance between the K⁺ and Cl⁻ ions in KCl is $2.80\times10^{-10}$ m. Calculate the energy required to separate the two ions to an infinite distance apart, assuming them to be point charges initially at rest. Express your answer in eV.
   $W$ in eV = $-U = kq^2/4\pi\epsilon_0 r^2$
   $W = 8.99\times10^9 \times 1.6\times10^{-19}/2.8\times10^{-10} \text{ eV} = 5.14 \text{ eV}$

9*. Two identical masses $m$ that carry equal charges $q$ are separated by a distance $d$. Show that if both are released simultaneously their speeds when they are separated a great distance are $v\sqrt{2}$, where $v$ is the speed that one mass would have at a great distance from the other if it were released and the other held fixed.
   If both are released, $2(1/2mv_b^2) = \Delta U$. If only one is released, $1/2mv^2 = \Delta U$. Hence $v_b = v\sqrt{2}$.

10. Protons from a Van de Graaff accelerator are released from rest at a potential of 5 MV and travel through a vacuum to a region at zero potential. (a) Find the final speed of the 5-MeV protons. (b) Find the accelerating electric field if the same potential change occurred uniformly over a distance of 2.0 m.
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(a) \( K = \frac{1}{2}mv^2 = e \Delta V; \quad v = \sqrt{2e \Delta V/m} \)

\[
v = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 5 \times 10^6}{1.67 \times 10^{-27}}} \text{ m/s} = 3.1 \times 10^7 \text{ m/s}
\]

(b) \( E = \frac{\Delta V}{\Delta x} \)

\( E = 2.5 \text{ MV/m} \)

11  
An electron gun fires electrons at the screen of a television tube. The electrons start from rest and are accelerated through a potential difference of 30,000 V. What is the energy of the electrons when they hit the screen (a) in electron volts and (b) in joules? (c) What is the speed of impact of electrons with the screen of the picture tube?

(a), (b) \( K = q\Delta V \)

(a) \( K = 30 \text{ keV} \); (b) \( K = 4.8 \times 10^{-15} \text{ J} \)

(c) \( v = \sqrt{2e \Delta V/m} \)

(c) \( v = 1.03 \times 10^8 \text{ m/s} \equiv c/3 \)

12  
(a) Derive an expression for the distance of closest approach of an \( \alpha \) particle with kinetic energy \( E \) to a massive nucleus of charge \( Ze \). Assume that the nucleus is fixed in space. (b) Find the distance of closest approach of a 5.0- and a 9.0-MeV \( \alpha \) particle to a gold nucleus; the charge of the gold nucleus is 79\( e \). Neglect the recoil of the gold nucleus.

(a) Use energy conservation; \( \Delta P + \Delta K = 0 \)

\[ K = 2kZe^2/r \text{ J}; \quad K = 2kZe/r \text{ eV}; \quad r = 2kZe/K(\text{eV}) \]

(b) Substitute \( K = 5 \text{ MeV} \) and \( 9 \text{ MeV} \) in (a)

\[ r_3 = 4.54 \times 10^{-14} \text{ m} = 45.4 \text{ fm}; \quad r_9 = 25.2 \text{ fm} \]

13\*  
Four 2-\( \mu \)C point charges are at the corners of a square of side 4 m. Find the potential at the center of the square (relative to zero potential at infinity) if (a) all the charges are positive, (b) three of the charges are positive and one is negative, and (c) two are positive and two are negative.

(a), (b), (c) Use Equ. 24-10; note that \( r_i = r = 4/\sqrt{2} \) m is the same for all charges.

\( a) \quad V = (8.99 \times 10^9/2.83)(8 \times 10^{-6}) \text{ V} = 25.4 \text{ kV} \)

(b) \( V = 3.18 \times 10^9 \times 4 \times 10^{-6} \text{ V} = 12.7 \text{ kV} \); (c) \( V = 0 \).

14  
Three point charges are on the x axis: \( q_1 \) is at the origin, \( q_2 \) is at \( x = 3 \) m, and \( q_3 \) is at \( x = 6 \) m. Find the potential at the point \( x = 0, y = 3 \) m if (a) \( q_1 = q_2 = q_3 = 2 \mu \)C, (b) \( q_1 = q_2 = 2 \mu \)C and \( q_3 = -2 \mu \)C, and (c) \( q_1 = q_3 = 2 \mu \)C and \( q_2 = -2 \mu \)C.

(a) Find distances from \( (0, 3) \) to \( q_1, q_2, \) and \( q_3 \)

Use Equ. 24-10; \( V = kq_1(r_1^{-1} + r_2^{-1} + r_3^{-1}) \)

\( r_1 = 3 \) m, \( r_2 = 4.24 \) m, \( r_3 = 6.71 \) m

\( V = 12.9 \text{ kV} \)

(b) Now \( V = kq_1(r_1^{-1} + r_2^{-1} - r_3^{-1}) \)

\( V = 7.55 \text{ kV} \)

(c) \( V = kq_1(r_1^{-1} - r_2^{-1} + r_3^{-1}) \)

\( V = 4.44 \text{ kV} \)

15  
Points \( A, B, \) and \( C \) are at the corners of an equilateral triangle of side 3 m. Equal positive charges of \( 2 \mu \)C are at \( A \) and \( B \). (a) What is the potential at point \( C \)? (b) How much work is required to bring a positive charge of 5 \( \mu \)C from infinity to point \( C \) if the other charges are held fixed? (c) Answer parts (a) and (b) if the charge at \( B \) is replaced by a charge of \(-2 \mu \)C.
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(a) Use Equ. 24-10
(b) \( W = \Delta U = qV_C \)
(c) Use Equ. 24-10

\[
V_C = 2 \times 8.99 \times 10^9 \times 2 \times 10^{-6}/3 \quad V = 12 \text{ kV}
\]

\[
W = 0.06 \text{ J}
\]

\[
V_C = 0; \quad W = 0
\]

16  A sphere with radius 60 cm has its center at the origin. Equal charges of 3 \( \mu \text{C} \) are placed at 60° intervals along the equator of the sphere. (a) What is the electric potential at the origin? (b) What is the electric potential at the north pole?

(a) Use Equ. 24-10; \( r = 0.6 \text{ m} \)
(b) Here \( r = 0.6 \sqrt{2} \text{ m} \)

\[
V = 6 \times 8.99 \times 10^9 \times 3 \times 10^{-6}/0.6 \quad V = 268 \text{ kV}
\]

\[
V = 268/\sqrt{2} \quad \text{kV} = 190 \text{ kV}
\]

17*  Two point charges \( q \) and \( q' \) are separated by a distance \( a \). At a point \( a/3 \) from \( q \) and along the line joining the two charges the potential is zero. Find the ratio \( q/q' \).

\[
q/(a/3) + q'/(2a/3) = 0; \quad q/q' = -1/2
\]

18  Two positive charges \(+q\) are on the \( x \) axis at \( x = +a \) and \( x = -a \). (a) Find the potential \( V(x) \) as a function of \( x \) for points on the \( x \) axis. (b) Sketch \( V(x) \) versus \( x \). (c) What is the significance of the minimum on your curve?

(a) For the two charges, \( r = |x - a| \) and \( |x + a| \), respectively. So

\[
V(x) = q\left(\frac{1}{|x - a|} + \frac{1}{|x + a|}\right)
\]

(b) \( V(x) \) versus \( x \) is shown below.

(c) At \( x = 0 \), \( dV/dx = 0 \) and the electric field, \( E_x = 0 \).

19  A point charge of \(+3e\) is at the origin and a second point charge of \(-2e\) is on the \( x \) axis at \( x = a \). (a) Sketch the potential function \( V(x) \) versus \( x \) for all \( x \). (b) At what point or points is \( V(x) \) zero? (c) How much work is needed to bring a third charge \(+e\) to the point \( x = \frac{1}{2}a \) on the \( x \) axis?

(a) The potential \( V(x) \) is given by

\[
V(x) = k(3e)/|x| - k(2e)/|x - a|
\]

The function \( V(x) \) is shown below.

For \( x > a \), \( V(x) = 0 \) at \( 3a \);
for \( 0 < x < a \), \( V(x) = 0 \) at \( x = 0.6a \).

