Tutorial Sheet 22
(Conditional Probability)

1. Suppose E and F are two events such that

$$P(E) = \frac{1}{3}, \quad P(F) = \frac{1}{5}, \quad \text{and} \quad P(E \text{ or } F \text{ but not both}) = \frac{1}{4}.$$

Calculate $P(E \cap F)$, $P(\overline{E} \cap F)$, $P(E \mid F)$, $P(F \mid \overline{E})$.
State whether E and F are (a) independent, (b) mutually exclusive.

2. Suppose E and F are two events such that

$$P(E) = \frac{1}{3}, \quad P(\overline{E} \mid F) = \frac{1}{4}, \quad \text{and} \quad P(E \cup F) = \frac{3}{7}.$$

Calculate $P(F \mid \overline{E})$, $P(F \cap E)$, $P(E \mid \overline{F})$.

The event G is independent of E and $P(E \cap G) = \frac{1}{6}$. Determine $P(G \mid \overline{E})$.

3. John never plays tennis on Monday, Tuesday, Wednesday and Thursday. The probability that he plays tennis on a given Friday is $1/3$. For each on the remaining two days, Saturday and Sunday, the conditional probability that he plays tennis, given that he played tennis on the previous days, is $1/2$. The conditional probability that he plays tennis, given that he did not play tennis on the previous days, is $3/4$.

Suppose E is the event that John plays tennis on Sunday, F is the event that he plays tennis on Saturday and G is the event that he plays tennis on Friday.

(a) Find the probabilities that he plays tennis on a given Saturday and on a given Sunday, that is, $P(F)$ and $P(E)$.

(b) Find the probability that he plays tennis on a given Sunday provided that he played tennis on Friday, that is $P(E \mid G)$. Similarly find the value of $P(E \mid \overline{G})$.