Tutorial Sheet 8
(Answers)

1. Suppose m and n are integers so that $m + n$ is even, i.e. $m + n = 2k$.
Therefore,

$$\begin{aligned} m - n &= (2k - n) - n \\ &= 2k - 2n \\ &= 2(k - n) \end{aligned}$$

where $k - n$ is integer.
Hence, by definition of even, $m - n$ is even.

3. Proof (by contraposition)
Suppose that p and q are two numbers less than 10.
Then

$$\begin{aligned} pq &< 10 \times 10 \\ &< 100 \end{aligned}$$

Hence, the product is less than 100.

4. Proof (by contraposition)
Suppose that p and q are two numbers greater than 25.
Then

$$\begin{aligned} p + q &> 25 + 25 \\ &> 50 \end{aligned}$$

Hence, the sum is greater than 50.

5. Since a divides b, there is an integer q such that $b = aq$. So $bc = (aq)c$, and hence qc is an integer such that $bc = a(qc)$. Therefore a divides bc, so the proof is complete. (a direct proof).

6. We can prove that $n^3 + n$ is even by cases.
Case (i) Suppose that n is even. Then $n = 2k$ for some $k \in N$, so

$$n^3 + n = 8k^3 + 2k = 2(4k^3 + k),$$

which is even.

Case (ii) Suppose that n is odd. Then $n = 2k + 1$ for some $k \in N$, so

$$n^3 + n = (8k^3 + 12k^2 + 6k + 1) + (2k + 1) = 2(4k^3 + 6k^2 + 4k + 1)),$$

which is even.

Here is a more elegant proof by cases. Given n in N. We have $n^3 + n = n(n^2 + 1)$. If n is even, so is $n(n^2 + 1)$. If n is odd, then n^2 is odd, hence $n^2 + 1$ is even, and so $n(n^2 + 1)$ is even