There are no other points where \( V(x) = 0 \), as can be seen from the graph.

(c) \( V(a/2) = 2ke/a \). So \( W = 2ke^2/a \).
**20** · If the electric potential is constant throughout a region of space, what can you say about the electric field in that region?

If $V$ is constant, its gradient is zero; consequently $E = 0$.

**21** · If $E$ is known at just one point, can $V$ be found at that point?

No

**22** · In what direction can you move relative to an electric field so that the electric potential does not change?

Move always perpendicular to the field.

**23** · A uniform electric field is in the negative $x$ direction. Points $a$ and $b$ are on the $x$ axis, $a$ at $x = 2$ m and $b$ at $x = 6$ m. (a) Is the potential difference $V_b - V_a$ positive or negative? (b) If the magnitude of $V_b - V_a$ is $10^5$ V, what is the magnitude $E$ of the electric field?

(a) Since $E_x = -(dV/dx)$, $V$ is greater the larger is $x$. So $V_b - V_a$ is positive.

(b) $E_x = \Delta V/\Delta x$

$E_x = 25$ kV/m

**24** · The potential due to a particular charge distribution is measured at several points along the $x$ axis as shown in Figure 24-21. For what value(s) in the range $0 < x < 10$ m is $E_x = 0$?

$dV/dx = 0$ at $x = 4.5$ m; $E_x = 0$ at $4.5$ m.

**25** · A point charge $q = 3.00 \mu C$ is at the origin. (a) Find the potential $V$ on the $x$ axis at $x = 3.00$ m and at $x = 3.01$ m. (b) Does the potential increase or decrease as $x$ increases? Compute $-\Delta V/\Delta x$, where $\Delta V$ is the change in potential from $x = 3.00$ m to $x = 3.01$ m and $\Delta x = 0.01$ m. (c) Find the electric field at $x = 3.00$ m, and compare its magnitude with $-\Delta V/\Delta x$ found in part (b). (d) Find the potential (to three significant figures) at the point $x = 3.00$ m, $y = 0.01$ m, and compare your result with the potential on the $x$ axis at $x = 3.00$ m. Discuss the significance of this result.

(a) Use Equ. 24-8

$V(3.00) = 8.99$ kV; $V(3.01) = 8.96$ kV

(b) From (a), $V$ decreases with $x$

$-\Delta V/\Delta x = 3.0$ kV/m

(c) $E = kq/r^2$

$E = 3.0$ kV/m, in agreement with (b)

(d) $V = kq/r = kq/(x^2 + y^2)^{1/2}$

$V(3.0, 0.01) = 8.99$ kV = $V(3.0, 0)$

For $y << x$, $V$ is independent of $y$ and the points $(x, 0)$ and $(x, y)$ are at the same potential, i.e., on an equipotential surface.

**26** · A charge of $+3.00 \mu C$ is at the origin, and a charge of $-3.00 \mu C$ is on the $x$ axis at $x = 6.00$ m. (a) Find the potential on the $x$ axis at $x = 3.00$ m. (b) Find the electric field on the $x$ axis at $x = 3.00$ m. (c) Find the potential on the $x$ axis at $x = 3.01$ m, and compute $-\Delta V/\Delta x$, where $\Delta V$ is the change in potential from $x = 3.00$ m to $x = 3.01$ m and $\Delta x = 0.01$ m. Compare your result with your answer to part (b).

(a) Use Equ. 24-10; $V(x) = k(q_1/r_1 + q_2/r_2)$

$V(3.00) = 0$

(b) $E_x = k(q_1/r_1^2 - q_2/r_2^2)$

$E_x(3.00) = 5.99$ kV/m

(c) Use Equ. 24-10; $r_1 = 3.01$ m, $r_2 = 2.99$ m

$V(3.01) = -59.9$ V; $-\Delta V/\Delta x = 5.99$ kV/m = $E_x$

**27** · A uniform electric field is in the positive $y$ direction. Points $a$ and $b$ are on the $y$ axis, $a$ at $y = 2$ m and $b$ at $y = 6$ m. (a) Is the potential difference $V_b - V_a$ positive or negative? (b) If the magnitude of $V_b - V_a$ is $2 \times 10^4$ V, what is the magnitude $E$ of the electric field?

(a) Since $E_y = -(dV/dy)$, $V$ is smaller the larger is $y$. So $V_b - V_a$ is negative.
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(b) \( E_y = \Delta V/\Delta y \)  
\( E_y = 5.0 \text{kV/m} \)

Note: In the first printing of the textbook this problem is identical to problem 23.

28  
In the following, \( V \) is in volts and \( x \) is in meters. Find \( E_x \) when (a) \( V(x) = 2000 + 3000x; \)  
(b) \( V(x) = 4000 + 3000x; \)  
(c) \( V(x) = 2000 – 3000x; \)  
and (d) \( V(x) = -2000, \) independent of \( x. \)

(a), (b), (c), (d) \( E_x = -dV/dx \)  
(a) \( E_x = -3 \text{kV/m}; \)  
(b) \( E_x = -3 \text{kV/m}; \)  
(c) \( E_x = 3 \text{kV/m}; \)  
(d) \( E_x = 0 \)

29*  
The electric potential in some region of space is given by \( V(x) = C_1 + C_2 x^2, \) where \( V \) is in volts, \( x \) is in meters, 
and \( C_1 \) and \( C_2 \) are positive constants. Find the electric field \( E \) in this region. In what direction is \( E? \)

\( E_x = -dV/dx = -2C_2 x; \)  
with \( C_2 > 0, \) \( E \) points in the negative \( x \) direction.

30  
A charge \( q \) is at \( x = 0 \) and a charge \(-3q \) is at \( x = 1 \text{ m}. \) (a) Find \( V(x) \) for a general point on the \( x \) axis. (b) Find the points on the \( x \) axis where the potential is zero. (c) What is the electric field at these points? (d) Sketch \( V(x) \) versus \( x. \)

(a) Use Equ. 24-10  
(b) \( V(\pm \infty) = 0; \)  
also set \( 1/|x| = 3/|x-1| \)  
(c) For \( x = 0.25 \text{ m}, \) \( V(x) = kq[1/x + 3/(x-1)] \)  
For \( x = -0.5 \text{ m}, \) \( V(x) = -kq[3/(1-x) + 1/x] \)

(d) A sketch of \( V(x) \) versus \( x \) is shown.

31  
An electric field is given by \( E_x = 2.0 \text{x}^3 \text{kN/C.} \) Find the potential difference between the points on the \( x \) axis at \( x = 1 \text{ m} \) and \( x = 2 \text{ m}. \)

\( \Delta V = V_2 - V_1 = -\int_{x_1}^{x_2} E_x \, dx \)  
\( -2 \int x^3 \, dx \, kV = \frac{1}{2} (2^4 - 1) \text{kV} = -7.5 \text{kV} \)

32  
Three equal charges lie in the \( xy \) plane. Two are on the \( y \) axis at \( y = -a \) and \( y = +a, \) and the third is on the \( x \) axis at \( x = a. \) (a) What is the potential \( V(x) \) due to these charges at a point on the \( x \) axis? (b) Find \( E_x \) along the \( x \)
axis from the potential function $V(x)$. Evaluate your answers to (a) and (b) at the origin and at $x = \infty$ to see if they yield the expected results.

(a) Determine the distances of $q_i$ from point $(x, 0)$
Use Equ. 24-10

$$r_1 = r_2 = \sqrt{x^2 + a^2}; r_3 = |x - a|$$

(b) For $x > a$, $|x - a| = (a - x)$; apply Equ. 24-14
For $x < a$, $|x - a| = (x - a)$; apply Equ. 24-14

At $x = 0$, the fields due to $q_1$ and $q_2$ cancel, so $E_x(0) = -kq/a^2$; this is also obtained from (b) if $x = 0$.

For $x \to \infty$, i.e., $x \gg a$, the three charges appear as a point charge $3q$, so $E_x = 3kq/x^2$; this is also the result one obtains from (b) for $x \gg a$.

33* • The electric potential in a region of space is given by $V = (2 \text{ V/m}^2)x^2 + (1 \text{ V/m}^2)yz$. Find the electric field at the point $x = 2 \text{ m}, y = 1 \text{ m}, z = 2 \text{ m}$.

