Tutorial Sheet 15
(Paths, Circuits & Shortest Path Algorithm)

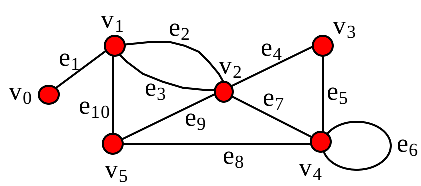
1. (a) Let $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}$. Find A^2 and A^3.

(b) Let G be the graph with vertices v_1, v_2, and v_3 and with A as its adjacency matrix. Use the answers in part (a) to find the number of walks of length 2 from v_1 to v_3 and the number of walks of length 3 from v_1 to v_3. Do not draw G to solve this problem.

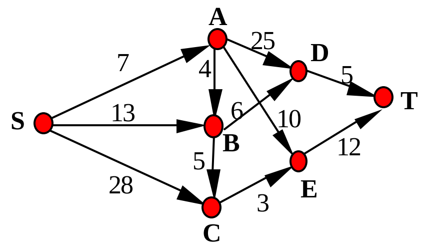
(c) Examine the calculations you performed in answering part (a) to find five walks of length 2 from v_3 to v_3. Then draw G and find the walks.

2. In the graph below, determine whether the following walks are paths, simple paths, closed walks, circuits, simple circuits, or are just walks.

(a) $v_0e_1v_1e_{10}v_5e_9v_2e_2v_1$
(b) $v_4e_7v_2e_9v_5e_{10}v_1e_3v_2e_9v_5$
(c) v_2
(d) $v_2v_3v_4v_5v_2v_4v_3v_2$
(e) $e_5e_8e_{10}e_3$



3. Find the shortest path from S to T in the following network:

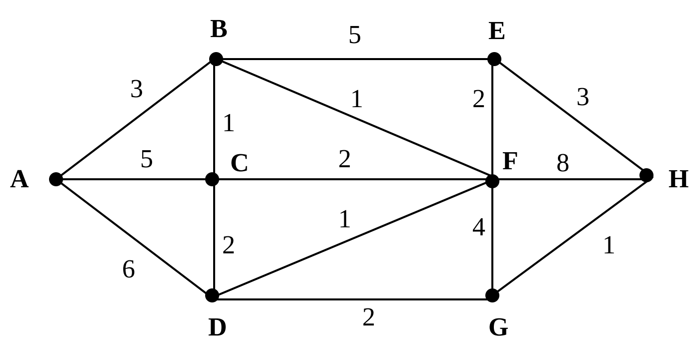


4. A company has branches in five cities A, B, C, D, and E. The fares for direct flights between these cities are as follows:

	A	**B**	**C**	**D**	**E**
A	-	50	40	25	10
B	50	-	20	90	25
C	40	20	-	10	25
D	25	90	10	-	55
E	10	25	25	55	-

What is the cost of travelling between each pair of cities by the cheapest route?

5. The graph in the following figure shows the communication channels and the communication time delays in the channels among eight communication centers. The centers are represented by vertices, the channels are represented by edges, and the communication time delay (in minutes) in each channel is represented by the weight of the edge. Suppose that at 3:00 p.m. communication center **A** broadcasts the news through all its channels. Other communication centers will then broadcast this news through all their channels as soon as they receive it.



For the communication centers B, C, D, E, F, G, and H, determine the earliest time each receives the news.