

Tutorial Sheet 9 (Matrix Arithmetic)

1. Find AB if

a) $A = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 4 \\ 1 & 3 \end{pmatrix}$.

b) $A = \begin{pmatrix} 1 & -1 \\ 0 & 1 \\ 2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -2 & -1 \\ 1 & 0 & 2 \end{pmatrix}$.

c) $A = \begin{pmatrix} 4 & -3 \\ 3 & -1 \\ 0 & -2 \\ -1 & -5 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 3 & 2 & -2 \\ 0 & -1 & 4 & -3 \end{pmatrix}$

2. Let $A = \begin{pmatrix} 2 & -3 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 3 \\ 5 \\ 6 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -3 \\ 9 & 10 \end{pmatrix}$ and $D = \begin{pmatrix} 8 & -3 & -5 \\ 9 & 11 & 8 \end{pmatrix}$, evaluate the following if exists.

a) $4A$

b) $5D$

c) $A^T + 3B$

d) AB

e) $3BA$

f) $C(3D)$

g) DC

h) $D^T C$

i) BCD

3. Let $A = \begin{pmatrix} -1 & 2 \\ 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} -3 & 2 \\ 1 & 1 \end{pmatrix}$, find A^3 and B^3 .

4. What is the most efficient way to multiply the matrices A_1, A_2, A_3 with sizes

a) $20 \times 50, 50 \times 10, 10 \times 40$?

b) $10 \times 5, 5 \times 50, 50 \times 1$?

*5. Show that $(A^T)^T = A$.

*6. Let A be a matrix. Show that the matrix AA^T is symmetric.

*Optional