Use Equ. 24-18
Set $x = 2 \text{ m}, y = 1 \text{ m}, z = 2 \text{ m}$

$$E_x = -4x, E_y = -z, E_z = -y$$

$E = -8 \text{ V/m} i - 2 \text{ V/m} j - 1 \text{ V/m} k$

34 • A potential is given by

$$V(x, y, z) = \frac{kQ}{\sqrt{(x - a)^2 + y^2 + z^2}}$$

(a) Find the components $E_x, E_y,$ and $E_z$ of the electric field by differentiating this potential function. (b) What simple charge distribution might be responsible for this potential?

(a) Use Equ. 24-18

$$E_x = \frac{kQ(x - a)}{[(x - a)^2 + y^2 + z^2]^{3/2}}$$

$$E_y = \frac{kQy}{[(x - a)^2 + y^2 + z^2]^{3/2}}$$

$$E_z = \frac{kQz}{[(x - a)^2 + y^2 + z^2]^{3/2}}$$

(b) It is the field due to a point charge $Q$ at $(a, 0, 0)$

35 • In the calculation of $V$ at a point $x$ on the axis of a ring of charge, does it matter whether the charge $Q$ is uniformly distributed around the ring? Would either $V$ or $E_x$ be different if it were not?

$V$ along the axis of the ring does not depend on the charge distribution. The electric field, however, does depend on the charge distribution, and the result given in Chapter 22 is valid only for a uniform distribution.

36 • (a) Sketch $V(x)$ versus $x$ for the uniformly charged ring in the $yz$ plane given by Equation 24-20. (b) At what point is $V(x)$ a maximum? (c) What is $E_x$ at this point?
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(a) $V(x) = \frac{kQ}{\sqrt{x^2 + a^2}}$ is shown in arbitrary units.

(b) $V(x)$ is a maximum at $x = 0$.

(c) $E_x$ is zero at $x = 0$ by symmetry.

37* A charge of $q = +10^8$ C is uniformly distributed on a spherical shell of radius 12 cm. (a) What is the magnitude of the electric field just outside and just inside the shell? (b) What is the magnitude of the electric potential just outside and just inside the shell? (c) What is the electric potential at the center of the shell? What is the electric field at that point?

(a) Use Gauss’s law: $E = 0$ for $r < 12$ cm; $E = kq/r^2$ for $r > 12$ cm. Just outside the shell, $E = 6.24$ kV/m

(b) Use Equ. 24-23: $V = 749$ V just outside and inside the shell

(c) See Figure 24-12 for $V$; use Gauss’s law for $E$: $V(r = 0) = 749$ V; $E(r = 0) = 0$

38 A disk of radius 6.25 cm carries a uniform surface charge density $\sigma = 7.5$ nC/m$^2$. Find the potential on the axis of the disk at a distance from the disk of (a) 0.5 cm, (b) 3.0 cm, and (c) 6.25 cm.

(a), (b), (c) Use Equ. 24-21:

\[
V = 2\pi \times 8.9 \times 10^9 \times 7.5 \times 10^{-9} \left[ \sqrt{0.005^2 + 0.0625^2} - 0.005 \right] V = 24.4$ V for $x = 0.05$ cm; $V = 16.7$ V for $x = 3.0$ cm; $V = 11.0$ V for $x = 6.25$ cm

39 An infinite line charge of linear charge density $\lambda = 1.5$ $\mu$C/m lies on the z axis. Find the potential at distances from the line charge of (a) 2.0 m, (b) 4.0 m, and (c) 12 m, assuming that $V = 0$ at 2.5 m.

Use Equ. 24-25; $V = -2\pi k \ln(r/a)$

(a), (b), (c) Evaluate $V$

\[
V(2$ m) = 6.02$ kV; $V(4$ m) = $-12.7$ kV; $V(12$ m) = $-42.3$ kV

Note: $\ln(1) = 0$; therefore $a = 2.5$ m

40 Derive Equation 24-21 by integrating the electric field $E_z$ along the axis of the disk. (See Equation 23-11.)

\[
V = -2\pi k \sigma \int_0^x \left[ 1 - \frac{x}{\sqrt{x^2 + R^2}} \right] dx = 2\pi k \sigma \left( \sqrt{x^2 + R^2} - x \right)
\]

41* A rod of length $L$ carries a charge $Q$ uniformly distributed along its length. The rod lies along the y axis with its center at the origin. (a) Find the potential as a function of position along the x axis. (b) Show that the result obtained in (a) reduces to $V = kQ/x$ for $x >> L$. 
The charge per unit length is \( \lambda = \frac{Q}{L} \). Consider an element of length \( dy \). The element of potential \( dV \) due to that line element is \( dV = (k \lambda / r) \, dy \), where \( r = \sqrt{\frac{x^2 + y^2}{2}} \). Then \( V(x, 0) \) is obtained by integrating \( dV \) from \(-L/2\) to \( L/2\).

\[
V(x, 0) = \frac{kQ}{L} \int_{-L/2}^{L/2} \frac{dx}{\sqrt{x^2 + y^2}} = \frac{kQ}{L} \ln \left( \frac{\sqrt{x^2 + L^2/4} + \frac{L}{2}}{\sqrt{x^2 + L^2/4} - \frac{L}{2}} \right).
\]

(b) Divide numerator and denominator within the parentheses by \( x \) and recall that \( \ln(x/y) = \ln x - \ln y \). Use the binomial expansion for \((1 + \epsilon)^{1/2} = 1 + \frac{1}{2}\epsilon - \frac{1}{8}\epsilon^2 + \cdots \) and \( \ln(1 + \Delta) = \Delta - \frac{\Delta^2}{2} + \cdots \). Keeping only the lowest order terms in \( L/x \) one obtains \( V = \frac{kQ}{x} \).

42. A disk of radius \( R \) carries a surface charge distribution of \( \sigma = \sigma_0 R/r \). (a) Find the total charge on the disk. (b) Find the potential on the axis of the disk a distance \( x \) from its center.

(a) To find \( Q \), integrate the charge on a ring of radius \( r \) and thickness \( dr \). \( Q = \int_0^R 2\pi r (\sigma_0 / r) \, dr = 2\pi \sigma_0 R^2 / 2 \).

(b) The element of charge is \( 2\pi \sigma_0 R \, dr \). The potential due to that ring of charge is given by Equ. 24-20. Thus

\[
V = 2\pi \sigma_0 kR \int_0^R \frac{dr}{\sqrt{x^2 + r^2}} = 2\pi \sigma_0 kR \ln \left( \frac{R + \sqrt{x^2 + R^2}}{x} \right)
\]

43. Repeat Problem 42 if the surface charge density is \( \sigma = \sigma_0 r^2 / R^2 \).

(a) Follow the procedure of Problem 42(a). \( Q = \int_0^R 2\pi \sigma_0 (r^3 / R^2) \, dr = \pi \sigma_0 R^2 / 2 \).

(b) Follow the procedure of Problem 42(b). \( V = \frac{2\pi \sigma_0 k}{R^2} \int_0^R \frac{r^3 \, dr}{\sqrt{x^2 + r^2}} = \frac{2\pi \sigma_0 k}{R^2} \left( \frac{R^2 - 2x^2}{3} \sqrt{x^2 + R^2} + \frac{2x^3}{3} \right) \)

44. A rod of length \( L \) carries a charge \( Q \) uniformly distributed along its length. The rod lies along the \( y \) axis with one end at the origin. Find the potential as a function of position along the \( x \) axis.

This problem is similar to Problem 41, except that here the integral extends from \( 0 \) to \( L \). Thus,

\[
V(x, 0) = \frac{kQ}{L} \int_0^L \frac{dy}{\sqrt{x^2 + y^2}} = \frac{kQ}{L} \ln \left( \frac{\sqrt{x^2 + L^2} + L}{x} \right)
\]

45*. A disk of radius \( R \) carries a charge density \( +\sigma_0 \) for \( r < a \) and an equal but opposite charge density \( -\sigma_0 \) for \( a < r < R \). The total charge carried by the disk is zero. (a) Find the potential a distance \( x \) along the axis of the disk. (b) Obtain an approximate expression for \( V(x) \) when \( x \gg R \).

