Searching for a proof for Goldbach 's Conjecture

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Chapter 1. Preamble, theorems and definitions

The Goldbach conjecture states that every even number in the set of natural numbers (or positive integers) can be represented as the sum of two prime numbers.
At the end of this document it will be demonstrated that this conjecture is true.
This demonstration will be given by first discussing a conjecture (for convenience called the ℕ-Conjecture) that was directly deduced from Goldbachs assertion, namely

Every natural positive number n where n is greater than 2, belongs to a (large) set of a pair of two primes p and q of which the mean value of each pair equals n.

A more formal definition is : $\forall n \in \mathbb{N}, \exists p , q \in \wp, n = (p + q)/2$.
A special case arises when p = q then $\forall n \in \mathbb{N}, \exists p , q \in \wp, n = p = q$

Theorem:
Fundamental theorem of arithmetic:
$\forall k \in \mathbb{N}$ has a unique factorization
$k = p_1^{a1} p_2^{a2} p_3^{a3} \ldots\ldots\ldots p_k^{ak}$ where the exponents a^1 ...... a^k are positive integers and
$p_1 < p_2 < p_3 \ldots\ldots < p_k$ are primes.

Definitions:
1. $\mathbb{N} = (0, 1, 2, 3, 4, 5, 6, 7, 8, \ldots.)$ is the set of natural numbers.
2. $\wp = (2, 3, 5, 7, 11 \ldots..)$ is the set of all prime numbers and $\wp \subset \mathbb{N}$
3. $M = \mathbb{N} \setminus \{0,1,2\}$
4. $\Re = (\{3,3\},\{3,5\},\{5,5\},\{5,7\},\{p_7,q_7\},\{p_8,q_8\},\ldots\ldots\ldots)$ Each pair is a pair of primes and the value of each pair equals the mean value of the two rpimes

Chapter 2. Defining natural numbers as a combination of primes.

A method was developed to determine whether any random natural number n greater than 2 can be defined as the mean value of two primes. The most straightforward method is described below and will be used for proving the ℕ-Conjecture.

To find two primes whose sum divided by 2 equals the mean value of two primes we annotate these primes as $p \in \wp$, and $q \in \wp$. This is reflected in formula (1):

(1) $$n \in \mathbb{N} = (p+q)/2$$

Since the difference between two primes is always an even number, and since we may also include the value 0 (zero) as the difference between p and q, we can now calculate for $\forall n \in M$ two numbers a and b so that $a,b \in \mathbb{N} | b \geq a$ and a value $v \in \mathbb{N} | v=0,1,2,3,......$with the following algorithm:

integer **$a,b,n \in M$; integer $v \in \mathbb{N}$;** ***prime*** **$p,q \in \wp$;** ***boolean*** **bool;** ***boolean_function*** **$\mathscr{F}(x)$;**
process **A**
bool = ***false***
for **v = 0** ***step*** **1** ***repeat*** **A** ***until*** **bool =** ***true***
begin **A**
a = n-v; b = n + v;
if **$\neg\mathscr{F}(a)$** ***repeat*** **A**
if **$\mathscr{F}(a) \wedge \mathscr{F}(b)$** ***then*** **p = a; q = b; bool =** ***true***
end **A**
record **{p, q}**

and add the constructed pair {p,q} to set $\mathfrak{R}$ in the same order as those of $n \in M$.

The question is: can we develop a method that can serve as a proof for the the mathematical deductive method and be accepted as a proof. If it can be considered a formal proof for the $\mathbb{N}$-Conjecture then:
$\forall n \in \mathbb{N} | n > 2, \exists\, p, q \in \wp | n = (p + q)/2$

Chapter 3. Proof of the ℕ-conjecture.

$\mathfrak{R}$ is the set of pairs of primes of M determined with the algorithm in chapter 2 for all elements of set M. The set consists of elements that are constituted as a pair of prime numbers p and q {p,q} for each $n \in M$.
$\mathfrak{R}$ = ({3,3},{3,5},{5,5},{5,7},$\{p_7,q_7\}$,$\{p_8,q_8\}$,.........) Each pair is a pair of primes and the value of each pair equals the mean value of the two primes.

