

Tutorial Sheet 12 (Properties of Binary Relations)

1. For each of the following relationships on the set $\{1, 2, 3, 4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

- a) $\{(2, 4), (4, 2)\}$
- b) $\{(1, 2), (2, 3), (3, 4)\}$
- c) $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$

2. Determine whether the relation R on the set of all integers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if

- a) $x \neq y$.
- b) $xy \geq 1$.
- c) $x = y + 1$ or $x = y - 1$.

3. Determine whether the relations represented by the following matrices are reflexive, symmetric, antisymmetric, and/or transitive.

a) $M_R = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ b) $M_R = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ c) $M_R = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$