(a) First, find the relation between \( a \) and \( R \). Since the surface charge density is uniform, the magnitudes of the two charges are equal if \( a^2 = R^2 - a^2 \), or \( a = R / \sqrt{2} \). The potential due to the positive charge is given by Equ. 24-20, where \( a^2 = R^2 / 2 \), i.e., \( V_r = 2\pi k \sigma_0 [(x^2 + R^2 / 2) - x] \). To find \( V_r \) we integrate \( dV \).
\[ V = -2\pi\sigma_0 k \int_{R/\sqrt{2}}^{R} \frac{r \, dr}{\sqrt{x^2 + r^2}} = -2\pi\sigma_0 k \left( \sqrt{x^2 + R^2} - \sqrt{x^2 + R^2/2} \right). \]

\[ V = 2\pi\sigma_0 k \left( 2\sqrt{x^2 + R^2/2} - \sqrt{x^2 + R^2} - x \right). \]

(b) To determine \( V \) for \( x \gg R \), factor out \( x \) from the square roots and expand using the binomial expansion. Keeping only the lowest order terms, the expression in parentheses reduces to \( R^4/16x^3 \), and one obtains \( V = \pi\sigma_0 k R^4/8x^3 \) for \( x \gg R \).

46 Use the result obtained in Problem 45(a) to calculate the electric field along the axis of the disk. Then calculate the electric field by direct integration using Coulomb's law.

For convenience we shall use the constant \( a \) rather than \( R \). \( E_x = -(dV/dx) \), and performing this operation one obtains

\[ E_x = -2\pi\sigma_0 k \left( \frac{2x}{\sqrt{x^2 + a^2}} - \frac{x}{\sqrt{x^2 + 2a^2}} - l \right) \]

The field \( E_x = E_{x+} + E_{x-} \). \( E_{x+} \) is given by Equ. 23-11. To determine \( E_{x-} \) we integrate the field due to a ring charge:

\[ E_{x-} = -2\pi\sigma_0 k \int_{a}^{2a} \frac{r \, dr}{(x^2 + r^2)^{3/2}} = -2\pi\sigma_0 k \left( \frac{1}{\sqrt{x^2 + 2a^2}} - \frac{1}{\sqrt{x^2 + a^2}} \right). \] \( E_{x+} + E_{x-} \) gives the above result.

47 A rod of length \( L \) has a charge \( Q \) uniformly distributed along its length. The rod lies along the \( x \) axis with its center at the origin. (a) What is the electric potential as a function of position along the \( x \) axis for \( x > L/2 \)? (b) Show that for \( x \gg L/2 \), your result reduces to that due to a point charge \( Q \).

(a) As before, we consider an element of charge \( dq = \lambda \, du = (Q/L) \, du \). \( dV \) due to \( dq \) is \( dV = (kQ/L)du/r \), where \( r = x - u \). To find \( V \) we integrate \( dV \) between \( u = -L/2 \) and \( L/2 \). The result is \( V = (kQ/L) \ln[(x + L/2)/(x - L/2)]. \)

(b) We divide numerator and denominator of the argument in the logarithm by \( x \) and use the binomial expansion to obtain the approximate argument \( 1 + L/x \), for \( x \gg L \). For \( x \ll 1 \), \( \ln(1 + x) \approx x \), and so \( V \approx kQ/x \) for \( x \gg L \).

48 A conducting spherical shell of inner radius \( a \) and outer radius \( b \) is concentric with a small metal sphere of radius \( a < b \). The metal sphere has a positive charge \( Q \). The total charge on the conducting spherical shell is \(-Q\). (a) What is the potential of the spherical shell? (b) What is the potential of the metal sphere?

(a) Use Gauss’s law: \( E \) for \( r > b = 0 \)

\[ V = -\int E_r \, dr = 0 \]

(b) A sphere acts like a point charge for \( a < r < b \)

\[ V_o = kQ \left( 1/a - 1/b \right) \]

49 Two very long, coaxial cylindrical shell conductors carry equal and opposite charges. The inner shell has radius \( a \) and charge \(+q\); the other shell has radius \( b \) and charge \(-q\). The length of each cylindrical shell is \( L \). Find the potential difference between the shells.

For \( a < r < b \), \( E_r \) is given by Equ. 23-9, i.e., \( E_r = 2kq/Lr \). The potential difference \( V_b - V_a \) is obtained by integration.
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\[ V_b - V_a = -\int_a^b \frac{2kq}{L} \frac{dr}{r} = -\frac{2kq}{L} \ln \left( \frac{b}{a} \right). \]

50  A uniformly charged sphere has a potential on its surface of 450 V. At a radial distance of 20 cm from this surface, the potential is 150 V. What is the radius of the sphere, and what is the charge of the sphere?

1. Write relations for the data given \( kq/R = 450 \) V (1); \( kq/(R + 0.2 \) m) = 150 V (2)
2. Divide (1) by (2) and solve for \( R \)
3. Find \( q \) using (1) or (2)

\[ R = 0.1 \text{ m} \]
\[ q = 5.01 \text{ nC} \]

51  Consider two infinite parallel planes of charge, one in the \( yz \) plane and the other at distance \( x = a \). (a) Find the potential everywhere in space when \( V = 0 \) at \( x = 0 \) if the planes carry equal positive charge densities \( +\sigma \). (b) Repeat the problem with charge densities equal and opposite, and the charge in the \( yz \) plane positive.

(a) We consider three regions, region I for \( x < 0 \), region II for \( 0 < x < a \), and region III for \( x > a \). In I the field is that due to the two infinite planes, so \( E = -\sigma/\varepsilon_0 i \), and the potential is \( V = (\sigma/\varepsilon_0)x \), consistent with \( V(0) = 0 \). In II the electric field is zero, and so \( V = 0 \). In region III, \( E = \sigma/\varepsilon_0 i \), so \( V \) in region III is \( V = -(\sigma/\varepsilon_0)x + C \), where \( C \) is a constant of integration. Since \( V(a) = V(0) = 0 \), \( C = a\sigma/\varepsilon_0 \) and \( V = (\sigma/\varepsilon_0)(a - x) \).

(b) Now \( E = 0 \) in regions I and III. In II, \( E = \sigma/\varepsilon_0 i \), and \( V = -(\sigma/\varepsilon_0)x \). In region I, \( V = 0 \); in region III, \( V = -\sigma x/a/\varepsilon_0 \).

52  Show that for \( x \gg R \) the potential on the axis of a disk charge approaches \( kQ/x \), where \( Q = \sigma\pi R^2 \) is the total charge on the disk. (Hint: Write \((x^2 + R^2)^{1/2} = x(1 + R^2/x^2)^{1/2}\) and use the binomial expression.)

Start with Eq. 24-21 and factor out the variable \( x \) from the expression in the square brackets. This gives \( V = 2\pi k \sigma x \left[ (1 + e)^{1/2} - 1 \right] \), where \( e = R^2/x^2 \ll 1 \). Then \( (1 + e)^{1/2} \simeq 1 + 1/2e \); thus, \( V = \pi k\sigma R^2/x^2 = kQ/x^2 \).

53  In Example 24-12 you derived the expression

\[ V(r) = \frac{kQ}{2R} \left( 3 - \frac{r^2}{R^2} \right) \]

for the potential inside a solid sphere of constant charge density by first finding the electric field. In this problem you derive the same expression by direct integration. Consider a sphere of radius \( R \) containing a charge \( Q \) uniformly distributed. You wish to find \( V \) at some point \( r < R \). (a) Find the charge \( q' \) inside a sphere of radius \( r \) and the potential \( V_1 \) at \( r \) due to this part of the charge. (b) Find the potential \( dV_2 \) at \( r \) due to the charge in a shell of radius \( r' \) and thickness \( dr' \) at \( r' > r \). (c) Integrate your expression in (b) from \( r' = r \) to \( r' = R \) to find \( V_2 \). (d) Find the total potential \( V \) at \( r \) from \( V = V_1 + V_2 \).

(a) Since the volume charge density is constant, \( q' = Qr^3/R^3 \). \( V_1 = kq'/r = kQr^2/R^3 \).

