Solution of Question 4
Tutorial Sheet 3

4* Consider the following solitaire games: for every integer i , there is an unlimited supply of balls marked with the number i. Initially, we are given a tray of balls, and we throw away the balls in the tray one at a time. If we throw away a ball that is marked with i, we can replace it by any finite number of balls marked 1, 2, … $i - 1$. (Thus, no replacement will be made if we throw away a ball marked with 1.) The game ends when the tray is empty. Determine whether the game always terminates for any tray of balls given initially.

Solution

For $n = 1$, there is a finite number of balls marked with 1 in the tray initially.

Since there is no replacement after a ball marked 1 is thrown away, the game terminates after a finite number of moves.

We assume that the game terminates if the largest number that appears on the balls is k.

Consider the case when the largest number that appears on the ball is $k + 1$.

According to the induction hypothesis, we eventually have to throw away a ball marked $k + 1$.

If we throw away only balls marked 1,2,….k, they will be exhausted in a finite number of moves.

Repeating this argument, we would have thrown away all balls marked with $k + 1$ in a finite number of moves.

Again, by induction hypothesis, the game terminates after a finite number of moves from then on.