Tutorial Sheet 3
(Mathematical Induction)

Prove that the following statements are true for all positive integers n.

1. $\dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \ldots\ldots + \dfrac{1}{2^n} = 1 - \dfrac{1}{2^n}$

2.   Prove that any positive integer n greater than or equal to 2 is either a prime or a product of primes.

3. $1^2 + 3^2 + 5^2 + \ldots\ldots + (2n-1)^2 = \dfrac{1}{3}n(4n^2 - 1)$

4*   Consider the following solitaire games: for every integer i, there is an unlimited supply of balls marked with the number i. Initially, we are given a tray of balls, and we throw away the balls in the tray one at a time. If we throw away a ball that is marked with i, we can replace it by any finite number of balls marked 1, 2, … $i-1$. (Thus, no replacement will be made if we throw away a ball marked with 1.) The game ends when the tray is empty. Determine whether the game always terminates for any tray of balls given initially.

*Optional