(b) \( dV_2 = (k/r) dq' \), where now \( dq' = 4\pi r^2 \rho \ dr' \) \((3Q/R^3)r^2 \ dr' \) since \( \rho = 3Q/4\pi R^3 \).
(c) Integrating $dV_2$ we have
\[ V_2 = \frac{3kQ}{R^3} \int_{r}^{R} r' \, dr' = \frac{3kQ}{2R^3} \left( R^2 - r^2 \right). \]

(d) $V = V_1 + V_2 = \frac{kQ}{2R} \left( 3 - \frac{r^2}{R^2} \right).$

---

54 A nonconducting sphere of radius $R$ has a volume charge density $\rho = \rho_0 r/R$, where $\rho_0$ is a constant. (a) Show that the total charge is $Q = \pi R^3 \rho_0$. (b) Show that the total charge inside a sphere of radius $r < R$ is $q = qr^4/R^4$. (c) Use Gauss's law to find the electric field $E_r$ everywhere. (d) Use $dV = -E_r \, dr$ to find the potential $V$ everywhere, assuming that $V = 0$ at $r = \infty$. (Remember that $V$ is continuous at $r = R$.)

\[
\begin{align*}
(a) & \quad Q = 4\pi \int_0^R \rho(r) r^2 \, dr = \frac{4\pi \rho_0}{R} \int_0^R r^3 \, dr = \pi \rho_0 R^3 \\
(b) & \quad q(r) = 4\pi \int_0^r \rho(r) \, dr = \frac{4\pi \rho_0}{R} \int_0^r r^3 \, dr = \frac{\pi \rho_0 r^4}{R} = \frac{Q r^4}{R^4} \\
(c) & \quad \text{For } r < R, \text{ and using Gauss’s law, we have } 4\pi r^2 E_r = q(r)/\varepsilon_0, \text{ so } E_r = \frac{q r^2}{4\pi \varepsilon_0 R^4} = \frac{kQ r^2}{R^4}; \text{ for } r > R, \text{ } E_r = \frac{kQ}{r^2}. \\
(d) & \quad \text{For } r > R, \text{ the potential is that of a point charge } Q: \quad V(r) = \frac{kQ}{r}. \text{ To find } V(r) \text{ for } r < R \text{ we integrate } -E_r \, dr \text{ from } R \text{ to } r.
\end{align*}
\]

\[ -\frac{kQ}{R^4} \int_{R}^{r} r^2 \, dr = \frac{kQ}{3R} \left( R^3 - r^3 \right) = V(r) - V(R). \text{ Since } V(R) = kQ/R, \quad V(r) = \frac{kQ}{3R} \left( 4 - \frac{r^3}{R^3} \right). \]

---

55 Two charged metal spheres are connected by a wire, and sphere $A$ is larger than sphere $B$ (Figure 24-22). The magnitude of the electric potential of sphere $A$ is

(a) greater than that at the surface of sphere $B$.
(b) less than that at the surface of sphere $B$.
(c) the same as that at the surface of sphere $B$.
(d) greater than or less than that at the surface of sphere $B$, depending on the radii of the spheres.
(e) greater than or less than that at the surface of sphere $B$, depending on the charge on the spheres.

(c) Figure 24-23 shows two parallel metal plates maintained at potentials of 0 and 60 V. Midway between the plates is a copper sphere. Sketch the equipotential surfaces and the electric field lines between the two plates.
The electric field lines, shown as solid lines, and the equipotential surfaces (intersecting the plane of the paper), shown as dashed lines, are sketched in the adjacent figure. The electric field lines point from the plane $V = 0$ to the plane $V = 60$ V, but some first terminate on the copper sphere, which is an equipotential surface. Note that the electric field lines are perpendicular to the conductors at their surfaces.

57* 

Figure 24-24 shows a metal sphere carrying a charge $-Q$ and a point charge $+Q$. Sketch the electric field lines and equipotential surfaces in the vicinity of this charge system. The electric field lines, shown as solid lines, and the equipotential surfaces (intersecting the plane of the paper), shown as dashed lines, are sketched in the adjacent figure. The point charge $+Q$ is the point at the right, and the metal sphere with charge $-Q$ is at the left. Near the two charges the equipotential surfaces are spheres, and the field lines are normal to the metal sphere at the sphere’s surface.

58 

Repeat Problem 57 with the charge on the metal sphere changed to $+Q$. The electric field lines, shown as solid lines, and the equipotential surfaces (intersecting the plane of the paper), shown as dashed lines, are sketched in the adjacent figure. The point charge $+Q$ is the point at the right, and the metal sphere with charge $+Q$ is at the left. Near the two charges the equipotential surfaces are spheres, and the field lines are normal to the metal sphere at the sphere’s surface. Very far from both charges, the equipotential surfaces and field lines approach those of a point charge $2Q$ located at the midpoint.

59 

Sketch the electric field lines and the equipotential surfaces both near and far from the conductor shown in Figure 24-20a, assuming that the conductor carries some charge $Q$. 
The equipotentials are shown with dashed lines, the field lines are shown in solid lines. It is assumed that the conductor carries a positive charge. Near the conductor the equipotentials follow the conductor’s contours; far from the conductor, the equipotentials are spheres centered on the conductor. The electric field lines are perpendicular to the equipotentials.

60 Two equal positive charges are separated by a small distance. Sketch the electric field lines and the equipotential surfaces for this system. The equipotentials are shown with dashed lines, the electric field lines are shown with solid lines. Near each charge, the equipotentials are spheres centered on each charge; far from the charges, the equipotential is a sphere centered at the midpoint between the charges. The electric field lines are perpendicular to the equipotential surfaces.

61 An infinite plane of charge has surface charge density $3.5 \, \mu C/m^2$. How far apart are the equipotential surfaces whose potentials differ by 100 V?

$$E = \sigma/2\varepsilon_0 = -\Delta V/\Delta x; \quad |\Delta x| = 2\varepsilon_0\Delta V/\sigma$$

$$\Delta x = 0.506 \, \text{mm}$$

62 A point charge $q = +1.7 \times 10^{-8} \, \text{C}$ is at the origin. Taking the potential to be zero at $r = \infty$, locate the equipotential surfaces at 20-V intervals from 20 to 100 V, and sketch them to scale. Are these surfaces equally spaced?

The equipotentials are spheres centered at the origin with radii $r_i = kq/V_i$.

For $V = 20 \, \text{V}$, $r = 0.499 \, \text{m}$; $V = 40 \, \text{V}$, $r = 0.25 \, \text{m}$;

$V = 60 \, \text{V}$, $r = 0.166 \, \text{m}$; $V = 80 \, \text{V}$, $r = 0.125 \, \text{m}$;

$V = 100 \, \text{V}$, $r = 0.01 \, \text{m}$. The figure shows the equipotentials.

63 (a) Find the maximum net charge that can be placed on a spherical conductor of radius 16 cm before dielectric breakdown of the air occurs. (b) What is the potential of the sphere when it carries this maximum charge?

(a) $E_{\text{max}} = 3 \, \text{MV/m} = kq_{\text{max}}/r^2$; $q_{\text{max}} = E_{\text{max}} r^2/k$

(b) $V_{\text{max}} = \pm kq_{\text{max}}/r$
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\[ q_{\text{max}} = 8.54 \, \mu\text{C} \quad V_{\text{max}} = \pm 480 \, \text{kV} \]

64  Find the greatest surface charge density \( \sigma_{\text{max}} \) that can exist on a conductor before dielectric breakdown of the air occurs.

\[ E_{\text{max}} = 3 \, \text{MV/m} = \sigma / \varepsilon_0; \quad \text{solve for} \sigma \]

\( \sigma_{\text{max}} = 26.6 \, \mu\text{C/m}^2 \)

65* Charge is placed on two conducting spheres that are very far apart and connected by a long thin wire (Figure 24-25). The larger sphere has a diameter twice that of the smaller. Which sphere has the largest electric field near its surface? By what factor is it larger than that at the surface of the other sphere?

Both spheres are at the same potential so \( V_L = V_S \). Hence, \( kQ_L/R_L = kQ_S/R_S \). The fields near the surfaces are \( E_S = kQ_S/R_S^2 \), \( E_L = kQ_L/R_L^2 \). It follows that \( E_S/E_L = R_L/R_S \) and that, therefore, the smaller sphere has the larger field near its surface, with \( E_S = E_L(R_L/R_S) = 2E_L \).