Applying formula (1) on every element of $\mathfrak{R}$ produces a set $\mathfrak{R}$+ of which the values of its elements are equal to those of set M by determining the mean value of each pair of primes. This defined in the following formula which is valid for all $n \in M$

(2) $\forall n \in M | \exists p,q \in \wp | n = (p + q)/2$.

If it is assumed that a random number $s \in M$ can not be produced in the set $\mathfrak{R}$+ then s can be divided in two earlier constructed numbers $u \in M$ and $w \in M$ where the sum of u and w equals s and which values in the set $\mathfrak{R}$+ equals those in set M. This means that u equals $(p_u + q_u)/2$ and w equals $(p_w + q_w)/2$ while $p_u, \in \wp$, $q_u \in \wp$, $p_w \in \wp$ and $q_w \in \wp$.
The natural number s can thus be considered as being equal to $(p_u + q_u + p_w + q_w)/2$ and the sum of these four primes can be substituted by two primes with a simple algorithm. The

assumption that s is not properly represented in $\Re+$ is therefore false. It can thus be concluded that all natural numbers belong to the set $\Re+$.

Using this method of proving that the $\mathbb{N}$-Conjecture is true and results in the $\mathbb{N}$-theorem:

Every natural positive number n where n is greater than 2, belongs to a (large) set of a pair of two primes p and q of which the mean value of each pair equals n.

This proof is a combination of logicism and constructivism schools of thought in proving mathematical conjectures and constructions.

Chapter 4. Proof of the Goldbach conjecture.

The step from proving the $\mathbb{N}$-conjecture to the Goldbach conjecture is simple and consists of adding the number 2 to the natural numbers of the $\mathbb{N}$-Conjecture. We only have to multiply the values of the positive numbers found in chapter 3 with a factor 2 to create the complete set of even numbers that are defined by adding the two primes that were determined in the $\mathbb{N}$-theorem.
Formally this can be expressed as:
$\forall n \in \mathbb{N} \mid n > 2, \exists\, p, q \in \wp \mid 2n = p + q$
and according to the Goldbach conjecture this may prove that:

Any even number greater than 2 can always be expressed as the sum of two prime numbers

and as such results in the Goldbach theorem.

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SOURCES:

1. The Princeton Companion to Mathematics.
Timothy Gowers, editor
ISBN 978-0-691-11880-2
2. Handbook of Mathematical Functions
Milton Abramowitz and Irene A. Stengun
Library of Congress Catalog Card Number: 65-12253
3. Mathematical Mountaintops
John L. Casti
Oxford University Press
ISBN 0-19-514171-7

ALGORITHMS

Based on the programming languages Algol60/68 and AEDjr a pseudo-code developed by the author was used to describe the algorithmic way to determine two primes p and q whose sum divided by 2 yields a natural number n. The first pseudo-code calculates the pair of primes nearest to n. The second pseudo-code calculates all pairs of primes whose mean value equals n. (A complete description of my pseudo code was defined and described as an appendix to my PhD-thesis presented to the executive staff of the Electronic Systems Lab of MIT in 1972).

integer **a, b, v, n;** ***prime*** **p, q;** ***boolean*** **bool;** ***boolean_function*** **$\mathscr{F}$(x);** ***process*** **A**
bool = ***false***
remark **n = any natural number > 2**
for **v = 0** ***step*** **1** ***repeat*** **A** ***until*** **bool =** ***true***
begin **A**
a = n-v; b = n + v;
if **¬$\mathscr{F}$(a)** ***repeat*** **A**
if **$\mathscr{F}$(a) $\wedge$ $\mathscr{F}$(b)** ***then*** **p = a; q = b; bool =** ***true***
end **A**
record **n, p, q**
remark **$\mathscr{F}$(x) is true if this function yields a prime; every aspect of** ***process*** **A is controlled by the** ***for*** **statement; the fastest known method to evaluate $\mathscr{F}$(x) as a prime is an extended Atkin's sieve (like primegen).**