Section A: (5% each)

A1. Using $f^{-1}(p) = (p-10) \bmod 26$, we have

	Z K B D S K V	P B K M D S Y X
p:	25-10-1-3-18-10-21	15-1-10-12-3-18-24-23
$f^{-1}(p)$	15-0-17-19-8-0-11	5-17-0-2-19-8-14-13
	P A R T I A L	F R A C T I O N

Thus, the original message is "PARTIAL FRACTION".

A2. i) The truth set of a propositional function is the set of elements in its domain that are assigned true propositions by the function.

ii) $x^3 - 2x^2 - x + 2 = (x-1)(x+1)(x-2) = 0$

The three roots are given as follows:

$x_1 = 1$

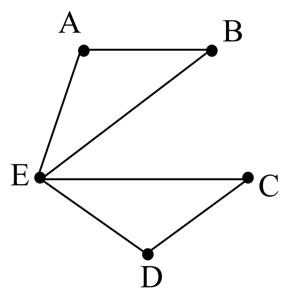
$x_2 = -1$

$x_2 = 2$

The set of truth values of the propositional function $Q(x)$ is $\{1, -1, 2\}$.

A3. Number of one-digit numbers formed is $C(4, 1) = 4$.
Number of two-digit numbers formed is $P(4, 2) - C(3, 1) = 12 - 3 = 9$.
Number of three-digit numbers formed is $P(4, 3) - P(3, 2) = 24 - 6 = 18$.
Number of four-digit numbers formed is $4! - 3! = 24 - 6 = 18$.
Then total numbers formed are $4 + 9 + 18 + 18 = 49$.

A4. Express the problem in terms of graph by taking the land areas as vertices and the bridges as edges joining the corresponding pairs of vertices



Since every vertex of G has even degree, therefore G is Eulerian.
One of the possible routes is ABECDEA.

A5. i) The required probability is $= \dfrac{{}_8C_3}{{}_{10}C_5} = \dfrac{56}{252} = \dfrac{2}{9}$ #

ii) The required probability is

$$= \frac{{}_4C_3 \cdot {}_6C_2}{{}_{10}C_5} + \frac{{}_4C_4 \cdot {}_6C_1}{{}_{10}C_5} = \frac{60}{252} + \frac{6}{252}$$

$$= \frac{66}{252} = \frac{11}{42}$$ #

A6. We can prove that $n^3 + n$ is even by cases.

Case (i) Suppose that n is even. Then $n = 2k$ for some $k \in N$, so

$$n^3 + n = 8k^3 + 2k = 2(4k^3 + k),$$

which is even.

Case (ii) Suppose that n is odd. Then $n = 2k + 1$ for some $k \in N$, so

$$n^3 + n = (8k^3 + 12k^2 + 6k + 1) + (2k + 1) = 2(4k^3 + 6k^2 + 4k + 1)),$$

which is even.

Here is a more elegant proof by cases. Given n in N. We have $n^3 + n = n(n^2 + 1)$. If n is even, so is $n(n^2 + 1)$. If n is odd, then n^2 is odd, hence $n^2 + 1$ is even, and so $n(n^2 + 1)$ is even.

A7. i) Assuming it is a Poisson Distribution,

$P(0) = e^{-3}$
$= 0.04979$

ii) $P(X > 3) = 1 - P(0) - P(1) - P(2) - P(3)$
$= 1 - e^{-3} - e^{-3}(3) - e^{-3}(3^2)/2! - e^{-3}(3^3)/3!$
$= 0.3528$

A8. i) $AB = \begin{bmatrix} 2 & 7 \\ 7 & 13 \end{bmatrix}$ $\quad BA = \begin{bmatrix} 4 & 7 & 3 \\ 4 & 2 & 6 \\ 7 & 6 & 9 \end{bmatrix}$

ii) $\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$

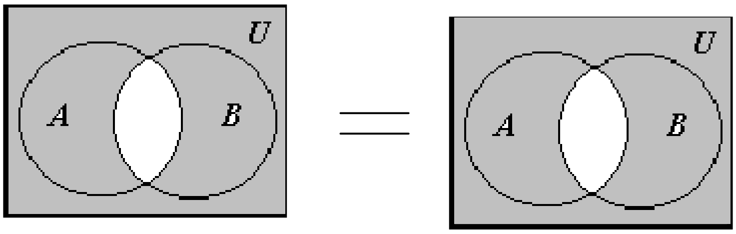
$\Rightarrow \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}^{-1} = \dfrac{1}{5}\begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$

$\Rightarrow \mathbf{A} = \dfrac{1}{5}\begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix} . \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 9/5 & -6/5 \\ -1/5 & 4/5 \end{bmatrix}$

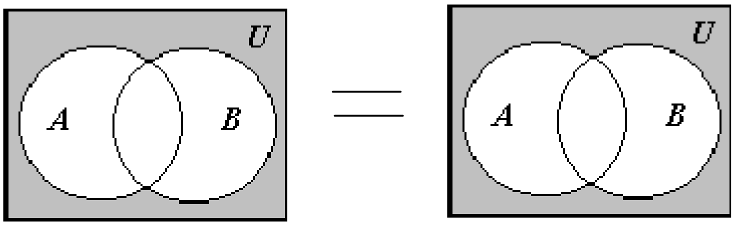
Section B: (12% each)