66 Charge is placed on two conducting spheres that are very far apart and connected by a long thin wire. The radius of the smaller sphere is 5 cm and that of the larger sphere is 12 cm. The electric field at the surface of the larger sphere is 200 kV/m. Find the surface charge density on each sphere.

1. Use the result of Problem 65 to find \( E_5 \)

\[ E_5 = 200(12/5) \, \text{kV/m} = 480 \, \text{kV/m} \]

2. \( \sigma = \varepsilon_0 E \)

\( \sigma_1 = 1.77 \, \mu\text{C/m}^2; \quad \sigma_2 = 4.25 \, \mu\text{C/m}^2 \)

67 Two concentric spherical shell conductors carry equal and opposite charges. The inner shell has radius \( a \) and charge \( +q \); the outer shell has radius \( b \) and charge \( -q \). Find the potential difference between the shells, \( V_a - V_b \).

See Problem 48. Since \( V_b = 0 \), \( V_a - V_b = kq(1/a - 1/b) \).

68 Two identical uncharged metal spheres connected by a wire are placed close by two similar conducting spheres with equal and opposite charges as shown in Figure 24-26. (a) Sketch the electric field lines between spheres 1 and 3 and between spheres 2 and 4. (b) What can be said about the potentials \( V_1, V_2, V_3, \) and \( V_4 \) of the spheres? (c) If spheres 3 and 4 are connected by a wire, prove that the final charge on each must be zero.

(a) The field lines are shown on the figure. The charged spheres induce charges of opposite sign on the spheres near them so that sphere 1 is negatively charged, and sphere 2 is positively charged. The total charge of the system is zero.

(b) \( V_1 = V_2 \) since they are connected. From the directions of the electric field lines it follows that \( V_3 > V_1 \) and \( V_4 < V_2 \).

(c) If 3 and 4 are connected, \( V_3 = V_4 \) and the conditions of part (b) can only be met with all potentials are zero.

Consequently the charge on each sphere is zero.

69* Two equal positive point charges \( +Q \) are on the x axis. One is at \( x = -a \) and the other is at \( x = +a \). At the origin,

(a) \( E = 0 \) and \( V = 0 \).
(b) $E = 0$ and $V = 2kQ/a$.
(c) $E = (2kQ^2/a^2)i$ and $V = 0$.
(d) $E = (2kQ^2/a^2)i$ and $V = 2kQ/a$.
(e) none of the above is correct.

(b) $E = 0$ and $V = 2kQ/a$.
(c) $E = (2kQ^2/a^2)i$ and $V = 0$.
(d) $E = (2kQ^2/a^2)i$ and $V = 2kQ/a$.
(e) none of the above is correct.

(b) The electrostatic potential is measured to be $V(x, y, z) = 4x + V_0$, where $V_0$ is a constant. The charge distribution responsible for this potential is

(a) a uniformly charged thread in the $xy$ plane.
(b) a point charge at the origin.
(c) a uniformly charged sheet in the $yz$ plane.
(d) a uniformly charged sphere of radius $1/\pi$ at the origin.

(c) Two point charges of equal magnitude but opposite sign are on the $x$ axis; $+Q$ is at $x = -a$ and $-Q$ is at $x = +a$. At the origin,

(a) $E = 0$ and $V = 0$.
(b) $E = 0$ and $V = 2kQ/a$.
(c) $E = (2kQ^2/a^2)i$ and $V = 0$.
(d) $E = (2kQ^2/a^2)i$ and $V = 2kQ/a$.
(e) none of the above is correct.

(c) True or false:

(a) If the electric field is zero in some region of space, the electric potential must also be zero in that region.
(b) If the electric potential is zero in some region of space, the electric field must also be zero in that region.
(c) If the electric potential is zero at a point, the electric field must also be zero at that point.
(d) Electric field lines always point toward regions of lower potential.
(e) The value of the electric potential can be chosen to be zero at any convenient point.
(f) In electrostatics, the surface of a conductor is an equipotential surface.
(g) Dielectric breakdown occurs in air when the potential is $3 \times 10^6$ V.

(a) False (b) True (c) False (d) True (e) True (f) True (g) False

(a) $V$ is constant on a conductor surface. Does this mean that $\sigma$ is constant? (b) If $E$ is constant on a conductor surface, does this mean that $\sigma$ is constant? Does it mean that $V$ is constant?

(a) No (b) Yes; Yes

An electric dipole has a positive charge of $4.8 \times 10^{-19}$ C separated from a negative charge of the same magnitude by $6.4 \times 10^{-10}$ m. What is the electric potential at a point $9.2 \times 10^{-10}$ m from each of the two charges? (a) $9.4$ V (b) Zero (c) $4.2$ V (d) $5.1 \times 10^9$ V (e) $1.7$ V

An electric field is given by $E = axi$, where $E$ is in newtons per coulomb, $x$ is in meters, and $a$ is a positive constant. (a) What are the SI units of $a$? (b) How much work is done by this field on a positive point charge $q_0$
when the charge moves from the origin to some point \( x \)? (c) Find the potential function \( V(x) \) such that \( V = 0 \) at \( x = 0 \).

(a) \( [a] = [F]/([C][L]) = ([M][L]/[T]^2)/([C][L]) = \text{kg/C.s}^2 \)

(b) \( W = \int F \, dx = ax^2 q_0/2 \)

(c) \( \Delta V = \Delta U/q_0 = -W/q_0 = -ax^2/2 \)

### 76

Two positive charges \( +q \) are on the \( y \) axis at \( y = +a \) and \( y = -a \). (a) Find the potential \( V \) for any point on the \( x \) axis. (b) Use your result in (a) to find the electric field at any point on the \( x \) axis.

(a) Use Equ. 24-20; potentials add

\[
V(x) = \frac{2kq}{\sqrt{x^2 + a^2}}
\]

(b) \( E(x) = -(dV/dx) \hat{i} = \frac{2kq x}{(x^2 + a^2)^{3/2}} \hat{i} \)

### 77*

If a conducting sphere is to be charged to a potential of 10,000 V, what is the smallest possible radius of the sphere such that the electric field will not exceed the dielectric strength of air?

For a sphere, \( E_r = V(r)/r; \ r_{\text{min}} = V/E_{\text{max}} \)

\( r_{\text{min}} = 10^{9}/3 \times 10^6 \text{ m} = 3.33 \text{ mm} \)

### 78

An isolated aluminum sphere of radius 5.0 cm is at a potential of 400 V. How many electrons have been removed from the sphere to raise it to this potential?

\( Q = VR/k = Ne \), where \( N \) is the number removed

\( N = VR/k = 1.39 \times 10^{10} \)

### 79

A point charge \( Q \) resides at the origin. A particle of mass \( m = 0.002 \text{ kg} \) carries a charge of 4.0 \( \mu \text{C} \). The particle is released from rest at \( x = 1.5 \text{ m} \). Its kinetic energy as it passes \( x = 1.0 \text{ m} \) is 0.24 J. Find the charge \( Q \).

\( K = -q\Delta V = -kQ(1/x_f - 1/x_i) \); solve for \( Q \)

\( Q = \frac{0.24 \times 3}{8.99 \times 10^9 \times 4 \times 10^{-6}} \text{ C} = -20 \text{ \mu C} \)

### 80

A conducting wedge is charged to a potential \( V \) with respect to a large conducting sheet (Figure 24-27).

(a) Sketch the electric field lines and the equipotentials for this configuration. Where along the \( x \) axis is \( |E| \) greatest? (b) An electron of mass \( m_e \) leaves the sheet with zero velocity. What is its speed \( v \) when it arrives at the wedge? (Ignore the effect of gravity.)

(a) The equipotentials (dashed lines) and electric field lines (solid lines) are shown in the sketch. The field is greatest at \( x = 0 \).

(b) Since the wedge is at the higher potential, the electron is accelerated toward the wedge.

Its kinetic energy is \( eV = 1/2m_e v^2 \), so

\( v = (2eV/m_e)^{1/2} \).

