

## Tutorial Sheet 23 (Random Variables)

1. Check whether the following function can serve as the probability distribution of some random variable:

$$f(x) = \frac{x+5}{21} \quad \text{for } x = 1, 2, 3.$$

2. State whether each of the following is a valid probability distribution of a random variable that can take on the values 1, 2, 3, 4, and 5. Explain your answer.

- |    |              |              |                |               |               |
|----|--------------|--------------|----------------|---------------|---------------|
| a) | $p(1) = 0.5$ | $p(2) = 0.2$ | $p(3) = 0.15$  | $p(4) = 0.1$  | $p(5) = 0.05$ |
| b) | $p(1) = 0.1$ | $p(2) = 0.1$ | $p(3) = 0.15$  | $p(4) = 0.2$  | $p(5) = 0.55$ |
| c) | $p(1) = 0.2$ | $p(2) = 0.1$ | $p(3) = -0.15$ | $p(4) = 0.45$ | $p(5) = 0.4$  |

3. A box contains 3 black balls and 5 white balls. A ball is drawn without replacement until a white ball is drawn. Let  $X$  be the number of draws required to draw a white ball.

- a) Write down the possible values of  $X$ .
- b) Find the probability distribution of  $X$ .

4. There are 4 black balls and 6 white balls in a bag. 5 balls are drawn randomly from the bag. Let  $X$  be the number of black balls drawn.

- a) Write down the possible values of  $X$ .
- b) Write down the number of different ways of choosing 5 balls from the bag.
- c) Find the number of different ways of choosing  $x$  black balls and  $(5 - x)$  white balls from the bag.
- d) Write down  $p(x)$  in terms of  $x$ , assuming that  $x$  is one of the values in (a).
- e) Write down the probability distribution of  $X$ .
- f) In general, if there are  $b$  black balls and  $w$  white balls in the bag and  $n$  balls are drawn, express  $p(x)$  in terms of  $b$ ,  $w$  and  $n$ .