B1. i) (a) 1+2=3 books
(b) 1+2+3+4=10 books
(c) 3 books

ii) The De Morgan's laws of algebra of sets are $\overline{A \cap B} = \overline{A} \cup \overline{B}$



and $\overline{A \cup B} = \overline{A} \cap \overline{B}$



B2. i)

p	q	r	$q \vee r$	$p \vee (q \vee r)$	$\neg r$	$p \vee q$	$[(p \vee (q \vee r)) \wedge (\neg r)] \to (p \vee q)$	
T	T	T	T	T	F	T	F	T
T	T	F	T	T	T	T	T	T
T	F	T	T	T	F	T	F	T
T	F	F	F	T	T	T	T	T
F	T	T	T	T	F	T	F	T
F	T	F	T	T	T	T	T	T
F	F	T	T	T	F	F	F	T
F	F	F	F	F	T	F	F	T

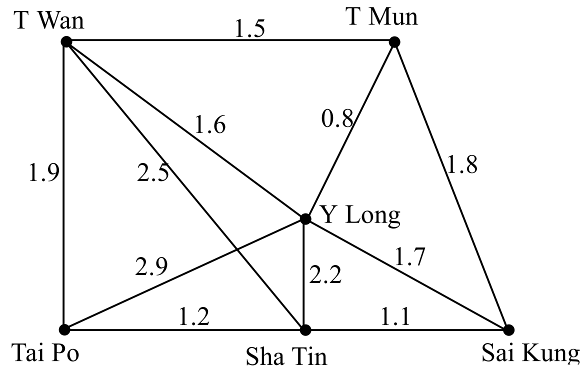
The statement form $[(p \vee (q \vee r)) \wedge (\neg r)] \to (p \vee q)$ is a tautology, hence the argument is valid.

ii) (a) Let $p =$ interest rates go up.
$q =$ stock market prices will go down,
$p \to q$
$\neg p$
$\therefore \neg q$

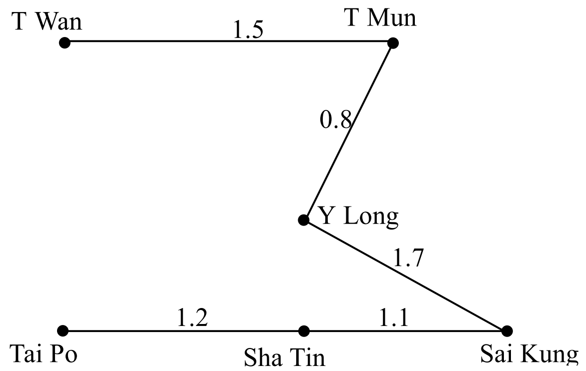
(b) The argument in ii)(a) is invalid.

B3. i)

(a)

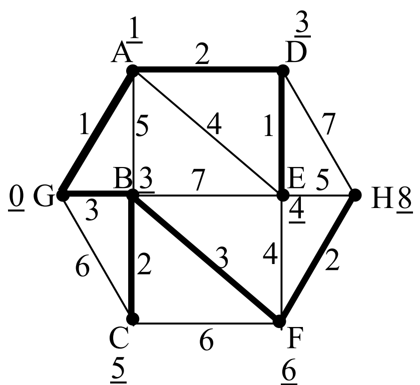


b)



c) min cost = 1.5 + 0.8 + 1.7 + 1.1 + 1.2 = 6.3 (millions)

ii)



Therefore, the shortest distance from G to D = 3
E = 4
F = 6
H = 8

B4 i) (a) $\mu = 100$, $\sigma = 16$,

$P(X > 120) = P[Z > (120-100)/16]$
$= P[Z > 1.25]$
$= 0.1057$

(b) $P[Z > (X-100)/16] = 0.2$
$(X - 100)/16 = 0.84$
$X = 113.4$

ii) (a) P (exactly 2 defective) = ${}_{10}C_2\ (0.02)^2\ (1 - 0.02)^8$
$= 0.01531$

(b) P (2 or more will be defective)
= 1 - P (no defective) - P (exactly 1 defective)
$= 1 - 0.98^{10} - 10*0.02*0.98^9$
$= 0.0162$

B5. i) (a) (2,1), (3,2), (4,3), (5,4).

(b)

R	1	2	3	4
1				
2	X			
3		X		
4			X	
5				X

(c) $R = \{(a_1, b_2), (a_2, b_1), (a_3, b_1), (a_4, b_3)\}$.

ii)

(a) The required probability is

$$= \frac{(0.36)\cdot\left(\frac{1}{6}\right)}{(0.64)\cdot\left(\frac{1}{4}\right) + (0.36)\cdot\left(\frac{1}{6}\right)} = \frac{0.06}{0.22} = \frac{3}{11}_{\#}$$

(ii) The required probability is

$$= 1 - \left[(0.64)\cdot\left(\frac{1}{4}\right) + (0.36)\cdot\left(\frac{1}{6}\right)\right] = 1 - \frac{11}{50} = \frac{39}{50}_{\#}$$