### 81*

A Van de Graaff generator has a potential difference of 1.25 MV between the belt and the outer shell. Charge is supplied at the rate of 200 \( \mu \text{C}/\text{s} \). What minimum power is needed to drive the moving belt?

\( W = qAV; P = dW/dt = \Delta V dq/dt \)

\( P = (200 \times 10^{-6} \times 1.25 \times 10^6) \text{ W} = 250 \text{ W} \)

### 82

A positive point charge \( +Q \) is located at \( x = -a \). (a) How much work is required to bring a second equal positive point charge \( +Q \) from infinity to \( x = +a? \) (b) With the two equal positive point charges at \( x = -a \) and \( x = +a \), how much work is required to bring a third charge \( -Q \) from infinity to the origin? (c) How much work is required to move the charge \( -Q \) from the origin to the point \( x = 2a \) along the semicircular path shown (Figure 24-28)?
(a) \( W = QV(a) = kQ^2/2a \)  
(b) \( V(0) = 2kQ/a; \ W = -2kQ^2/a \)  
(c) \( V(2a) = kQ^2(1/3a + 1/a) \) so \( \Delta V = (kQ/a)(-2/3) \) and \( W = 2kQ^2/3a \)

83 A charge of 2 nC is uniformly distributed around a ring of radius 10 cm that has its center at the origin and its axis along the \( x \) axis. A point charge of 1 nC is located at \( x = 50 \) cm. Find the work required to move the point charge to the origin. Give your answer in both joules and electron volts.

1. Use Equ. 24-20 to find \( V(0.5 \text{ m}) \) and \( V(0) \)  
   \( V(0.5 \text{ m}) = 35.3 \text{ V}, \ V(0) = 179.8 \text{ V} \)

2. \( W = q\Delta V \)  
   \( W = 1.45 \times 10^{-7} \text{ J} = 9.03 \times 10^{11} \text{ eV} \)

84 The centers of two metal spheres of radius 10 cm are 50 cm apart on the \( x \) axis. The spheres are initially neutral, but a charge \( Q \) is transferred from one sphere to the other, creating a potential difference between the spheres of 100 V. A proton is released from rest at the surface of the positively charged sphere and travels to the negatively charged sphere. At what speed does it strike the negatively charged sphere?

\[ \frac{1}{2}mv^2 = e\Delta V; \ v = \left( \frac{2e\Delta V}{m_p} \right)^{1/2} = (2 \times 1.6 \times 10^{-19}) \times 100 \times \left( \frac{1.6 \times 10^{-19}}{1.67 \times 10^{-27}} \right)^{1/2} \text{ m/s} = 1.38 \times 10^5 \text{ m/s} \]

85* A spherical conductor of radius \( R_1 \) is charged to 20 kV. When it is connected by a long, fine wire to a second conducting sphere far away, its potential drops to 12 kV. What is the radius of the second sphere?

1. Write the initial and final conditions  
   \( 20 \text{ kV} = k(Q_1 + Q_2)/R_1; \ 12 \text{ kV} = kQ_1/R_1 = kQ_2/R_2 \)

2. Solve for \( R_1/R_2 \)  
   \( R_1/R_2 = 3/2; \ R_2 = (2/3)R_1 \)

86 A uniformly charged ring of radius \( a \) and charge \( Q \) lies in the \( yz \) plane with its axis along the \( x \) axis. A point charge \( Q' \) is placed on the \( x \) axis at \( x = 2a \). (a) Find the potential at any point on the \( x \) axis due to the total charge \( Q + Q' \). (b) Find the electric field for any point on the \( x \) axis.

(a) \( V = V_0 + V_\theta; \ V_\theta = kQ/\sqrt{x^2 + a^2}; \ V(x) = kQ/\sqrt{x^2 + a^2} + kQ'/|x - 2a| \)

(b) By symmetry \( E = E_z \). For \( x > 2a, E_{x\theta} \) and \( E_{z\theta} \) are positive; for \( x < 2a, E_{x\theta} \) is negative. Since \( E_{x\theta} = kQx/(x^2 + a^2)^{3/2} \), the sign is given correctly by that expression for all \( x \).

For \( x > 2a, E_z = kQx(x^2 + a^2)^{3/2} + kQ'(x - 2a)^2; \) for \( x < 2a, E_z = kQx(x^2 + a^2)^{3/2} - kQ'(x - 2a)^2 \).

87 A metal sphere centered at the origin carries a surface charge of charge density \( \sigma = 24.6 \text{ nC/m}^2 \). At \( r = 2.0 \text{ m} \), the potential is 500 V and the magnitude of the electric field is 250 V/m. Determine the radius of the metal sphere.

1. Use the data to determine \( Q \)  
   \( Q/4\pi\varepsilon_0r = V(r); \ Q = 4\pi\varepsilon_0rV(r) \)

2. \( Q = 4\pi\sigma r^2; \) find \( R \)  
   \( R = [rV(r)\varepsilon_0\sigma]^{1/2} = 0.60 \text{ m} \)

88 Along the axis of a uniformly charged disk, at a point 0.6 m from the center of the disk, the potential is 80 V and the magnitude of the electric field is 80 V/m; at a distance of 1.5 m, the potential is 40 V and the magnitude of the electric field is 23.5 V/m. Find the total charge residing on the disk.

1. Write \( V(0.6)/V(1.5), \) using Equ. 24-21  
   \[ 2 = \frac{\sqrt{0.36 + R^2} - 0.6}{\sqrt{2.25 + R^2} - 1.5}; \ R = 0.8 \text{ m} \]

2. Use Equ. 23-11 to find \( \sigma \)  
   \[ \sigma = 2\varepsilon_0(23.5/0.118) \text{ C/m}^2 = 3.54 \text{ nC/m}^2 \]

3. \( Q = 4\pi\sigma R^2 \)

89* When you touch a friend after walking across a rug on a dry day, you typically draw a spark of about 2 mm. Estimate the potential difference between you and your friend before the spark. \( E \) at breakdown is 3000 V/mm. So the potential difference is about 6000 V.

90 When \(^{235}\text{U}\) captures a neutron, it fissions (splits) into two nuclei, in the process emitting several neutrons that can cause other uranium nuclei to fission. Assume that the fission products are two nuclei of equal charges of +46e and that these nuclei are at rest just after fission and are separated by twice their radius, \( 2R = 1.3 \times 10^{-14} \text{ m} \).

(a) Calculate the electrostatic potential energy of the fission fragments. This is approximately the energy released per fission. (b) About how many fissions per second are needed to produce 1 MW of power in a reactor?

(a) \( U = kq^2/2r; \ r = 2R \)  
(b) \( U = 8.99 \times 10^9 \times 46^2 \times 1.6 \times 10^{-19}/1.3 \times 10^{-14} \text{ eV} = 234 \text{ MeV} \)

91 A radioactive \(^{210}\text{Po}\) nucleus emits an \( \alpha \) particle of charge +2e and energy 5.30 MeV. Assume that just after
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the $\alpha$ particle is formed and escapes from the nucleus, it is a distance $R$ from the center of the daughter nucleus $^{208}\text{Pb}$, which has a charge $+82e$. Calculate $R$ by setting the electrostatic potential energy of the two particles at this separation equal to 5.30 MeV. (Neglect the size of the $\alpha$ particle.)

$$U = kq_1q_2/R; \quad R = kq_1q_2/U$$

92  Two large, parallel, nonconducting planes carry equal and opposite charge densities of magnitude $\sigma$. The planes have area $A$ and are separated by a distance $d$. (a) Find the potential difference between the planes. (b) A conducting slab having thickness $a$ and area $A$, the same area as the planes, is inserted between the original two planes. The slab carries no net charge. Find the potential difference between the original two planes and sketch the electric field lines in the region between the original two planes.

(a) The field due to each plane is $\sigma/2\varepsilon_0$, and in the region between the planes the fields add. So $E = \sigma/\varepsilon_0$ and $\Delta V = \sigma d/\varepsilon_0$ since the field is uniform.

93*  A uniformly charged ring with a total charge of 100 $\mu$C and a radius of 0.1 m lies in the $yz$ plane with its center at the origin. A meterstick has a point charge of 10 $\mu$C on the end marked 0 and a point charge of 20 $\mu$C on the end marked 100 cm. How much work does it take to bring the meterstick from a long distance away to a position along the $x$ axis with the end marked 0 at $x = 0.2$ m and the other end at $x = 1.2$ m.

1. Use Equ. 24-20 to find $V(0.2)$ and $V(1.2)$

\[ V(0.2) = 4.02 \text{ MV}; \quad V(1.2) = 0.747 \text{ MV} \]

2. $W = q_1V(1.2) + q_2V(0.2)$

\[ W = 14.9 \text{ J} + 40.2 \text{ J} = 55.1 \text{ J} \]

94  Three large conducting plates are parallel to one another with the outer plates connected by a wire. The inner plate is isolated and carries a charge density $\sigma_1$ on the upper surface and $\sigma_0$ on the lower surface, where $\sigma_1 + \sigma_0 = 12 \mu$C/m$^2$. The inner plate is 1 mm from the top plate and 3 mm from the bottom plate. Find the surface charge densities $\sigma_1$ and $\sigma_0$.

The configuration of the conducting plates is shown in the figure. Since the top and bottom plates are connected, they are at the same potential. The field in the 1-mm region is $E_x = \sigma_1/\varepsilon_0$, that in the 3-mm region is $E_y = \sigma_0/\varepsilon_0$. So $\sigma_1 = 3\sigma_0$.

Also, $\sigma_1 + \sigma_0 = 12 \mu$C. Solving for $\sigma_1$ and $\sigma_0$, one obtains $\sigma_1 = 9 \mu$C, $\sigma_0 = 3 \mu$C.

95  A point charge $q_1$ is at the origin and a second point charge $q_2$ is on the $x$ axis at $x = a$ as in Example 24-5.

(a) Calculate the electric field everywhere on the $x$ axis from the potential function given in that example. (b) Find the potential at a general point on the $y$ axis. (c) Use your result from (b) to calculate the $y$ component of the electric field on the $y$ axis.

Compare your result with that obtained directly from Coulomb's law.

(a) We need to consider three regions, as in Example 24-5. Region I, $x > a$; region II, $0 < x < a$; and region III, $x < 0$. The potentials in each of these regions are given in Example 24-5. To find $E_x$ we take derivatives of $V(x)$ and obtain: Region I, $E_x = kq_1/x^2 + kq_2/(x-a)^2$; region II, $E_x = -kq_1/x^2 - kq_2/(x-a)^2$; and region III, $E_x = -kq_1/x^2 - kq_2/(x-a)^2$.

(b) The distance between $q_1$ and a point on the $y$ axis is $y$; the distance between a point on the $y$ axis and $q_2$ is $(y^2 + a^2)^{1/2}$. The potential at a point on the $y$ axis is given by $V(y) = kq_1/|y| + kq_2/(y^2 + a^2)^{1/2}$, where we use only the positive value for the square root.

(c) To obtain the $y$ component of $E$ at a point on the $y$ axis we take the derivative of $V(y)$. For $y > 0$ one obtains $E_y = kq_1/[y^3 + kq_2y/(y^2 + a^2)^{3/2}]$; for $y < 0$, $E_y = -kq_1/[y^3 + kq_2y/(y^2 + a^2)^{3/2}]$. These are the components of the fields due to $q_1$ and $q_2$ that one obtains using Coulomb’s law.
A particle of mass $m$ carrying a positive charge $q$ is constrained to move along the $x$ axis. At $x = -L$ and $x = L$ are two ring charges of radius $L$ (Figure 24-29). Each ring is centered on the $x$ axis and lies in a plane perpendicular to it. Each carries a positive charge $Q$. (a) Obtain an expression for the potential due to the ring charges as a function of $x$ for $-L < x < L$. (b) Show that in this region, $V(x)$ is a minimum at $x = 0$. (c) Show that for $x << L$, the potential is of the form $V(x) = V(0) + \alpha x^2$. (d) Derive an expression for the angular frequency of oscillation of the mass $m$ if it is displaced slightly from the origin and released

(a) Along the $x$ axis, for $-L < x < L$, $V(x) = \frac{kQ}{\sqrt{(x + L)^2 + L^2}} + \frac{kQ}{\sqrt{(x - L)^2 + L^2}}$.

(b) To show that $V(x)$ is a minimum at $x = 0$, we must show that the first derivative of $V(x) = 0$ at $x = 0$ and that the second derivative is positive. One finds $dV/dx = (kQ) \left\{ \frac{3(L - x)^2}{(L - x)^2 + L^2}^{3/2} + \frac{3(L + x)^2}{(L + x)^2 + L^2}^{3/2} - \frac{1}{(L - x)^2 + L^2}^{3/2} - \frac{1}{(L + x)^2 + L^2}^{3/2} \right\}$. For $x = 0$, this expression reduces to $kQ/2\sqrt{2}L^3 = 0.3536kQ/L^3$.

(c) Since at $x = 0$, $dV/dx = 0$ and $d^2V/dx^2 = 0.3536/L^3$, it follows that $V(x)$ must be of the form $V(x) = V(0) + \alpha x^2$, where $V(0) = \sqrt{2} kQ/L$ and the constant $\alpha = kQ/\sqrt{2} L^3$.

(d) The potential energy of the harmonic oscillator is $1/2 kx^2$, where $k$ is the spring constant, and the angular frequency $\omega = \sqrt{k/m}$ . In this situation, the spring constant is $2\alpha$ and $\omega = \sqrt{2kQ/mL^3}$.

Three concentric conducting spherical shells have radii $a$, $b$, and $c$ such that $a < b < c$. Initially, the inner shell is uncharged, the middle shell has a positive charge $Q$, and the outer shell has a negative charge $-Q$. (a) Find the electric potential of the three shells. (b) If the inner and outer shells are now connected by a wire that is insulated as it passes through the middle shell, what is the electric potential of each of the three shells, and what is the final charge on each shell?

(a) Since the total charge is zero, $V(r = c) = 0$, so $V(c) = 0$. Between the outer and middle shells the field is $E_r = kQ/r^2$, so the potential difference between $c$ and $b$ is $kQ(1/b - 1/c)$, and since $V(c) = 0$, $V(b) = kQ(1/b - 1/c)$. The inner shell carries no charge, so the field between $r = a$ and $r = b$ is zero and $V(a) = V(b)$.

(b) When the inner and outer shell are connected their potentials are equal. Also $Q_a + Q_c = -Q$, the initial charge on the outer shell. As before, $V(c) = 0 = V(a)$. In the region between the region where $r = a$ and $r = b$, the field is $kQ_a/r^2$ and the potential at $r = b$ is then $V(b) = kQ_a (1/b - 1/a)$. The enclosed charge for $b < r < c$ is $Q_a + Q$, and by Gauss’s law the field in this region is $k(Q_a + Q)/r^2$ and the potential difference between $b$ and $c$ is $V(c) - V(b) = k(Q_a + Q)(1/c - 1/b) = -V(b)$ since $V(c) = 0$.

We now have two expressions for $V(b)$ which can be used to determine $Q_a$. One obtains

$Q_b = Q$, $Q_a = -Q \frac{a(c - b)}{b(c - a)}$ and $Q_c = -Q \frac{b(a - c)}{b(c - a)}$ and $V(b) = kQ \frac{(c - b)(b - a)}{b^2(c - a)}$.

Consider two concentric spherical metal shells of radii $a$ and $b$, where $b > a$. The outer shell has a charge $Q$, but the inner shell is grounded. This means that the inner shell is at zero potential and that electric field lines leave the outer shell and go to infinity but other electric field lines leave the outer shell and end on the inner shell. Find the charge on the inner shell.

Let the charge on the inner shell be $q$. Then the potential at the outer shell is $V(b) = k(Q + q)/b$. We can also determine $V(b)$ by considering the potential difference between $a$, i.e., $0$ and $b$, which is $V(b) = kq(1/b - 1/a)$.

Solving for $q$ one obtains $q = -Q(a/b)$. 

97* Consider two concentric spherical metal shells of radii $a$ and $b$, where $b > a$. The outer shell has a charge $Q$, but the inner shell is grounded. This means that the inner shell is at zero potential and that electric field lines leave the outer shell and go to infinity but other electric field lines leave the outer shell and end on the inner shell. Find the charge on the inner shell.

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